CS 572(Assignment 5)

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Problem 7.2

(a) We are going to use the following syntax and semantics as our knowledge base:
K_1 = the unicorn is mythical
K_2 = the unicorn is mortal
K_3 = the unicorn is mammal
K_4 = the unicorn has horn
K_5 = the unicorn is magical

NOTE: $\neg$ symbol means "NOT"
Now we can represent all the conditions as follows:

(a) $P_1 : K_1 \Rightarrow \neg K_2$
(b) $P_2 : \neg K_1 \Rightarrow (K_2 \wedge K_3)$
(c) $P_3 : (\neg K_2 \vee K_3) \Rightarrow K_4$
(d) $P_4 : K_4 \Rightarrow K_5$

Mythical:
Assume that the Unicorn is Mythical is $True$ i.e. K_1 is true.
Now we can easily say that:
K_2 is $True$, K_3 is $True$, K_4 is $True$, K_5 is $True$
Thus, The Knowledge base is satisfied.

Assume that the Unicorn is Mythical is $False$ i.e. K_1 is False.
Now we can easily say that:
K_2 is $False$, K_3 is $True$, K_4 is $True$, K_5 is $True$
Thus, The Knowledge base is satisfied.
As the knowledge base is satisfied for both the statemets i.e. Unicorn being mythical and not mythical, we cannot infer anything. Hence, Proved!

Magical:
Transforming the Knowledge Base into CNF
$P_1 : \neg K_1 \vee K_2$
$P_2 : (K_1 \vee \neg K_2) \wedge (K_1 \vee K_3)$
$P_3 : (\neg K_2 \vee K_4) \wedge (\neg K_3 \vee K_4)$
$P_4 : \neg K_4 \vee K_5$

Now, lets separate the conjuct:
$C_1 : \urcorner K_1 \vee K_2$
$C_2 : K_1 \vee \urcorner K_2$
$C_3 : K_1 \vee K_3$
$C_4 : \urcorner K_2 \vee K_4$
$C_5 : \urcorner K_3 \vee K_4$
$C_6 : \urcorner K_4 \vee K_5$

Now let's add $\urcorner P_5$, in order to prove or disprove that the Unicorn is Magical.
$C_7 : \urcorner P_5$

Using one CNF rule on other we solve:
$C_8 : K_2 \vee K_3$ —— from C_1 and C_3
$C_9 : K_3 \vee K_4$ —— from C_8 and C_4
$C_{10} : K_4$ —— from C_9 and C_5
$C_{11} : K_5$ —— from C_{10} and C_6
$C_{12} : \{\}$ —— from C_{11} and C_7

So, by Proof of contradiction, we can say that Unicorn is Magical.

Again let's use the same CNF $C_1 : \urcorner K_1 \vee K_2$
$C_2 : K_1 \vee \urcorner K_2$
$C_3 : K_1 \vee K_3$
$C_4 : \urcorner K_2 \vee K_4$
$C_5 : \urcorner K_3 \vee K_4$
$C_6 : \urcorner K_4 \vee K_5$

Now let's add $\urcorner P_4$, in order to prove that the Unicorn is horned.
$C_7 : \urcorner P_4$

Using one CNF rule on other we solve:
$C_8 : K_2 \vee K_3$ —— from C_1 and C_3
$C_9 : K_3 \vee K_4$ —— from C_8 and C_4
$C_{10} : K_4$ —— from C_9 and C_5
$C_{11} : \{\}$ —— from C_{10} and C_7

So, by Proof of contradiction, we can say that Unicorn is Horned.

Problem 7.18
Given: $[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]$

a. Lets use a Truth Table, with the notations as:

T	T	T	T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T	T	T	T	F	F
T	T	T	T	F	T	T	T	T	F	F	T	T
T	F	F	T	F	T	F	T	T	F	F	T	F
T	T	T	T	T	T	T	T	F	F	T	T	T
T	T	F	T	T	F	F	T	F	F	T	T	F
T	T	T	T	F	T	T	T	F	F	F	T	T
T	T	F	T	F	T	F	T	F	F	F	T	F

Figure 1: Truth Table

$[(F \Rightarrow P) \vee (D \Rightarrow P)] \Rightarrow [(F \wedge D) \Rightarrow P]$
So, the above truth table shows that the sentence is true, hence Valid.

b.
Converting the LHS of main implication to CNF:
- $(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)$
- $(\neg Food \vee Party) \vee (\neg Drinks \vee Party)$
- $(\neg Food \vee Party \vee \neg Drinks \vee Party)$
- $(\neg Food \vee \neg Drinks \vee Party)$

Converting the RHS of main implication to CNF:
- $(Food \wedge Drinks) \Rightarrow Party$
- $\neg(Food \wedge Drinks) \vee Party$
- $(\neg Food \vee \neg Drinks) \vee Party$
- $\neg Food \vee \neg Drinks \vee Party$

Since we have $LHS = RHS$, we know that it is a valid statement and thus confirms to answer in Part a.

c.
Proving this with a resolution:
$(\neg Food \vee Party) \vee (\neg Drinks \vee Party) \Rightarrow (\neg Food \vee Party) \vee (\neg Drinks \vee Party)$

Knowledge Base(KB) = $(\neg Food \vee Party) \vee (\neg Drinks \vee Party)$
a = $(\neg Food \vee Party) \vee (\neg Drinks \vee Party)$

Getting a Contradiction: (KB $\wedge$ $\neg$ a)

$\rightarrow [(\neg Food \vee Party) \vee (\neg Drinks \vee Party)] \wedge [\neg((\neg Food \vee Party) \vee (\neg Drinks \vee Party))]$
$\rightarrow [(\neg Food \vee Party) \vee (\neg Drinks \vee Party)] \wedge [(Food \vee \neg Party) \vee (Drinks \vee \neg Party))]$
$\rightarrow [\neg Food \vee Party \vee \neg Drinks] \wedge [Food \vee Drinks \vee \neg Party]$
$\rightarrow \{\}$

Everything here gets cancelled out which means that only empty clause remains, which implied Knlowledge Base(KB), entails a.
Hence, Proved.