Exam Seat No.______

THAKUR COLLEGE OF SCIENCE & COMMERCE

NAAC Accredited with Grade "A" (3rd Cycle)

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ISO 9001 : 2015 Certified

Degree College

Computer Journal

CERTIFICATE

SEMESTER II **UID No.** ______

Class F·Y·BSC-(CS) **Roll No.** 1823 **Year** 2019-20

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AB 01/02/2020

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Examiner

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Practical No. 01

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$= \lim_{x \to a} \dfrac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \lim_{x \to a} \dfrac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \dfrac{1}{3} \lim_{x \to a} \dfrac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \dfrac{1}{3} \dfrac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$= \dfrac{1}{3} \times \dfrac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$= \dfrac{1}{3} \times \dfrac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$= \dfrac{1}{3} \times \dfrac{4\sqrt{a}}{2\sqrt{3a}}$

$= \dfrac{2}{3\sqrt{3}}$

2. $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

$$= \lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$$

$$= \lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$$

$$= \lim_{y \to 0} \frac{y}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a+2}\,(\sqrt{a+0} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\,(\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\,(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

3. $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$

By substituting $x - \dfrac{\pi}{6} = h$

$x = h + \dfrac{\pi}{6}$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$$

$$= \lim_{h \to 0} \frac{\cos h \cdot \cos \pi/6 - \sin h \cdot \sin \pi/6 - \sqrt{3}\sin h \cos \pi/6 + \cos h \sin \pi/6}{\pi - 6\left(\dfrac{6h + \pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h \cdot \sqrt{3}/2 - \sin h \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h \sqrt{3}/2 + \cos h /\sqrt{2}\right)}{\pi - 6h + \pi}$$

$$= \lim_{h \to 0} \frac{\cos \sqrt{3}/2\, h - \sin h/2 - \sin 3h/2 - \cos \frac{\sqrt{3}h}{2}}{-6h}$$

$$= \lim_{h \to 0} \frac{-\dfrac{\sin 4h}{2}}{-6h} = \lim_{h \to 0} \frac{\sin 4h}{3 + 2h}$$

$$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = 1/3 \times 1 = \frac{1}{3}.$$

4) $\lim \left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator & Denominator both

$= \lim\limits_{x\to\infty} \left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$

$= \lim\limits_{x\to\infty} \left[\dfrac{(x^2+5-x^2+3)\cdot(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)\,(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$

$= \lim\limits_{x\to\infty} \dfrac{8\,(\sqrt{x^2+3}+\sqrt{x^2+1})}{2\,(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$= 4 \lim\limits_{x\to\infty} \dfrac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$

After applying limit we get,

$= 4$

5) $F(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$

at $x = \pi/2$

$F(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore F(\pi/2) = 0$

$\therefore$ at $x = \pi/2$ define.

1) $\lim_{x \to \pi/2} F(x) = \lim_{x \to \pi/2} \frac{\cos x}{\pi - 2x}$

By substituting method

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$$

$$= \lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2(2h + \pi/2)}$$

$$= \lim_{h \to 0} \frac{\cos(h + \frac{\pi}{2})}{-2h}$$ using $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$.

$$= \lim_{h \to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$$

$$= \lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$= \lim_{h \to 0} \frac{-\sin h}{-2h} = \frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{2}$$

b) $\lim_{x \to \pi/2} F(x) = \lim_{x \to \pi/2} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$

$= \lim_{x \to \pi/2} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$

$= \lim_{x \to \pi/2} \frac{2 \cdot \sin x \cdot \cos x}{\sqrt{2} \cdot \sin x}$

$= \lim_{x \to \pi/2} \frac{2 \cos x}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} \lim_{x \to \pi/2} \cos x$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ F is not continous at $x = \pi/2$

5) ii) $F(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & 0 < x < 3 \\ = x + 3 & 3 \le x \le 6 \\ \cancel{\frac{x^2 + 9}{x + 3}} & 6 \le x < 9 \end{cases}$ at $x = 3$ & $x = 6$

at $x = 3$.

i) $F(3) = \frac{x^2-9}{x-3} = 0$

f at $x = 3$ define

ii) $\lim_{x \to 3^+} F(x) = \lim_{x \to 3} x + 3.$

$F(3) = x + 3 = 3 + 3 = 6$

f is defined at $x = 3$

$\lim_{x \to 3^+} F(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} F(x) = \lim_{x \to 3} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ L.H.L = R.H.L

F is continous at $x = 3$

For $x = 6$

$F(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = \underline{\underline{3}}$

i) $\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x \to 6^+} (x-3) = 6-3 = 3$

$\lim_{x \to 6^+} x + 3 = 3+6 = 9$

$\therefore$ L.H.S $\neq$ R.H.L

6) $F(x) = \frac{1-\cos 4x}{x^2}$ $\quad x \neq 0$ $\Big\}$ at $x=0$

$= k$ $\quad x = 0$

$\rightarrow$ f is continuous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$

$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$

$8 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$\therefore k = 8$

ii) $F(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$ $\Big\}$ at $x = 0$.

$= k$ $\quad x = 0$

$\rightarrow F(x) = (\sec^2 x)^{\cot^2 x}$

using

$\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$ &

$\cot^2 x = \frac{1}{\tan^2 x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x \to 0} (1 + \tan^2 x)^{1/\tan^2 x}$

we know that

$\lim_{x \to 0} (1 + p(x))^{1/p(x)} = e$

$= e$

$\therefore K = e.$

iii) $p(x) = \begin{cases} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & x = \pi/3 \\ = K & x \neq \pi/3 \end{cases}$ at $x = \pi/3$

$x - \pi/3 = h$

$x = h + \pi/3$

where $h \to 0$

$f(\pi/3) + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim_{h \to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

Using

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - \tan \pi/3 + \tan h}{1 - \tan \pi/3 \ \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}\,(1 - \tan \pi/3 \cdot \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$= \lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3 \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \tan h}}{-3h}$$

$$= \lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3 \tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$= \lim_{h \to 0} \frac{-4 \tan h}{-3h\,(1 - \sqrt{3} \tan h)}$$

$$= \lim_{h \to 0} \frac{4 \tan h}{3h\,(1 - \sqrt{3} \tan h)}$$

$$= \frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \ \lim_{h \to 0} \frac{1}{(1 - \sqrt{3} \tan h)} \qquad \frac{\tan h}{h} = 1$$

$$= \frac{4}{3} \ \frac{1}{(1 - \sqrt{3}\,(0))}$$

$$= \frac{4}{3}$$

7) $F(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ 9 & x = 0 \end{cases}$ at $x=0$

$\rightarrow f(0) = 9$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos 3x}{x\tan x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 \frac{3x}{2}}{x\tan x} \quad \ldots \left\{\because 1-\cos x = 2\sin^2 \tfrac{x}{2}\right\}$$

$$= \lim_{x\to 0} \frac{\dfrac{2\sin^2 3x/2}{x^2}}{\dfrac{x\tan x}{x^2}}$$

$$= 2\lim_{x\to 0} \frac{\dfrac{\sin^2 \frac{3x}{2}}{9/4\, x^2} \times 9/4}{\dfrac{\tan x}{x}}$$

$$= \frac{9}{4} \times 2 \frac{\left(\lim\limits_{x\to 0} \dfrac{\sin \frac{3x}{2}}{\frac{3x}{2}}\right)^2}{\lim\limits_{x\to 0} \dfrac{\tan x}{x}}$$

$$= \frac{9}{2} \frac{(1)^2}{1} = 9/2$$

$\therefore \lim\limits_{x\to 0} f(x) \neq f(0)$

$\therefore f$ is discontinuous at $x=0$

i) $f(x) = \frac{1-\cos 3x}{x \tan x}$ $\quad x \neq 0$, $x = 0$ } at $x=0$

$= 9/2$

ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\quad x \neq 0$, $x=0$ } at $x=0$

$= \pi/60$

$\rightarrow f(0) = \frac{\pi}{60}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{3x}-1 \sin x^\circ}{x^2}$$

$$= \lim_{x\to 0} \left(\frac{e^{3x}-1}{x}\right) \times \left(\frac{\sin x^\circ}{x}\right)$$

$$= \lim_{x\to 0} \left(\frac{e^{3x}-1}{3x} \times 3\right) \times \left(\frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \times \pi/180\right)$$

$$= 3\left(\lim_{x\to 0} \frac{e^{3x}-1}{3x}\right) \times \frac{\pi}{180} \left(\lim_{x\to 0} \frac{\sin \frac{\pi x}{180}}{\pi x/180}\right)$$

$= 3 \log e \times \frac{\pi}{180} (1)$ $\quad \left\{ \because \lim_{x\to 0} \frac{e^x-1}{x} = \log e \right.$ & $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$= 3 \times \frac{\pi}{180}$

$= \frac{\pi}{60} = f(0)$

$\therefore f$ is continuous $\because \lim f(x) = f(0)$ at $x = 0$.

iii) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x = 0$, find $f(0)$.

→ f is continuous at $x = 0$

$$f(0) = \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \frac{1 - \cos x}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \frac{2 \sin^2 x/2}{x^2/4 \cdot 4}$$

$$= \left(\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} \right) + \frac{1}{2} \lim_{x \to 0} \left(\frac{\sin x/2}{x/2} \right)^2$$

$$= \log e + \frac{1}{2}(1)^2 \qquad \left[\because \lim_{x \to 0} \frac{e^x - 1}{x} = 1 \right]$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

iv) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$\rightarrow$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{1 + \sin x}{(1-\sin x)(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2\,(2\sqrt{2})}$$

$$\cancel{= \frac{1}{4(\sqrt{2})}}$$

$$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

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PRACTICAL No. 2

Derivative

Q.1) show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to a} \frac{\tan x - \tan a}{(x-a) \tan x \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a + h - a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times (\tan(a+h) \tan a}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \tan B)$

$$\lim_{h\to 0} \frac{(a-a+h) - (1+\tan a \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a} = -\frac{\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \quad \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$f(a) = -\operatorname{cosec}^2 a$

$\therefore$ f is differentiable $\forall a \in R$

ii) $\operatorname{cosec} x$

$$f(x) = \operatorname{cosec} x$$

$$f(a) = \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \frac{\sin a + \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$$f(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

$$= \lim_{h \to 0} -\frac{\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2 \cdot \cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2 \cos(2a+0/2)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \cdot \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$f(a) = \frac{\lim_{x \to a} f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$\cos C + \cos D =$

Formula:- $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a+h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos a \cos(a+h) \times (-h/2)} \times -1/2$$

$$= \frac{-1}{2} \times -2 \frac{\sin\left(2a+0/2\right)}{\cos a \cos(a+0)}$$

$$= \frac{-1}{2} \times (-2) \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \sec a$

Q.2) $f(x) = 4x+1, \quad x \le 2$
$\quad = x^2+5, \quad x > 0$, at $x = 2$.

$\rightarrow$ $\lim_{x\to 2^-} \frac{f(x) - f(2)}{x-2}$

$$\lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$\lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x - 8}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{x-2} = 4$$

$\therefore D(2^-) = 4$

R.H.D =

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$$= 2 + 2 = 4.$$

$Df(2^+) = 4.$

RHD = LHD

f is differentiable at $x = 2$

3) $f(x) = 4x + 7 \quad , \quad x < 3$

$\quad = x^2 + 3x + 1 \quad , \quad x \geq 3$ at $x - 3$.

RHD:

$$Df(3^+) = \cancel{\lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 \times 3 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$\lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$$

$$\lim_{x \to 3^+} \frac{(x+6)(x-6)}{x-3} = 3+6 = 9$$

$Df(3^+) = 9.$

$LHD = Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}$$

$Df(3^+) = 4$

$RHD = LHD$

f is not differentiable at $x = 3$

5) $f(x) = 8x - 5$, $x \le 2$
$= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$

→ $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

R.H.D =

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$$= 3 \times 2 + 2 = 8$$

$Df(2^+) = 8$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)} = 8.$$

$\therefore$ LHD = RHD

$\therefore Df(2^-) = 8$

AK
06/02/2020

Practical No. 3

Aim :- Application of Derivation

Q.1) Find the interval in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

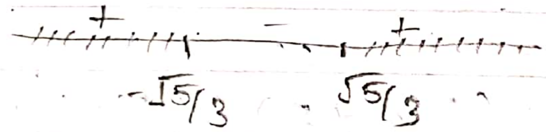
$\therefore f'(x) = 3x^2 - 5$

$\therefore f$ is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$



$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x - 2) > 0$

$\therefore x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$x \in (-2, 5/3)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

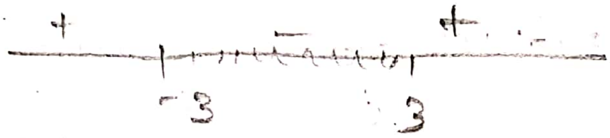
$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$



$\therefore x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore (x-4)(x+1) > 0$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

$\therefore x \in (-1, 4)$

Q2)

1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upwards if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64.$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2$

$\therefore x \in (3/2, \infty).$

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5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\therefore f''(x) \not> 0$

$\therefore$ There exisit no interval.

Practical No:- 4

Q.1)

i) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

Now consider,

$f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2+6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = \cancel{2^2} + 16/2^2$

$= 4 + 16/4$

$= 4+4$

$= 8$

$f''(-2) = 2 + 96/-2^4$

$= 2 + 96/16$

$= 2+6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum values at $x = 2$, and $x = -2$

2) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider,

$f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$\therefore f''(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + \cancel{60(-1)^3}$

$= 30 - \cancel{60}$

$= -30 < 0$

$\therefore f$ has maximum values at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

3) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0, \; x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

4) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2 \times 8 - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 > 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$

Q.2) Sol:

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \rightarrow$ Given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = 0 + \frac{9.5}{55}$$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{55.9467}$$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.1712 + \frac{0.0011}{55.9393}$$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712.

2) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newton's Method,

$x_{n+1} = x_n - \frac{f(x_0)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{6}{23}$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9861 - 4$

$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7156 - 10.806 - 9$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 21.8943 - 4$

$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7015 + 0.0050$

$= 2.7065$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation

By (ii) Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{2.2}{5.2}$$

$$= 2 - 0.4230$$

$$= 1.577$$

$$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$$

$$= 3.9219 - 4.4764 - 15.77 + 17$$

$$= 0.6755$$

$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$

$$= 7.4608 - 5.6772 - 10$$

$$= -8.2164$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + \frac{0.6755}{8.2164}$$

$$= 1.577 + 0.0822$$

$$= 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$

$$= 4.5677 - 6.9653 - 16.592 + 17$$

$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$

$$= 8.2588 - 5.97312 - 10$$

$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= \cancel{1.6518}\ 1.6592 + \frac{0.0204}{7.7143}$$

$$= 1.6592 + 0.0026$$

$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$

$$= 4.5892 - 4.9708 - 16.618 + 17$$

$$= 0.0004$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$

$$= 8.2847 - 5.9824 - 10$$

$$= -7.6977$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.6618 + \frac{0.0004}{7.6977}$$

$$= 1.6618$$

The root of equation is 1.6618.

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PRACTICAL No:- 5

1) Solve the following integration.

1) $\int \frac{dx}{\sqrt{x^2+2x-3}} \, dx$

$I = \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$I = \int \frac{1}{\sqrt{x^2+2x+1-4}} \, dx$

$= \int \frac{1}{\sqrt{(x-1)^2-4}} \, dx = \int \frac{1}{\sqrt{(x+1)^2-(2)^2}} \, dx$

$\because a^2+2ab+b^2 = (a+b)^2$

substitute,

$x+1=t$

$dx = \frac{1}{t} \times dt$ where, $t=1$; $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$

$= \log(|t+\sqrt{t^2-4}|)$

$\left[\because \int \frac{1}{\sqrt{x^2-a^2}} \, dx = \log|x+\sqrt{x^2-a^2}|\right]$

$t=x+1$

$= \log(|x+1+\sqrt{(x+1)^2-4}|)$

$= \log(|x+1+\sqrt{x^2+2x-3}|)+c$

2) $\int (4e^{3x}+1) \, dx$

$I = \int (4e^{3x}+1) \, dx$

$= \int 4e^{3x} dx + \int 1 \, dx$

$= 4\int e^{3x} dx + \int 1 \, dx$

$= \frac{4e^{3x}}{3} + x$ $\left[\because \int e^{ax} dx = \frac{1}{a} e^{ax}\right]$

$= \frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x} \, dx$

$I = \int 2x^2 - 3\sin x + 5\sqrt{x} \, dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2} dx$

$= \int 2x^2 dx - 3\int \sin x \, dx + 5\int x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt[3]{x}}{3} + c$

$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c \quad \int \sin x \, dx = -\cos x \, dx\right]$

$= \frac{2x^3 + 10\sqrt[3]{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$I = \int \frac{x^3+3x+4\,dx}{\sqrt{x}}$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right) dx$

$= \int \frac{x^3}{x^{1/2}}\,dx + \int \frac{3x}{x^{1/2}}\,dx + \int \frac{4x}{x^{1/2}}\,dx$

$= \int x^{5/2}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/2}\,dx$

$= \frac{x^{5/2+1}}{5/2+1} + 3\,\frac{x^{1/3+1}}{1/2+1} + 4\,\frac{x^{-1/2+1}}{-1/2+1}$

$= \frac{x^{7/2}}{7/2} + \frac{3x^{3/2}}{3/2} + 4\,\frac{x^{1/2}}{1/2}$

$= \frac{2x^{7/2}}{7} + 2x^{3/2} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\,du$

$= \int t^4\, \cancel{\sin(2t^4)} \times \frac{1}{2\times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du$

$= \frac{t^4 \times \sin(2t^4)}{8}\,du$

substitute t^4 with $\frac{u}{2}$

$\therefore \int \frac{u/2 \times \sin(2\,u/2)}{8}\,du$

$= \int \frac{u/2 \times \sin(u)/2}{8}\,du$

$= \int \frac{u \times \sin(u)\,du}{16}$

$= \frac{1}{16}\int u \times \sin(u)\,du$

$= \frac{1}{16}\left(u \times (-\cos(u)) - \int -\cos(u)\,du\right)$

$[\because \int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1\,du$ $\quad v = -\cos(u)$

$= \frac{1}{16} \times \left(u \times (-\cos u)\right) + \int \cos(u)\,du$

$= \frac{1}{16} \times \left(u \times (-\cos(u)) + \sin(u)\right)$ $[\because \int \cos x\,dx = \sin x\,dx]$

Resubstituting $\quad u = 2t^4.$

$= \frac{1}{16} \times (8t^4 x(-\cos(2t^4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

(1) $\int \sqrt{x}\,(x^2-1)\,dx$

$I = \int \sqrt{x}\,(x^2-1)\,dx$

$= \int x^{1/2}(x^2-1)\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= \frac{x^{5/2+1}}{5/2+1} - \frac{x^{1/2+1}}{1/2+1}$

$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2}$

$= \frac{2x^{7/2}}{2} - \frac{2x^3}{3} + C$

(2) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$I = \int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \frac{\cos x}{\sin x^{3/2}}\,dx$

put $t = \sin x$

$dt = \cos x\,dx$

$= \frac{1}{(t)^{2/3}}\,dt$

$(t)^{-2/3}\,dt$

$= \frac{t^{-2/3+1}}{-2/3+1}$

$= \frac{t^{1/3}}{1/3}$

$= 3\sqrt[3]{t} + C$

$= 3\sqrt[3]{\sin x} + C$

9) $\int e^{\cos^2 x} \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

put $\cos^2 x = t$

$2\cos x(-\sin x)\, dx = dt$

$-\sin 2x \, dx = dt$

$\sin 2x \, dx = -dt$

$\therefore \int e^t (-dt)$

$-\int e^t dt$

$= -e^t + c \quad [\because \int e^x dx = e^x + c]$

$= -e^t + c$

Resubstituting $\cos^2 x = t$

$\therefore -e^{\cos^2 x} + c$

10) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \, dx$

put $x^3 - 3x^2 + 1 = t$

$\therefore (3x^2 - 6x)\, dx = dt$

$3(x^2 - 2x)\, dx = dt$

$(x^2 - 2x)\, dx = \frac{dt}{3}$

$\int \left(\frac{1}{t}\right) \frac{dt}{3}$

$\frac{1}{3} \int \left(\frac{1}{t}\right) dt$

$= \frac{1}{3} \log |t| + c \quad [\because \int \left(\frac{1}{x}\right) dx = \log |x| + c]$

Resubstituting $x^3 - 3x^2 + 1 = t$

$\therefore \frac{1}{3} \log |x^3 - 3x^2 + 1| + c$

AK

02/01/2020

PRACTICAL No:- 6

AIM:- Application of integration & numerical Integration

Q1) Find the length of the following :-

(i) $x = t - \sin t;\ y = 1 - \cos t,\ t \in [0, 2\pi]$

Soln:- arc length = $\int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^3}$

$$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2}\ dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t + dt} = \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\ dt$$

$$= \int_0^{2\pi} 2\sin {}^1/_2\ dt$$

$$= \left[-4\cos {}^1/_2 \right]_0^{2\pi}$$

$$= (-4\cos\pi) + 4\cos 0$$

$= 8$ units

2) $y = \sqrt{4 - x^2}$. $\quad x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x}}\ 2$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$l = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\ dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\ dx$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 2\pi$$

3) $y = x^{3/2}$ in $[0, 4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$L = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1+ 9/4}\; dx$$

$$= \frac{1}{2} \int_0^4 \sqrt{4+9x}\; dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$$

$$= \frac{-1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$\frac{1}{27}\left(40^{3/2} - 8\right) \text{ units.}$$

4) $x = 3\sin t$, $y = 3\cos t$, $t \in (0, 2\pi)$

$$\frac{dx}{dt} = 3\cos t \; ; \; \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\; dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\; dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\; dt$$

$$= \int_0^{2\pi} 3\sqrt{1}\; dt$$

$$= 3\int_0^{2\pi} 3\cdot dt$$

$$= 6\pi \text{ units.}$$

5) $x = \frac{1}{6}y^3 + \frac{1}{2}y$ or $y \in (1, 2)$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \frac{(y^4-1)^2}{4y^4}}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1) + 4xy^4}{4y^4}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12} \text{ units.}$$

Q-II] $\int_0^4 x^2\, dx$ $x = 4$

$$L = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2\, dx = \frac{1}{3}\left[(y_0 + y_3) + 4(y_1 + y_3)2(y_2)\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$\cancel{= \frac{64}{3}}$$

$$\int_0^4 x^2\, dx = 21.533$$

Q.3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n=6$

$h = \dfrac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4166	0.58	0.70	0.80087	0.8727	0.9924

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{1}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$$

09/01/...

PRACTICAL No. 7

Topic :- Differential Equation

Q.1)
1) $x \dfrac{dy}{dx} + y = e^x$

Sol. Dividing by x

$\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{e^x}{x}$

by comparing with

$\dfrac{dy}{dx} + p(x)y = Q(x)$

$IF = e^{\int f\, dx}$

$= e^{\int 1/x\, dx}$

$= \int e^{\log x}\, dx = x$

$y(IF) = \int Q\,(I.F)\, x\, dx + c$

$y(x) = \int \frac{e^x}{x} \cdot x\, dx + c$

$y(x) = \int e^x + c$

$y(x) = e^x + c$.

2) $e^x\, dy/dx + 2xy = 1$

Dividing by e^x.

$\dfrac{dy}{dx} + 2y = \dfrac{1}{e^x}$

By comparing with
$\frac{dy}{dx} + p(x)y = Q(x)$

$I_f = e^{\int p\,dx}$
$= e^{\int 2\,dx}$
$= e^{2x}$

$y(I\cdot F) = \int Q(I\cdot F)\,x\,dx + c$
$y(e^{2x}) = \int \frac{1}{e^{x}} \cdot e^{2x} \cdot x\,dx + c$
$= \int e^{2x} - e^{x}\,dx + c$
$= \int e^{x}\,dx + C.$
$ye^{2x} = e^{x} + c.$

3) $x\,dy/dx = \cos x/x - 2y.$
Sol^n:- $x\,dy/dx = \cos x/x - 2y$
$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$
comparing with
$\frac{dy}{dx} + p(x)\,y = Q(x)$
$I_f = e^{\int p\,dx}$
$= e^{\int 2/x\,dx}$
$= e^{2/x}\ x = \sin x^2 = x^2$

$y(I.f) = \int Q(x)\,(I.F)\,dx + c$
$= \int \frac{\cos x}{x^2} - x^2\,dx + c$
$= \int \cos x + c$
$x^2 y = \sin x + c$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Sol^n:- $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both sides)
$p(x) = 3/x$ $Q(x) = \sin x/x^3$
$I.F = e^{\int p\,dx}$
$= e^{\int 3/x\,dx}$
$= e^{3\,x}\ x$
$= e^{\ln x^3}\ x$
$I_f = x^3$
$y(I.f) = \int Q(x)\,(I.F)\,dx + c$
$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$
$= \int \sin x\,dx + c$

$x^3 y = -\cos x + c$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

Sol^n:- $\frac{dy}{dx} + 2y = \frac{2x}{e^{x}}$

$P(x) = 2$ $Q(x) = 2x/e^{2x} = 2xe^{-2x}$
$I_f = e^{\int P(x)\,dx}$
$= e^{\int 2\,dx}$
$= e^{2x}$

$$y(I.F) = \int Q(x)\,(I\cdot F)\,dx + C$$
$$= \int 2x e^{-2x}\, e^{2x}\, dx + C$$
$$= \int 2x\, dx + C$$
$$y e^{2x} = x^2 + C$$

6) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

Sol^n:- $\sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$

$$\frac{\sec^2 x\, dx}{\tan x} = -\frac{\sec^2 y\, dy}{\tan y}$$

$$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$$

$\therefore \log|\tan x| = -\log|\tan y| + C$

$\log|\tan x \cdot \tan y| = C$

$\tan x \cdot \tan y = e^C$

7) $\frac{dy}{dx} = \sin^2(x - y + 1)$

Sol^n:- put $x - y + 1 = v$

Differentiating on both sides

$\cancel{x} - y + 1 = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \frac{dy}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v\, dv = \int dx$$

$\tan v = x + C$

$\tan(x + y - 1) = x + C$

8) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

put $2x + 3y = v$

$$2 + 3\frac{dy}{dx} = \cancel{\frac{dv}{dx}}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

:8

$$\frac{1}{3}\left(\frac{dv}{dx}-2\right)=\frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx}=\frac{v-1}{v+2}+2$$

$$\frac{dv}{dx}=\frac{v-1+2v+4}{v+2}$$

$$=\frac{3v+3}{v+2}$$

$$=\frac{3(v+1)}{v+2}$$

$$\int\left(\frac{v+2}{v+1}\right)dv=3dx$$

$$=\int\frac{v+1}{v}dx+\int\frac{1}{v+1}dv=3x$$

$v+\log(x)=3x+c$

$2x+\cancel{3y}+\log|2x+3y+1|=3x+c$

$\cancel{3y}=x-\log|2x+3y+1|+c$

PRACTICAL No. 8

* Using Euler's Method find the following :-

1) $\frac{dy}{dx}=y+e^{x}-2,\ y(0)=2,\ h=0.5$ find $y(2)$

$f(x)=y+e^{x}-2,\quad x_0=0$

$y(0)-2,\ h=0.5$

$y(0.2)=?$

n	x_n	$y(0)$	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.14787	3.5743
2	1	3.5443	4.2425	5.7205
3	1.5	5.7205	8.2021	9.8215

$y(2)=9.8215$

2) $\frac{dy}{dx} = 1+y^2$, $y(0)=0$, $h=0.2$

Find $y(1)$

→ $x_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$

find $y(1)$. $y(0) = 1$, $x_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.355
4	0.8	1.3313	0.7694	1.5051
5	1	1.5051		

$y(1) = 1.6051$

4) $\frac{dy}{dx} = 3x^2+1$, $y(1) = 2$

Find $y(2)$ for $h = 0.5$ & $h = 0.5$

→ $h = 0.5$ & $h = 0.25$ $y_0 = 2$, $x_0 = 1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	2	4
1	1.5	1.5	4	7.875
2	2	2	7.875	

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.8875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	122.6482	299.9966
4	2	299.9966		

$y(1) = 299.9966$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$

Find $y(1.2)$ with $h = 0.2$

$y(0) = 1$ $x(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1) = 3.6$

23/01/2020

PRACTICAL No. 9

AIM :- Limits & Partial-order Derivative

Q.1) Evaluate the following limits :-

1) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$$\frac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$$

$$\frac{64+3+1-1}{4+5}$$

$$\frac{67}{9}$$

ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$$\frac{(0+1)\,((2)^2+(0)^2-4(2))}{2+3(0)}$$

$$\frac{1\,(4+0-8)}{8}$$

$$\frac{-4}{2} = -2$$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{(x)^2-(yz)^2}{x^2(x-yz)}$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)(x-yz)}{x^2(x-yz)} \quad \left[\because (a)^2-b^2=(a+b)(a-b)\right]$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x}$$

$$\frac{1+(1)(1)}{(1)^2} = 2$$

Q.2) Find f_x, f_y for each of the following

i) $f(x,y) = xy\,e^{x^2+y^2}$

$$f(x) = \frac{\partial f}{\partial x}$$

$$= \frac{\partial\,(xy\,e^{x^2+y^2})}{\partial x}$$

$$= y\,\cancel{\frac{\partial\,(x\cdot e^{x^2+y^2})}{\partial x}}$$

$$= y\left[x\cdot\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\cdot\frac{d}{dx}(x)\right]$$

$$\left[\because \frac{d}{dx}(uv) = u\cdot v' + v\cdot u'\right]$$

$= y\left[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2} \cdot (1)\right]$

$= y \cdot e^{x^2+y^2}[2x+1]$

Now, $f(y) = \frac{\partial f}{\partial y}$

$= \frac{\partial (xy e^{x^2+y^2})}{\partial y}$

$= x \cdot \frac{\partial}{\partial y}(y \cdot e^{x^2+y^2})$

$= x\left[y \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dy}(y)\right]$

$\because \frac{d}{dx}(uv) = uv' + vu'$

$= x \cdot [2y^2 \cdot e^{x^2+y^2} + e^{x^2+y^2}]$

$= x \cdot e^{x^2+y^2}[2y^2+1]$

ii) $f(x,y) = e^x \cos y$

$\therefore f(x) = e^x \cos y$

$f(y) = e^x \frac{d}{dy}(\cos y)$

$= e^x(-\sin y)$

$= -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$\cancel{f(x) = \frac{\partial f}{\partial y}}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial (x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (x^3y^2 - 3x^2y + y^3 + 1)}{\partial}$

$= x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q3) Using defination find value for $f(x,y) = \frac{2x}{1+y^2}$

$fx(a,b) = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$

where $(a,b) = (0,0)$

$\therefore fx(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$

Similarly

$fy(a,b) = \lim \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{0-0}{h} = 0$

Q.4) Find all sec... ...vaties of $\partial\partial f$.

i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial(y^2 - xy)}{x^2}$

$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy) \cdot \frac{d}{dx}(x^2)}{(x^2)^2}$

$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}\right]$

$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$

$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4} = \frac{x(xy - 2y^2)}{x^4}$

$fx = \frac{xy - y^2}{x^3}$

$fy = \frac{\partial f}{\partial y} = \partial\left(\frac{y^2 - xy}{x^2}\right) = \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$

$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right) / \partial y$

$= \frac{1}{x^2} 2y - \frac{1}{x}$

$\therefore fy = \frac{2y - x}{x^2}$

$f(x, xS) = \frac{\partial(xy - 2y^2 / x^3)}{\partial x}$

$= \frac{x^3 \frac{d}{dx}(xy - 2y^2) - (xy - 2y^2)\frac{d}{dx}(x^2)}{(x^3)^2}$

$= \frac{x^3(y) - (xy)y^2(3x^2)}{6}$

$= x^2y - 3x^2y + 6x^2y^2$

$= \frac{6x^2y^2 - 2x^3y}{x^6}$

$= \frac{x^2(6y^2 - 2xy)}{x^6}$

$= \frac{6y^2 - 2xy}{x^4}$

$= \frac{1}{x^2} \quad -\frac{1}{x^3} \quad 2(2y)$

$= \frac{1}{x^2} = -\frac{4}{x^3} = \frac{x^3 - 4y x^2}{x^4}$

$= x^2 \frac{(x-4y)}{x^6}$

$= \frac{x-4y}{x^4}$

$f(xy) = \frac{\partial \left(\frac{2y-x}{x^2}\right)}{\partial x}$

$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2} - \frac{1}{x}\right)}{\partial x}$

$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$

$= -\frac{4y}{x^3} + \frac{1}{x^2}$

$= -\frac{4yx^2 + x^3}{x^4}$

$= \cancel{x^2} \frac{(x-4y)}{x^6}$

$= \frac{x-4y}{x^4}$

$\therefore f(xy) = f(yx) = \frac{x-4y}{x^4}$

there verified

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial\left((x^3+3x^2y^2) - \log(x^2+1)\right)}{\partial x}$

$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$

$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$= 0 \cdot 3(2y)(x^2) + 0$

$f(y) = 6x^2y$

$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right]$

$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$

$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$

$= 6x^2(1) = 6x^2$

$$f(xy) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$$

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$= 0 + 6x(2y)$

$= 12xy$

$f(xy) = f(yx) = 12xy$

theorem verified.

iii) $f(x, y) = \sin(xy) + e^{x+y}$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$$

$= \cos(xy)(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$$

$= \cos(xy)(x) + e^{x+y}(1)$

$= x\cos xy + e^{x+y}$

$$f(xx) = \frac{\partial}{\partial x} fx$$

$$= \frac{\partial(y\cos xy + e^{x+y})}{\partial x}$$

$= y\cos xy(y) + e^{x+y}(1)$

$= y^2\cos xy + e^{x+y}$

$$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial(x\cos xy + e^{x+y})}{\partial y}$$

$= x\cos xy(x) + e^{x+y}(1)$

$= x^2\cos xy + e^{x+y}$

$$f(xy) = \frac{\partial f(x)}{\partial y} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$$

$= y[-\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1)$

$[\because d/dx\, uv = u \cdot v' + v \cdot u']$

$= -xy\sin(xy) + \cos xy + e^{x+y}$

$$f(yx) = \frac{\partial fy}{\partial x} = \frac{\partial(x\cos xy + e^{x+y})}{\partial x}$$

$= \cos xy(1) + x(-\sin(xy))(y) + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e$

$f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$

Q.5) Find the linearization of $f(x,y)$ at given pt.

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2}$
$= \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$

$fx(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$fy(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \frac{2+x-1+(y-1)}{\sqrt{2}}$

$= \frac{2+x-1+(y-1)}{\sqrt{2}}$

$= \frac{2-1x+y-2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0 \cdot (\sin \pi/2)$
$= 1 - \pi/2$

$f(x) = -1 + y\cos x$

$f(y) = 1$

$fx(\pi/2, 0) = -1 + 0 \cdot \cos(\pi/2)$
$= -1$

$fy(\pi/2, 0) = 1$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \frac{\pi}{2}) + 1(y-0)$

$= 1 - \frac{\pi}{2} - x + \pi/2 + y$

$= y - x + 1$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1)$
$= 0$

$fx(1,1) = 1$

$f(y) = 1/y$

$fy(1,1) = 1$

$\therefore$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

PRACTICAL No. 10

Q.1) Find the directional derivative of the following function at given points & in the direction of given vector.

Ans: $f(x,y) = x + 2y - 3$ $\quad a = (1,-1)$ $\quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$\therefore D_u f(a) = \frac{1}{\sqrt{10}}$

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2) $f(x) = y^2 - 4x + 1$ $a = (3, 4)$ $u = i + 5j$

Here $u = i + 5j$ is not a unit vector

$|\vec{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3, 4) = (4)^2 - 4(3) + 1 = 5$

$f(a + hu) = f(3, 4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f_{x,y}(a + hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + \cancel{5} - \cancel{5}}{h}$

$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

3) $2x + 3y$ $a = (1, 2)$, $u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$|\vec{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

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unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1, 2) = 2(1) + 3(2) = 8$

$f(a + hu) = f(1, 2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a + hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{18h}{5} \cancel{+8} \cancel{-8} / h$

$= \frac{18h}{5}$

Q.2) Find gradient vector for the following funct

i) $f(x, y) = x^y + y^x$, $a = (1, 1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x, y) = (f_x, f_y)$

$= \left(y x^{y-1} + y^x \log y, x^y \log x + x y^{x-1}\right)$

$f(1, 1) = (1 + 0, 1 + 0)$

$= (1, 1)$

2) $f(x, y) = (\tan^{-1} x) \cdot y^2$. $a = (1, -1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x, y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1, -1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

3) $f(x, y, z) = xyz - e^{x+y+z}$, $a = (1, -1, 0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x, y, z) = f_x, f_y, f_z$

$= f_z - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1, -1, 0) = ((-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0})$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q.3) Find the eq^n of tangent & normal for each of the following using curves at given points.

i) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$f_x = \cos y \cdot 2x + e^{xy} \cdot y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0) = \therefore x_0 = 1, y_0 = 0$

eq^n tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \cdot 2(1) + e^0 \cdot 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ $\therefore$ It is the required eq^n of tangent.

eq^n of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$= 1 + 2(0) + d = 0$ at $(1, 0)$.

$d + 1 = 0$

$\therefore d = -1$

2) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f_x = 2x+0-2+0+0$

$= 2x-2$

$f_y = 0+2y-0+3+0$

$= 2y+3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2,\ y_0 = -2$

$f_x(x_0,y_0) = 2(2)-2 = 2$

$f_y(x_0,y_0) = 2(-2)+3 = -\cancel{8}1$

Eq^n of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (1(y+2)) = 0$

$2x-2-y-2=0$

$2x-y-4=0 \rightarrow$ It is required eq^n of tangent.

eq^n of Normal

$= ax+by+c=0$

$bx+ay+d=0$

$= -1(x) + 2(y) + d = 0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\therefore d=6$

Q.4) Find the eq^n of tangent & normal line for each of the following:-

1) $x^2-2yz+3y+xz=7$ at $(2,1,0)$

$f_x = 2x-0+0+z$

$f_x = 2x+z$

$f_y = 0-2z+3+0$

$= 2z+3$

$f_z = 0-2y+0+x$

$= -2y+x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0=2,\ y_0=1,\ z_0=0$

$f_x(x_0,y_0,z_0) = 2(2)+0 = 4$

$f_y(x_0,y_0,z_0) = 2(0)+3 = 3$

$f_z(x_0,y_0,z_0) = -2(1)+2 = 0$

Eq^n of tangent

$f_x(x_0-x_0) + f_y(y_0-y_0) + f_z(z_0-z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x-8+3y-3=0$

$4x+3y-11=0 \rightarrow$ This is required eq^n of tangent.

Eq^n of normal at $(4,3,-11)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

2) $3xy^2 - x - y + z = -4$ at $(1, -1, 2)$

at $(1, -1, 2)$

$= 3xyz - x - y + z + 4 = 0$

$= f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2) \therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqⁿ of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

This is required eqⁿ of tangent

Eqⁿ of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{\cancel{f_x}} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q3) 1) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \rightarrow ①$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \rightarrow ②$

Multiply eqⁿ 1 by 2

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

substitute ~~value~~ of x in eqⁿ ①

$2(0) - y = \cancel{-2}$

$y = 2$

$\therefore$ Critical points are $(0, 2)$

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$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $x > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= \underline{-4}$

2) $f(x, y) = 2x^4 + 3x^2 y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \rightarrow (1)$

$f_y = 0 \quad 3x^2 - 2y = 0 \longrightarrow (2)$

Multiply eq^n (1) with 3, (2) with 4 we get,

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$y = 0$

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0, 0)$

$= 24(0)^2 + 6(0) = 0$

$\therefore r = 0$

$f(x, y)$ at $(0, 0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 \cdot 0$

$= \underline{0}$

$rt - s^2 = 0(-2) - (0)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$ (nothing to say)

3) $f(x, y) = -x^2 - y^2 + 2x + 8y - 70$

$\therefore f_x = -2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore -2x + 2 = 0$

$x = 1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{8}{2} = 4 \quad \therefore y = 4$

$\therefore$ Critical point is $(1, 4)$

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$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 47 - 70$

$= \cancel{23}$

$= -23$

AK
06/12/2024