

# Assignment

EE23BTECH11001 - Aashna Sahu

Q: Which one of the options given is the inverse Laplace transform of  $\frac{1}{s^3-s}$ ?  
 $u(t)$  denotes the unit-step function.

(A)  $\left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t\right)u(t)$

(B)  $\left(\frac{1}{3}e^{-t} - e^t\right)u(t)$

(C)  $\left(-1 + \frac{1}{2}e^{-(t-1)} + \frac{1}{2}e^{(t-1)}\right)u(t-1)$

(D)  $\left(-1 - \frac{1}{2}e^{-(t-1)} - \frac{1}{2}e^{(t-1)}\right)u(t-1)$

(GATE ME 2023)

**Solution:**

Using partial fraction

$$\frac{1}{s^3-s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} \quad (1)$$

On Solving

$$\Rightarrow A = -1 \quad B = \frac{1}{2} \quad C = \frac{1}{2} \quad (2)$$

$$X(s) = \frac{-1}{s} + \frac{1}{2(s+1)} + \frac{1}{2(s-1)} \quad (3)$$

$$\text{As } e^{-at}u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+a} \quad (4)$$

Now

$$\mathcal{L}^{-1}(X(s)) = x(t) \quad (5)$$

$$x(t) = -1e^0u(t) + \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^tu(t) \quad (6)$$

$$\Rightarrow x(t) = \left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^t\right)u(t) \quad (7)$$