Assignment 2

no late homework would be accepted

1 (Exercise 14.3-8) Let $G = (V, E)$ be a weighted, directed graph with nonnegaive weight function $w : E \to \{0, 1, ..., W\}$ for some nonnegative integer W. Modify Dijkstra's algorithm to compute the shortest paths from source vertex s in $O(W \cdot |V| + |E|)$ time.

2 Modify Dijkstra's algorithm in order to solve the bottleneck path problem: Given a directed graph $G = (V, E)$ with edge weight $c : E \to R$, and two nodes $s, t \in V$, find an s-t-path whose longest edge is shortest possible. Describe whole algorithm and show the correctness of your algorithm.

3 (Exercise 25.1-9) Modify Faster-All-Pairs-Shortest-Paths so that it can determine whether the graph contains a negative-weight cycle.

4 Let $a_1, \ldots, a_n$ be a sequence of positive integers. A *labeled tree* for this sequence is a binary tree T of n leaves named $v_1, \ldots, v_n$, from left to right. We label v_i by a_i, for all i, $1 \leq i \leq n$. Let D_i be the length of the path from v_i to the root of T. The *cost* of T is given by

$$cost(T) = \sum_{i=1}^{n} a_i D_i.$$

The problem is: Given a sequence of n positive integers $a_1, \ldots, a_n$, construct a labeled tree for this sequence that has the lowest cost. Your algorithm should run in $O(n^3)$ time. (Hint: Use Dynamic Programming.)

Your answer should include: (i) The main ideas (in words) behind the algorithm which makes the correctness self-evident, (ii) pseudocode, and (iii) an analysis of the running time and space.

5 (Exercise 15.1-15) The Fibonacci number are defined by recurrence

$$F_0 = 0, F_1 = 1, F_i = F_{i-1} + F_{i-2}.$$

Give a $O(n)$-time dynamic-programming algorithm to compute the nth Fibonacci number. Draw the subproblem graph. How many vertices and edges are in the graph?

6 Assume that you have an unlimited supply of coins in each of the integer denominations $d_1, d_2, \ldots, d_n$, where each $d_i > 0$. Given an integer amount $m \geq 0$, we wish to make change for m using the minimum number of coins drawn from the above denominations.

Give a dynamic programming algorithm for this problem. You need only determine the minimum number of coins required, not the actual denominations that are used.

Your answer must include (a) a brief description of the main ideas (from which the correctness of the method should be evident), (b) pseudocode, and (c) an analysis of the running time and space as a function of n and m.

7 (Exercise 24.2-4) Give an efficient algorithm to count the total number of paths in a directed acyclic graph. Analyze your algorithm.

8 Given a directed graph $G = (V, E)$, with nonnegative weight on its edges, and in addition, each edge is colored red or blue. A path from u to v in G is characterized by its total length, and the number of times it switches colors. Let $\delta(u, k)$ be the length of a shortest path from a source node s to u that is allowed to change color at most k times. Design a dynamic program to compute $\delta(u, k)$ for all $u \in V$. Explain why your algorithm is correct and analyze its running time.