Quick-find [eager approach]

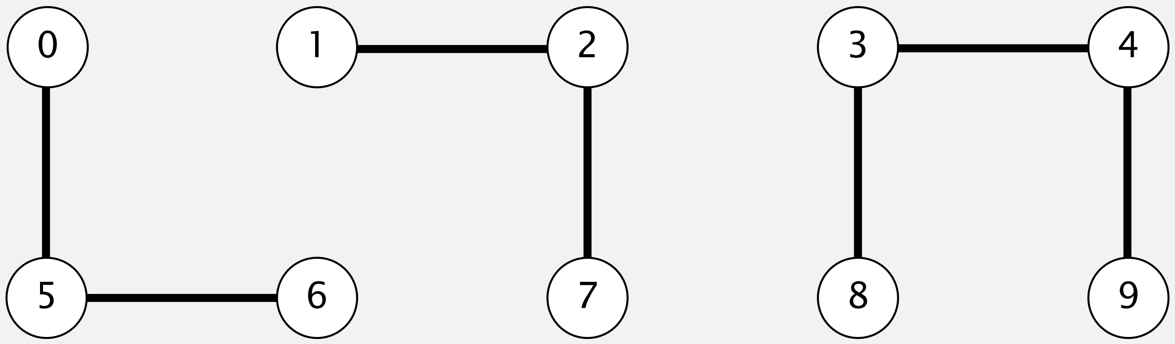
Data structure.

- Integer array `id[]` of length `N`.
- Interpretation: `p` and `q` are connected iff they have the same id.

if and only if

	0	1	2	3	4	5	6	7	8	9
`id[]`	0	1	1	8	8	0	0	1	8	8

0, 5 and 6 are connected
1, 2, and 7 are connected
3, 4, 8, and 9 are connected



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`id[]`	0	1	1	8	8	0	0	1	8	8

Find. Check if `p` and `q` have the same id.

`id[6] = 0; id[1] = 1`
6 and 1 are not connected

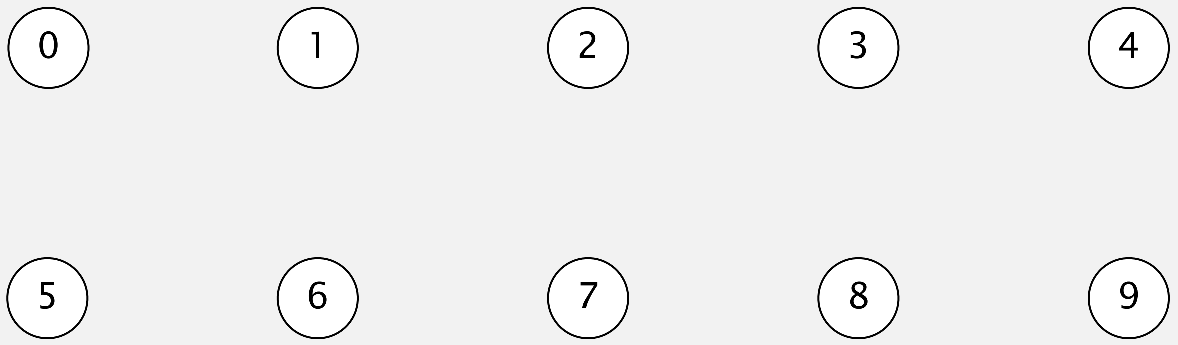
Union. To merge components containing `p` and `q`, change all entries whose id equals `id[p]` to `id[q]`.

	0	1	2	3	4	5	6	7	8	9
`id[]`	1	1	1	8	8	1	1	1	8	8

after union of 6 and 1

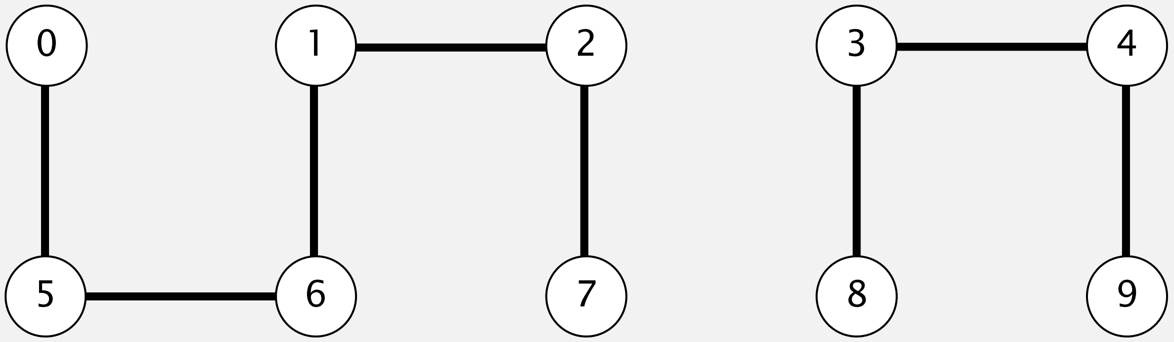
problem: many values can change

Quick-find demo



	0	1	2	3	4	5	6	7	8	9
id[]	0	1	2	3	4	5	6	7	8	9

Quick-find demo



	0	1	2	3	4	5	6	7	8	9
id[]	1	1	1	8	8	1	1	1	8	8

Quick-find: Java implementation

```
public class QuickFindUF
{
   private int[] id;

   public QuickFindUF(int N)
   {
      id = new int[N];
      for (int i = 0; i < N; i++)
         id[i] = i;
   }

   public boolean connected(int p, int q)
   {  return id[p] == id[q];  }

   public void union(int p, int q)
   {
      int pid = id[p];
      int qid = id[q];
      for (int i = 0; i < id.length; i++)
         if (id[i] == pid) id[i] = qid;
   }
}
```

set id of each object to itself
(N array accesses)

check whether p and q
are in the same component
(2 array accesses)

change all entries with id[p] to id[q]
(at most 2N + 2 array accesses)

Quick-find is too slow

Cost model. Number of array accesses (for read or write).

algorithm	initialize	union	find
quick-find	N	N	1

order of growth of number of array accesses

quadratic

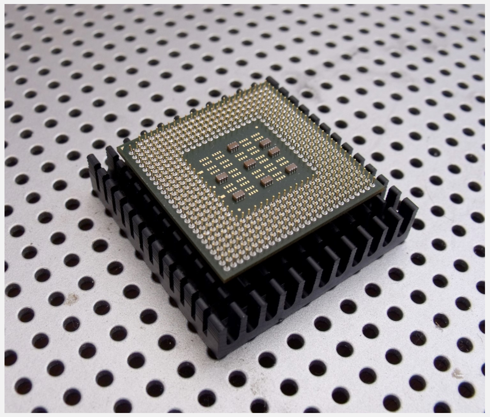
Union is too expensive. It takes N^2 array accesses to process a sequence of N union commands on N objects.

Quadratic algorithms do not scale

Rough standard (for now).

- 10^9 operations per second.
- 10^9 words of main memory.
- Touch all words in approximately 1 second.

a truism (roughly) since 1950!

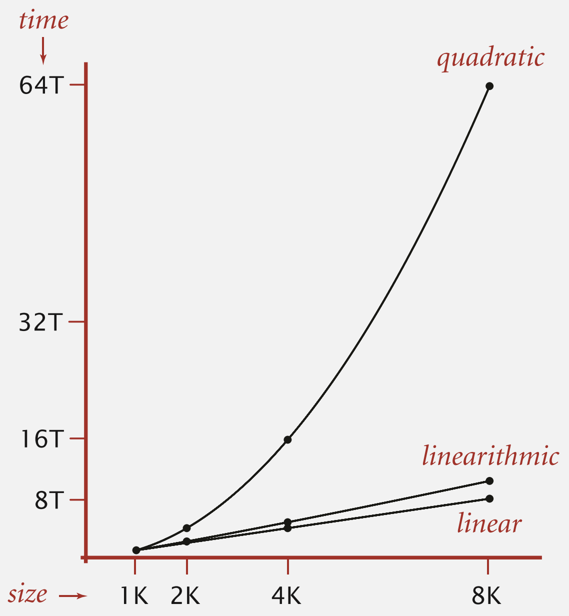


Ex. Huge problem for quick-find.

- 10^9 union commands on 10^9 objects.
- Quick-find takes more than 10^{18} operations.
- 30+ years of computer time!

Quadratic algorithms don't scale with technology.

- New computer may be 10x as fast.
- But, has 10x as much memory $\Rightarrow$ want to solve a problem that is 10x as big.
- With quadratic algorithm, takes 10x as long!


time
64T
32T
16T
8T
quadratic
linearithmic
linear
size
1K
2K
4K
8K