

$T(n) = 2T(n/4) + 1$
 $a=1, f(n)=1, \epsilon=.5$
 $b=4$
 $n^{\log_4 2} = n^{.5} \neq O(n^{.5-.5})$
 $1) \neq O(n^{.5-.5})$
 $2) 1 = \Omega(n^{.5})$
 $3) 1 = \Omega(n^{.5})$
 $T(n) = \Theta(n^{.5})$
 $= \Theta(\sqrt{n})$

4.5-1b)
 $T(n) = 2T(n/4) + \sqrt{n}$
 $a=1, f(n)=\sqrt{n}$
 $b=4$
 $1) \sqrt{n} = O(n^{\log_4 2}) = O(n^{.5})$
 $\text{false } n \in \mathbb{Z}^+$
 to satisfy
 $a) \sqrt{n} = n^{.5}$
 $2) \sqrt{n} = \Theta(n^{\log_4 2})$ true
 $\text{possible case as } \sqrt{n} = n^{.5}$
 $3) \sqrt{n} = \Omega(n^{\log_4 2}) \in \Omega(n^{.5})$
 false
 $n \in \mathbb{Z}^+$
 $\sqrt{n} = n^{.5}$

$T(n) = \sqrt{n} \log n$
4.5-1-c)
 $T(n) = 2T(n/4) + n$
 $a=2, f(n)=n$
 $b=4$
 $n^{\log_4 2} = n^{.5}$
 $1) n = O(n^{\log_4 2 - \epsilon})$
 $= O(n^{.5 - \epsilon})$
 false
 no epsilon
 $2) n = \Theta(n^{.5})$
 false
 $3) n = \Omega(n^{.5 + \epsilon})$
 $\epsilon = .5$
 $n = \Omega(n^{.5 + .5})$

~~$T(n)$~~ check regularity
 $a f(n/b) \leq c f(n/b)$
 $2 \frac{n}{4} \leq c \frac{n}{4}$
 $c=2$
true
 $T(n) = \Omega(n^{.5})$
 $= \Omega(\sqrt{n})$

4.5-1-d)
 $T(n) = 2T(n/4) + n^2$
 ~~$a=2, f(n)=n^2$~~
 ~~$b=4$~~
 $1) n^2 = O(n^{.5 - \epsilon})$
 false
 no epsilon
 $2) n^2 = \Theta(n^{.5})$
 false
 $3) n^2 = \Omega(n^{.5 + \epsilon})$
 $\epsilon = 1.5$
 $= \Omega(n^{.5 + 1.5})$
 $a f(n/b) \leq c f(n/b)$
 $2 \frac{n^2}{4} \leq c \frac{n^2}{4}$
 $c=2$
true

~~$T(n)$~~
 $T(n) = \Omega(n^{.5})$

4.5-2)
 $2.8073 = \log_4(a)$
 $a = 4^{2.8073}$
 $a = 4$
 4.8
4.5-3)
 ~~$a=1, f(n)=1$~~
 ~~$b=2$~~
 ~~$T(n) = \log_2(n) + \epsilon$~~
 ~~$\epsilon = .3$~~