

Assignment

EE23BTECH11001 - Aashna Sahu

Q: Which one of the options given is the inverse Laplace transform of $\frac{1}{s^3-s}$?
 $u(t)$ denotes the unit-step function.

- (A) $\left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t\right)u(t)$
- (B) $\left(\frac{1}{3}e^{-t} - e^t\right)u(t)$
- (C) $\left(-1 + \frac{1}{2}e^{-(t-1)} + \frac{1}{2}e^{(t-1)}\right)u(t-1)$
- (D) $\left(-1 - \frac{1}{2}e^{-(t-1)} - \frac{1}{2}e^{(t-1)}\right)u(t-1)$

(GATE ME 2023)

Solution:

Using partial fraction,

$$\frac{1}{s^3-s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} \quad (1)$$

On Solving,

$$\Rightarrow A = -1 \quad B = \frac{1}{2} \quad C = \frac{1}{2} \quad (2)$$

$$X(s) = \frac{-1}{s} + \frac{1}{2(s+1)} + \frac{1}{2(s-1)} \quad (3)$$

$$\text{As } e^{-at}u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+a} \quad \text{Re}(s) > -a \quad (4)$$

$$\text{And } -e^{-at}u(-t) \xrightarrow{\mathcal{L}} \frac{1}{s+a} \quad \text{Re}(s) < -a \quad (5)$$

Now,

$$\mathcal{L}^{-1}(X(s)) = x(t) \quad (6)$$

There are 4 cases possible,

$$x(t) = \begin{cases} \left(1 - \frac{1}{2}e^{-t} - \frac{1}{2}e^t\right)u(-t) & \text{Re}(s) < -1 \\ -1u(-t) + \frac{1}{2}e^{-t}u(t) - \frac{1}{2}e^t u(-t) & \text{Re}(s) \in (-1, 0) \\ -u(t) + \frac{1}{2}e^{-t}u(t) - \frac{1}{2}e^t u(-t) & \text{Re}(s) \in (0, 1) \\ \left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t\right)u(t) & \text{Re}(s) > 1 \end{cases} \quad (7)$$

Thus the correct option is (A) $\left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t\right)u(t)$ for $\text{Re}(s) > 1$