CS 572(Assignment 9)

Aashish Dhakal
aashish@iastate.edu

Problem 1: Given:
$P(w_1) = 1/3$ —— (1)
$P(w_2) = 2/3$ —— (2)
$p(x|w_1) \sim N(\theta_1, 1)$ —— (3)
$p(x|w_2) \sim N(\theta_2, 1)$ —— (4)
$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$ —— (5)

Now we have, $D_1 = \{1, 2, 3\}$ and $D_2 = \{3, 7\}$
From (3) and (5), we have
$L(x|\theta_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\theta_1)^2}$ ——— (6)

Now, $L(\theta_1) = P(D_1|\theta_1) = P(x = 1|\theta_1) * P(x = 2|\theta_1) * P(x = 3|\theta_1)$
Taking log on both sides:
$L\ L(\theta_1) = ln\ P(x = 1|\ \theta_1) * ln\ P(x = 2|\ \theta_1) * ln\ P(x = 3|\ \theta_1)$

$L\ L(\theta_1) = log \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(1-\theta_1)^2} * log \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(2-\theta_1)^2} * log \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(3-\theta_1)^2}$ —— (7)

Taking partial derivative of (7) and set it equals to 0.
$= (1 - \theta_1)\ +\ (2 - \theta_1)\ +\ (1 - \theta_1)$
$\Rightarrow 6\ = 3\theta_1$
$\theta_1\ =\ 2$

Similarly, for θ_2:
$=\ (3 - \theta_2)\ +\ (7 - \theta_2)$
$\Rightarrow 10\ = 2\theta_2$
$\theta_2\ =\ 5$

Therefore, we rewrite: $p(x|w_1) \sim N(2, 1);\ p(x|w_2) \sim N(5, 1)$.

Problem 2
The Bayes decision rule for a data point x assigns class (label) w_1 to it, if $p(w_1|x) > p(w_2|x)$, and assigns label w_2 otherwise.

$$p(w_1|x) = \frac{p(x|w_1)p(w_1)}{C}; p(w_2|x) = \frac{p(x|w_2)p(w_2)}{C},$$

Where $C = p(x)$, which can be treated as a normalization constant, and is not relevant for the decision.

Let's find a condition at which $p(w_1|x) > p(w_2|x)$:

$$\frac{p(x|w_1)}{3} > \frac{p(x|w_2)2}{3}$$
$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1(x-2)^2}{2}} > 2\frac{1}{\sqrt{2\pi}}e^{-\frac{1(x-5)^2}{2}}$$
$$e^{-\frac{1(x-2)^2}{2}} > e^{-\frac{1(x-5)^2}{2}}$$

Lets' take a log of each part of the inequality:

$$(-\tfrac{1}{2})(x^2 - 4x + 4) > log(2) + (-\tfrac{1}{2})(x^2 - 10x + 25)$$
$$(-\tfrac{1}{2})(6x - 21) > log(2)$$
$$x < \frac{-2log(2)+21}{6} \approx 3.27$$

Therefore, for $x \ <\sim \ 3.27$, Bayes decision rule assigns label w_1 to x, w_2 otherwise.