Adversarial Search

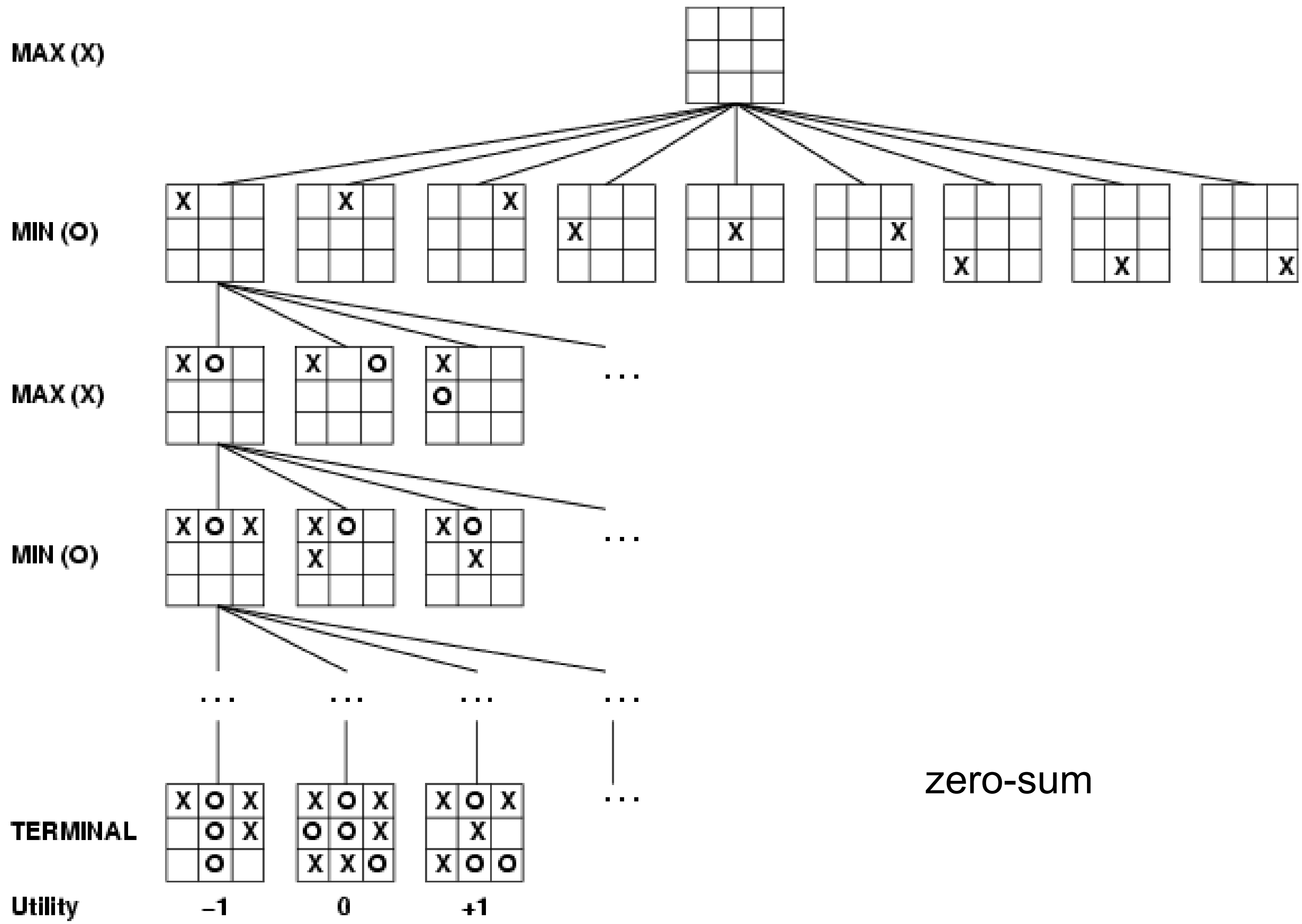
Outline

- Games
- Minimax search (foundation)
- α-β pruning (adding intelligence into Minimax)

Games vs. search problems

- "Unpredictable" opponent → specifying a move for every possible opponent reply
- Time limits → unlikely to find goal, must approximate
- Game tree: 2-player, deterministic, turns.
- Formulate game problems
  - Initial state: the board position and identifies the player to move.
  - Successor function: return a list of (move, state) pairs.
  - Goal test: determine terminal states (where the game has ended).
  - Utility function: give a numeric value for each terminal state.

Game Tree of Tic-tac-toe



zero-sum

Minimax

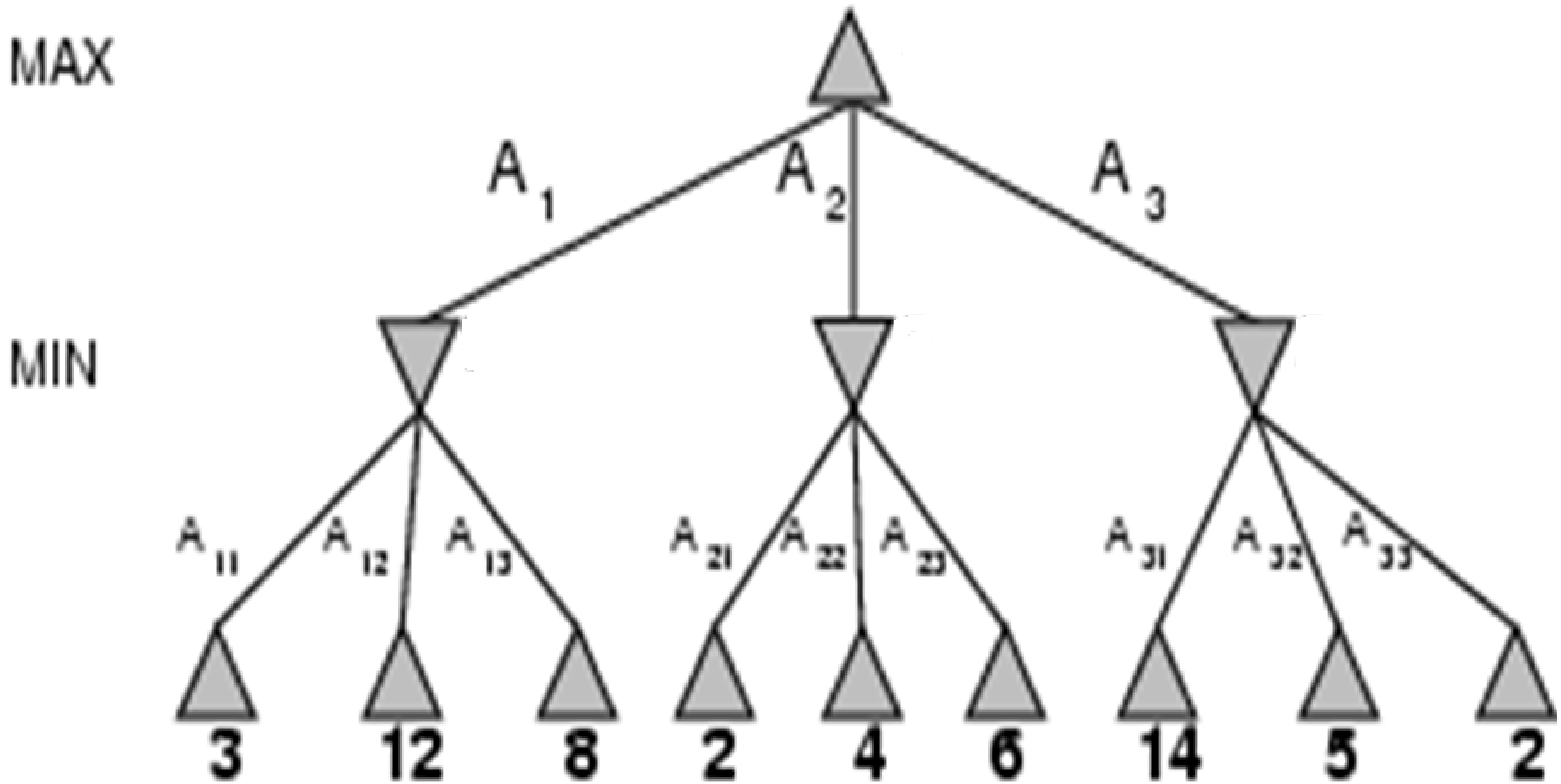
- Perfect play for deterministic games.
- Ideas
  - assume the opponent always chooses a move that minimizes the desirability of states (from my perspective).
  - The heuristic evaluation at level i depends on the move (by the other player) at level i+1.

Minimax algorithm

- Perform a fix-level look ahead (depth-first) search.
- Evaluate the heuristic values of leaf nodes.
- Back up the heuristic values from the leaf nodes to the root by choosing the **maximal** values when it’s turn, and choosing the **minimal** values when it’s opponent’s turn.

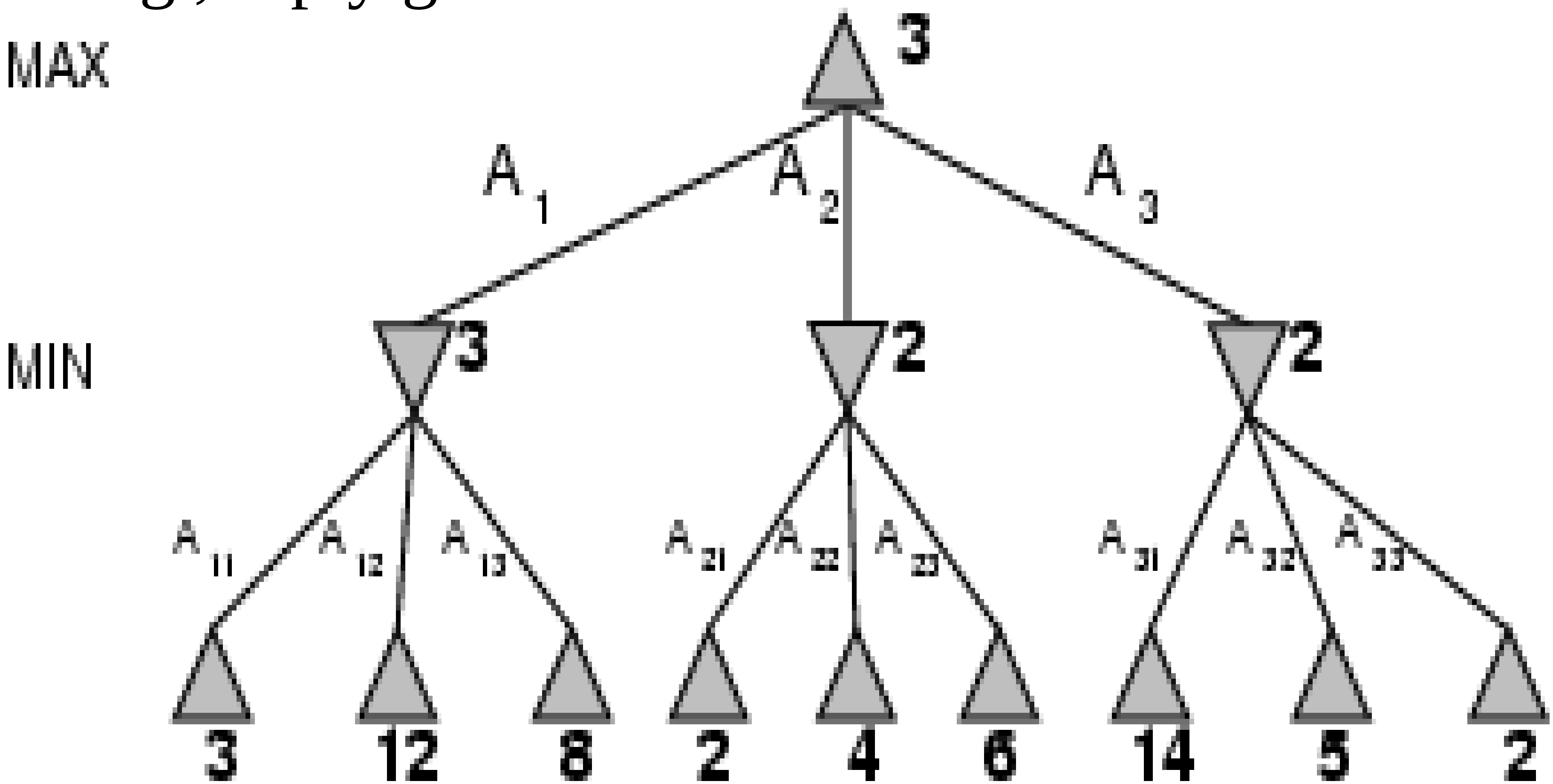
Example

E.g., 2-ply game:



Result

E.g., 2-ply game:



Properties of minimax (DFS)

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$ (depth-first exploration)

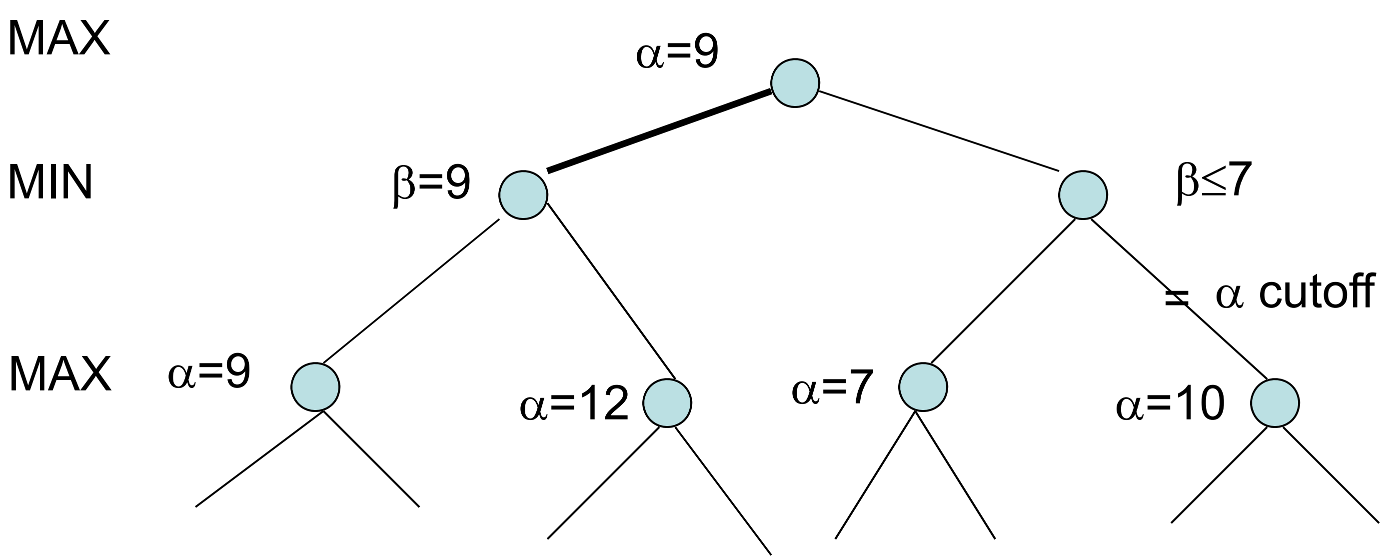
- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
→ exact solution completely infeasible

α-β Pruning

- Each MAX node (your ply) has an α value to keep track of the current maximum of its back-up values.
- Each MIN node (opponent's ply) has a β value to keep track of the current minimum of its back-up values.
- Use **α** and **β values** to detect two kinds of opportunities for pruning the search tree without affecting the root node's decision.
  - **α** cutoff (cut min)
  - **β** cutoff (cut max)

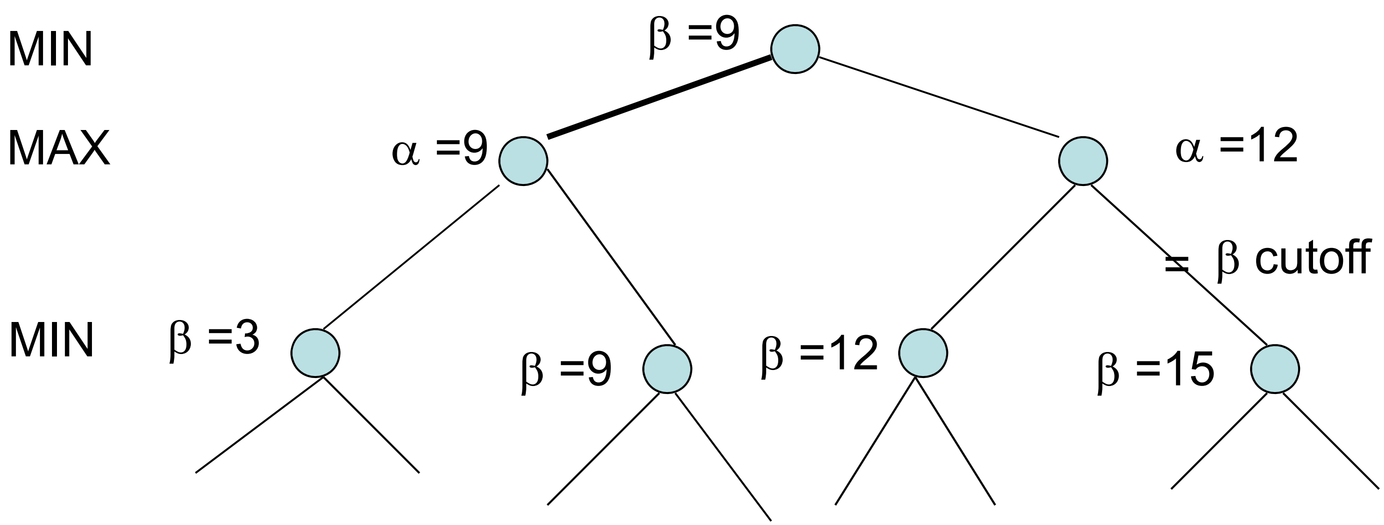
α cutoff

- When β value of a MIN node ≤ α value of an ancestor MAX node, all branches below the MIN node can be pruned.

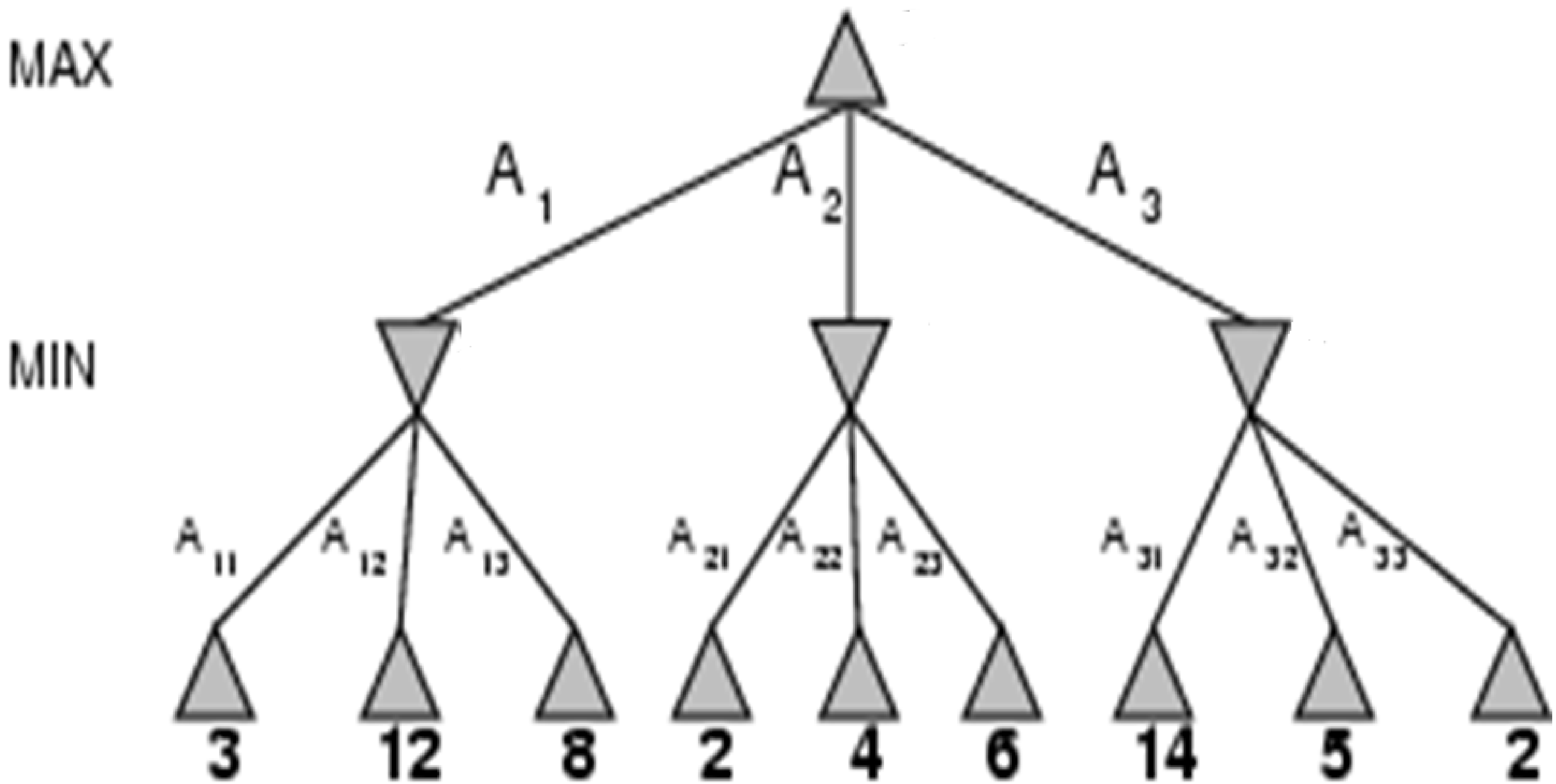


β cutoff

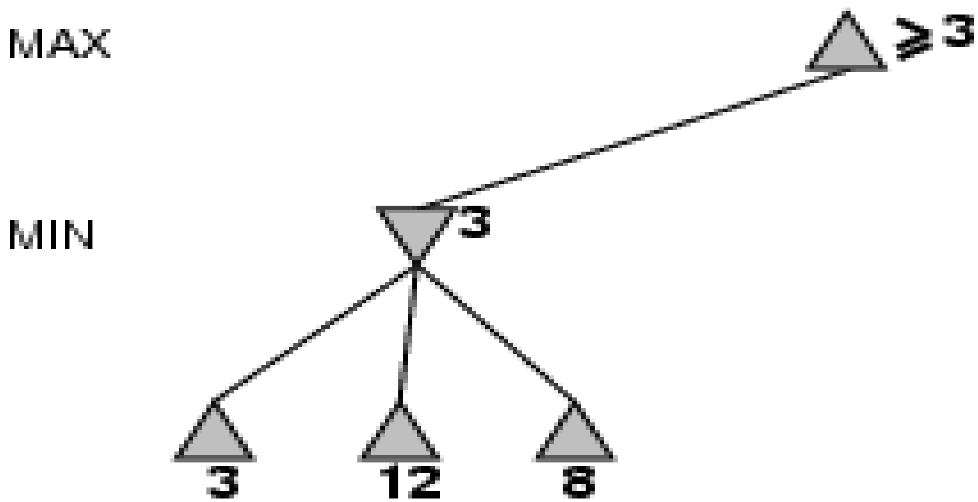
- When α value of a MAX node ≥ β value of an ancestor MIN node, all branches below the MAX node can be pruned.

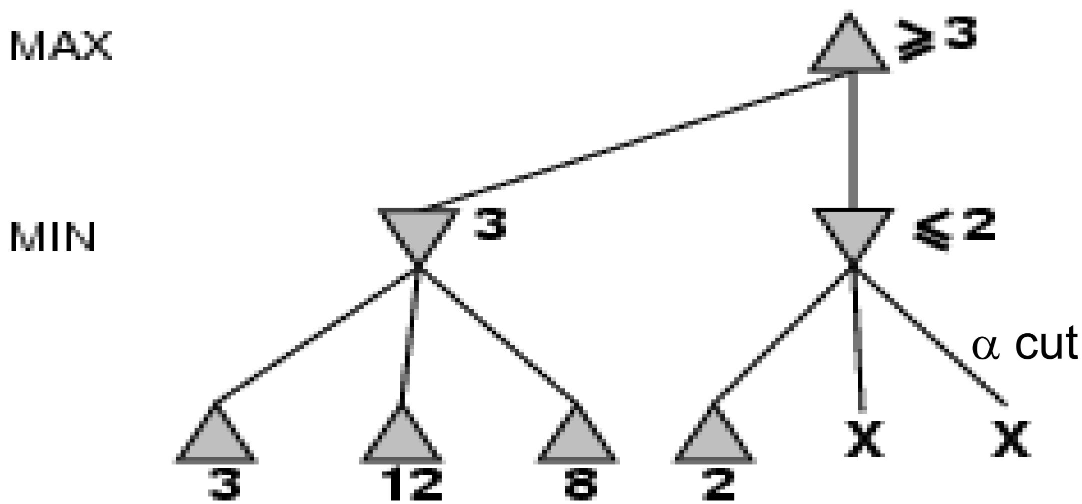


α-β pruning example

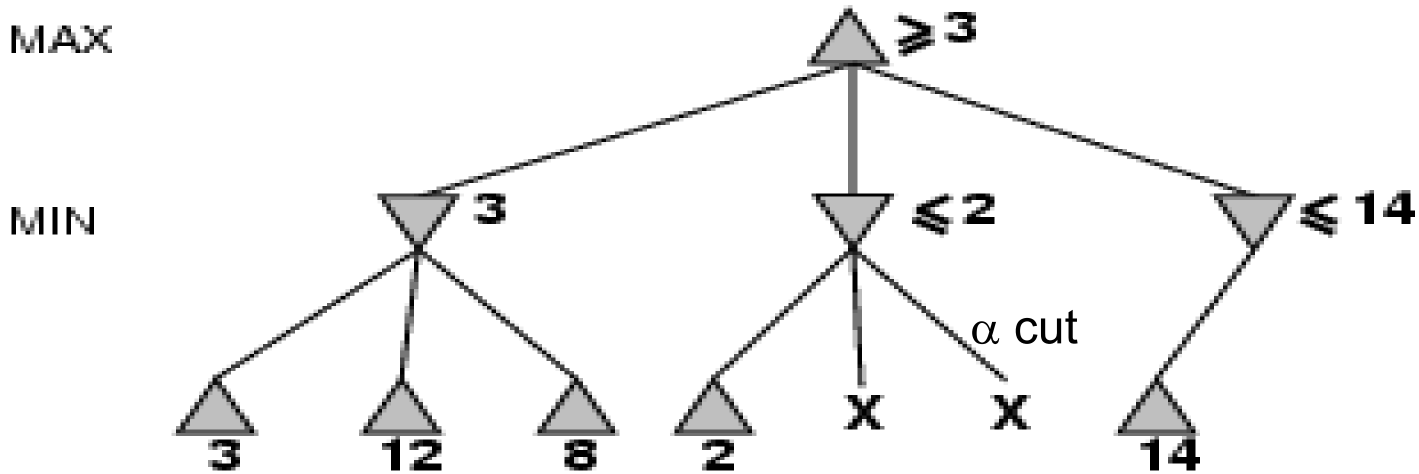


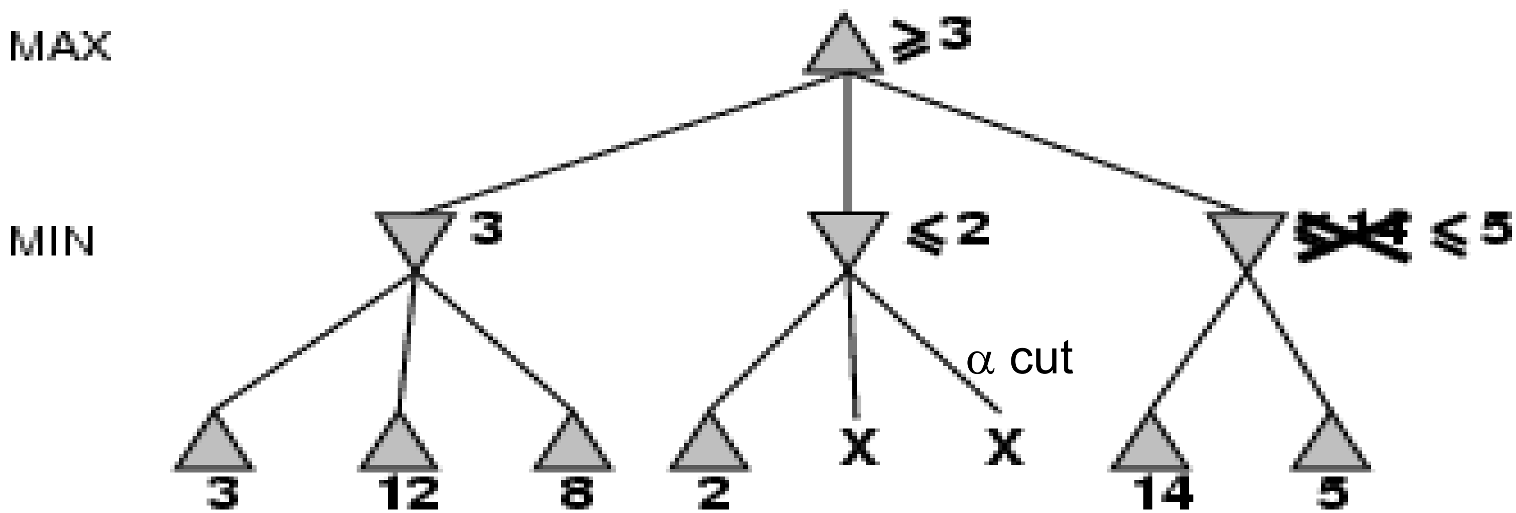
α-β pruning example (cont'd)



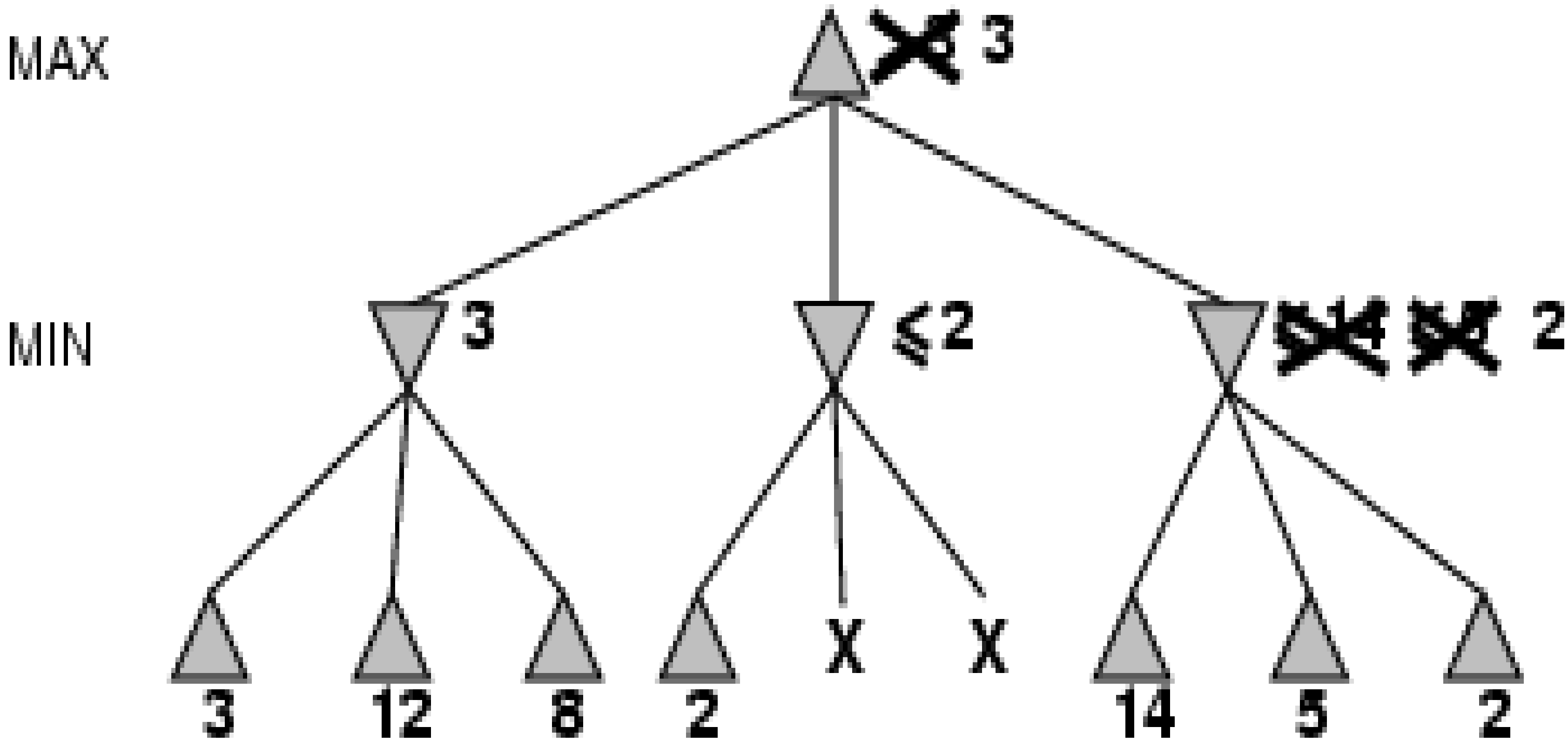


α-β pruning example (cont'd)



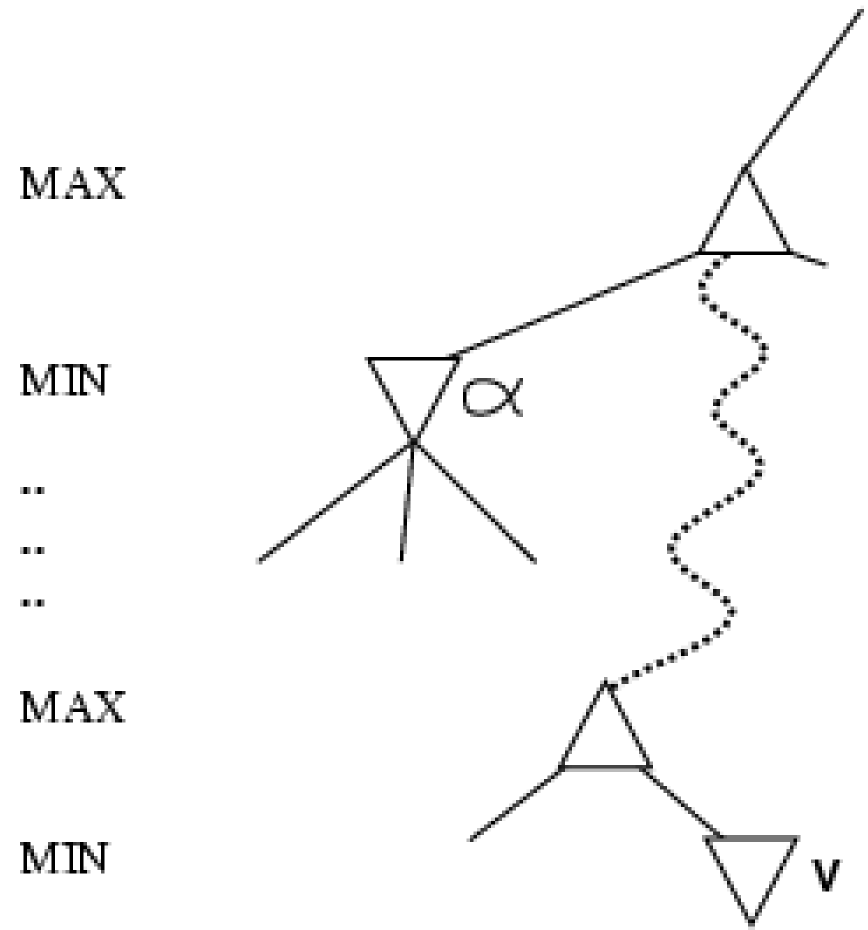


α-β pruning example (cont’d)



Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*
- If *v* is worse than α, *max* will avoid it
  → prune that branch
- Define β similarly for *min*



Properties of α-β

- Back up values from an intermediate node if
  - All children of the node has the values sent up
  - An α cut or a β cut is detected at the node
- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
  - With “best first”, time complexity = $O(b^{m/2})$
    - → $b^{1/2}$ breadth of search
  - With “random walk”, time complexity = $O(b^{3m/4})$

Deterministic games in practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.
- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.
- Othello: human champions refuse to compete against computers, who are too good.
- Go: AlphaGo Master defeated Ke Jie, current world No. 1 ranking player, by three to zero in 2017. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

Summary

- Games are fun to work on!
- They illustrate several important points about AI.
- Perfection is unattainable → must approximate.
- Good idea to think about what to think about.