Assignment

EE23BTECH11001 - Aashna Sahu

Q:Which one of the options given is the inverse Laplace transform of $\frac{1}{s^3-s}$?
$u(t)$ denotes the unit-step function.

(A) $\left(-1+\frac{1}{2}e^{-}t+\frac{1}{2}e^{t}\right)u(t)$

(B) $\left(\frac{1}{3}e^{-}t-e^{t}\right)u(t)$

(C) $\left(-1+\frac{1}{2}e^{-(t-1)}+\frac{1}{2}e^{(t-1)}\right)u(t-1)$

(D) $\left(-1-\frac{1}{2}e^{-(t-1)}-\frac{1}{2}e^{(t-1)}\right)u(t-1)$

(GATE ME 2023)

Solution:
Using partial fraction,

$$\frac{1}{s^3-s}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s-1} \tag{1}$$

On Solving,

$$\implies A=-1 \quad B=\frac{1}{2} \quad C=\frac{1}{2} \tag{2}$$

$$X(s)=\frac{-1}{s}+\frac{1}{2(s+1)}+\frac{1}{2(s-1)} \tag{3}$$

$$\text{As} \quad e^{-at}u(t)\xrightarrow{\mathcal{L}}\frac{1}{s+a} \quad \Re(s)>-a \tag{4}$$

$$\text{And} \quad -e^{-at}u(-t)\xrightarrow{\mathcal{L}}\frac{1}{s+a} \quad \Re(s)<-a \tag{5}$$

Now,

$$\mathcal{L}^{-1}(X(s))=x(t) \tag{6}$$

There are 4 cases possible,

$$x(t)=\begin{cases}\left(1-\frac{1}{2}e^{-t}-\frac{1}{2}e^{t}\right)u(-t) & \Re(s)<-1\\ -1u(-t)+\frac{1}{2}e^{-t}u(t)-\frac{1}{2}e^{t}u(-t) & \Re(s)\in(-1,0)\\ -u(t)+\frac{1}{2}e^{-t}u(t)-\frac{1}{2}e^{t}u(-t) & \Re(s)\in(0,1)\\ \left(-1+\frac{1}{2}e^{-}t+\frac{1}{2}e^{t}\right)u(t) & \Re(s)>1\end{cases} \tag{7}$$

Thus the correct option is (A) $\left(-1+\frac{1}{2}e^{-}t+\frac{1}{2}e^{t}\right)u(t)$ for $\Re(s)>1$