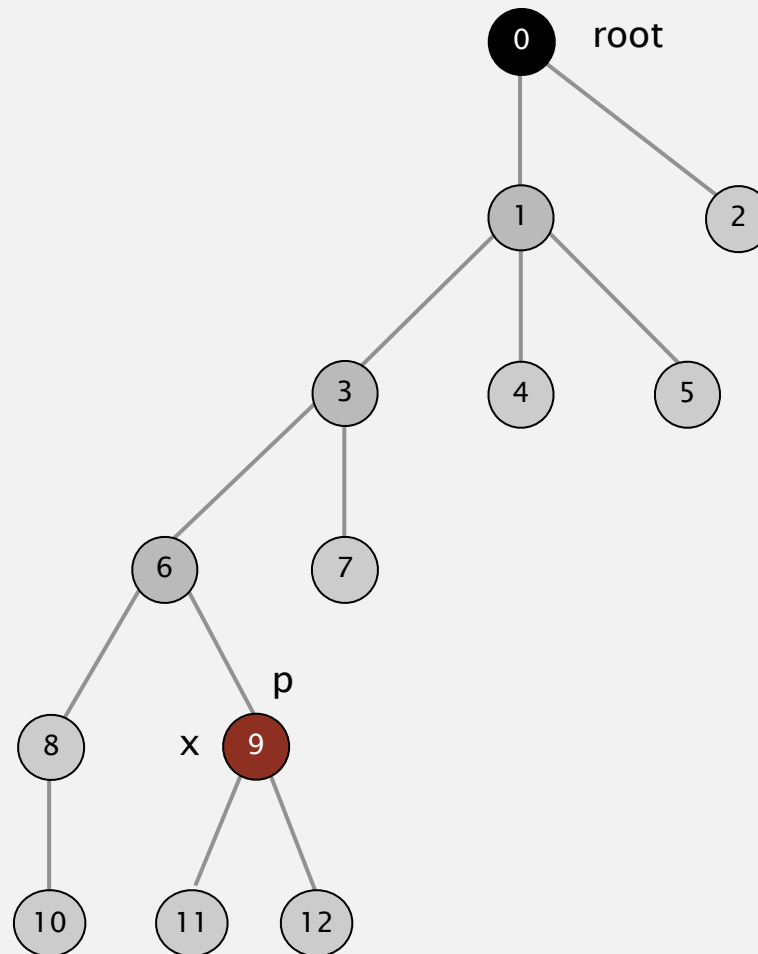


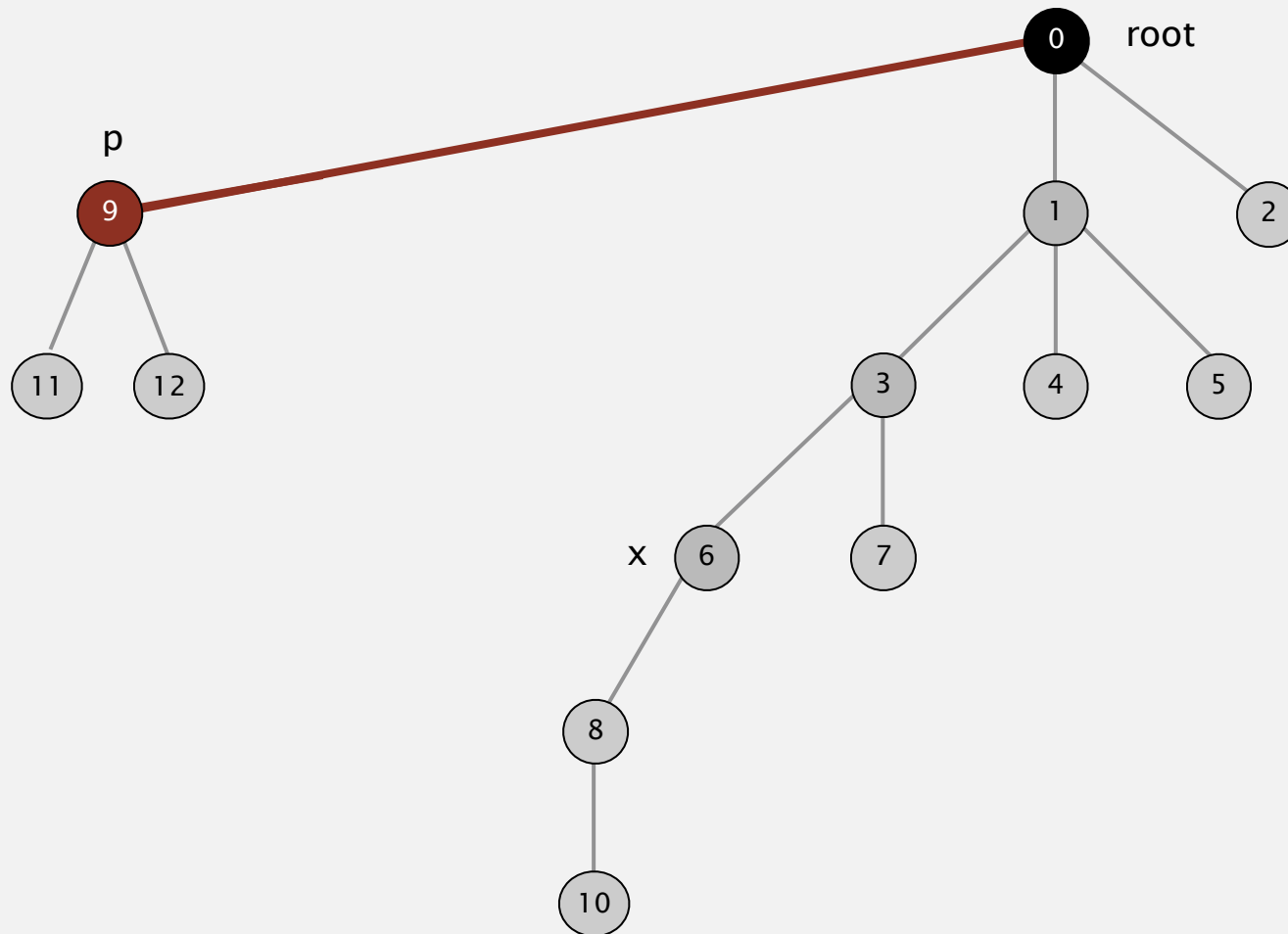
Improvement 2: path compression

Quick union with path compression. Just after computing the root of p , set the id of each examined node to point to that root.



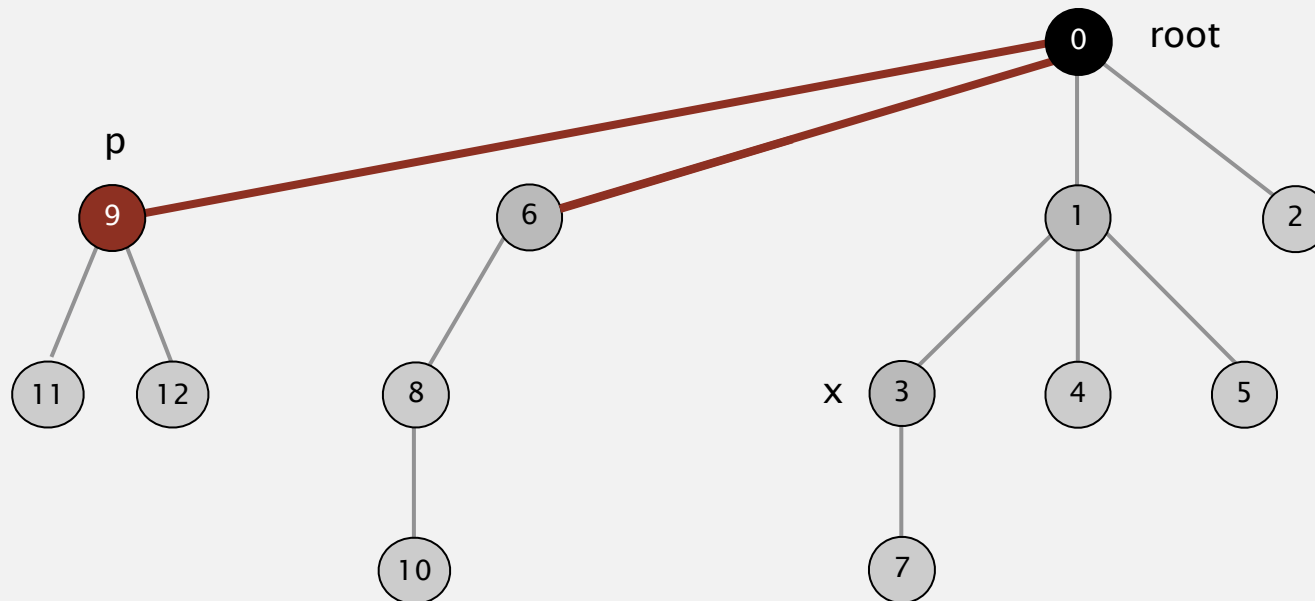
Improvement 2: path compression

Quick union with path compression. Just after computing the root of p , set the id of each examined node to point to that root.



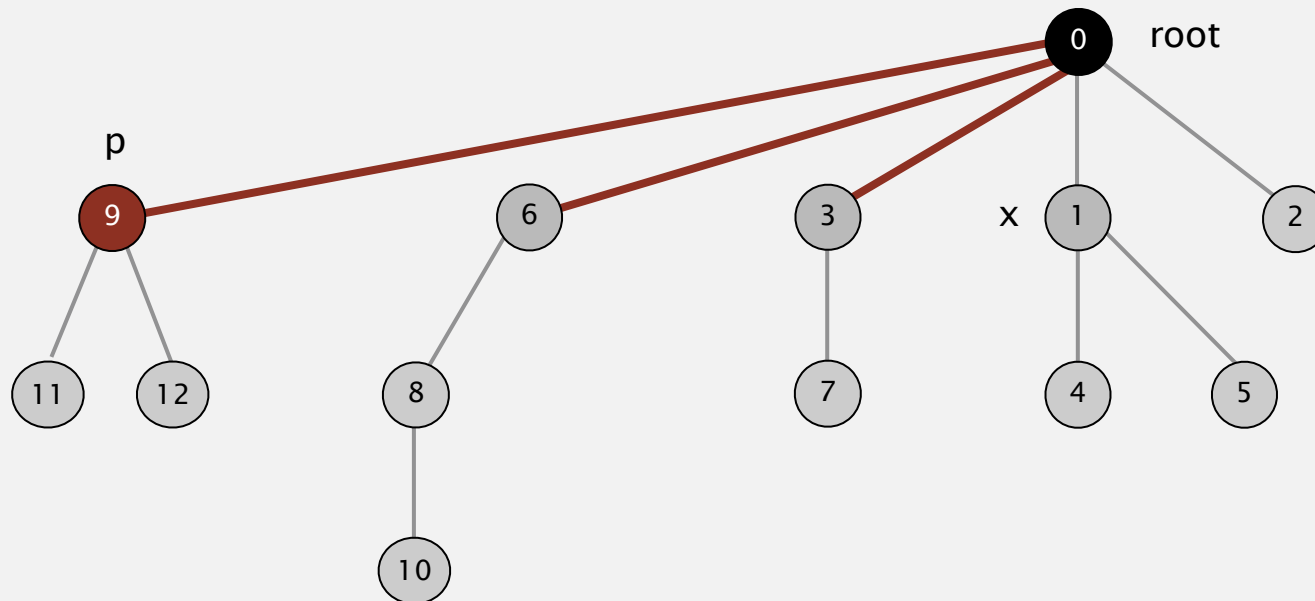
Improvement 2: path compression

Quick union with path compression. Just after computing the root of p , set the id of each examined node to point to that root.



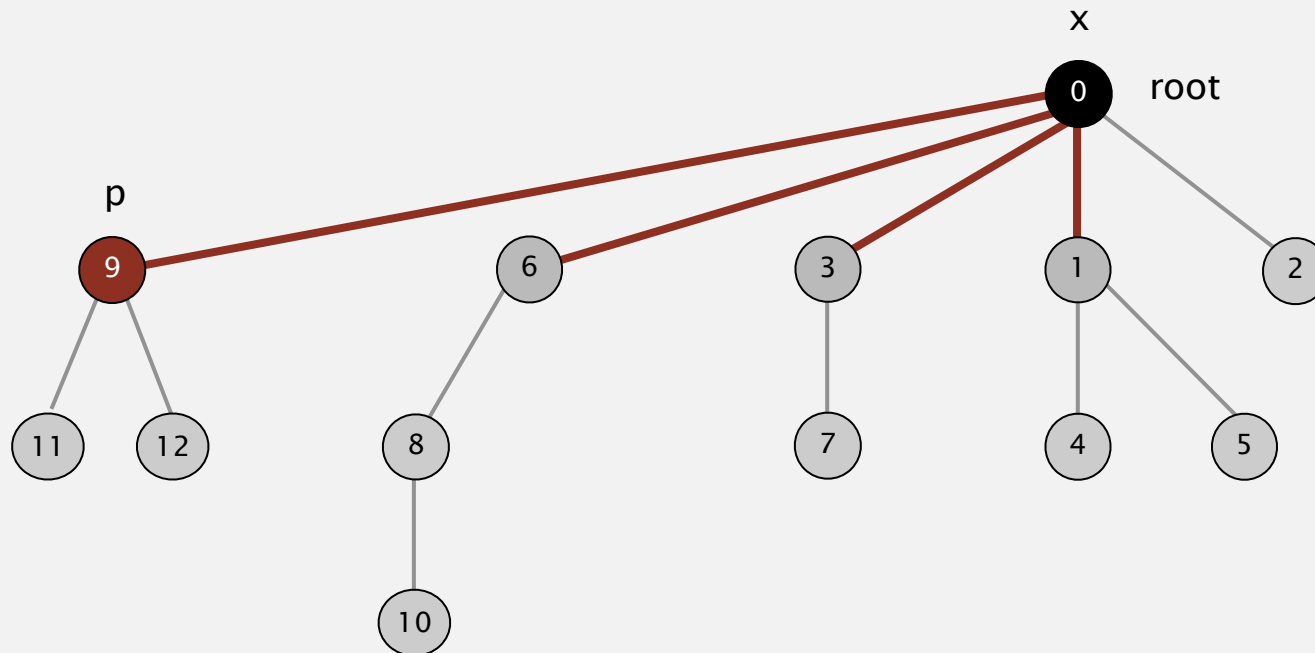
Improvement 2: path compression

Quick union with path compression. Just after computing the root of p , set the id of each examined node to point to that root.



Improvement 2: path compression

Quick union with path compression. Just after computing the root of p , set the id of each examined node to point to that root.



Path compression: Java implementation

Two-pass implementation: add second loop to `root()` to set the `id[]` of each examined node to the root.

Simpler one-pass variant: Make every other node in path point to its grandparent (thereby halving path length).

```
private int root(int i)
{
    while (i != id[i])
    {
        id[i] = id[id[i]];
        i = id[i];
    }
    return i;
}
```

← only one extra line of code !

In practice. No reason not to! Keeps tree almost completely flat.

Weighted quick-union with path compression: amortized analysis

Proposition. [Hopcroft-Ulman, Tarjan] Starting from an empty data structure, any sequence of M union-find ops on N objects makes $\leq c (N + M \lg^* N)$ array accesses.

- Analysis can be improved to $N + M \alpha(M, N)$.
- Simple algorithm with fascinating mathematics.


N	$\lg^* N$
1	0
2	1
4	2
16	3
65536	4
2^{65536}	5

iterate log function

Linear-time algorithm for M union-find ops on N objects?

- Cost within constant factor of reading in the data.
- In theory, WQUPC is not quite linear.
- In practice, WQUPC is linear.

Amazing fact. [Fredman-Saks] No linear-time algorithm exists.


in "cell-probe" model of computation