



EICoM

Electrical | Computer | Mechatronics

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اِرَادَةٌ .. ثُقَّةٌ .. تَحْيِيرٌ

1. (a) $-0.7822; -0.544; 1.681$

(b) $4\cos 2t: 4; -1.665; -3.960$

$4\sin(2t + 90^\circ): 4; -1.665; -3.960$

(c) $3.2\cos(6t + 15^\circ): 3.091; 1.012; 2.429$

$3.2\cos(6t + 105^\circ): 3.091; 1.012; 2.429$

2. (a)

$$5 \sin 300t = [5 \cos(300t - 90^\circ)]$$

$$1.95 \sin(\pi t - 92^\circ) = [1.95 \cos(\pi t - 182^\circ)]$$

$$2.7 \sin(50t + 5^\circ) - 10 \cos 50t$$

$$= 2.7 \sin 50t \cos 5^\circ + 2.7 \cos 50t \sin 5^\circ - 10 \cos 50t$$

$$= 2.6897 \sin 50t - 9.7647 \cos 50t = [10.13 \cos(50t + 15.4^\circ)]$$

(b)

$$66 \cos(9t - 10^\circ) = [66 \sin(9t + 80^\circ)]$$

$$4.15 \cos 10t = [4.15 \sin(10t + 90^\circ)]$$

$$10 \cos(100t - 9^\circ) + 10 \sin(100t + 19^\circ)$$

$$= 11.0195 \sin 100t + 13.1325 \cos 100t$$

$$= 17.14 \cos(100t - 40^\circ)$$

$$= [17.14 \sin(100t + 50^\circ)]$$

3. (a) v_1 leads i_1 by -45°
- (b) v_1 leads by $-45 + 80 = 35^\circ$
- (c) v_1 leads by $-45 + 40 = -5^\circ$
- (d) $5\sin(10t - 19^\circ) = 5\cos(10t - 19^\circ)$ therefore v_1 leads by $-45 + 109 = 64^\circ$

4. $v_1 = 34 \cos(10t + 125^\circ)$

- (a) $i_1 = 5 \cos 10t$; v_1 lags i_1 by -235° ($-360^\circ + 125^\circ$)
- (b) $i_1 = 5 \cos(10t - 80^\circ)$; v_1 lags i_1 by -155° ($-235^\circ + 80^\circ$)
- (c) $i_1 = 5 \cos(10t - 40^\circ)$; v_1 lags i_1 by -195° ($-235^\circ + 40^\circ$)
- (d) $i_1 = 5 \cos(10t + 40^\circ)$; v_1 lags i_1 by -275° ($-235^\circ - 40^\circ$)
- (e) $i_1 = 5 \sin(10t - 19^\circ) = 5 \cos(10t - 109^\circ)$; v_1 lags i_1 by -126° ($-235^\circ + 109^\circ$)

5. (a) $\cos 4t$ leads $\sin 4t$; $\sin 4t$ lags $\cos 4t$
- (b) the first is lagging by 80°
- (c) the second is lagging by 80°
- (d) the second is lagging by 88°
- (e) Neither term lags

6. (a) $\cos 3t - 7 \sin 3t = 0$

$$7.07 \cos(3t + 1.4289) = 0$$

$$3t + 1.4289 = 1.5708$$

$$3t = 0.1419$$

$$t = \boxed{0.0473 \text{ s}}$$

Also, $3t = 0.1419 + \pi$

$$t = \boxed{1.0945 \text{ s}}$$

and, $3t = 0.1419 + 2\pi$

$$t = \boxed{2.1417 \text{ s}}$$

(b) $\cos(10t + 45^\circ) = 0$

$$10t + 0.7854 = 1.5708$$

$$10t = 0.7854 \Rightarrow t = \boxed{0.0785 \text{ s}}$$

Also, $10t = 0.7854 + \pi \Rightarrow t = \boxed{0.3927 \text{ s}}$

and, $10t = 0.7854 + 2\pi \Rightarrow t = \boxed{0.7069 \text{ s}}$

(c) $\cos 5t - \sin 5t = 0$

$$5t + 0.7854 = 1.5708$$

$$5t = 0.7854 \Rightarrow t = \boxed{0.1571 \text{ s}}$$

Also, $5t = 0.7854 + \pi \Rightarrow t = \boxed{0.7854 \text{ s}}$

and, $5t = 0.7854 + 2\pi \Rightarrow t = \boxed{1.4137 \text{ s}}$

(d) $\cos 2t + \sin 2t - \cos 5t + \sin 5t = 0$

$$1.4142 \cos(1.5t - 0.7854) = 0$$

$$1.5t - 0.7854 = 1.5708$$

$$1.5t = 2.3562 \Rightarrow t = \boxed{1.5708 \text{ s}}$$

Also, $1.5t = 2.3562 + \pi \Rightarrow t = \boxed{3.6652 \text{ s}}$

and, $1.5t = 2.3562 + 2\pi \Rightarrow t = \boxed{5.7596 \text{ s}}$

7. (a) $t = 0$; $t = 550 \text{ ms}$; $t = 0$; $t = 126 \text{ ms}$

8. (a) $v(t) = 2t; 0 \leq t < 0.5s$

$\therefore v(0.25s) = \boxed{0.5 \text{ V}}$

(b) Using the first term of the Fourier series only,

$$v(t) = \frac{8}{\pi^2} \sin \pi t$$

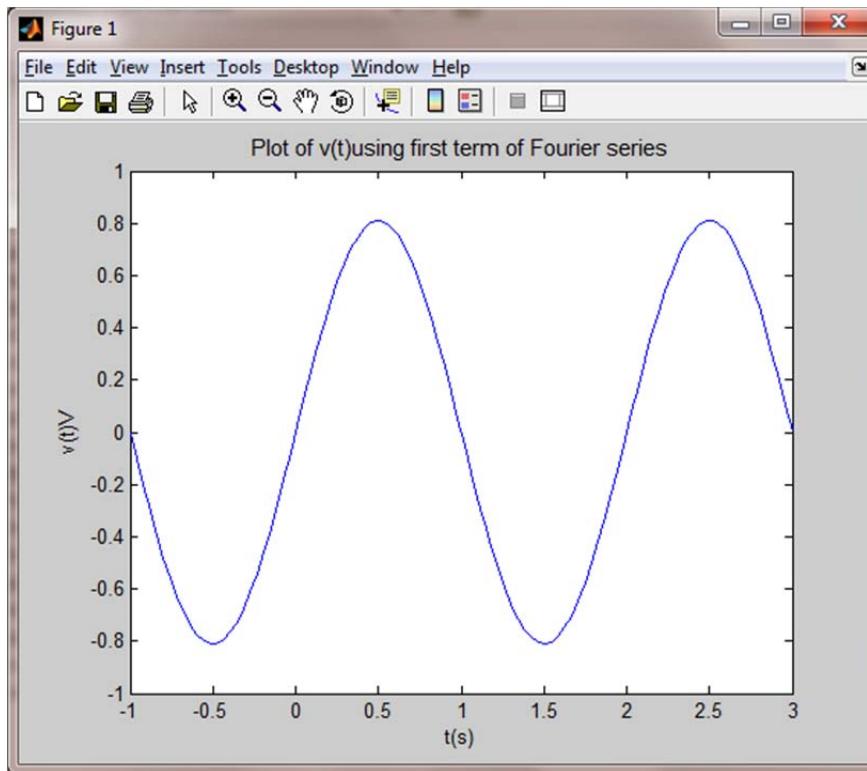
$$v(0.25s) = \frac{8}{\pi^2} \sin 45^\circ = \boxed{0.5732 \text{ V}}$$

(c) Using the first three terms of the Fourier series,

$$v(t) = \frac{8}{\pi^2} \sin \pi t - \frac{8}{3^2 \pi^2} \sin 3\pi t + \frac{8}{5^2 \pi^2} \sin 5\pi t$$

$$v(0.25s) = \boxed{0.4866 \text{ V}}$$

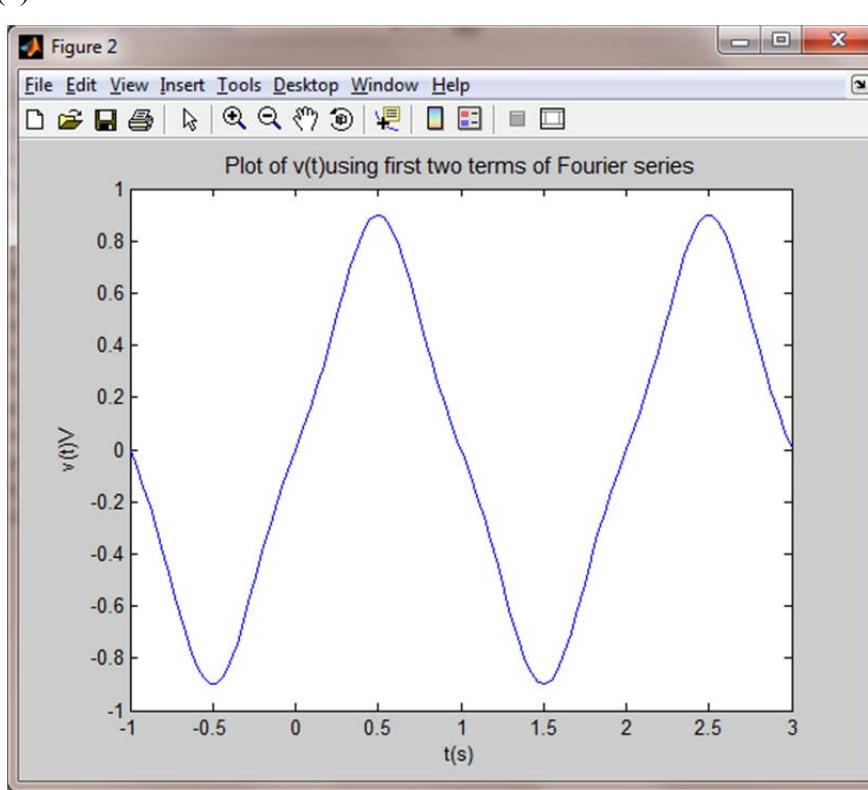
(d)



```
t=linspace(-1,3);
v = 8/(pi^2)*sin(pi*t);
figure(1);
plot(t,v);
xlabel('t(s)')
```

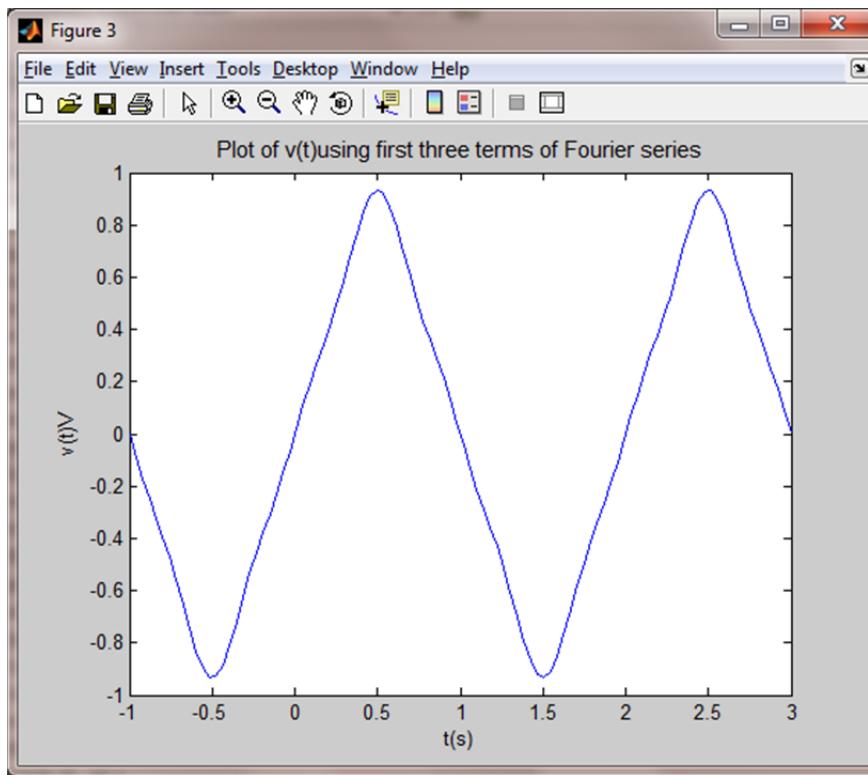
```
ylabel('v(t)V')
title('Plot of v(t)using first term of Fourier series','FontSize',11)
```

(e)



```
t=linspace(-1,3);
v = 8/(pi^2)*(sin(pi*t)-1/(3^2)*sin(3*pi*t));
figure(2);
plot(t,v);
xlabel('t(s)')
ylabel('v(t)V')
title('Plot of v(t)using first two terms of Fourier series','FontSize',11)
```

(f)



```
t=linspace(-1,3);
v = 8/(pi^2)*(sin(pi*t)-1/(3^2)*sin(3*pi*t)+ 1/(5^2)*sin(5*pi*t));
figure(3);
plot(t,v);
xlabel('t(s)')
ylabel('v(t)V')
title('Plot of v(t)using first three terms of Fourier series','FontSize',11)
```

9. (b) V_{rms} V_m
- 110 V 156 V
- 115 V 163 V
- 120 V 170 V

10. In this problem, when we apply Thevenin's theorem with the inductor as the load, we get,

$$v_{oc} = v_s \cdot \frac{1}{1+10} = (4.53 \cos(0.333 \times 10^{-3} t + 30^\circ)) \frac{1}{11} = 0.4118 \cos(0.333 \times 10^{-3} t + 30^\circ) \text{ V}$$

$$R_{th} = \frac{1 \times 10}{1+10} = \frac{10}{11} = 0.909 \Omega$$

Now for a series RL circuit with $L = 3mH$, $R_{th} = 0.909\Omega$ and a source voltage of $0.4118 \cos(0.333 \times 10^{-3} t + 30^\circ)$ V, we get,

$$\begin{aligned} i_L(t) &= \frac{V_m}{\sqrt{R_{th}^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R} + 30^\circ\right) \\ &= \frac{0.4118}{\sqrt{(0.909)^2 + (0.333 \times 10^{-3} \times 3 \times 10^{-3})^2}} \cos\left(0.333 \times 10^{-3} t - \tan^{-1} \frac{(0.333 \times 10^{-3} \times 3 \times 10^{-3})}{0.909} + 30^\circ\right) \\ &= 0.453 \cos(0.333 \times 10^{-3} t + 30^\circ) A \\ \therefore i_L(t = 0) &= 0.453 \cos(30^\circ) = 392.3 \text{ mA} \end{aligned}$$

Now,

$$v_L(t) = L \frac{di_L}{dt} = -3 \times 10^{-3} \times 0.333 \times 10^{-3} \times 0.453 \sin(0.333 \times 10^{-3} t + 30^\circ)$$

$$= 0.4526 \cos(0.333 \times 10^{-3} t + 120^\circ) \mu\text{V}$$

$$v_L(t = 0) = -0.2262 \mu\text{V}$$

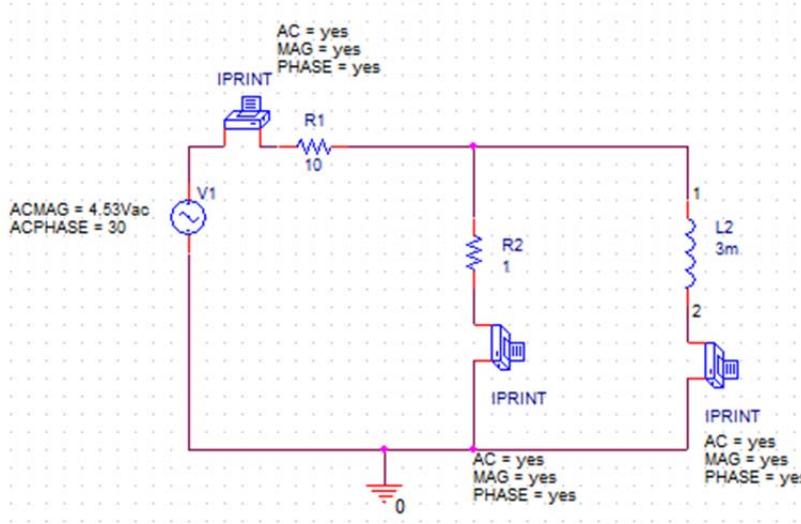
$$i_R(t = 0) = \frac{v_L(t = 0)}{R} = -0.2262 \mu\text{A}$$

$$i_s(t = 0) = i_R + i_L = 392.3 \text{ mA}$$

(b) $v_L(t) = 0.4526 \cos(0.333 \times 10^{-3} t + 120^\circ) \mu\text{V}$

Pspice Verification:

This has been verified using the phasor in Pspice.



FREQ IM(V_PRINT1)IP(V_PRINT1)

5.300E-05 4.530E-01 3.000E+01

***** 06/14/12 15:54:20 ***** PSpice Lite (April 2011) ***** ID# 10813 *****

** Profile: "SCHEMATIC1-prob10_10" [C:\ORCAD\ORCAD_16.5_LITE\examples\prob10_1

***** AC ANALYSIS TEMPERATURE = 27.000 DEG C

FREQ IM(V_PRINT2)IP(V_PRINT2)

5.300E-05 4.526E-07 1.200E+02

11. $8.84\cos(100t - 0.785)$ A

12. In this problem, when we apply Thevenin's theorem with the inductor as the load, we get,

$$v_{oc} = 25 \cos 100t \times \frac{1}{(1+2)} \times 2 = 12.5 \cos 100t \text{ V}$$

$$R_{th} = (1+1)\Omega \parallel 2\Omega = 1\Omega$$

Now for a series RL circuit with $L = 10mH$, $R_{th} = 1\Omega$ and a source voltage of $12.5 \cos 100t \text{ V}$, we get,

$$\begin{aligned} i_L(t) &= \frac{V_m}{\sqrt{R_{th}^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right) \\ &= \frac{12.5}{\sqrt{1^2 + (100 \times 10 \times 10^{-3})^2}} \cos\left(100t - \tan^{-1} \frac{(100 \times 10 \times 10^{-3})}{1}\right) \\ &= 8.84 \cos(100t - 45^\circ) \text{ A} \end{aligned}$$

Now,

$$\begin{aligned} v_L(t) &= L \frac{di_L}{dt} = -8.84 \times 10 \times 10^{-3} \times 100 \sin(100t - 45^\circ) \\ &= 8.84 \cos(100t + 45^\circ) \text{ V} \end{aligned}$$

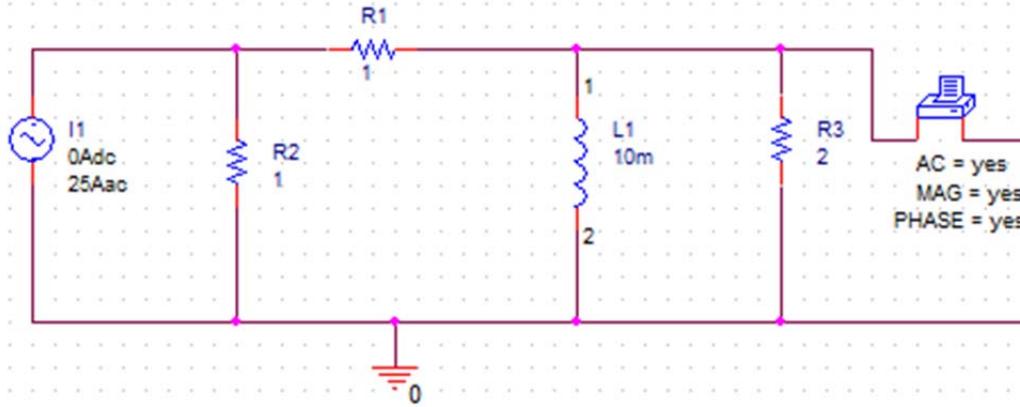
Voltage across the 2Ω resistor is equal to $v_L(t)$.

Power dissipated in 2Ω resistor is given by,

$$p_R(t) = \frac{v_R^2}{R} = \frac{v_L^2}{R} = \boxed{39.07 \cos^2(100t + 45^\circ) \text{ W}}$$

Pspice Verification:

This has been verified using the phasor in Pspice.



FREQ VM(N00183,0)VP(N00183,0)

1.592E+01 8.839E+00 4.500E+01

13. 1.92cos(40t - 0.876) V

14. Let 'i' be the current flowing in the circuit in the clockwise direction. Then, on applying KVL, we get,

$$15i + v_C = 3 \cos 40t$$

Substituting, $i = i_C = C \frac{dv_C}{dt}$ on the above KVL equation, we get,

$$30 \times 10^{-3} \frac{dv_C}{dt} + v_C = 3 \cos 40t$$

Let us choose to express the response as,

$$v_C(t) = A \cos(40t + \theta)$$

$$\frac{dv_C}{dt} = -40A \sin(40t + \theta)$$

On rewriting the KVL equation, we get,

$$-1.2A \sin(40t + \theta) + A \cos(40t + \theta) = 3 \cos 40t$$

$$\sqrt{A^2 + (-1.2A)^2} \cos\left((40t + \theta) + \tan^{-1}\left(\frac{1.2A}{A}\right)\right) = 3 \cos 40t$$

On equating the terms, we get,

$$A = 1.92$$

$$\theta = -\tan^{-1}(1.2) = -50.19^\circ$$

$$\therefore v_C(t) = 1.92 \cos(40t - 50.19^\circ) \text{ V}$$

Energy stored in a capacitor is given by,

$$w_C = \frac{1}{2} C v_C^2(t)$$

At t=10 ms,

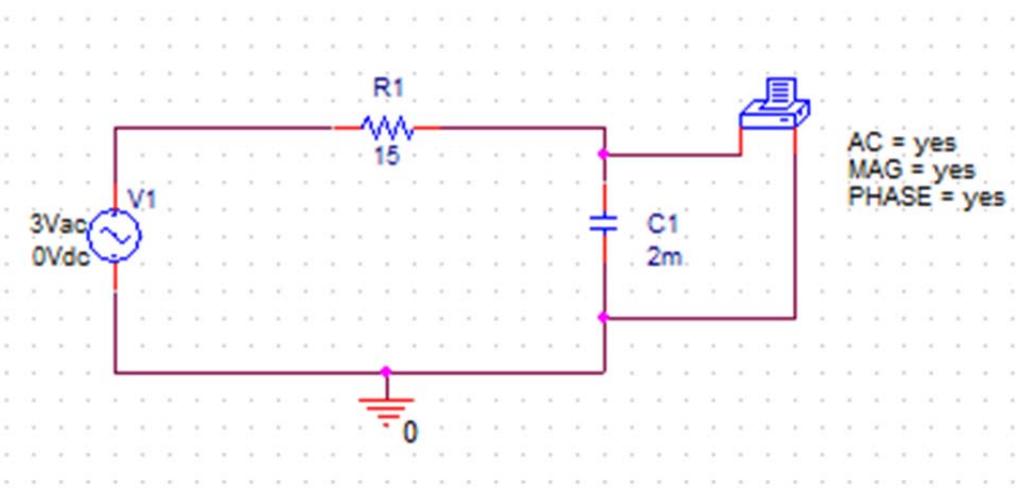
$$w_C(10\text{ms}) = \frac{1}{2} \times 2 \times 10^{-3} \times 1.71^2 = \boxed{2.92 \text{ mJ}}$$

At t=40 ms,

$$w_C(40\text{ms}) = \frac{1}{2} \times 2 \times 10^{-3} \times 1.45^2 = \boxed{2.1 \text{ mJ}}$$

Pspice Verification:

Phasor method is used to verify the solution.



AC = yes
MAG = yes
PHASE = yes

FREQ VM(N00132,0)VP(N00132,0)

6.366E+00 1.921E+00 -5.019E+01

15. 7.02cos(6t - 0.359) A

16. (a) $50\angle -75^\circ = 50 \cos(-75^\circ) + j50 \sin(-75^\circ) = [12.94 - j48.29]$

(b)

$$19e^{j30^\circ} = 19 \cos(30^\circ) + j19 \sin(30^\circ) = [16.45 + j9.5]$$

$$\begin{aligned} 2.5\angle -30^\circ + 0.5\angle 45^\circ &= 2.5 \cos(-30^\circ) + j2.5 \sin(-30^\circ) + 0.5 \cos(45^\circ) + j0.5 \sin(45^\circ) \\ &= [2.52 - j0.89] \end{aligned}$$

(c) $(2+j2)(2-j2) = 8 = [8\angle 0^\circ]$

(d) $(2+j2)(5\angle 22^\circ) = (2.82\angle 45^\circ)(5\angle 22^\circ) = [14.14\angle 67^\circ]$

17. (a) $2.88 \angle 11.5^\circ$

(b) $1 \angle 90^\circ$

(c) $1 \angle 0^\circ$

(d) $2.82 + j0.574$

(e) $2.87 - j4.90$

18. (a) $4(8 - j8) = \boxed{45.25 \angle -45^\circ}$
- (b) $4\angle 5^\circ - 2\angle 15^\circ = (3.98 + j0.35) - (1.93 + j0.52) = 2.05 - j0.17 = \boxed{2.05 \angle -4.74^\circ}$
- (c) $(2 + j9) - 5\angle 0^\circ = -3 + j9 = \boxed{9.5 \angle 108.44^\circ}$
- (d) $\frac{-j}{10 + j5} - 3\angle 40^\circ + 2 = \frac{-j}{10 + j5} - (2.3 + j1.93) + 2 = -0.34 - j2.01 = \boxed{2.03 \angle 260.4^\circ}$
- (e) $(10 + j5)(10 - j5)(3\angle 40^\circ) + 2 = 225\angle 40^\circ + 2 = 174.36 + j144.63 = \boxed{226.54 \angle 39.68^\circ}$

19. (a) $7.79 + j4.50$ (b) $6.74 - j0.023$ (c) $7.67 + j87.5$ (d) $2.15 + j2.50$ (e) $2.89 + j241$

20. (a) $\frac{2+j3}{1+8\angle 90^\circ} - 4 = \frac{2+j3}{1+j8} - 4 = -3.6 - j0.2 = \boxed{3.6\angle 183.18^\circ}$

(b) $\left(\frac{10\angle 25^\circ}{5\angle -10^\circ} + \frac{3\angle 15^\circ}{3-j5} \right)j2 = (2\angle 35^\circ + 0.52\angle -44.04^\circ)(2\angle 90^\circ) = -1.58 + j4.02$
 $= \boxed{4.32\angle 111.46^\circ}$

(c) $\left[\frac{(1-j)(1+j)+1\angle 0^\circ}{-j} \right] (3\angle -90^\circ) + \frac{j}{5\angle -45^\circ}$
 $= 9\angle 0^\circ + 0.2\angle 135^\circ = 8.86 + j0.14 = \boxed{8.86\angle 0.91^\circ}$

21. $88.7\sin(20t - 27.5^\circ)$ mA; $2.31\sin(20t + 62.5^\circ)$ V

22. Let i_L in the complex form be $i_L = Ae^{j(35t+\theta^\circ)}$ A.

Given, $i_s = 5 \sin(35t - 10^\circ) = 5e^{j(35t-100^\circ)} A$

$$v_L = L \frac{di_L}{dt} = 0.4 \frac{d}{dt} (Ae^{j(35t+\theta^\circ)}) = j14Ae^{j(35t+\theta^\circ)}$$

$$v_R = i_L R = 6Ae^{j(35t+\theta^\circ)}$$

$$v_S = v_C = v_R + v_L = A(6 + j14)e^{j(35t+\theta^\circ)}$$

$$i_C = C \frac{dv_C}{dt} = 0.01 \frac{d}{dt} (A(6 + j14)e^{j(35t+\theta^\circ)}) = A(-4.9 + j2.1)e^{j(35t+\theta^\circ)}$$

Applying KCL, we get,

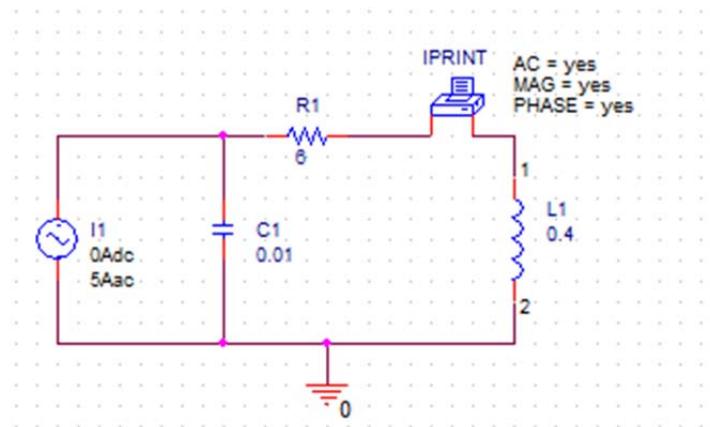
$$i_s = i_C + i_L$$

$$5e^{j(35t-100^\circ)} = 4.43Ae^{j(35t+\theta^\circ+151.69^\circ)}$$

$$\Rightarrow A = 1.129 \text{ and } \theta = -251.69^\circ = 108.31^\circ$$

$$\therefore i_L = 1.129e^{j(35t+108.31^\circ)} = \boxed{1.129 \cos(35t + 108.31^\circ) A}$$

Pspice Verification:



FREQ IM(V_PRINT1) IP(V_PRINT1)

5.570E+00 1.129E+00 1.083E+02

23.

$$\left[(62.5)^2 + (1.25\omega)^2 \right]^{1/2} \cos\left(\omega t + 31.3^\circ + \tan^{-1} \frac{1.25\omega}{62.5}\right) \text{ mA}$$

24. This problem can be easily solved by performing a source transformation which results in a circuit with voltage source, resistance and inductance.

Given, $i_s = 5e^{j10t}$
 $v_s = 10e^{j10t}$

The steady-state expression for $i_L(t)$ can be found as:

$$\begin{aligned} i_L(t) &= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) \\ &= \frac{10}{\sqrt{2^2 + 10^2 \times 0.4^2}} \cos\left(10t - \tan^{-1}\left(\frac{10 \times 0.4}{2}\right)\right) \\ &= \boxed{2.24 \cos(10t - 63.44^\circ)} \text{ A} \end{aligned}$$

25. (a) $75.9 \angle 0^\circ$

(b) $5 \angle -42^\circ$

(c) $1 \angle 104^\circ$

(d) $8.04 \angle -78.4^\circ$

26. (a) $11\sin 100t = 11\cos(100t - 90^\circ) = \boxed{11\angle -90^\circ}$
- (b) $11\cos 100t = \boxed{11\angle 0^\circ}$
- (c) $11\cos(100t - 90^\circ) = \boxed{11\angle -90^\circ}$
- (d) $3\cos 100t - 3\sin 100t = 3\angle 0^\circ - 3\angle -90^\circ = 3 + j3 = \boxed{4.24\angle 45^\circ}$

27. (a) $9\cos(2\pi \times 10^3 t + 65^\circ)$ V

(b) $500\cos(2\pi \times 10^3 t + 6^\circ)$ mA

(c) $14.7\cos(2\pi \times 10^3 t + 4^\circ)$ V

28. (a) $\frac{2-j}{5\angle 45^\circ} V = \frac{2.24\angle -26.56^\circ}{5\angle 45^\circ} = \boxed{0.45\angle -71.56^\circ V}$

(b) $\frac{6\angle 20^\circ}{1000} - j V = 0.00564 - j1 = \boxed{1\angle -89.67^\circ V}$

(c) $(j)(52.5\angle -90^\circ) V = \boxed{52.5\angle 0^\circ V}$

29. (a) 0; 11
- (b) -11; 0
- (c) 0; 11
- (d) -3; -3

30. (a) $v(t) = 9 \cos(100\pi t + 65^\circ)$ V

At $t = 10$ ms: $v(t) = 9 \cos 245^\circ =$ -3.8 V

At $t = 25$ ms: $v(t) = 9 \cos 515^\circ =$ -8.16 V

(b) $v(t) = 2 \cos(100\pi t + 31^\circ)$ V

At $t = 10$ ms: $v(t) = 2 \cos 211^\circ =$ -1.71 V

At $t = 25$ ms: $v(t) = 2 \cos 481^\circ =$ -1.03 V

(c) $v(t) = 22 \cos(100\pi t + 14^\circ) - 8 \cos(100\pi t + 33^\circ)$ V

At $t = 10$ ms: $v(t) = 22 \cos 194^\circ - 8 \cos 213^\circ =$ -14.64 V

At $t = 25$ ms: $v(t) = 22 \cos 464^\circ - 8 \cos 483^\circ =$ -0.97 V

31. (a) $2 \angle 0^\circ$
(b) $400 \angle -90^\circ \text{ mV}$
(c) $10 \angle 90^\circ \mu\text{V}$

32. (a) Phasor current through the resistor:

Using ohm's law, we get:

$$\mathbf{V}_R = \mathbf{I}R$$

$$\mathbf{I} = \frac{\mathbf{V}_R}{R} = \boxed{1\angle 30^\circ \text{ A}}$$

(b) At $\omega=1$ rad/s, the voltage across the capacitor-inductor combination is 0 as their equivalent impedance is 0.

$$Z_{eq} = -j + j = 0$$

$$\therefore \frac{\mathbf{V}_R}{\mathbf{V}_{C-L}} = \boxed{\infty}$$

(c) At $\omega = 2$ rad/s,

$$Z_{eq} = -j0.5 + j2 = j1.5$$

$$\mathbf{V}_{C-L} = (1\angle 30^\circ)(1.5\angle 90^\circ) = 1.5\angle 120^\circ \text{ V}$$

$$\therefore \frac{\mathbf{V}_R}{\mathbf{V}_{C-L}} = \frac{1\angle 30^\circ}{1.5\angle 120^\circ} = \boxed{0.67\angle -90^\circ}$$

33. (a) $20 \angle 0^\circ \text{ mV}$
(b) $31.8 \angle -90^\circ \mu\text{V}$
(c) $3.14 \angle 90^\circ \text{ V}$
(d) $20 \angle -0.1^\circ \text{ V}$
(e) $3.14 \angle 89.6^\circ$
(f) $20 \text{ mV}; 0; 0; 20 \text{ mV}; 21.9 \text{ mV}$

34. (a) Given:

$$\mathbf{I}_{10} = 2\angle 42^\circ \text{ mA}$$

$$\mathbf{V} = 40\angle 132^\circ \text{ mV}$$

$$\omega = 1000 \text{ rad/s}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}_{10}} = \frac{40\angle 132^\circ}{2\angle 42^\circ} = 20\angle 90^\circ \Omega$$

The phase angle of 90 degrees shows that it is an inductor.

(b) $\omega = 1000 \text{ rad/s}$

$$Z_L = j\omega L = j20 \Omega$$

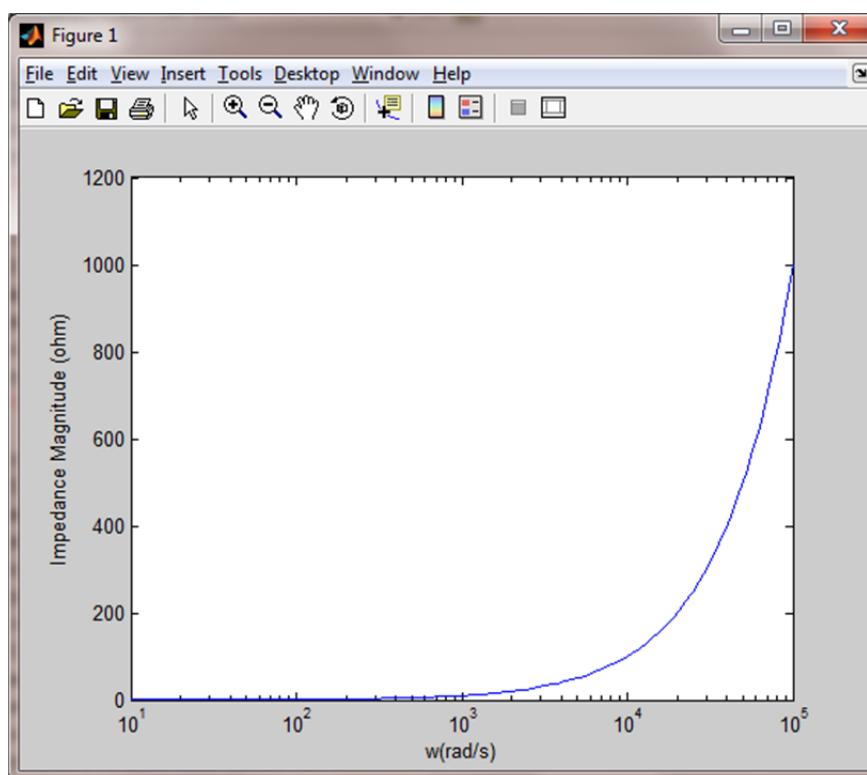
$$\Rightarrow L = \boxed{20 \text{ mH}}$$

35. (a) 2.5Ω ; (b) $50 \angle 35^\circ$; $100 \angle 35^\circ$

36. (a) The equivalent impedance of a 1Ω resistor in series with a 10mH inductor as a function of ω is given by,

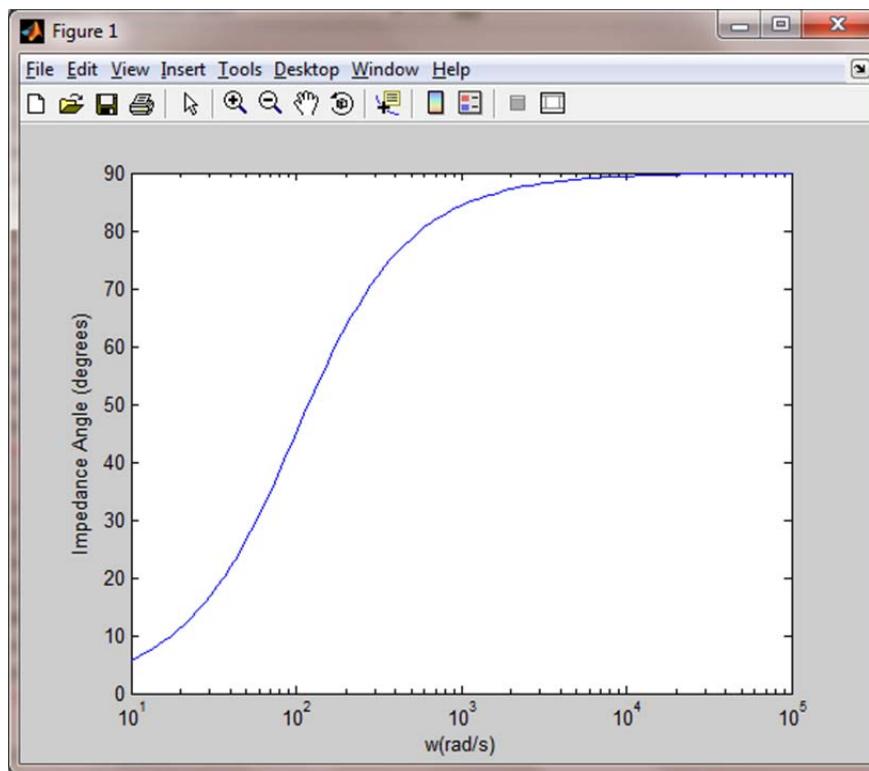
$$\mathbf{Z}_{eq} = R + j\omega L = 1 + j\omega 0.01 \Omega$$

(b)



```
w = logspace(1,5,100);
Z = 1+i*w*0.01;
mag = abs(Z);
semilogx(w, mag);
xlabel('w(rad/s)');
ylabel('Impedance Magnitude (ohm)');
```

(c)



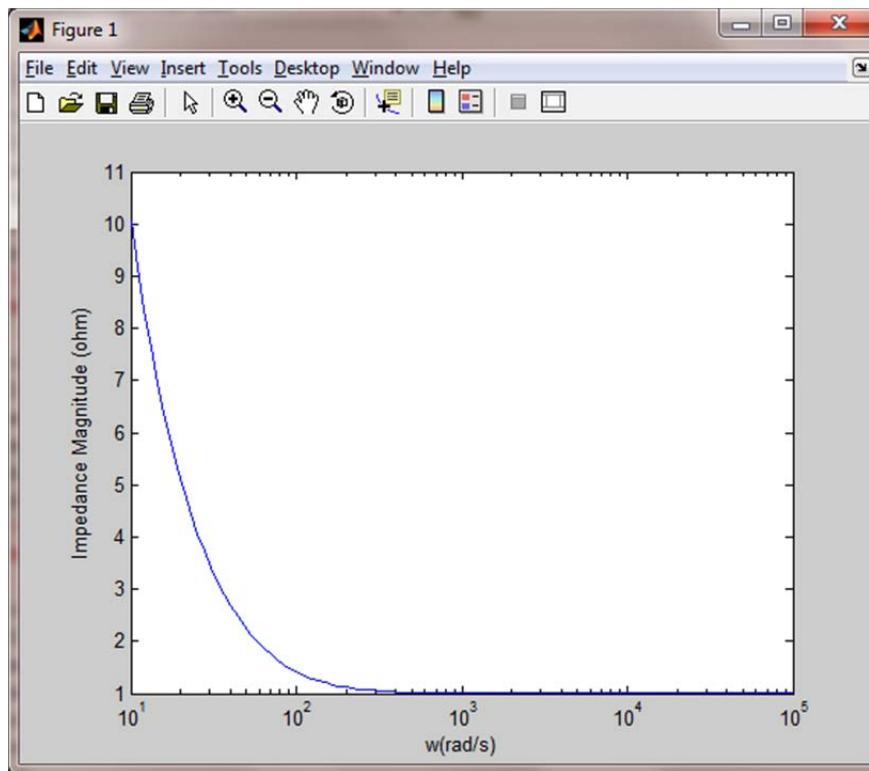
```
w = logspace(1,5,100);
Z = 1+i*w*0.01;
theta = angle(Z);
theta_degrees = angledim(theta,'radians','degrees');
semilogx(w, theta_degrees);
xlabel('w(rad/s)');
ylabel('Impedance Angle (degrees);')
```

37. (a) $1001 \angle -2.9^\circ \Omega$ (b) $20 \angle 90^\circ \Omega$ (c) $20 \angle 88.8^\circ \Omega$

38. (a) The equivalent impedance of a 1Ω resistor in series with a 10mF capacitor as a function of ω is given by,

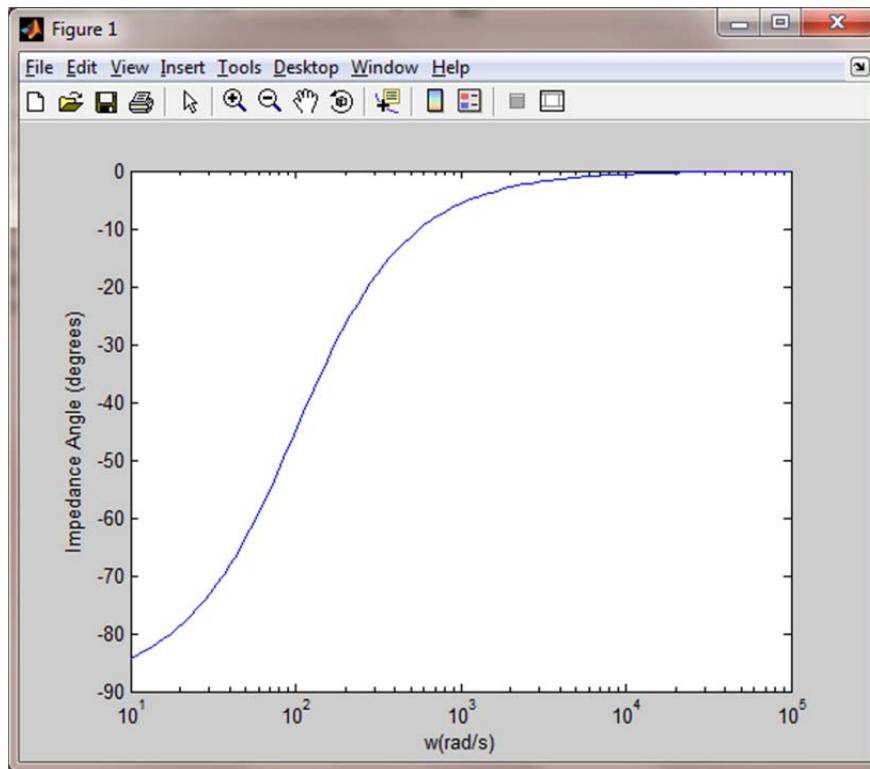
$$\mathbf{Z}_{eq} = R - \frac{j}{\omega C} = \boxed{1 - \frac{j100}{\omega}} \Omega$$

(b)



```
w = logspace(1,5,100);
Z = 1-i*100*w.^-1;
mag = abs(Z);
semilogx(w, mag);
xlabel('w(rad/s)');
ylabel('Impedance Magnitude (ohm)');
```

(c)



```
w = logspace(1,5,100);
Z = 1-i*100*w.^-1;
theta = angle(Z);
theta_degrees = angledim(theta,'radians','degrees');
semilogx(w, theta_degrees);
xlabel('w(rad/s)');
ylabel('Impedance Angle (degrees)').
```

39. (a) $31.2 \angle -38.7^\circ \text{ mS}$ (b) $64.0 \angle -51.3^\circ \text{ mS}$ (c) $20 \angle 89.9^\circ \text{ S}$ (d) $1 \angle -89.9^\circ \text{ mS}$ (e) $1000 \angle 89.9^\circ \text{ S}$

40. Looking into the open terminals we see that the parallel combination of 20 mH and 55 Ω is in series with the series combination of 10 mF and 20Ω , this combination is in parallel with 25 Ω.

(a) $\omega = 1 \text{ rad/s}$

$$\mathbf{Z}_L = j\omega L = j0.02 \Omega$$

$$\mathbf{Z}_C = -\frac{j}{\omega C} = -j100 \Omega$$

$$\mathbf{Z}_{eq} = \frac{\left(\frac{55 \times j0.02}{55 + j0.02} + 20 - j100 \right) \times 25}{\left(\frac{55 \times j0.02}{55 + j0.02} + 20 - j100 \right) + 25} = 22.66 - j5.19 = \boxed{23.24 \angle -12.9^\circ \Omega}$$

(b) $\omega = 10 \text{ rad/s}$

$$\mathbf{Z}_L = j\omega L = j0.2 \Omega$$

$$\mathbf{Z}_C = -\frac{j}{\omega C} = -j10 \Omega$$

$$\mathbf{Z}_{eq} = \frac{\left(\frac{55 \times j0.2}{55 + j0.2} + 20 - j10 \right) \times 25}{\left(\frac{55 \times j0.2}{55 + j0.2} + 20 - j10 \right) + 25} = 11.74 - j2.88 = \boxed{12.08 \angle -13.78^\circ \Omega}$$

(c) $\omega = 100 \text{ rad/s}$

$$\mathbf{Z}_L = j\omega L = j2 \Omega$$

$$\mathbf{Z}_C = -\frac{j}{\omega C} = -j \Omega$$

$$\mathbf{Z}_{eq} = \frac{\left(\frac{55 \times j2}{55 + j2} + 20 - j \right) \times 25}{\left(\frac{55 \times j2}{55 + j2} + 20 - j \right) + 25} = 11.14 - j0.30 = \boxed{11.14 \angle -1.54^\circ \Omega}$$

41. $11.3 \angle -5.3^\circ \Omega$

42. (a) 3Ω in series with 2mH

$$\mathbf{Z}_{eq} = 3 + j4 = 5\angle 53.13^\circ \Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20^\circ)(5\angle 53.13^\circ) = 15\angle -33.13^\circ \text{ V}$$

(b) 3Ω in series with $125\mu\text{F}$

$$\mathbf{Z}_{eq} = 3 - j4 = 5\angle -53.13^\circ \Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20^\circ)(5\angle -53.13^\circ) = 15\angle -73.13^\circ \text{ V}$$

(c) 3Ω , 2mH , and $125\mu\text{F}$ in series

$$\mathbf{Z}_{eq} = 3 + j4 - j4 = 3\angle 0^\circ \Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20^\circ)(3) = 9\angle -20^\circ \text{ V}$$

(d) 3Ω , 2mH and $125\mu\text{F}$ in series but $\omega = 4$ krad/s

$$\mathbf{Z}_{eq} = 3 + j8 - j2 = 6.71\angle 63.44^\circ \Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20^\circ)(6.71\angle 63.44^\circ) = 20.13\angle -43.44^\circ \text{ V}$$

43. (a) $30 - j0.154 \Omega$
(b) $23.5 + j9.83 \Omega$
(c) $30 + j0.013 \Omega$
(d) $30 + j1.3 \times 10^{-5} \Omega$
(e) $30 + 1.3 \times 10^{-8} \Omega$

44. One method is to use the current divider rule in order to calculate $i(t)$. In the given circuit, there are three parallel branches.

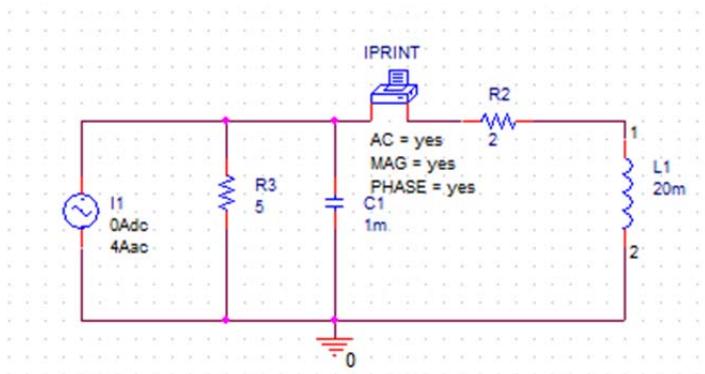
$$\mathbf{Z}_{eq} = \frac{1}{5^{-1} + (-j10)^{-1} + (2+j2)^{-1}} = 2 + j0.67 = 2.11\angle 18.52^\circ \Omega$$

$$\mathbf{Z} = (2+j2)\Omega = 2.83\angle 45^\circ \Omega$$

$$\mathbf{I} = \mathbf{I}_s \frac{\mathbf{Z}_{eq}}{\mathbf{Z}} = \frac{(4\angle -20^\circ)(2.11\angle 18.52^\circ)}{2.83\angle 45^\circ} = 2.98\angle -46.48^\circ A$$

$$\therefore i(t) = 2.98 \cos(100t - 46.48^\circ) A$$

PSpice Verification:



FREQ IM(V_PRINT1)IP(V_PRINT1)

1.592E+01 2.981E+00 -4.656E+01

45. (a) *One possible solution:* A 1Ω resistor in series with 1 H and 10^{-4} F .
- (b) *One possible solution:* A 6.894Ω resistor in series with 11.2 mH .
- (c) *One possible solution:* A 3Ω resistor in series with 2.5 mF .

46. One out of many possible design solutions:

(a) At $\omega = 10 \text{ rad/s}$, the equivalent admittance is given as, $\mathbf{Y} = 1 \text{ S}$. We can construct this using a 1 S conductance (1Ω resistor) in parallel with an inductor L and a capacitor C

such that $\omega C - \frac{1}{\omega L} = 0$. Selecting L as 5H arbitrarily yields the value of a capacitor as 2mF.

Thus, one design can be 1Ω resistor in parallel with 5H inductor and 2mF capacitor.

(b) At $\omega = 10 \text{ rad/s}$, the equivalent admittance is given as,

$\mathbf{Y} = 12 \angle -18^\circ \text{ S} = 11.4127 - j3.7082 \text{ S}$. We can construct this using a 11.4127 S conductance ($87.6 \text{ m}\Omega$ resistor) in parallel with an inductor L such that

$-\frac{j}{\omega L} = -j3.7082 \text{ S}$. This yields the value of the inductor as 26.9 mH.

Thus, one design can be $87.6 \text{ m}\Omega$ resistor in parallel with 26.9 mH inductor.

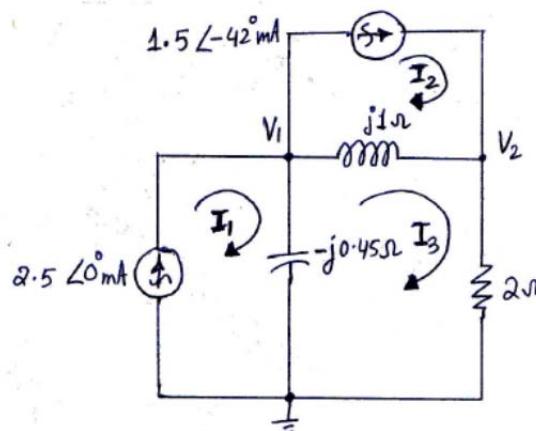
(c) At $\omega = 10 \text{ rad/s}$, the equivalent admittance is given as, $\mathbf{Y} = 2 + j \text{ mS}$. We can construct this using a 2 mS conductance (500Ω resistor) in parallel with a capacitor C such that $j\omega C = j0.001 \text{ S}$. This yields the value of the capacitor as 0.1 mF.

Thus, one design can be 500Ω resistor in parallel with 0.1 mF capacitor.

47. **BOTH SOURCES ARE SUPPOSED TO OPERATE AT 100 rad/s.** Then,

$$v_1(t) = 2.56\cos(100t + 139.2^\circ) \text{ V}; v_2(t) = 4.35\cos(100t + 138.3^\circ) \text{ V}.$$

48. (a)

(b) In mesh 1, we have $\mathbf{I}_1 = 2.5\angle 0^\circ \text{ mA}$.In mesh 2, we have $\mathbf{I}_2 = 1.5\angle -42^\circ \text{ mA}$.

In mesh 3, we have,

$$(\mathbf{I}_3 - \mathbf{I}_1)\mathbf{Z}_C + (\mathbf{I}_3 - \mathbf{I}_2)\mathbf{Z}_L + 2\mathbf{I}_3 = 0$$

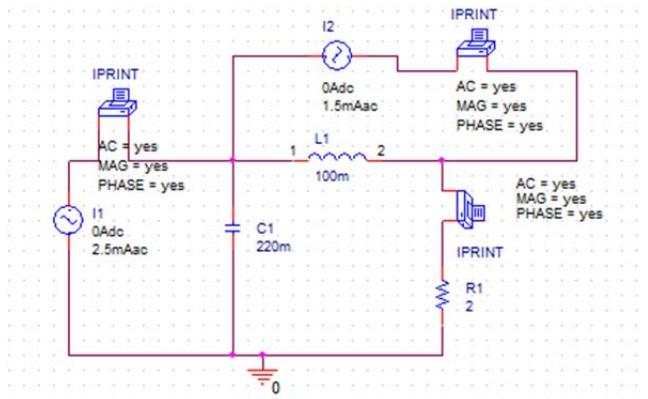
$$\begin{aligned} \mathbf{I}_3 &= \frac{\mathbf{I}_1\mathbf{Z}_C + \mathbf{I}_2\mathbf{Z}_L}{2 + \mathbf{Z}_C + \mathbf{Z}_L} = \frac{2.5 \times 10^{-3} \times (-j0.4545) + (1.1147 - j1.0037) \times 10^{-3} \times j}{2 - j0.4545 + j} \\ &= \frac{1.004\angle -1.23^\circ}{2.073\angle 15.25^\circ} = 0.4843\angle -16.48^\circ \text{ mA} \end{aligned}$$

$$\therefore i_1(t) = 2.5 \cos 10t \text{ mA}$$

$$\therefore i_2(t) = 1.5 \cos(10t - 42^\circ) \text{ mA}$$

$$\therefore i_3(t) = 0.4843 \cos(10t - 16.48^\circ) \text{ mA}$$

Pspice Verification:



```
FREQ      IM(V_PRINT1)IP(V_PRINT1)
1.592E+00  4.843E-04 -1.649E+01
*** 06/16/12 23:24:09 ***** PSpice Lite (April 2011) ***** ID# 10813 ****
** Profile: "SCHEMATIC1-prob10_48"  [ C:\ORCAD\ORCAD_16.5_LITE\examples\prob10_48.sch ]
***** AC ANALYSIS          TEMPERATURE = 27.000 DEG C
*****
FREQ      IM(V_PRINT2)IP(V_PRINT2)
1.592E+00  2.500E-03  2.120E-11
*** 06/16/12 23:24:09 ***** PSpice Lite (April 2011) ***** ID# 10813 ****
** Profile: "SCHEMATIC1-prob10_48"  [ C:\ORCAD\ORCAD_16.5_LITE\examples\prob10_48.sch ]
***** AC ANALYSIS          TEMPERATURE = 27.000 DEG C
*****
FREQ      IM(V_PRINT3)IP(V_PRINT3)
1.592E+00  1.500E-03 -4.200E+01
```

49. $v_1(t) = 928\cos(10t - 86.1^\circ) \mu V$; $v_2(t) = 969\cos(10t - 16.5^\circ) \mu V$.

50. In the circuit given by Fig. 10.60, we have,

$$\mathbf{V}_1 - \mathbf{I}_1(j30) = \mathbf{V}_2 + 55(\mathbf{I}_1 - \mathbf{I}_2) \text{ and}$$

$$\mathbf{V}_1 - \mathbf{I}_1(j30) = \mathbf{V}_3 + \mathbf{I}_2(-j20)$$

On simplification, we get,

$$\mathbf{I}_1(55 + j30) - 55\mathbf{I}_2 = -2.2635 + j9.848$$

$$\mathbf{I}_1(j30) - \mathbf{I}_2(-j20) = -0.1045 + j9.0665$$

Solving for \mathbf{I}_1 and \mathbf{I}_2 , we get,

$$\mathbf{I}_1 = 0.6247 + j0.3339 = \boxed{0.71\angle 28.12^\circ \text{ A}}$$

$$\mathbf{I}_2 = 0.4838 + j0.4956 = \boxed{0.69\angle 45.69^\circ \text{ A}}$$

51.

$$0.809 \angle -4.8^\circ$$

52. Using phasor domain, in mesh 1, we get,

$$\begin{aligned} 2\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) &= 2.5\angle 9^\circ \\ \Rightarrow \mathbf{I}_1(2 + j10) - \mathbf{I}_2(j10) &= 2.4692 + j0.3911 \end{aligned} \quad [1]$$

In mesh 2, we get,

$$\begin{aligned} j10(\mathbf{I}_2 - \mathbf{I}_1) - j0.3\mathbf{I}_2 + 5\mathbf{I}_1 &= 0 \\ \Rightarrow \mathbf{I}_1(5 - j10) + \mathbf{I}_2(j9.7) &= 0 \end{aligned} \quad [2]$$

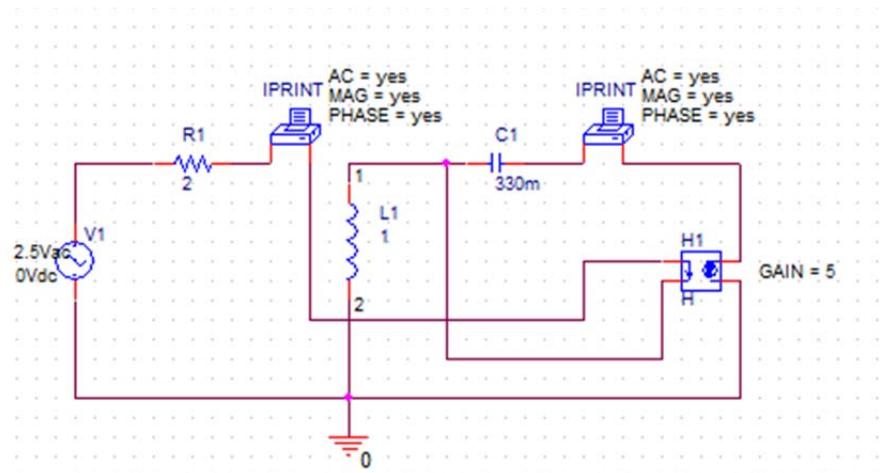
On solving eqns [1] and [2] we get,

$$\mathbf{I}_1 = 0.3421 + j0.0695 = 0.35\angle 11.48^\circ \text{ A}$$

$$\mathbf{I}_2 = 0.3169 + j0.2479 = 0.4\angle 38.04^\circ \text{ A}$$

$$\begin{aligned} \therefore i_1(t) &= 0.35 \cos(10t + 11.48^\circ) \text{ A and} \\ i_2(t) &= 0.4 \cos(10t + 38.04^\circ) \text{ A} \end{aligned}$$

Pspice verification:



```
FREQ            IM(V_PRINT1)IP(V_PRINT1)

1.592E+00    3.490E-01    1.150E+01

*** 06/17/12 14:13:22 **** PSpice Lit

** Profile: "SCHEMATIC1-prob10_52" [ C:

****        AC ANALYSIS
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```
*****
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```
FREQ            IM(V_PRINT2)IP(V_PRINT2)

1.592E+00    4.024E-01    3.807E+01
```

53.

 $2.73 \angle 152^\circ \text{ A}$

54. Using node voltage analysis in phasor domain, we get the nodal equations as,

$$\mathbf{I}_1 + \frac{\mathbf{V}_1}{j2} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j4} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{1+j3.8} = 0 \quad [1]$$

$$\mathbf{I}_2 - \frac{\mathbf{V}_2}{2} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j4} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{1+j3.8} = 0 \quad [2]$$

$$\mathbf{I}_1 = 15\angle 0^\circ = 15A$$

$$\mathbf{I}_2 = 25\angle 131^\circ = -16.4015 + j18.8677A$$

On simplifying the equations [1] and [2], we get,

$$\mathbf{V}_1(0.0648 - j0.4961) + \mathbf{V}_2(-0.0648 - j0.0039) = -15$$

$$\mathbf{V}_1(0.0648 + j0.0039) + \mathbf{V}_2(-0.5648 - j0.0039) = -16.4015 + j18.8677$$

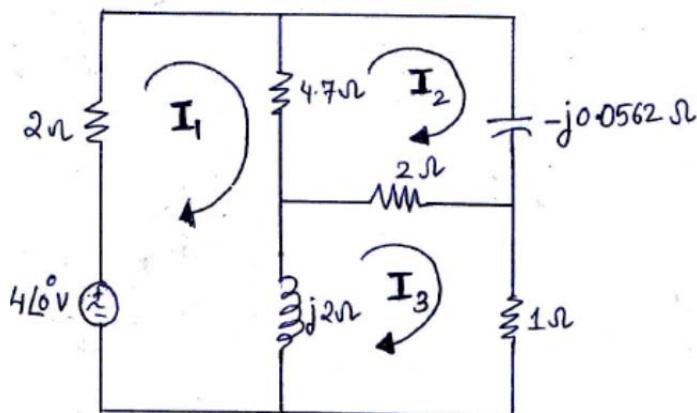
$$\mathbf{V}_2 = -29.5221 + j29.7363 = \boxed{41.9\angle 134.8^\circ V}$$

Matlab Verification:

```
>> syms v1 v2;
eqn1 = (15+v1/(2i)+(v1-v2)/(-4i)+(v1-v2)/(1+3.8i));
eqn2 = (-16.4015+18.8677i+(v1-v2)/(1+3.8i)+(v1-v2)/(-4i)-v2/2);
answer=solve(eqn1, eqn2, 'v1', 'v2');
digits(4);
V2 = vpa(answer.v2)
V2 = -29.52+29.74*i
```

55. 1.14cos(20t + 12°) V

56.



Using phasor domain, in mesh 1, we get,

$$\begin{aligned} 2\mathbf{I}_1 + 4.7(\mathbf{I}_1 - \mathbf{I}_2) + j2(\mathbf{I}_1 - \mathbf{I}_2) &= 4 \\ \Rightarrow (6.7 + j2)\mathbf{I}_1 - 4.7\mathbf{I}_2 - j2\mathbf{I}_3 &= 4 \end{aligned} \quad [1]$$

In mesh 2, we get,

$$\begin{aligned} 4.7(\mathbf{I}_2 - \mathbf{I}_1) - j0.0562\mathbf{I}_2 + 2(\mathbf{I}_2 - \mathbf{I}_3) &= 0 \\ \Rightarrow -4.7\mathbf{I}_1 + (6.7 - j0.0562)\mathbf{I}_2 - 2\mathbf{I}_3 &= 0 \end{aligned} \quad [2]$$

In mesh 3, we get,

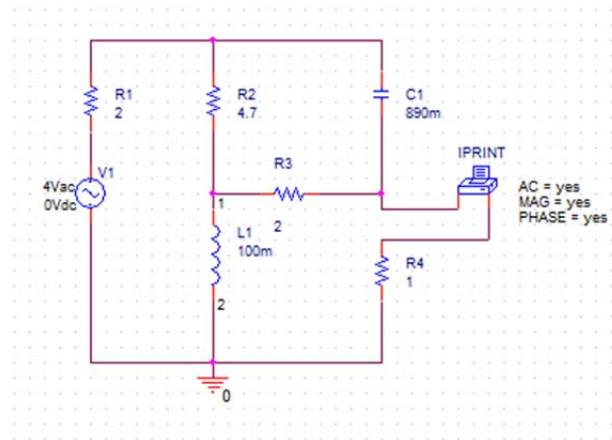
$$\begin{aligned} j2(\mathbf{I}_3 - \mathbf{I}_1) + 2(\mathbf{I}_3 - \mathbf{I}_2) + \mathbf{I}_3 &= 0 \\ \Rightarrow -j2\mathbf{I}_1 - 2\mathbf{I}_2 + (3 + j2)\mathbf{I}_3 &= 0 \end{aligned} \quad [3]$$

Here, $\mathbf{I}_x = \mathbf{I}_3$. On solving we get,

$$\mathbf{I}_x = \mathbf{I}_3 = 1.1104 + j0.2394 = 1.136 \angle 12.16^\circ \text{ A}$$

$$\therefore i_x(t) = 1.136 \cos(20t + 12.16^\circ) \text{ A}$$

Pspice Verification:

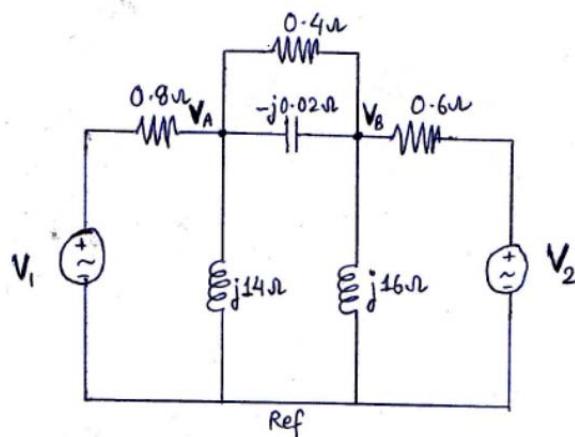


FREQ IM(V_PRINT1)IP(V_PRINT1)

3.183E+00 1.136E+00 1.216E+01

57. $155\cos(14t + 37^\circ)$ A; $82.2\cos(14t - 101^\circ)$ A; $42.0\cos(14t - 155^\circ)$ A;
 $71.7\cos(14t + 50^\circ)$ A.

58.



Using node voltage analysis in phasor domain, we get the nodal equations as,

At node A,

$$\frac{(\mathbf{V}_1 - \mathbf{V}_A)}{0.8} - \frac{(\mathbf{V}_A - \mathbf{V}_B)}{0.4} - \frac{(\mathbf{V}_A - \mathbf{V}_B)}{-j0.02} - \frac{\mathbf{V}_A}{j14} = 0 \quad [1]$$

At node B,

$$\frac{(\mathbf{V}_2 - \mathbf{V}_B)}{0.6} + \frac{(\mathbf{V}_A - \mathbf{V}_B)}{0.4} + \frac{(\mathbf{V}_A - \mathbf{V}_B)}{-j0.02} - \frac{\mathbf{V}_B}{j16} = 0 \quad [2]$$

Given,

$$\mathbf{V}_1 = 0.009\angle 0.5^\circ = 0.009 + j0.000078 \text{ V}$$

$$\mathbf{V}_2 = 0.004\angle 1.5^\circ = 0.004 + j0.0001 \text{ V}$$

On simplifying the nodal equations [1] and [2], we get,

$$\mathbf{V}_A (-3.75 - j49.9286) + \mathbf{V}_B (2 + j50) = -\frac{0.009 + j0.000078}{0.8}$$

$$\mathbf{V}_A (2 + j50) + \mathbf{V}_B (-4.1667 - j49.9375) = -\frac{0.004 + j0.0001}{0.6}$$

and on solving, we get,

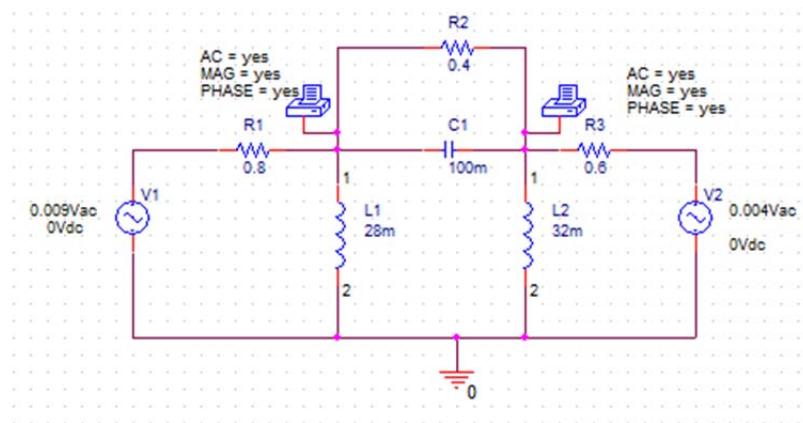
$$\mathbf{V}_A = 0.00613 + j0.00033 = 0.00613\angle 3.09^\circ \text{ V}$$

$$\mathbf{V}_B = 0.00612 + j0.00040 = 0.00613\angle 3.75^\circ \text{ V}$$

$$\therefore v_A = 0.00613 \cos(500t + 3.09^\circ) \text{ V and}$$

$$v_B = 0.00613 \cos(500t + 3.75^\circ) \text{ V}$$

Pspice Verification:



FREQ VM(N00227) VP(N00227)

7.957E+01 6.138E-03 3.122E+00

*** 06/18/12 09:52:20 ***** PSpice Li

** Profile: "SCHEMATIC1-prob10_58" [C

***** AC ANALYSIS

FREQ VM(N00296) VP(N00296)

7.957E+01 6.136E-03 3.787E+00

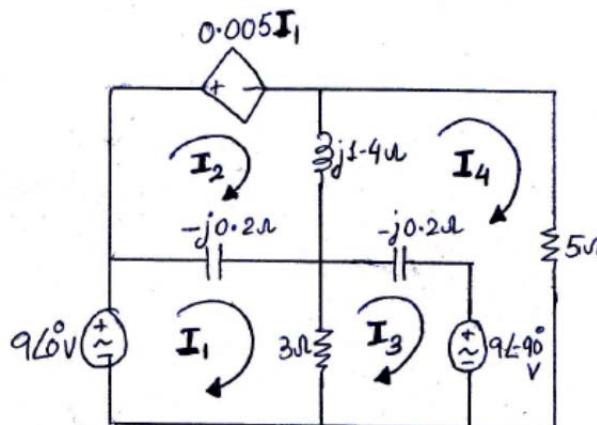
59. (a)

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-R_f}{R_f - j/\omega C_1 - \frac{j}{\omega C_1}}$$

(b)

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\omega R_f C_1}{\frac{1}{A} [\omega R_f C_1 + (R_f C_f - j)] + (\omega R_f C_f - j)}$$

60.



Using phasor domain, in mesh 1, we get,

$$\begin{aligned} (-j0.2)(\mathbf{I}_1 - \mathbf{I}_2) + 3(\mathbf{I}_1 - \mathbf{I}_3) &= 9 \\ \Rightarrow (3 - j0.2)\mathbf{I}_1 + j0.2\mathbf{I}_2 - 3\mathbf{I}_3 &= 9 \end{aligned} \quad [1]$$

In mesh 2, we get,

$$\begin{aligned} 0.005\mathbf{I}_1 + j1.4(\mathbf{I}_2 - \mathbf{I}_3) - j0.2(\mathbf{I}_2 - \mathbf{I}_1) &= 0 \\ \Rightarrow (0.005 + j0.2)\mathbf{I}_1 + j1.2\mathbf{I}_2 - j1.4\mathbf{I}_3 &= 0 \end{aligned} \quad [2]$$

In mesh 3, we get,

$$\begin{aligned} -j0.2(\mathbf{I}_3 - \mathbf{I}_4) + 3(\mathbf{I}_3 - \mathbf{I}_1) + \mathbf{I}_3 - j9 &= 0 \\ \Rightarrow -3\mathbf{I}_1 + (3 - j0.2)\mathbf{I}_3 + j0.2\mathbf{I}_4 &= j9 \end{aligned} \quad [3]$$

In mesh 4, we get,

$$\begin{aligned} 5\mathbf{I}_4 - j0.2(\mathbf{I}_4 - \mathbf{I}_3) + j1.4(\mathbf{I}_4 - \mathbf{I}_2) &= -j9 \\ \Rightarrow -j1.4\mathbf{I}_2 + j0.2\mathbf{I}_3 + \mathbf{I}_4(5 + j1.2) &= -j9 \end{aligned} \quad [4]$$

On solving we get,

$$\mathbf{I}_1 = -18.33 + j20 = 27.13 \angle 132.5^\circ \text{ A}$$

$$\mathbf{I}_2 = 5.092 - j3.432 = 6.14 \angle -33.98^\circ \text{ A}$$

$$\mathbf{I}_3 = -19.76 + j21.55 = 29.24 \angle 132.5^\circ \text{ A}$$

$$\mathbf{I}_4 = 1.818 - j0.02 = 1.82 \angle -0.63^\circ \text{ A}$$

Therefore,

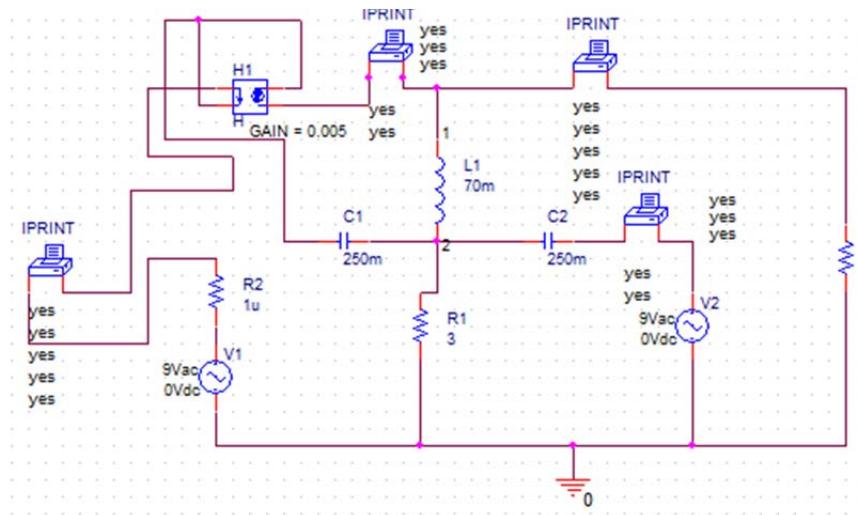
$$i_1(t) = 27.13 \cos(20t + 132.5^\circ) \text{ A}$$

$$i_2(t) = 6.14 \cos(20t - 33.98^\circ) \text{ A}$$

$$i_3(t) = 29.24 \cos(20t + 132.5^\circ) \text{ A}$$

$$i_4(t) = 1.82 \cos(20t - 0.63^\circ) \text{ A}$$

Pspice Verification:



```
FREQ      IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)

3.183E+00  2.712E+01  1.325E+02  -1.833E+01  1.999E+01
*** 06/21/12 13:48:06 ***** PSpice Lite (April 2011) *****
** Profile: "SCHEMATIC1-prob10_60_old" [ C:\ORCAD\ORCAD_16.5_LI

****      AC ANALYSIS                      TEMPERATURE = 27.000
*****
*****
```



```
FREQ      IM(V_PRINT3)IP(V_PRINT3)IR(V_PRINT3)II(V_PRINT3)

3.183E+00  3.052E+01  1.350E+02  -2.158E+01  2.157E+01
-----
```

```
FREQ      IM(V_PRINT4)IP(V_PRINT4)IR(V_PRINT4)II(V_PRINT4)

 3.183E+00   1.818E+00  -6.301E-01   1.818E+00  -2.000E-02
*** 06/21/12 13:48:06 ***** PSpice Lite (April 2011) *****
** Profile: "SCHEMATIC1-prob10_60_old" [ C:\ORCAD\ORCAD_16.5\

***** AC ANALYSIS           TEMPERATURE = 27.0

*****
```



```
FREQ      IM(V_PRINT5)IP(V_PRINT5)IR(V_PRINT5)II(V_PRINT5)

 3.183E+00   6.141E+00  -3.398E+01   5.092E+00  -3.432E+00
```

61. Left hand source contributions: $5.58 \angle -91.8^\circ \text{ V}; 1.29 \angle -75.9^\circ \text{ V}$

Right hand source contributions: $1.29 \angle -75.9^\circ \text{ V}; 9.08 \angle -115^\circ \text{ V}$

62. Using node voltage analysis in phasor domain, we get the nodal equations as,

At node 1,

$$\mathbf{I}_1 - \mathbf{I}_2 - \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j5} - \frac{\mathbf{V}_1}{j3} = 0 \quad [1]$$

At node 2,

$$\mathbf{I}_2 + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j5} - \frac{\mathbf{V}_2}{2} = 0 \quad [2]$$

$$\mathbf{I}_1 = 33 \times 10^{-3} \angle 3^\circ \text{ mA}$$

$$\mathbf{I}_2 = 51 \times 10^{-3} \angle -91^\circ \text{ mA}$$

On simplifying the nodal equations [1] and [2], we get,

$$\mathbf{V}_1(j0.1333) + \mathbf{V}_2(-j0.2) = \mathbf{I}_2 - \mathbf{I}_1$$

$$\mathbf{V}_1(j0.2) + \mathbf{V}_2(-0.5 - j0.2) = -\mathbf{I}_2$$

and on solving these we get,

$$\boxed{\mathbf{V}_1 = -0.4694 - j0.1513 = 493.18 \angle -162.14^\circ \text{ mV}}$$

$$\boxed{\mathbf{V}_2 = -0.0493 - j0.27 = 274.46 \angle -100.34^\circ \text{ mV}}$$

63. $7.995\cos(40t + 2.7^\circ) + 0.343\cos(30t + 90.1^\circ)$ mV; $7.995\cos(40t + 2.4^\circ) + 1.67\cos(30t - 180^\circ)$ mV

64. Calculate Thevenin Impedance

$$Z_{\text{thevenin}} = j2 + 4\angle 10^\circ = 3.94 + j2.69 = \boxed{4.77\angle 34.32^\circ \Omega}$$

Calculate Thevenin voltage:

$$\mathbf{V}_1 = (1.5\angle 24^\circ)(j2) = (1.5\angle 24^\circ)(2\angle 90^\circ) = 3\angle 114^\circ \text{ V}$$

$$\mathbf{V}_2 = -(2\angle 38^\circ)(4\angle 10^\circ) = -8\angle 48^\circ \text{ V}$$

$$\mathbf{V}_{TH} = \mathbf{V}_1 - \mathbf{V}_2 = (3\angle 114^\circ) + (8\angle 48^\circ) = 4.13 + j8.68 = \boxed{9.61\angle 64.55^\circ \text{ V}}$$

Current \mathbf{I}_1 through the impedance $(2-j2) \Omega$ is found as:

$$\mathbf{Z}_{\text{total}} = 3.94 + j2.69 + 2 - j2 = 5.94 + j0.69 = 5.97\angle 6.626^\circ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{TH}}{\mathbf{Z}_{\text{total}}} = \frac{9.61\angle 64.55^\circ}{5.97\angle 6.63^\circ} = \boxed{1.6\angle 57.92^\circ \text{ A}}$$

65.

$$1.56 \angle 27.8^\circ \text{ A}$$

66. (a) Thevenin Equivalent

$$\mathbf{Z}_{\text{thevenin}} = (12 - j34) \parallel (j10) = \frac{340 + j120}{12 - j24} = 1.67 + j13.33 = 13.43 \angle 82.86^\circ \Omega$$

Using current divider to find the current through $j10$ branch,

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{I}_s \cdot \mathbf{Z}_{eq}}{\mathbf{Z}} = \frac{22 \angle 30^\circ (340 + j120)}{12 - j24} \\ &= 29.55 \angle 22.87^\circ \text{ A}\end{aligned}$$

$$\mathbf{V}_{TH} = \mathbf{V}_{oc} = (29.55 \angle 22.87^\circ)(10 \angle 90^\circ) = 295.46 \angle 112.87^\circ \text{ V}$$

(b) Norton Equivalent

$$\mathbf{Z}_{norton} = \mathbf{Z}_{\text{thevenin}} = 1.67 + j13.33 = 13.43 \angle 82.86^\circ \Omega$$

$$\mathbf{I}_N = \mathbf{I}_{sc} = 22 \angle 30^\circ \text{ A}$$

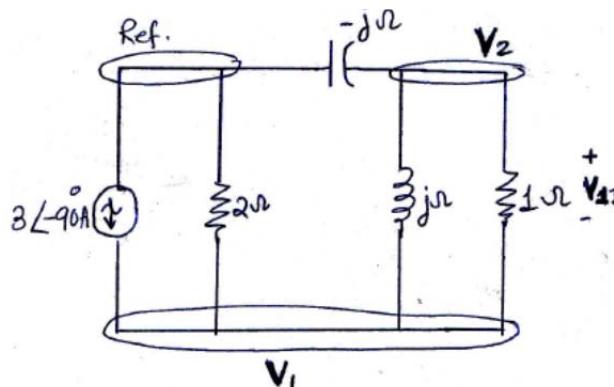
(c) Current flowing from a to b

$$\mathbf{Z}_{total} = 1.67 + j13.33 + 7 - j2 = 8.67 + j11.33 = 14.26 \angle 52.57^\circ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{TH}}{\mathbf{Z}_{total}} = \frac{295.46 \angle 112.87^\circ}{14.26 \angle 52.57^\circ} = 20.72 \angle 60.3^\circ \text{ A}$$

67. (b) $259 \angle 84.5^\circ \mu\text{V}$

68. Let us first consider the current source only.



Using node voltage analysis in phasor domain, we get the nodal equations as,

At node 1,

$$\mathbf{I}_s + \frac{(\mathbf{V}_2 - \mathbf{V}_1)}{1} = \frac{\mathbf{V}_1}{2} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{j1}$$

$$\Rightarrow \mathbf{V}_1(1.5 - j) + \mathbf{V}_2(-1 + j) = -j3 \quad [1]$$

At node 2,

$$\frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j1} - \frac{\mathbf{V}_2}{-j} = \frac{(\mathbf{V}_2 - \mathbf{V}_1)}{1}$$

$$\Rightarrow \mathbf{V}_1(1 - j) - \mathbf{V}_2 = 0 \quad [2]$$

$$\mathbf{I}_s = 3\angle -90^\circ A$$

On solving the nodal equations [1] and [2], we get,

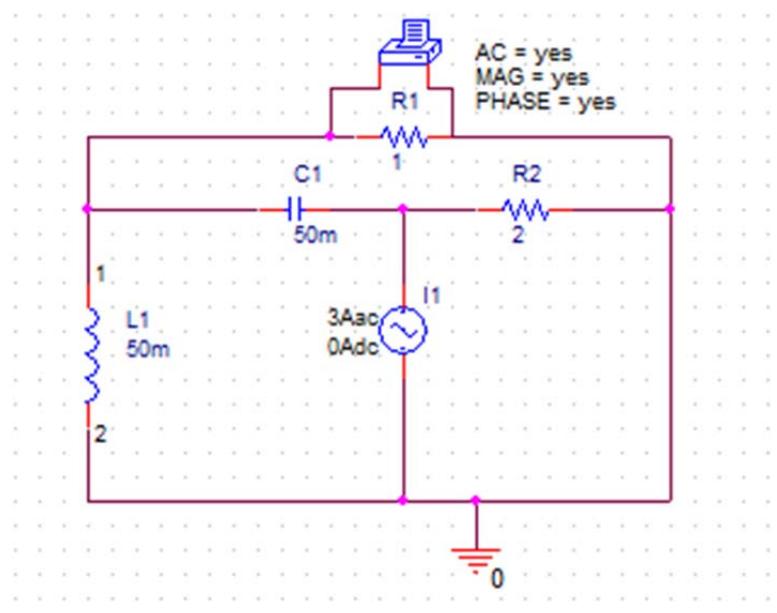
$$\mathbf{V}_{1I} = -0.9231 - j1.3846 \text{ V}$$

$$\mathbf{V}_{2I} = -2.3077 - j0.4615 \text{ V}$$

$$\mathbf{V}_I = \mathbf{V}_{2I} - \mathbf{V}_{1I} = -1.3846 + j0.9231 = 1.66\angle 146.3^\circ \text{ V}$$

$$v_{1I}(t) = 1.66 \cos(20t + 146.3^\circ) \text{ V}$$

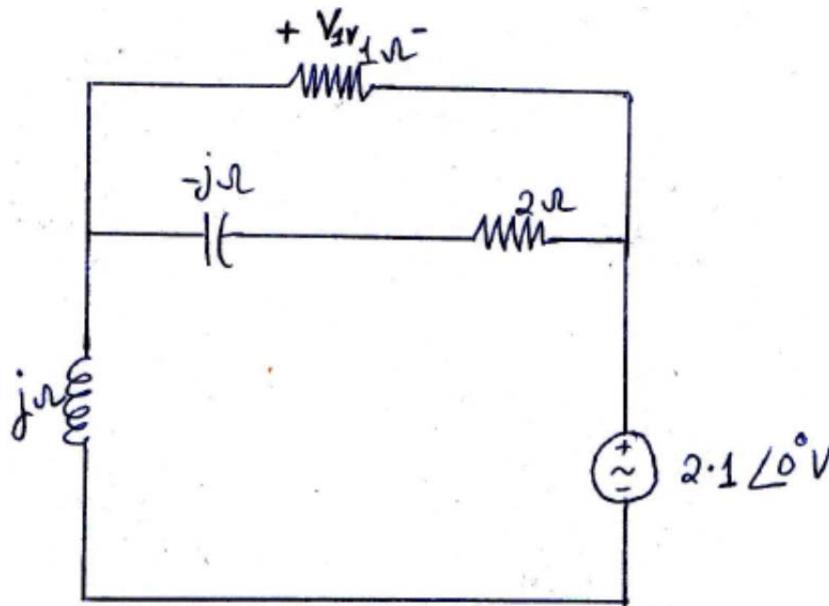
Pspice Verification:



FREQ VM(N00166,0) VP(N00166,0)

3.183E+00 1.664E+00 1.463E+02

Now let us consider the voltage source only.



Then the current flowing in the circuit will be,

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{2.1}{\frac{2-j}{3-j} + j} = 1.1307 - j1.4538 \text{ A}$$

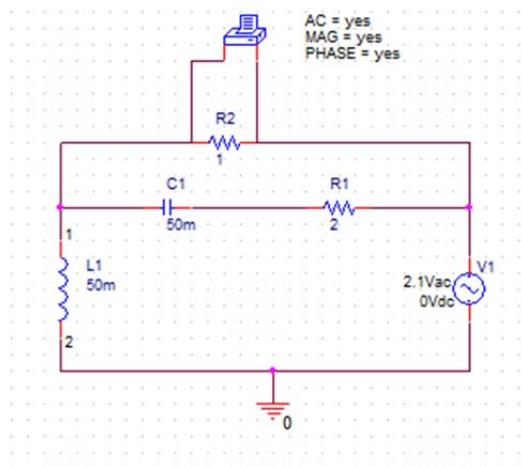
Using current divider to find the current through 1Ω branch,

$$\mathbf{I}_1 = \frac{\mathbf{I}_s \mathbf{Z}_{eq}}{\mathbf{Z}_R} = (1.1307 - j1.4538)(0.7 - j0.1)$$

$$\mathbf{V}_{1V} = -\mathbf{I}_1 \mathbf{Z}_R = -0.6461 + j1.1307 = 1.3 \angle 119.74^\circ \text{ V}$$

$$v_{1V}(t) = 1.3 \cos(20t + 119.74^\circ) \text{ V}$$

Pspice Verification:



FREQ VM(N00211,N00222) VP(N00211,N00222)

3.183E+00 1.302E+00 1.197E+02

69. (a) $24\cos^2(20t - 163^\circ)$ W

70. Using phasor analysis, we get the open circuit voltage as,

$$\mathbf{V}_{oc} = 1\angle 0^\circ \text{ V}$$

For finding the short circuit current through terminal a-b, we can apply KVL,

$$1\angle 0^\circ = -j(0.25\mathbf{I}_N(j2) + \mathbf{I}_N) + j2\mathbf{I}_N = (0.5 + j)\mathbf{I}_N$$

$$\mathbf{I}_N = \frac{1\angle 0^\circ}{(0.5 + j)} = 0.4 - j0.8 = 0.89\angle -63.43^\circ \text{ A}$$

Now for finding the equivalent impedance,

$$\mathbf{Z}_N = \frac{\mathbf{V}_{oc}}{\mathbf{I}_N} = 0.5 + j\Omega$$

For a parallel combination of a resistor and a capacitor or an inductor,

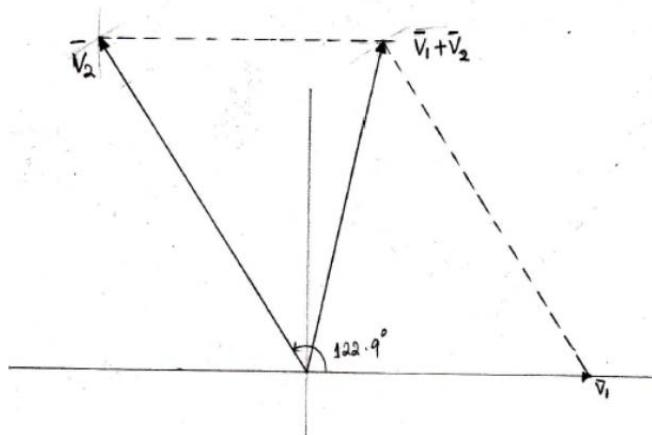
$$\mathbf{Z}_{eq} = \frac{1}{R^{-1} + (jX)^{-1}} = 0.5 + j$$

$$\Rightarrow R = \frac{1}{0.4} = 2.5 \Omega \text{ and } X = \frac{1}{0.8} = 1.25 \text{ from which at } \omega = 1 \text{ rad / s , we get,}$$

the value of $L = 1.25 \text{ H}$

71. (b) I_S leads I_R by 83° ; I_C by -7° ; I_x by 146°

72. Taking $50 \text{ V} = 1 \text{ inch}$, from the figure, we get the angle as $\pm 122.9^\circ$. (The figure below shows for the angle $+122.9^\circ$ only.)



Analytical Solution: On solving,

$$|100 + 140\angle\alpha| = 120$$

$$|100 + 140 \cos \alpha + j140 \sin \alpha| = 120$$

$$\alpha = \pm 122.88^\circ$$

73. (a) $\mathbf{V}_R = 51.2 \angle -140^\circ \text{ V}$; $\mathbf{V}_L = 143 \angle 13^\circ \text{ V}$; $\mathbf{I}_L = 57 \angle -85^\circ \text{ A}$; $\mathbf{I}_C = 51.2 \angle -50^\circ \text{ A}$;
 $\mathbf{I}_R = 25.6 \angle 26^\circ \text{ A}$

74. (a) $\mathbf{V}_s = 120\angle 0^\circ \text{ V}$

$$\mathbf{Z}_1 = 40\angle 30^\circ \Omega$$

$$\mathbf{Z}_2 = 50 - j30 = 58.31\angle -30.96^\circ \Omega$$

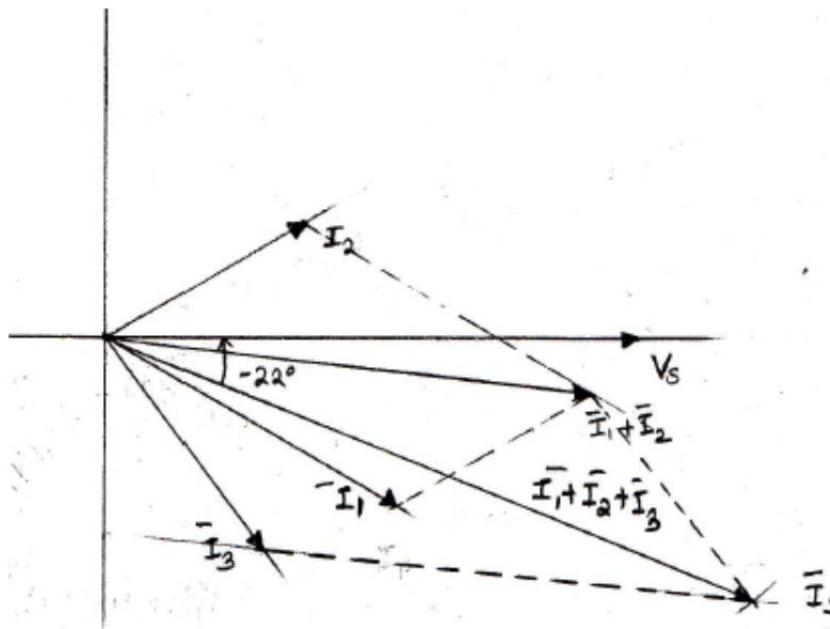
$$\mathbf{Z}_3 = 30 + j40 = 50\angle 53.13^\circ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{Z}_1} = \frac{120\angle 0^\circ}{40\angle 30^\circ} = 3\angle -30^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_s}{\mathbf{Z}_2} = \frac{120\angle 0^\circ}{58.31\angle -30.96^\circ} = 2.05\angle -30.96^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{\mathbf{V}_s}{\mathbf{Z}_3} = \frac{120\angle 0^\circ}{50\angle 53.13^\circ} = 2.4\angle -53.13^\circ \text{ A}$$

(b) Here the scale is : 50V = 1 inch and 2A = 1 inch.

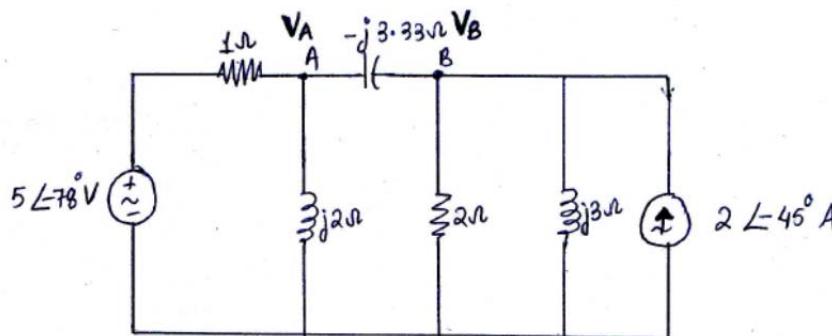


(c) From the graph, we find that,

$$\mathbf{I}_s = 6.2\angle -22^\circ \text{ A}$$

75. (b) $0.333 \angle 124^\circ$

76. (a)



(b) Thevenin Impedance

$$Z_{TH} = (1 \parallel j2) + (2 \parallel j3) \Omega$$

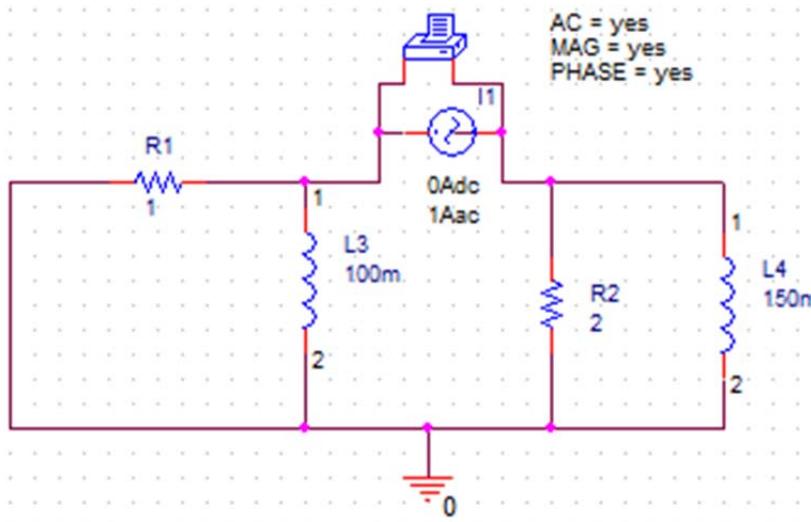
$$= \frac{j2}{1+j2} + \frac{j6}{2+j3}$$

$$= \frac{-18+j10}{-4+j7}$$

$$= \frac{20.6\angle 150.94^\circ}{8.06\angle 119.74^\circ}$$

$$= 2.55\angle 31.2^\circ \Omega$$

Pspice Verification:



FREQ VM(N00233,N00263)VP(N00233,N00263)

3.183E+00 2.554E+00 3.120E+01

Thevenin voltage:

In order to find the thevenin voltage, after removing the capacitor and on applying node voltage method, we can write the nodal equations as,

At node A,

$$\frac{5\angle -78^\circ - \mathbf{V}_{A'}}{1} = \frac{\mathbf{V}_{A'}}{j2} \quad [1]$$

At node B,

$$\frac{4\angle -45^\circ - \mathbf{V}_{B'}}{2} = \frac{\mathbf{V}_{B'}}{j3} \quad [2]$$

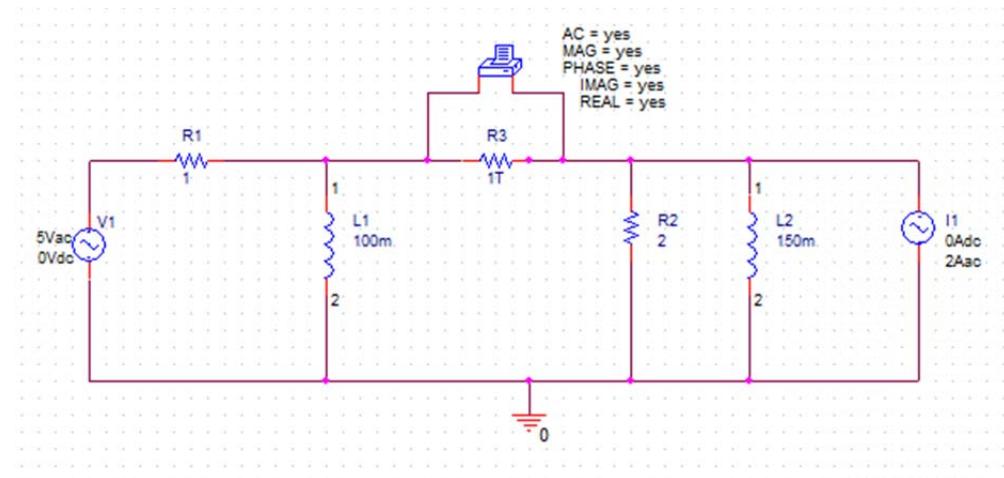
Solving the nodal equations [1] and [2], we get,

$$\mathbf{V}_{A'} = 4.472\angle -51.43^\circ \text{ V}$$

$$\mathbf{V}_{B'} = 3.328\angle -11.31^\circ \text{ V}$$

$$\begin{aligned} \mathbf{V}_{TH} &= \mathbf{V}_{A'} - \mathbf{V}_{B'} \\ &= 4.472\angle -51.43^\circ - 3.328\angle -11.31^\circ \\ &= -0.4752 - j2.8437 \\ &= \boxed{2.88\angle -99.49^\circ \text{ V}} \end{aligned}$$

Pspice Verification:



FREQ VM(N00225,N00247)VP(N00225,N00247)VR(N00225,N00247)VI(N00225,N00247)

3.183E+00 2.884E+00 -9.949E+01 -4.756E-01 -2.844E+00

Calculate $v_c(t)$:

$$\begin{aligned}\mathbf{Z}_c &= -j3.33 = 3.33\angle -90^\circ \Omega \\ \mathbf{Z}_{total} &= 2.18 - j2.01 = 2.96\angle -42.67^\circ \Omega \\ \mathbf{V}_c &= \frac{\mathbf{V}_{TH} \times \mathbf{Z}_c}{\mathbf{Z}_{total}} \\ &= \frac{(2.88\angle -99.49^\circ)(3.33\angle -90^\circ)}{2.96\angle -42.67^\circ} \\ &= 3.24\angle -146.82^\circ V\end{aligned}$$

$v_c(t) = 3.24 \cos(20t - 146.82^\circ) V$

Pspice Verification:

FREQ VM(N00225,N00247)VP(N00225,N00247)

3.183E+00 3.238E+00 -1.469E+02

(c) The current flowing out of the positive terminal of the voltage source is given by

$$\frac{5\angle -78^\circ - \mathbf{V}_A}{1} A. \text{ If we apply nodal voltage analysis, we get,}$$

At node A,

$$\frac{5\angle -78^\circ - \mathbf{V}_A}{1} = \frac{\mathbf{V}_A}{j2} + \frac{\mathbf{V}_C}{-j3.33}$$

From (b), we have, $\mathbf{V}_C = 3.24\angle -146.82^\circ V$

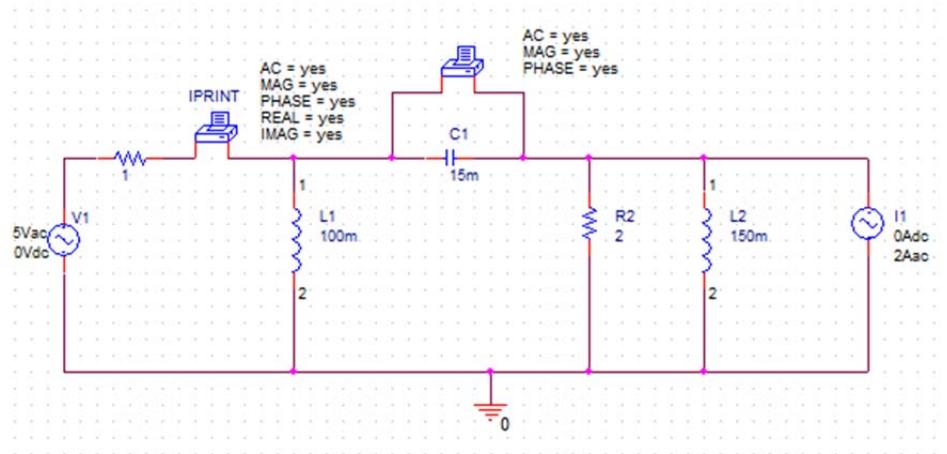
On solving, we get,

$$\mathbf{V}_A = 2.0348 - j3.057 = 3.67\angle -56.35^\circ V$$

$$\mathbf{I} = -0.995 - j1.833 = 2.08\angle -118.49^\circ A$$

$i(t) = 2.08 \cos(20t - 118.49^\circ) A$

Pspice Verification:



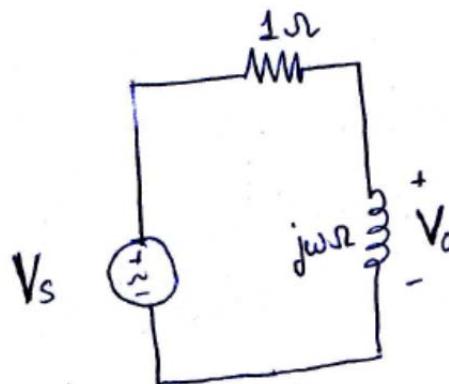
FREQ IM(V_PRINT2)IP(V_PRINT2)IR(V_PRINT2)II(V_PRINT2)

3.183E+00 2.087E+00 -1.186E+02 -9.984E-01 -1.832E+00

77. If both sources operate at 20 rad/s, $v_c(t) = 510\sin(20t - 124^\circ)$ mV. However, in the present case,

$$v_c(t) = 563\sin(20t - 77.3^\circ) + 594\sin(19t + 140^\circ) \text{ mV}.$$

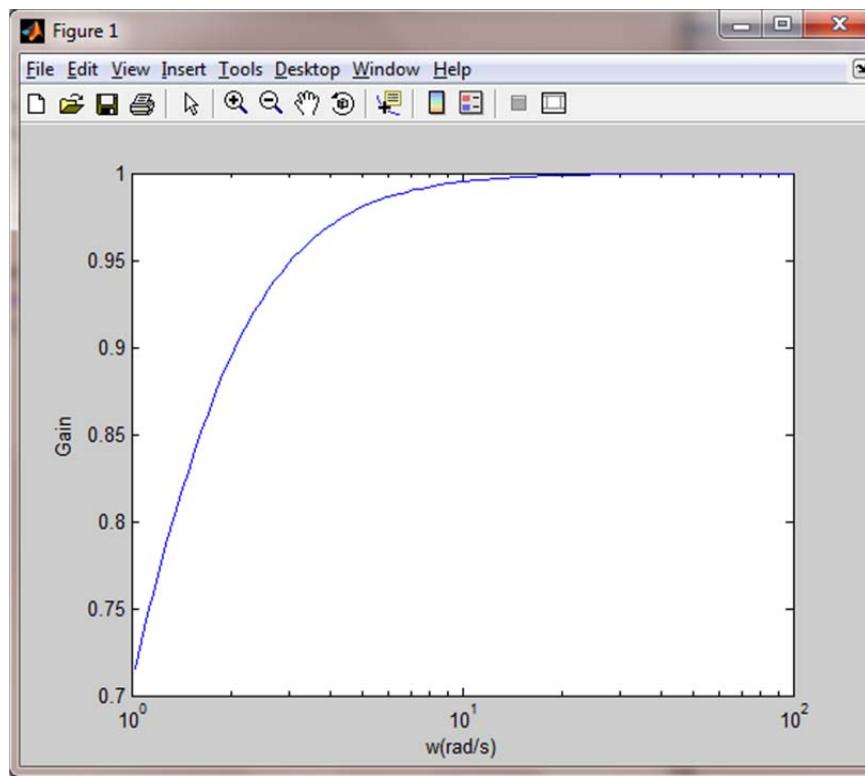
78. (a)



(b) Using the voltage divider rule, we get,

$$\frac{V_o}{V_s} = \frac{j\omega}{1+j\omega} = \boxed{\frac{\omega}{\sqrt{1+\omega^2}} \angle (90^\circ - \tan^{-1}(\omega))}$$

(c)



(d) From the plot of the gain, we see that the circuit transfers high frequencies more effectively to the output.

79. (b) $1 \angle -90^\circ$

(d)

$$\left| \frac{\mathbf{V}_o}{\mathbf{V}_s} \right| = \frac{1}{\sqrt{1 + \omega^2}}$$

The circuit transfers low frequencies to the output more effectively, as the “gain” approaches zero as the frequency approaches infinity.

80. One out of many possible design solutions:

Here, the impedance is given as,

$$\mathbf{Z} = \frac{(22 - j7)}{5\angle 8^\circ} = 4.618 \angle -25.65^\circ = 4.1629 - j2 \Omega$$

If $\mathbf{Z} = 4.1629 - j2 \Omega$ is constructed using a series combination of single resistor,

capacitor and an inductor, then, $R = 4.16 \Omega$ and $-j2 = j\omega L - \frac{j}{\omega C}$. Selecting L as 200nH

arbitrarily yields the value of the capacitor as 0.12pF.

Thus, one design will be 4.16Ω resistor in series with 200nH inductor and 0.12pF capacitor.

$$1. \quad v_1(t) = v_s(t)/5$$

$$p_1(t) = v_1(t)^2/1 = v_s(t)^2/25$$

$$(a) \quad v_s = 9 \text{ V}$$

$$v_1 = v_1(0) = v_1(1) = v_1(2)$$

$$p_1 = p_1(0) = p_1(1) = p_1(2) = v_1^2/1 = v_s^2/25 = 9^2/25 = 3.24 \text{ W}$$

$$(b) \quad v_s = 9\sin 2t \text{ V}$$

$$p_1(0) = v_s(0)^2/25 = [9\sin(2*0)]^2 = 0 \text{ W}$$

$$p_1(1) = v_s(1)^2/25 = [9\sin(2*1)]^2/25 = 2.68 \text{ W}$$

$$p_1(2) = v_s(2)^2/25 = [9\sin(2*2)]^2/25 = 1.86 \text{ W}$$

$$(c) \quad v_s = 9\sin(2t + 13^\circ) = 9\sin(2t + 0.227) \text{ V}$$

$$p_1(0) = v_s(0)^2/25 = [9\sin(0.227)]^2/25 = 0.164 \text{ W}$$

$$p_1(1) = v_s(1)^2/25 = [9\sin(2 + 0.227)]^2/25 = 2.03 \text{ W}$$

$$p_1(2) = v_s(2)^2/25 = [9\sin(4 + 0.227)]^2/25 = 2.53 \text{ W}$$

$$(d) \quad v_s = 9e^{-t} \text{ V}$$

$$p_1(0) = v_s(0)^2/25 = 9^2/25 = 3.24 \text{ W}$$

$$p_1(1) = v_s(1)^2/25 = (9e^{-1})^2/25 = 0.44 \text{ W}$$

$$p_1(2) = v_s(2)^2/25 = (9e^{-2})^2/25 = 59.3 \text{ mW}$$

2. KVL: $-v_s + 500i + v_c = 0 \Rightarrow i = (v_s - v_c)/500$

(a) $v_s(t) = 30 u(-t) V$

$$v_c(t) = 30e^{-t/RC} = 30e^{-t/0.002} V$$

$$i = -\frac{30}{500} e^{-t/0.002}, t > 0$$

$$p_s(1.5 \text{ ms}) = \boxed{0 \text{ W}}$$

$$p_R(1.5 \text{ ms}) = i^2(1.5 \text{ ms}) * 500 = \boxed{401.6 \mu W}$$

$$p_C(1.5 \text{ ms}) = v_c(1.5 \text{ ms}) * i(1.5 \text{ ms}) = \left(30e^{-0.0015/0.002}\right) * \left(-\frac{30}{500} e^{-0.0015/0.002}\right)$$

$$= \boxed{-401.6 \text{ mW}}$$

(b) $v_s(t) = 10 + 20 u(t) V$

$$v_c(t) = 10 + 20 \left(1 - e^{-t/RC}\right) = 30 - 20e^{-t/2ms}, t > 0$$

$$i = \frac{30 - (30 - 20e^{-t/2ms})}{500} = 40e^{-t/2ms} \text{ mA}, t > 0$$

$$i(1.5 \text{ ms}) = 18.9 \text{ mA}$$

$$p_s(1.5 \text{ ms}) = -(30)(0.0189)W = \boxed{-566.8 \text{ mW}}$$

$$p_R(1.5 \text{ ms}) = i^2(1.5 \text{ ms}) * 500 = \boxed{178.5 \text{ mW}}$$

$$p_C(1.5 \text{ ms}) = v_c(1.5 \text{ ms}) * i(1.5 \text{ ms}) = \boxed{388.3 \text{ mW}}$$

3. (a) $v_s = -10u(-t)$ V

$$i(0^-) = -10/1 = -10 \text{ A}$$

$$i(0^+) = -10/1 = -10 \text{ A}$$

$$i(t>0) = -10e^{-tR/L} = -10e^{-4t} \text{ A}, i(200\text{ms}) = -4.49 \text{ A}$$

$$p_s(0^-) = -(-10)(-10) = \boxed{-100 \text{ W}}, p_R(0^-) = (-10)^2(1) = \boxed{100 \text{ W}}, p_L(0^-) = v_L(0^-) i(0^-) = (0)(-10) = \boxed{0 \text{ W}}$$

$\boxed{\text{W}}$

$$p_s(0^+) = -(0)(-10) = \boxed{0 \text{ W}}, p_R(0^+) = (-10)^2(1) = \boxed{100 \text{ W}};$$

$$p_L(0^+) = v_L(0^+) i(0^+) = \left(L \frac{di(0^+)}{dt}\right) i(0^+) = \frac{1}{4}(-10)(-4)e^{-4t}(-10e^{-4t})|_{t=0} = \boxed{-100 \text{ W}}$$

$$p_s(200\text{ms}) = \boxed{0 \text{ W}};$$

$$p_R(200\text{ms}) = i^2(200\text{ms})(1) = \boxed{20.19 \text{ W}};$$

$$p_L(200\text{ms}) = v_L(200\text{ms})i(200\text{ms}) = \left(L \frac{di(200\text{ms})}{dt}\right) i(200\text{ms})$$

$$= \frac{1}{4}(-10)(-4)e^{-4t}(-10e^{-4t})|_{t=200\text{ms}} = \boxed{-20.19 \text{ W}}$$

(b) $v_s = 20 + 5u(t)$ V

$$i(t) = 20 + 5(1 - e^{-4t})u(t) \text{ A}; v_L(t) = L \frac{di}{dt} = 5e^{-4t}u(t) \text{ V}$$

$$p_s(0^-) = -(20)(20) = \boxed{-400 \text{ W}}, p_R(0^-) = (20)^2(1) = \boxed{400 \text{ W}}, p_L(0^-) = v_L(0^-) i(0^-) = (0)(20) = \boxed{0 \text{ W}}$$

$$p_s(0^+) = -(25)(20) = \boxed{-500 \text{ W}}, p_R(0^+) = i(0^+)^2(1) = [(25 - 5e^{-4t})|_{t=0}]^2 = 20^2 = \boxed{400 \text{ W}},$$

$$p_L(0^+) = v_L(0^+) i(0^+) = (5)(20) = \boxed{100 \text{ W}}$$

$$p_s(200\text{ms}) = -(25)(25 - 5e^{-4t})|_{t=\frac{1}{5}} = \boxed{-568.83 \text{ W}}$$

$$p_R(200\text{ms}) = i(200\text{ms})^2(1) = \left[(25 - 5e^{-4t})|_{t=\frac{1}{5}}\right]^2 = \boxed{517.71 \text{ W}}$$

$$p_L(200\text{ms}) = v_L(200\text{ms})i(200\text{ms}) = 5e^{-4t}(25 - 5e^{-4t})|_{t=\frac{1}{5}} = \boxed{51.12 \text{ W}}$$

4. We assume the circuit has reached sinusoidal steady state at $t = 10 \mu s$

$$1 \text{ k}\Omega \rightarrow 1 \text{ k}\Omega, 15 \text{ mH} \rightarrow j3\omega$$

$$Z_{eq} = 1k/j3\omega = 900 + j300 \Omega$$

$$V_{eq} = (900 + j300)(0.1 \angle 0^\circ) = (94.87 \angle 0.322 \text{ rad}) \text{ V}$$

$$p_R(10 \mu s) = \frac{[94.87 \cos(2 \times 10^5 (10 \times 10^6) + 0.322)]^2}{1000} = \boxed{4.2 \text{ W}}$$

$$I_L = \frac{94.87 \angle 0.322 \text{ rad}}{3000 \angle \pi/2 \text{ rad}} = (31.62 \angle -1.25 \text{ rad}) \text{ mA}$$

$$p_L(10 \mu s) = v_{eq}(10 \mu s)i_L(10 \mu s) =$$

$$(94.87 \cos(2 \times 10^5 (10 \times 10^6) + 0.322)) (0.03162 \cos(2 \times 10^5 (10 \times 10^6) + 0.322)) = \boxed{-1.5 \text{ W}}$$

$$p_S(10 \mu s) = v_{eq}(10 \mu s)i_S(10 \mu s)$$

$$= (94.87 \cos(2 \times 10^5 (10 \times 10^6) + 0.322)) 0.1 \cos(2 \times 10^5 (10 \times 10^6) + 0.322)$$

$$= \boxed{-2.7 \text{ W}}$$

5. $v_C(0^+) = (6)(4) = 24 \text{ V}$

(a) For $t > 0$, $v(t) = v_C(t) = 24e^{-\frac{t}{RC}} = 24e^{-\frac{t}{0.06}} \text{ V}$

$$p_R(t) = \frac{v^2(t)}{R} = \frac{\left(24e^{-\frac{t}{0.06}}\right)^2}{6} \Rightarrow p_R(t) = 96e^{-\frac{t}{0.03}} \text{ W}$$

$$p_C(t) = v_C(t)i_C(t) = v(t) \left[C \frac{dv(t)}{dt} \right]$$

$$= \left(24e^{-\frac{t}{0.06}}\right) \left(10 \times 10^{-3} \left(-\frac{24}{0.006}\right) e^{-\frac{t}{0.06}}\right) \Rightarrow p_C(t) = -96e^{-\frac{t}{0.03}} \text{ W}$$

(b) $p_R(60 \text{ ms}) = 96e^{-\frac{0.06}{0.03}} = 96e^{-2} \Rightarrow p_R(60 \text{ ms}) = 13 \text{ W}$

6. $v_C(0^-) = (6)(8) = 48 \text{ V}$, $i_c(0^-) = 0 \text{ A}$,
 $p_R(0^-) = v_C(0^-)^2/R = 48^2/6 = 384 \text{ W}$, $p_C(0^-) = v_C(0^-)(0) = 0 \text{ W}$,
 $p_s(0^-) = -v_C(0^-)i_s(0^-) = -384 \text{ W}$

$v_C(0^+) = 48 \text{ V}$, $i_R = 48/6 = 8 \text{ A}$, $i_C(0^+) = i_s(0^+) - i_R(0^+) = 1 - 8 = -7 \text{ A}$
 $p_R(0^+) = v_C(0^+)^2/R = 48^2/6 = 384 \text{ W}$, $p_C(0^+) = v_C(0^+) i_C(0^+) = (48)(-7) = -336 \text{ W}$,
 $p_s(0^+) = -v_C(0^+)i_s(0^+) = -(48)(1) = -48 \text{ W}$

For $t > 0$, $v_C(t) = K_1 + K_2 e^{-\frac{t}{RC}} = K_1 + K_2 e^{-\frac{t}{0.06}}$
 $t \rightarrow \infty$, $v_C(\infty) = K_1 = i_s(\infty)R = 1(6) = 6 \text{ V}$

$v_C(0) = K_1 + K_2 = 48 \Rightarrow K_2 = 42$
 $\therefore v_C(t) = 6 + 42e^{-\frac{t}{0.06}} \text{ V}$

$v_C(75 \text{ ms}) = 18 \text{ V}$
 $i_C(75 \text{ ms}) = i_s(75 \text{ ms}) - i_R(75 \text{ ms}) = 1 - \frac{18}{6} = -2 \text{ A}$
 $p_R(75 \text{ ms}) = \frac{18^2}{6} \Rightarrow p_R(75 \text{ ms}) = 54 \text{ W}$
 $p_C(75 \text{ ms}) = (18)(-2) \Rightarrow p_C(75 \text{ ms}) = -36 \text{ W}$
 $p_s(75 \text{ ms}) \Rightarrow p_s(75 \text{ ms}) = -18 \text{ W}$

7. We assume the circuit is in sinusoidal steady state

$$1 \Omega \rightarrow 1 \Omega, 4 \Omega \rightarrow 4 \Omega, 4 \mu F \rightarrow -j25k\Omega$$

$$i_{1\Omega} \approx i_s = 2.5 \cos(10t) A, i_{4\Omega} \approx 0$$

$$p_{1\Omega}(t) = i_s^2(t)(1) = \frac{6.25}{2}(1 + \cos(20t)) W$$

$$p_{1\Omega}(0) = 6.25 W, p_{1\Omega}(10 ms) = 6.19 W, p_{1\Omega}(20 ms) = 6 W$$

$p_{4\Omega}(t) \approx 0$ for all time

$p_{4\mu F}(t) \approx 0$ for all time

$$p_s(0) = -6.25 W, p_s(10 ms) = -6.19 W, p_s(20 ms) = -6 W$$

8.

$$i_L(0^-) = 0 \text{ A}$$

$$v_L(0^-) = v_C(0^-) = 0 \text{ V}$$

$$i_L(0^+) = 0 \text{ A}$$

$$v_L(0^+) = v_C(0^+) = 0 \text{ V}$$

$$R = 10 // 11 = \frac{10}{11} \Omega \quad , \alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 10 \cdot 11} = 0.275 \quad , \omega_0 = \frac{1}{\sqrt{LC}} = 1 \text{ rad/s}$$

$$\alpha < \omega_0 \Rightarrow \text{Underdamped case: } \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 0.9614 \frac{\text{rad}}{\text{s}}$$

Forced response general solution:

$$i_L(t) = i_f + i_{nat} = 10 + e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t) \text{ A}$$

$$i_L(0^+) = 10 + B_1 = 0 \Rightarrow B_1 = -10 \text{ A}$$

$$\frac{di_L(t)}{dt} = e^{-\alpha t} (-\omega_d B_1 \sin \omega_d t + \omega_d B_2 \cos \omega_d t) - \alpha e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{v_C(0^+)}{L} = 0 \text{ A} \Rightarrow$$

$$\omega_d B_2 - \alpha B_1 = 0 \Rightarrow B_2 = \frac{\alpha B_1}{\omega_d} = \frac{-10}{0.9614} = -2.86 \text{ A}$$

$$\therefore i_L(t) = 10 + e^{-0.275t} (-10 \cos 0.9614t - 2.86 \sin 0.9614t) \text{ A}$$

$$p_L(t) = v_L(t)i_L(t) = L i_L(t) \frac{di_L(t)}{dt}$$

$$p_L(0) = 0 \text{ W}$$

$$p_L(1) = 12.54 \text{ W}$$

9. For $t > 0$, $v_C(t) = V(0^+)e^{-\frac{t}{RC}} = V(0^+)e^{-\frac{t}{0.12}} V$

$$w_C(0^+) = \frac{1}{2}Cv_C^2(0^+) = 100 \text{ mJ} \Rightarrow v(0^+) = \sqrt{2} \text{ V}$$

$$\therefore v_C(t) = \sqrt{2}e^{-\frac{t}{0.12}} \text{ V} \Rightarrow v(120 \text{ ms}) = 0.52 \text{ V} \Rightarrow p_R(120 \text{ ms}) = \frac{(0.52)^2}{1.2} = \boxed{226 \text{ mW}}$$

Energy dissipated: $w_C(t) = \frac{1}{2}Cv_C^2(t) = 0.1e^{-\frac{t}{0.06}} W$

Energy remaining at $t = 1 \text{ s}$ is: $w_C(1 \text{ s}) = 0.1 - 0.1e^{-\frac{1}{0.06}} \approx 0 \text{ W}$
 or 100 mJ of energy dissipated in the first second.

$$T_f = T_i + \Delta T = 273.15 + 23 + \frac{100 \times 10^{-6} kJ}{(10^{-3} kg) \left(0.9 \frac{kJ}{kg \cdot K} \right)} = 273.15 + 23 + 0.111 = 296.261 K$$

or a temperature increase of 0.111 °C.

$$10. \text{ (a) } P = (30 \times 10^3)^2 (1.2 \times 10^{-3}) = 1.08 \text{ MW}$$

$$\text{(b) } W = (1.08 \times 10^6)(150 \times 10^{-6}) = 162 \text{ J}$$

$$11. \quad 18 \text{ k}\Omega \rightarrow 18 \text{ k}\Omega, 1 \mu\text{F} \rightarrow -j22.22 \text{ k}\Omega$$

$$Z_{eq} = 18 - j 22.2 \text{ k}\Omega = 28.6 \angle -51^\circ \text{ k}\Omega$$

$$V_{eq} = Z_{eq} I_s = (28.6 \times 10^3 \angle -51^\circ)(9 \times 10^{-3} \angle 15^\circ) = 257.4 \angle -36^\circ \text{ V}$$

$$(a) \quad p_l(t) = (257.4 \cos(45t - 36^\circ))(9 \times 10^{-3} \cos(45t + 15^\circ)) \\ = 0.73 + 1.16 \cos(90t - 21^\circ) \text{ W}$$

$$(b) \quad P_l = 0.73 \text{ W}$$

12. $1 \Omega \rightarrow 1 \Omega, 1 \text{ mH} \rightarrow j0.155 \Omega$

$$(a) P_R = \frac{V_m^2}{2R} = \frac{100^2}{2} = \boxed{5 \text{ kW}}$$

$$I_L = \frac{V}{Z_L} = \frac{100 \angle 45^\circ}{0.155 \angle 90^\circ} = 645.16 \angle -45^\circ \text{ A}$$

$$P_L = \frac{1}{2} V_m I_{Lm} \cos(\theta - \varphi) = \frac{1}{2} (100)(645.16) \cos(45 + 45) = \boxed{0 \text{ W}}$$

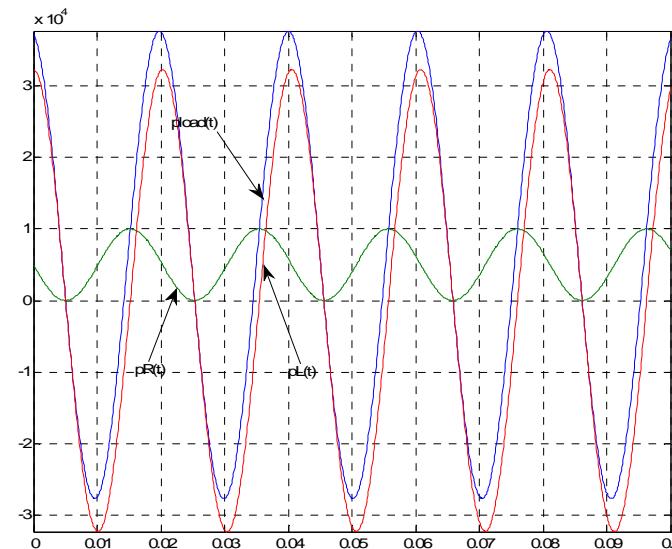
$$Z_{load} = 1/j0.155 = 0.1532 \angle 81.2^\circ \Omega$$

$$I_{load} = V/Z_{load} = 652.7 \angle -36.2^\circ \text{ A}$$

$$(b) p_{load}(t) = v(t)i_{load}(t) = (100 \cos(155t + 45^\circ))(652.7 \cos(155t - 36.2^\circ)) \\ \Rightarrow p_{load}(t) = 5 + 32.64 \cos(310t + 8.8^\circ) \text{ kW}$$

$$p_R(t) = \frac{v^2(t)}{R} = \frac{(100 \cos(155t + 45^\circ))^2}{1} = 5(1 + \cos(310t + 9090^\circ)) \text{ kW}$$

$$p_L(t) = v(t)i_L(t) = (100 \cos(155t + 45^\circ))(645.16 \cos(155t - 45^\circ)) \\ = 32.26 \cos(310t) \text{ kW}$$



$$13. I = 4 - j2 = \sqrt{20}\angle -26.6^\circ A$$

$$(a) Z = 9 \Omega, P = \frac{I_m^2}{2} R = \frac{20(9)}{2} = 90 W$$

$$(b) Z = -j1000 \Omega, V = ZI = 1000\sqrt{20}\angle -116.6^\circ V,$$

$$P = \frac{V_m I_m}{2} \cos(\theta - \varphi) = \frac{1000\sqrt{20}\sqrt{20}}{2} \cos(90^\circ) = 0 W$$

$$(c) Z = 1 - j2 + j3 = 1 + j = \sqrt{2}\angle 45^\circ \Omega, V = ZI = \sqrt{2}\sqrt{20}\angle 18.4^\circ V,$$

$$P = \frac{\sqrt{2}\sqrt{20}\sqrt{20}}{2} \cos(45^\circ) = 10 W$$

$$(d) Z = 6\angle 32^\circ \Omega, V = ZI = 6\sqrt{20}\angle -5.4^\circ V, P = \frac{6\sqrt{20}\sqrt{20}}{2} \cos(-5.4^\circ) = 59.73 W$$

$$(e) Z = \frac{1.5\angle -19^\circ}{2+j} k\Omega = 671\angle -45.6^\circ \Omega, V = ZI = 671\sqrt{20}\angle -72.2^\circ,$$

$$P = \frac{671\sqrt{20}\sqrt{20}}{2} \cos(-45.6^\circ) = 4.7 kW$$

14. Mesh equations:

$$-79\angle -40^\circ + 5I_1 + j7(I_1 - I_2) = 0 \quad 152 + j7(I_2 - I_1) - j3.1I_2 = 0$$

Solve mesh equations: $I_1 = 29.18\angle -118.5^\circ$ A, $I_2 = 25.96\angle -164.22^\circ$ A

$$P_R = \frac{I_m^2 R}{2} = \boxed{2129 \text{ W}}$$

$$P_{S1} = -\frac{V_m I_m}{2} \cos(\theta - \varphi) = -\frac{(79)(29.18)}{2} \cos(-40^\circ + 118.5^\circ) = \boxed{-230 \text{ W}}$$

$$P_{S2} = \frac{(152)(25.96)}{2} \cos(0^\circ + 164.22^\circ) = \boxed{-1899 \text{ W}}$$

$$\boxed{P_L = 0 \text{ W}}, \quad \boxed{P_C = 0 \text{ W}}$$

Power balance: $\boxed{2129 \text{ W} = (230 + 1899) \text{ W}}$

$$15. \text{ (a)} \quad Z_{in} = 1 + \frac{(3+j2.8)(-j1.5)}{3+j2.8-j1.5} = 2.4\angle -47.4^\circ \Omega$$

$$I_s = \frac{194\angle 3^\circ}{2.4\angle -47.4^\circ} = 80.5\angle 50.4^\circ A$$

$$P_S = -\frac{1}{2}(194)(80.5) \cos(3^\circ - 50.4^\circ) = -5.29 kW$$

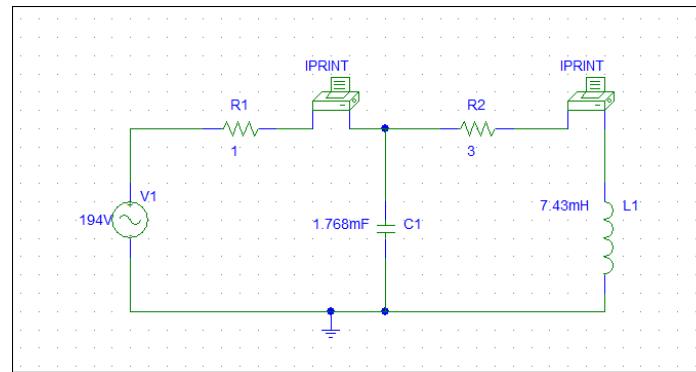
$$P_{1\Omega} = \frac{1}{2}(80.5)^2(1) = 3.24 kW$$

$$I_{3\Omega} = \frac{-j1.5}{3+j2.8-j1.5} (80.5\angle 50.4^\circ) = 37\angle -63^\circ A$$

$$P_{3\Omega} = \frac{1}{2}(37)^2(3) = 2.05 kW$$

$$P_L = 0 W, \quad P_C = 0 W$$

(b) Pspice simulation



FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000E+01	8.049E+01	5.040E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.000E+01	3.693E+01	-6.304E+01

16.

$$V_{th} = \frac{j700}{225 + j700} (15\angle 60^\circ) = 14.28\angle 78^\circ V$$

$$Z_{th} = \frac{225(j700)}{225 + j700} = 204 + j65.5 \Omega$$

(a) $Z_L = Z_{th}^* = 204 - j65.5 \Omega$

(b) $P_L = \frac{|V_{th}|^2}{8*R_{th}} = \frac{14.28^2}{8*204} = 12.5 \text{ mW}$

$$17. V_{th} = \frac{9000 - j8000}{9225 + j8000} (15\angle 60^\circ) = 14.8\angle 59.3^\circ V$$

$$Z_{th} = \frac{225(9000 - j8000)}{9225 - j8000} = 222 + j2.7 \Omega$$

(a) $Z_L = Z_{th}^* = 222 - j2.7 \Omega$

(b) $P_L = \frac{|V_{th}|^2}{8 * R_{th}} = \frac{14.8^2}{8 * 222} = 123.3 \text{ mW}$

18. KCLs:

$$\begin{aligned} \frac{V_x - 20}{2} - 2V_c + \frac{V_x - V_c}{3} &= 0 \\ \frac{V_c}{-j2} + \frac{V_c - V_x}{3} &= 0 \end{aligned} \Rightarrow \begin{aligned} 5V_x - 14V_c &= 60 \\ j2V_x + (3 - j2)V_c &= 0 \end{aligned} \Rightarrow \begin{aligned} V_x &= 9.223\angle -83.88^\circ V \\ V_c &= 5.122\angle -140.2^\circ V \end{aligned}$$

Power associated with dependent source:

$$P_{2Vc} = -\frac{1}{2}(9.223)(2 \times 5.122) \cos(-83.88^\circ + 140.2^\circ) = -26.22 W$$

So the power supplied by the dependent source is 26.22 W.

19.

(a)

$$\text{KVL: } j1.92I_x + 4.8(I_x - j2) + 8(I_x - 1.6I_x) = 0 \Rightarrow I_x = 5 \text{ A}$$

$$I_{4.8\Omega} = 5 - j2 = 5.385 \angle -21.8^\circ \text{ A}, \quad I_{8\Omega} = 0.6I_x = 3 \text{ A}$$

$$V_{4.8\Omega} = 4.8I_{4.8\Omega} = 25.85 \angle -21.8^\circ \text{ V}, \quad V_{8\Omega} = 8I_{8\Omega} = 24 \text{ V}$$

$$P_{4.8\Omega} = \frac{1}{2}(5.385)^2(4.8) = \boxed{69.6 \text{ W}}, \quad P_{8\Omega} = \frac{1}{2}(3)^2(8) = \boxed{36 \text{ W}}$$

$$(b) \quad P_{-j2A} = \frac{1}{2}(25.85)(2) \cos(-21.8^\circ - 90^\circ) = \boxed{-9.6 \text{ W}}$$

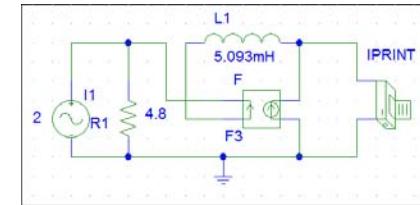
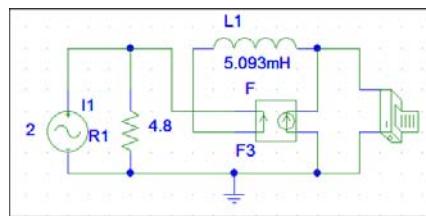
$$P_{1.6Ix} = \frac{1}{2}(24)(1.6 \times 5) \cos(180^\circ) = \boxed{-96 \text{ W}}$$

$$(c) \quad \text{Voc: } \left. \begin{aligned} & -(-j2) + \frac{V_1}{4.8} - I_x = 0 \\ & I_x = 1.6I_x \Rightarrow I_x = 0 \end{aligned} \right\} \Rightarrow V_{oc} = V_1 = -j9.6 = 9.6 \angle 90^\circ \text{ V}$$

$$\text{Isc: } \left. \begin{aligned} & I_{sc} = 1.6I_x - I_x = 0.6I_x \\ & I_x = -\frac{4.8}{4.8+j1.92}(-j2) = 1.857 \angle 68.2^\circ \text{ A} \end{aligned} \right\} \Rightarrow I_{sc} = 1.114 \angle 68.2^\circ \text{ A}$$

$$Z_{TH} = \frac{9.6 \angle 90^\circ}{1.114 \angle 68.2^\circ} = -8 - j3.2 \Omega \Rightarrow \boxed{Z_L = -8 + j3.2 \Omega}$$

(d) Pspice simulation:



FREQ	VM(\$N_0002,0)	VP(\$N_0002,0)
6.000E+01	9.600E+00	-9.000E+01

FREQ	IM(V_PRINT11)	IP(V_PRINT11)
6.000E+01	1.114E+00	6.820E+01

20.

(a) Waveform (a): $I_{avg} = \frac{(-2)(2)+(1)(2)+0(2)}{6} = -\frac{1}{3} = \boxed{-0.333 A}$

Waveform (b): $I_{avg} = \frac{\frac{1}{2}(5)(1)+0(3)}{4} = \frac{5}{8} = \boxed{0.625 A}$

(b) Waveform (a): $I_{avg}^2 = \frac{(-2)^2(2)+(1)^2(2)}{6} = \frac{5}{3} = \boxed{1.667 A^2}$

Waveform (b):

$$i(t) = 5 \times 10^3 t \text{ A}$$

$$i^2(t) = 25 \times 10^6 t^2 \text{ A}^2$$

$$I_{avg}^2 = \frac{1}{4ms} \int_0^{1ms} i^2(t) dt = \frac{1}{4ms} 25 \times 10^6 \frac{t^3}{3} \Big|_0^{1ms} = \boxed{2.083 A^2}$$

21.

(a) $P = \frac{5^2}{2.2} = \boxed{11.36 \text{ W}}$

(b) $v_s = 4 \cos 80t - 8 \sin 80t = 4 \cos 80t - 8 \cos(80t - 90^\circ)$
 $\rightarrow V_s = 4 \angle 0^\circ - 8 \angle -90^\circ = 4 + j8 = \sqrt{80} \angle 63.4^\circ \text{ V}$
 $P = \frac{1}{2} \left(\frac{80^2}{2.2} \right) = \boxed{18.18 \text{ W}}$

(c) $v_s = 10 \cos 100t + 12.5 \cos(100t + 19^\circ) \rightarrow 10 + 12.5 \angle 19^\circ = 23.18 \angle 10.11^\circ \text{ V}$
 $P = \frac{1}{2} \left(\frac{23.18^2}{2.2} \right) = \boxed{122.12 \text{ W}}$

22.

(a) $7\sin 30t \text{ V} \rightarrow \frac{7}{\sqrt{2}} = \boxed{4.95 \text{ V rms}}$

(b) $100\cos 80t \text{ mA} \rightarrow \frac{100}{\sqrt{2}} = \boxed{70.71 \text{ mA rms}}$

(c) $120\sqrt{2} \cos(5000t - 45^\circ) \text{ V} \rightarrow \frac{120\sqrt{2}}{\sqrt{2}} = \boxed{120 \text{ V rms}}$

(d) $\frac{100}{\sqrt{2}} \sin(2t + 72^\circ) \text{ A} \rightarrow \frac{100}{\sqrt{2}\sqrt{2}} = \boxed{50 \text{ A rms}}$

23.

(a) $62.5 \cos 100t \text{ mV} \rightarrow \frac{62.5}{\sqrt{2}} = \boxed{44.19 \text{ mV rms}}$

(b) $1.95 \cos 2t \text{ A} \rightarrow \frac{1.95}{\sqrt{2}} = \boxed{1.38 \text{ A rms}}$

(c) $208\sqrt{2} \cos(100\pi t - 29^\circ) \text{ V} \rightarrow \frac{208\sqrt{2}}{\sqrt{2}} = \boxed{208 \text{ V rms}}$

(d) $\frac{400}{\sqrt{2}} \sin(2000t - 14^\circ) \text{ A} \rightarrow \frac{400}{\sqrt{2}\sqrt{2}} = \boxed{200 \text{ A rms}}$

24.

(a) $i(t) = 3 \sin 4t A \rightarrow \frac{3}{\sqrt{2}} = \boxed{2.12 A rms}$

(b) $v(t) = 4 \sin 20t \cos 10t = \frac{1}{2} (\sin 30t + \sin 10t) V \rightarrow \sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2} = \boxed{\frac{1}{2} V rms}$

(c) $i(t) = 2 - \sin 10t mA \rightarrow \sqrt{2^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \boxed{2.12 mA rms}$

(d) $I_{eff} = \sqrt{\frac{1}{3} \int_1^3 (2.82 \times 10^{-3})^2 dt} = \sqrt{\frac{1}{3} (2.82 \times 10^{-3})^2 (3 - 1)} = \boxed{2.3 mA rms}$

25.

(a) $f_0 = 0.167 \text{ Hz}, T = 6 \text{ s}$

$$I_{eff} = \sqrt{\frac{1}{6} \left(\int_0^2 (-2)^2 dt + \int_2^4 (1)^2 dt \right)} = \sqrt{\frac{10}{6}} = 1.291 \text{ A rms}$$

(b) $f_0 = 250 \text{ Hz}, T = 4 \text{ ms}$

$$I_{eff} = \sqrt{\frac{1}{4 \text{ ms}} \int_0^{1 \text{ ms}} \left(\frac{5}{10^{-3}} t \right)^2 dt} = 144.3 \text{ A rms}$$

26.

(a) $I_{avg} = \frac{(-9)(1)}{3} = -3 \text{ mA}$

$$I_{eff} = \sqrt{\frac{1}{3} \int_2^3 (-9)^2 dt} = 5.196 \text{ mA rms}$$

(b) $F_{avg} = \frac{\frac{1}{2}(-1)(0.05ms) + \frac{1}{2}(1)(0.05ms)}{3} = 0$

$$F_{rms} = \sqrt{\frac{1}{0.3ms} (2) \int_0^{0.05ms} \left(-\frac{1}{0.05} t\right)^2 dt} = \frac{1}{3000} = 0.333 \times 10^{-3} \text{ rms}$$

27. $1 \text{ k}\Omega \rightarrow 1 \text{ k}\Omega, 2\text{H} \rightarrow j1\text{k}\Omega \rightarrow Z = 1 + j1 \text{ k}\Omega$

Let $i(t) = I_o \cos(500t + \varphi) A \rightarrow I = I_o \angle 0^\circ A$

$$V = ZI = 1k\sqrt{2}I_o \angle 45^\circ V \rightarrow v(t) = 1k\sqrt{2}I_o \cos(500t + \varphi + 45^\circ) V$$

$$p(t) = v(t)i(t) = \frac{1k\sqrt{2}I_o^2}{2} \left[\frac{1}{\sqrt{2}} + \cos(1000t + \varphi + 45^\circ) \right] W$$

$$\text{Require: } \max[p(t)] = \frac{1k\sqrt{2}I_o^2}{2} \left(\frac{1}{\sqrt{2}} + 1 \right) = \frac{1}{4} \Rightarrow I_o = \sqrt{\frac{2}{1k\sqrt{2} \left(\frac{1}{\sqrt{2}} + 1 \right)^2}} \Rightarrow I_o = 14.39 \text{ mA}$$

$$\text{Largest rms current tolerated is: } \frac{14.39 \text{ mA}}{\sqrt{2}} = \boxed{10.175 \text{ mA rms}}$$

28. (a) $v = 5 V$

$$\boxed{T = \infty s}$$

$$\boxed{f = 0 \text{ Hz}}$$

$$\boxed{5 V \text{ rms}}$$

(b) $v = 2\sin 80t - 7\cos 20t + 5 V$

$$\boxed{T = \frac{\pi}{10} s}$$

$$\boxed{f = \frac{10}{\pi} \text{ Hz}}$$

$$V_{eff} = \sqrt{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + 5^2} = \boxed{7.18 V \text{ rms}}$$

(c) $v = 5\cos 50t + 3\sin 50t = 5\cos 50t + 3 \sin(50t - \angle 90^\circ) V \rightarrow 5\angle 0^\circ + 3\angle -90^\circ = 5 - j3 = 5.831\angle -31^\circ V \rightarrow 5.831 \cos(50t - \angle -31^\circ) V$

$$\boxed{T = \frac{\pi}{25} s}$$

$$\boxed{f = \frac{25}{\pi} \text{ Hz}}$$

$$V_{eff} = \sqrt{\left(\frac{5.831}{\sqrt{2}}\right)^2} = 4.123 V \text{ rms} \text{ or just } V_{eff} = \sqrt{\left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = \boxed{4.123 V \text{ rms}}$$

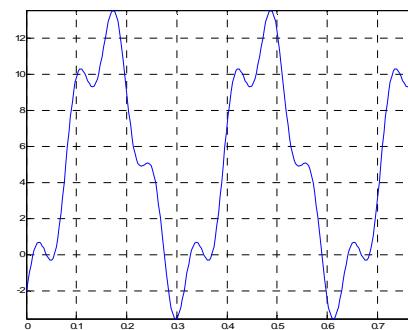
(d) $i = 8\cos^2 90t \text{ mA} = \frac{8}{2}(1 + \cos 180t) = 4 + 4\cos 180t \text{ mA}$

$$\boxed{T = \frac{\pi}{90} s}$$

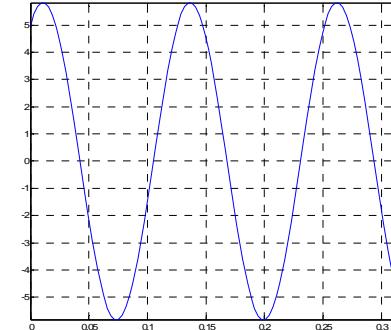
$$\boxed{f = \frac{90}{\pi} \text{ Hz}}$$

$$I_{eff} = \sqrt{4^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \boxed{4.9 \text{ mA rms}}$$

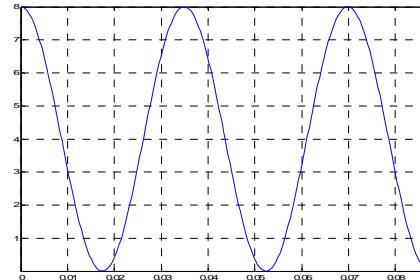
(e) Simulations:



(b)



(c)



(d)

$$29. \quad 14 \text{ mH} \rightarrow j0.56 \Omega, 28 \text{ mH} \rightarrow 1.12 \Omega$$

$$Z_{eq} = R//j1.12 = \frac{j1.12R}{R + j1.12}$$

$$V_R = \frac{Z_{eq}}{j0.56 + Z_{eq}} (120\angle 0^\circ) = \frac{j1.12R}{-0.6272 + j1.68R} (120\angle 0^\circ) V$$

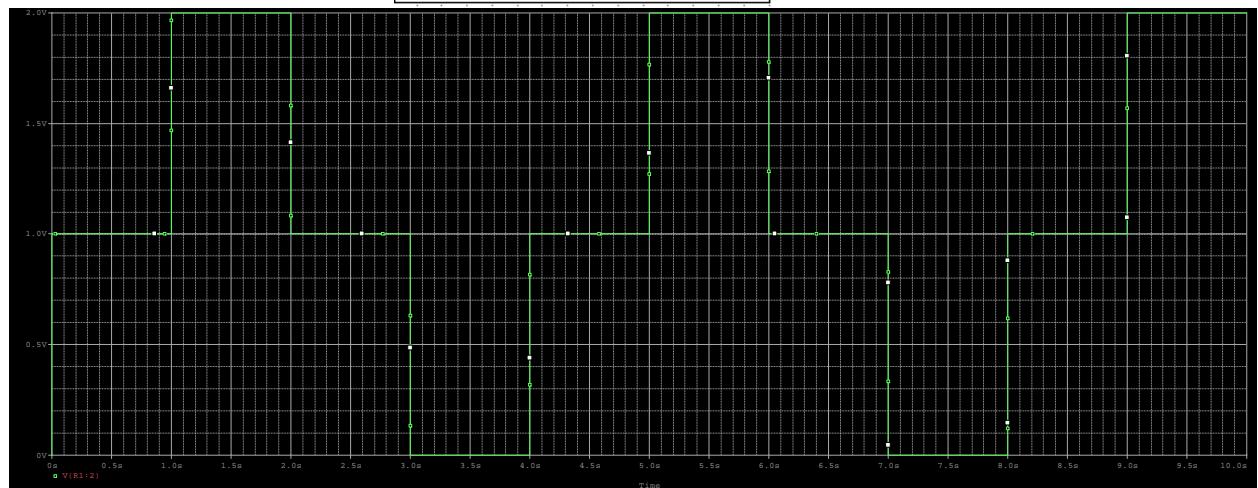
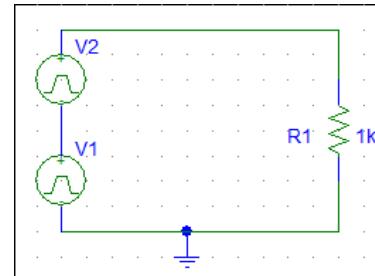
$$V_{14 \text{ mH}} = \frac{j0.56}{j0.56 + Z_{eq}} (120\angle 0^\circ) = \frac{-0.6272 + j0.56R}{-0.6272 + j1.68R} (120\angle 0^\circ) V$$

$$\begin{aligned} \text{Require: } V_R rms &= V_{14 \text{ mH}} rms \Leftrightarrow |V_R| = |V_{14 \text{ mH}}| \Leftrightarrow \frac{1.12R}{\sqrt{(0.6272)^2 + (1.68R)^2}} = \frac{\sqrt{(0.6272)^2 + (0.56R)^2}}{\sqrt{(0.6272)^2 + (1.68R)^2}} \\ &\Leftrightarrow 1.12R = \sqrt{(0.6272)^2 + (1.68R)^2} \Leftrightarrow R = \frac{0.6272}{\sqrt{0.9408}} = \boxed{0.6466 \Omega} \end{aligned}$$

$$V_R rms = V_{14 \text{ mH}} rms = \frac{120.1}{\sqrt{2}} = \boxed{84.92 \text{ V rms}}$$

30. $V_{avg} = \frac{(1)(1)+(2)(1)+(1)(1)}{4} = \boxed{1V}$

$$V_{eff} = \sqrt{\frac{1}{4}(1^2 + 2^2 + 1^2)} = \boxed{1.225 V rms}$$



31.

$$(a) \quad I = \frac{119\angle 3^\circ}{22+14\angle 32^\circ} = 3.43\angle -9.36^\circ A \text{ rms}$$

$$V_1 = Z_1 I = (14\angle 32^\circ)(3.43\angle -9.36^\circ) = 48\angle 22.64^\circ V \text{ rms}$$

$$P_1 = (48)(3.43) \cos(22.64^\circ + 9.36^\circ) = \boxed{139.7 W}$$

$$P_2 = (3.43)^2(22) = \boxed{259 W}$$

$$AP_s = (119)(3.43) = \boxed{408.17 VA}$$

$$PF = \cos(3^\circ + 9.36^\circ) = \boxed{0.997}$$

$$(b) \quad I = \frac{119\angle 3^\circ}{2+6-j} = 14.76\angle 10.125^\circ A \text{ rms}$$

$$V_2 = Z_2 I = (6-j)I = 89.8\angle 0.665^\circ V \text{ rms}$$

$$P_1 = (14.76)^2(2) = \boxed{435.5 W}$$

$$P_2 = (89.8)(14.76) \cos(0.665^\circ - 10.125^\circ) = \boxed{1307.5 W}$$

$$AP_s = (119)(14.76) = \boxed{1756.44 VA}$$

$$PF = \cos(3^\circ - 10.125^\circ) = \boxed{0.9923}$$

$$(c) \quad I = \frac{119\angle 3^\circ}{100\angle 70^\circ + 75\angle 90^\circ} = 0.69\angle -75.5^\circ A \text{ rms}$$

$$V_1 = Z_1 I = (100\angle 70^\circ)(0.69\angle -75.5^\circ) = 69.03\angle -5.5^\circ V \text{ rms}$$

$$V_2 = Z_2 I = (75\angle 90^\circ)(0.69\angle -75.5^\circ) = 51.8\angle 14.4^\circ V \text{ rms}$$

$$P_1 = (69.03)(0.69) \cos(-5.5^\circ + 75.5^\circ) = \boxed{16.3 W}$$

$$P_2 = (51.8)(0.69) \cos(14.4^\circ + 75.5^\circ) = \boxed{0 W}$$

$$AP_s = (119)(0.69) = \boxed{82.14 VA}$$

$$PF = \cos(3^\circ + 75.5^\circ) = \boxed{0.1984}$$

32.

- (a) $PF = \cos(0^\circ) = \textcolor{red}{1}$
- (b) $PF = \cos(90^\circ) = \textcolor{red}{0}$
- (c) $PF = \cos(-90^\circ) = \textcolor{red}{0}$
- (d) $PF = \cos(-56^\circ) = \textcolor{red}{0.53}$

33.

- (a) $\text{PF} = 1$: The load is purely resistive
- (b) $\text{PF} = 0.85$ lagging: The load is a resistor and inductor combination
- (c) $\text{PF} = 0.211$ leading: The load is a resistor and capacitor combination
- (d) $\text{PF} = \cos(-90^\circ)$: The load is a capacitor

34.

- (a) Voltage lags current by 126°
- (b) Voltage leads current by $\cos^{-1}(0.55) = 56.6^\circ$
- (c) Voltage lags current by $\cos^{-1}(0.685) = 46.76^\circ$
- (d) Voltage lags current by $\cos^{-1}\left(\frac{100}{500}\right) = 78.46^\circ$

35.

$$(a) P = V_{rms} I_{rms} PF = 25 W \Rightarrow I_{rms} = \frac{25}{V_{rms} PF} = \frac{25}{(120)(0.88)} = 0.237 A$$

$$Z_L = \frac{V_{rms}}{I_{rms}} \angle -\cos^{-1}(0.88) = 506.88 \angle -28.36^\circ = 446 - j241 \Omega$$

Therefore, the load may be realized with a 446Ω resistor in series with an $11 \mu F$ capacitor.

$$(b) P = V_{rms} I_{rms} PF = 150 W, AP = V_{rms} I_{rms} = 25 VA$$

$$PF = \frac{AP}{P} = \frac{25}{150} = 0.167 \text{ lagging}$$

$$I_{rms} = \frac{25}{110} = 0.227 A \text{ rms}$$

$$Z_L = \frac{110}{0.227} \angle \cos^{-1}(0.167) = 484 \angle 80.4^\circ = 80.83 + j477.2 \Omega$$

Therefore, the load may be realized with an 80.83Ω resistor in series with a 1.27 H capacitor.

36.

(a) $Z_{eq} = 1k/(50\angle - 100^\circ) = 50.38\angle - 97.156^\circ \Omega$

$$V = Z_{eq}I_s = (50.38\angle - 97.156^\circ)(0.275\angle 20^\circ) = 13.853\angle - 77.156^\circ V$$

$$I_{load} = \frac{V}{50\angle - 100^\circ} = 0.2711\angle 22.84^\circ A$$

$$p_{1k\Omega}(t) = \frac{v^2(t)}{1k\Omega} = \frac{(13.853 \cos(40t - 77.156^\circ))^2}{1k\Omega} \Rightarrow p_{1k\Omega}(20\text{ ms}) = 13\text{ mW}$$

$$p_{load}(t) = v(t)i_{load}(t) = 13.853 \cos(40t - 77.156^\circ) 0.2711 \cos(40t - 22.84^\circ)$$

$$\Rightarrow p_{load}(20\text{ ms}) = 1.192\text{ W}$$

(b) $P_{1k\Omega} = \frac{V_m^2}{2R} = \frac{(13.853)^2}{2000} = 98\text{ mW}$

$$I_{load} = \frac{1}{2}V_m I_{m,load} \cos(\theta - \varphi) = \frac{1}{2}(13.853)(0.2711) \cos(-77.156^\circ - 22.84^\circ) = -\frac{1}{3}W$$

$$= -333\text{ mW}$$

(c) $AP_{load} = \frac{1}{2}V_m I_{m,load} = 1.92\text{ VA}$

(d) $PF_{source} = \cos(-77.156^\circ - 20^\circ) = \cos(-97.156^\circ) = 0.125\text{ leading}$

37.

(a) Load is resistor, R_L . Then, $V = \frac{275\angle 20^\circ mA}{\frac{1k\Omega}{R_L}} = V_m\angle 20^\circ V$.

Therefore, $\text{PF}_{\text{source}} = \cos(0) = 1$

(b) Load: $1000+j900 \Omega$.

$$Z_{eq} = 1k//(1000 + j900) = 613.43\angle 17.76^\circ \Omega$$

$$V = Z_{eq}I_s = (613.43\angle -17.76^\circ)(0.275\angle 20^\circ) = 169\angle 37.76^\circ V$$

$$\text{PF}_{\text{source}} = \cos(37.76^\circ - 20^\circ) = \cos(17.76^\circ) = 0.9523 \text{ lagging}$$

(c) Load: $500\angle -5^\circ \Omega$

$$Z_{eq} = 1k//(500\angle -5^\circ) = 333.6\angle -3.33^\circ \Omega$$

$$V = Z_{eq}I_s = (333.6\angle -3.33^\circ)(0.275\angle 20^\circ) = 91.74\angle 16.67^\circ V$$

$$\text{PF}_{\text{source}} = \cos(16.67^\circ - 20^\circ) = \cos(-3.33^\circ) = 0.9983 \text{ leading}$$

38.

- (a) $PF_{source} = 0.95$ leading $\Rightarrow Z_{eq} = 1k//Z_{load}$ is capacitive.

$$\angle Z_{eq} = -\cos^{-1}(PF_{source}) = -\cos^{-1}(0.95) = -18.195^\circ$$

The combined load may be realized by the 1 kΩ resistor in parallel with capacitor C

$$\text{Then, } Z_{eq} = 1k//\frac{1}{j\omega C} = \frac{\frac{1k}{j\omega C}}{1k + \frac{1}{j\omega C}} = \frac{1k}{1 + j\omega kC} = \frac{1k - jk^2\omega C}{1 + (k\omega C)^2}$$

$$\text{Then, } \angle Z_{eq} = \tan^{-1}\left(\frac{-k^2\omega C}{1k}\right) = -\tan^{-1}(\omega kC) = -18.195^\circ \Rightarrow \omega kC = \tan(18.195^\circ) = 0.3287$$

$$\therefore C = \frac{328.6 \times 10^{-6}}{\omega} F$$

For $\omega = 40$ rad/s, the desired load may be realized with an 8.22 μF capacitor

- (b) $PF_{source} = 1$

\therefore Desired load must be a resistor with non-zero resistance

- (c) $PF_{source} = 0.45$ lagging $\Rightarrow Z_{eq} = 1k//Z_{load}$ is inductive.

$$\angle Z_{eq} = \cos^{-1}(PF_{source}) = \cos^{-1}(0.45) = 63.26^\circ$$

The combined load may be realized by the 1 kΩ resistor in parallel with an inductor L

$$\text{Then, } Z_{eq} = 1k//j\omega L = \frac{j\omega kL}{1k + j\omega L} = \frac{j\omega kL(1k + j\omega L)}{(1k)^2 + (\omega L)^2} = \frac{\omega^2 kL^2 + j\omega k^2 L}{(1k)^2 + (\omega L)^2}$$

$$\text{Then, } \angle Z_{eq} = \tan^{-1}\left(\frac{\omega k^2 L}{\omega^2 kL^2}\right) = \tan^{-1}\left(\frac{1k}{\omega L}\right) = 63.26^\circ \Rightarrow \frac{1k}{\omega L} = \tan(63.26^\circ) = 1.9845$$

$$\therefore L = \frac{1}{1.9845 \times 10^{-3} \omega} H$$

For $\omega = 40$ rad/s, the desired load may be realized with a 12.6 H inductor

39.

(a) Mesh equations:

$$\begin{aligned} -200\angle 0^\circ + Z_A I_1 + Z_B (I_1 - I_2) = 0 \\ Z_B (I_2 - I_1) + Z_C I_2 + Z_D I_2 = 0 \end{aligned} \} \Rightarrow \begin{bmatrix} Z_A + Z_B & -Z_B \\ -Z_B & Z_B + Z_C + Z_D \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 200\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 25.14\angle 12.2^\circ \\ 3.27\angle -42.5^\circ \end{bmatrix} A \text{ rms}$$

$$V_A = 135.4\angle -9.6^\circ V \text{ rms}, \quad V_B = 70.2\angle 18.7^\circ V \text{ rms}$$

$$V_C = 29.3\angle -16^\circ V \text{ rms}, \quad V_D = 49\angle 38.6^\circ V \text{ rms}$$

So,

$$AP_A = 3.4 \text{ kVA}, \quad AP_B = 1.64 \text{ kVA}, \quad AP_C = 95.8 \text{ VA}, \quad AP_D = 160.6 \text{ VA}$$

$$PF_{source} = \cos(\angle V_2 - \angle I_s) = \cos(0^\circ - 12.2^\circ) \Rightarrow PF_{source} = 0.9774 \text{ leading}$$

(b) Mesh analysis as in (a):

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 147\angle 118^\circ \\ 21.8\angle 77^\circ \end{bmatrix} A \text{ rms}$$

$$V_A = 294\angle -21^\circ V \text{ rms}, \quad V_B = 131\angle 124^\circ V \text{ rms}$$

$$V_C = 49\angle 103^\circ V \text{ rms}, \quad V_D = 87\angle 135^\circ V \text{ rms}$$

So,

$$AP_A = 43.3 \text{ kVA}, \quad AP_B = 17.3 \text{ kVA}, \quad AP_C = 1.07 \text{ kVA}, \quad AP_D = 1.91 \text{ kVA}$$

$$PF_{source} = \cos(\angle V_2 - \angle I_s) = \cos(0^\circ - 118^\circ) \Rightarrow PF_{source} = 0.465 \text{ leading}$$

40.

(a) $S = AP\angle \cos^{-1}(PF) = \frac{P}{PF} \angle \cos^{-1}(PF) = \frac{100}{0.75} \angle \cos^{-1}(0.75) = \boxed{133.3 \angle 41.41^\circ \text{ VA}}$

(b) $S = V_{eff}I_{eff}\angle\theta - \varphi = (120)(10.3)\angle 32^\circ - 29^\circ = \boxed{1236 \angle 3^\circ \text{ kVA}}$

(c) $S = P + jQ = 1000 + j10 = \boxed{1000.05 \angle -0.57^\circ \text{ VA}}$

(d) $S = AP\angle \cos^{-1}(PF) = \boxed{450 \angle 49.5^\circ \text{ VA}}$

41.

(a) $AP = |S| = 1.118 \text{ KVA}$

$$PF = \cos(\tan^{-1}(Q/P)) = \cos(\tan^{-1}(0.5)) = \cos(26.56^\circ) = 0.894 \text{ lagging}$$

$$Q = |S| \sin(26.56^\circ) = 500 \text{ VAR}$$

(b) $AP = 400 \text{ VA}$

$$PF = 1$$

$$Q = 0 \text{ VAR}$$

(c) $AP = 150 \text{ VA}$

$$PF = \cos(-21^\circ) = 0.93 \text{ leading}$$

$$Q = 150 \sin(-21^\circ) = -53.75 \text{ VAR}$$

(d) $AP = 75 \text{ VA}$

$$PF = \cos(25^\circ) = 0.906 \text{ lagging}$$

$$Q = 75 \sin(25^\circ) = 31.7 \text{ VAR}$$

42.

(a) $S = |3 + j1.5| \angle \tan^{-1}(1.5/3) = 3.354 \angle 26.56^\circ \text{ VA}$
 $PF = \cos(26.56^\circ) = 0.894 \text{ lagging}$

(b) $S = |1 + j4| \angle \tan^{-1}(1/4) = 4.123 \angle 14^\circ \text{ kVA}$
 $PF = \cos(14^\circ) = 0.97 \text{ lagging}$

43.

(a) $PF = \cos(24^\circ) = 0.913 \text{ lagging}$

(b) Current drawn by corrective device is:

$$I_c = \left(\frac{S_c}{V}\right)^* = \left(\frac{j(Q_{new} - Q_{old})}{V}\right)^* = -j\frac{P}{V}(\tan(\cos^{-1}(PF_{new})) - \tan(\cos^{-1}(PF_{old})))$$

$$\Rightarrow I_c = -j\frac{P}{V}(\tan(\cos^{-1}(0.98)) - \tan(\cos^{-1}(0.913))) = j118.9 = 118.9\angle 90^\circ \text{ A rms}$$

$$\therefore Z_c = \frac{V}{I_c} = \frac{230}{118.9\angle 90^\circ} = -j1.934 \Omega$$

(c) **Yes**. Selecting the impedance of the corrective device to be small enough can switch to a leading PF.

In this case, this can be achieved by setting

$$Z_c \leq -j\frac{V^2}{P(\tan(\cos^{-1}(PF_{old})))} = -j1.41$$

or, for operating frequency of 60 Hz by using a corrective capacitor value of

$$C \geq \frac{P(\tan(\cos^{-1}(PF_{old})))}{\omega V^2} = \frac{50 \times 10^3 (\tan(\cos^{-1}(0.8)))}{377(230)^2} = 1.88 \text{ mF}$$

44.

$$Z_{eq} = \left(\frac{1}{18+j10} + \frac{1}{-j5} + \frac{1}{1000} \right)^{-1} = 5.5 \angle -76^\circ \Omega$$

$$I_s = \frac{V_s}{18 + Z_{eq}} = \frac{240 \angle 45^\circ}{18 + Z_{eq}} = 12 \angle 60.5^\circ A \text{ rms}$$

$$V_1 = \frac{Z_{eq}}{18 + Z_{eq}} V_s = 66 \angle -16^\circ V \text{ rms}$$

$$S_{18\Omega,1} = I_s I_s^* 18 \angle 0^\circ = 2.58 \angle 0^\circ \text{ kVA}$$

$$I_2 = \frac{V_1}{18 + j10} = 3.2 \angle -44.7^\circ A \text{ rms}$$

$$S_{18\Omega,2} = I_2 I_2^* 18 \angle 0^\circ = 184.4 \angle 0^\circ \text{ VA}$$

$$V_{j10\Omega} = j10 I_2 = 32 \angle 45.2^\circ V \text{ rms}$$

$$S_{j10\Omega} = V_{j10\Omega} I_2^* = 102.45 \angle 90^\circ \text{ VA}$$

$$I_{-j5\Omega} = \frac{V_1}{-j5\Omega} = 13.2 \angle 74.3^\circ A \text{ rms}$$

$$S_{-j5\Omega} = V_1 I_{-j5\Omega}^* = 869 \angle -90^\circ \text{ VA}$$

$$S_{1000\Omega} = \frac{V_1 V_1^*}{1000} \angle 0^\circ = 4.34 \angle 0^\circ \text{ VA}$$

Also, $S_{source} = V_s I_s^* = (240 \angle 45^\circ)(12 \angle -60.5^\circ) = 2.874 \angle -15.46^\circ \text{ kVA}$

(Note: the sum of the complex power absorbed by the passive elements equals the complex power provided of the source.)

$$PF_{source} = \cos(-15.46^\circ) = 0.9638 \text{ leading}$$

45.

For the existing circuit,

$$Z_{in} = -j10 + (20//(j20 + 10)) = 11.435 \angle -19.65^\circ \Omega$$

$$PF_{source} = \cos(19.65^\circ) = 0.94 \text{ leading}$$

Let the required capacitor impedance be: $-jZ_C$. Then:

$$Z_{in,new} = -j10 + (20//(j20 + (10//(-jZ_C)))) , \text{ or}$$

$$Z_{in,new} = \frac{(2 \times 10^3 + 100Z_C) + j(2 \times 10^3 - 400Z_C)}{(200 + 20Z_C) + j(200 - 30Z_C)}$$

It is required that: $\angle Z_{in,new} = -\cos^{-1}(0.95) \Rightarrow$

$$\tan^{-1} \left(\frac{(2 \times 10^3 - 400Z_C)(200 + 20Z_C) - (2 \times 10^3 + 100Z_C)(200 - 30Z_C)}{(2 \times 10^3 + 100Z_C)(200 + 20Z_C) + (2 \times 10^3 - 400Z_C)(200 - 30Z_C)} \right) = \frac{-5Z_C}{14Z_C^2 - 80Z_C + 800} = \tan(-\cos^{-1}(0.95)) = A \Rightarrow (5 + 14A)Z_C^2 - 80AZ_C + 800A = 0$$

Solving the quadratic provides two values: $Z_C = 8.821 \Omega$ and $Z_C = -74.82 \Omega$

The positive value yields a capacitive correction, as required, so that:

$$Z_C = \frac{1}{\omega C} = 8.821 \Rightarrow C = \frac{1}{8.821(2\pi)50} = \boxed{360.85 \mu F}$$

46.

- (a) Benchmark reactive power value: $0.7(P) = 0.7(175) = 122.5 \text{ KVAR}$
Reactive power subject to penalty: $205 - 122.5 = 82.5 \text{ KVAR}$
Annual cost in penalties: $12(82.5)(\$0.15) = \148.5
- (b) Using a 100 KVAR capacitor compensation the total reactive power is reduced to:
 $205 - 100 = 105 \text{ kVAR}$, which is now below the penalty benchmark. For compensation at \$75/KVAR the savings are:
For the first year: $\$148.5 - \$75 = \$73.5$
For the second year (and subsequently): $\$148.5 - 0 = \148.5

47.

$$Z_{eq} = \left(\frac{1}{10} + \frac{1}{15 - j25} \right)^{-1} = 8.25 \angle -14^\circ \Omega$$

$$I_s = \frac{V_s}{j30 + Z_{eq}} = \frac{50 \angle -17^\circ}{j30 + Z_{eq}} = 1.72 \angle -91^\circ A \text{ rms}$$

$$V_{j30 \Omega} = j30I_s = 51.5 \angle -1^\circ V \text{ rms}$$

$$S_{j30 \Omega} = V_{j30 \Omega} I_s^* = 88.4 \angle 90^\circ \text{ VA}$$

$$V_1 = \frac{Z_{eq}}{j30 + Z_{eq}} V_s = 14.12 \angle -105.1^\circ V \text{ rms}$$

$$S_{10 \Omega} = \frac{V_1 V_1^*}{1000} = 20 \angle 0^\circ \text{ VA}$$

$$I_2 = \frac{V_1}{-j25 + 15} = 0.48 \angle -46^\circ A \text{ rms}$$

$$S_{15 \Omega} = I_2 I_2^* 15 = 3.54 \angle 0^\circ \text{ VA}$$

$$V_{j-25 \Omega} = -j25I_2 = 12.1 \angle -136^\circ V \text{ rms}$$

$$S_{-j25 \Omega} = V_{-j25 \Omega} I_2^* = 5.9 \angle -90^\circ \text{ VA}$$

$$\text{Also, } S_{source} = V_s I_s^* = (50 \angle -17^\circ)(1.72 \angle -91^\circ) = 85.85 \angle 74.05^\circ \text{ kVA}$$

(Note: the sum of the complex power absorbed by the passive elements equals the complex power provided of the source.)

$$PF_{source} = \cos(74.05^\circ) = 0.275 \text{ leading}$$

48.

(a) $200 \text{ mH} \rightarrow j2\Omega$

$$V_s = 50\angle -17^\circ \text{ V rms}$$

Mesh analysis:

$$\begin{aligned} -V_s + j30I_1 + j2(I_1 - I_2) &= 0 \\ j2(I_2 - I_1) - j25I_2 + 15I_2 &= 0 \end{aligned} \Rightarrow \begin{bmatrix} j32 & -j2 \\ -j2 & 15 - j23 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1.56\angle -107^\circ \\ 0.11\angle 40^\circ \end{bmatrix} \text{ A rms}$$

$$PF_{source} = \cos(-17^\circ + 107^\circ) = 0$$

(b) $AP_{source} = |V_s||I_1| = \boxed{78 \text{ VA}}$

(d) $Q_{source} = |V_s||I_1| \sin(-17^\circ + 107^\circ) = \boxed{78 \text{ VAR}}$

49.

(a) $V_s = 50\angle -17^\circ \text{ V rms}$

Mesh analysis:

$$\begin{aligned} -V_s + j30I_1 + 10(I_1 - I_2) &= 0 \\ 10(I_2 - I_1) + j30I_2 + 15I_2 &= 0 \end{aligned} \Rightarrow \begin{bmatrix} 10 + j30 & -10 \\ -10 & 25 + j30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1.5132\angle -92.3^\circ \\ 0.3875\angle -142.5^\circ \end{bmatrix} \text{ A rms}$$

$$S_{10\Omega} = (I_1 - I_2)(I_1 - I_2)^* 10 = 16.89 \text{ VA}$$

$$S_{15\Omega} = I_2 I_2^* 15 = 2.25 \text{ VA}$$

$$S_{j30,1\Omega} = j30 I_1 I_1^* = 68.7 \angle 90^\circ \text{ VA}$$

$$S_{j30,2\Omega} = j30 I_2 I_2^* = 4.5 \angle 90^\circ \text{ VA}$$

(b) $S_{source} = V_s I_1^* = 75.66 \angle 75.34^\circ \text{ VA}$

Note that the sum of the complex power absorbed by the passive elements equals the complex power supplied of the source.

(c) $PF_{source} = \cos(75.34^\circ) = 0.253 \text{ lagging}$

50. Let $S_{\text{old}} = P_{\text{old}} + jQ_{\text{old}}$ be the complex power associated with the original load conditions. Adding a capacitor in parallel results in total complex power: $S_{\text{old}} = P_{\text{old}} + jQ_{\text{old}}$.

Then, the capacitor reactive power is:

$$S_c = jQ_c = S_{\text{new}} - S_{\text{old}} = P_{\text{new}} + jQ_{\text{new}} - P_{\text{old}} - jQ_{\text{old}}$$

Since the load average power requirements remain unchanged: $P_{\text{new}} = P_{\text{old}}$, and so

$$S_c = jQ_c = jQ_{\text{new}} - jQ_{\text{old}} = jP(\tan\theta_{\text{new}} - \tan\theta_{\text{old}}) = VI_c^*$$

$$I_c = \left(\frac{S_c}{V}\right)^* = \frac{[jP(\tan\theta_{\text{new}} - \tan\theta_{\text{old}})]^*}{V} = \frac{jP(\tan\theta_{\text{old}} - \tan\theta_{\text{new}})}{V}$$

$$\text{Since, } Z_C = \frac{V}{I_c} = \frac{1}{j\omega C} \Rightarrow C = \boxed{\frac{P(\tan\theta_{\text{old}} - \tan\theta_{\text{new}})}{\omega V^2}}$$

51.

(a) $V_m = 1200\sqrt{2} = \boxed{1697 \text{ V}}$

(b) $(1 \text{ ms}) = 1200\sqrt{2} \cos(2\pi 50 \times 10^{-3}) 10\sqrt{2} \cos(2\pi 50 \times 10^{-3} + \cos^{-1}(0.9)) = \boxed{16.46 \text{ kW}}$

(c) $AP_s = 1200(10) = \boxed{12 \text{ kVA}}$

(d) $Q = 1200(20)\sin^{-1}(0.9) = \boxed{9.4 \text{ kVAR}}$

(e) $Z_L = \frac{1200}{10} \angle \cos^{-1}(0.9) = \boxed{120 \angle 25.84^\circ \Omega}$

(f) $S = 1200(20) \angle \cos^{-1}(0.9) = \boxed{12 \angle 25.84^\circ \text{ kVA}}$

52.

(a) $V_{50\Omega} = 50I_s = 250\angle 0^\circ V$

$$I_{80\Omega} = \frac{j60}{80 + j60} I_s = 3\angle 53.1^\circ A$$

$$V_{ab} = 80I_{80\Omega} = 240\angle 53.1^\circ V$$

$$V_s = V_{ab} + V_{50} = 438.3\angle 26^\circ V$$

$$PF_S = \cos(0 - 26) = 0.9 \text{ lagging}$$

(b) $I_{j60\Omega} = I_s - I_{80\Omega} = 4\angle -37^\circ A$

$$AP_{50\Omega} = 250(5) = 1250 VA$$

$$AP_{80\Omega} = 240(3) = 720 VA$$

$$AP_{j60\Omega} = 240(4) = 960 VA$$

(c) $P_s = \frac{438.29*(5)}{2} \cos(26^\circ) = 985 W$

(d) $V_{TH} = V_{ab} = 240\angle 53.1^\circ V$

$$Z_{TH} = 80//j60 = 48\angle 53.1^\circ \Omega$$

$$P_{100\Omega} = \frac{|V_{100\Omega}|^2}{2(100)} = \frac{\left| \frac{100}{100 + Z_{TH}} V_{TH} \right|^2}{200} = 159.43 W$$

53.

(a) The load is resistive (100Ω) and so $\text{PF}_{\text{load}} = 1$

(b) KCL:

$$\frac{V_s}{j60} - 5 + \frac{V_s}{80} + \frac{V_s}{100} = 0 \Rightarrow V_s = 178.6 \angle 36.5^\circ V$$

$$P_s = \frac{1}{2} V_s I_s \cos(36.5^\circ - 0^\circ) = 358.7 W$$

$$(c) I_{j60\ \Omega} = \frac{V_s}{j60} = 2.98 \angle -53.5^\circ A$$

$$p_{j60\ \Omega}(t) = v_{j60\ \Omega}(t)i_{j60\ \Omega}(t) = |V_s| \cos(2\pi 50t + \angle V_s) |I_{j60\ \Omega}| \cos(2\pi 50t + \angle I_{j60\ \Omega})$$

$$p_{j60\ \Omega}(t) = 178.6 \cos(2\pi 50t + 36.5^\circ) 2.98 \cos(2\pi 50t - 53.5^\circ)$$

$$\text{For } t = 2 \text{ ms, } p_{j60\ \Omega}(t) = 152.2 W$$

(d)

$$\text{In unaltered circuit, } Z = \frac{V_s}{I_s} = \frac{178.6 \angle 36.5^\circ}{5 \angle 0^\circ} = 35.7 \angle 36.5^\circ = A + jB = 28.7 + j21.3 \Omega$$

Including a capacitor C , with impedance $X_c = -j/\omega C$, in parallel with 100Ω load results in:

$$Z_{\text{new}} = Z // -jX_c$$

$$Z_{\text{new}} = \frac{-jX_c Z}{Z - jX_c} = \frac{-jX_c(A + jB)}{(A + jB) - jX_c} = \frac{AX_c^2 + j(-A^2 X_c + BX_c^2 - B^2 X_c)}{A^2 + (B - X_c)^2}$$

$$\text{It is required that } \angle Z_{\text{new}} = \tan^{-1} \left(\frac{-A^2 X_c + BX_c^2 - B^2 X_c}{AX_c^2} \right) = \cos^{-1}(0.95) \Rightarrow$$

$$\frac{-A^2 - B^2 + BX_c}{AX_c} = \tan(\cos^{-1}(0.95)) \Rightarrow X_c = \frac{A^2 + B^2}{B - \text{Atan}(\cos^{-1}(0.95))} = 107.86 \Omega$$

$$\text{and } C = \frac{1}{\omega X_c} \Rightarrow C = 185.4 \mu F$$

54. $20 \text{ mH} \rightarrow j0.64 \Omega$

$$(a) I_S = \frac{V_s}{5+j0.64} = 8.93 \angle -97.3^\circ A$$

$$Q_S = \frac{|V_s| |I_S|}{2} \sin(\theta - \varphi) = \frac{45(8.93)}{2} \sin(-90^\circ + 97.3^\circ) = 0.315 \text{ VAR}$$

$$(b) AP_{5\Omega} = \frac{I_S (I_S)^* 5}{2} = 199.24 \text{ VA}$$

$$AP_s = -\frac{|V_s| |I_S|}{2} = -200.86 \text{ VA} \quad (\text{i.e., supplied})$$

$$V_{20 \text{ mH}} = j0.64 I_S = 5.713 \angle -73^\circ V$$

$$AP_{20 \text{ mH}} = \frac{|V_{20 \text{ mH}}| |I_S|}{2} = 25.5 \text{ VA}$$

$$(c) S_s = V_s (I_S)^* = -200.86 \angle 7.3^\circ \text{ VA} \quad (\text{i.e., supplied})$$

$$S_{5\Omega} = AP_{5\Omega} \angle 0^\circ = 199.24 \angle 0^\circ \text{ VA}$$

$$S_{20 \text{ mH}} = V_{20 \text{ mH}} (I_S)^* = 25.5 \angle 90^\circ \text{ VA}$$

$$(d) PF_s = \cos(-90^\circ + 97.3^\circ) = 0.992 \text{ lagging}$$

55.

(a) $Z_{in} = jX_1 + (R//jX_2)$

$$Z_{in} = jX_1 + \left(\frac{1}{R} + \frac{1}{jX_2}\right)^{-1} = jX_1 + \frac{jRX_2}{R + jX_2} = \frac{jRX_2 + jX_1(R + jX_2)}{R + jX_2}$$

$$Z_{in} = \frac{RX_2^2 + j(X_1X_2^2 + R^2X_1 + R^2X_2)}{R^2 + X_2^2}.$$

$$S = V_{s,rms}I_{s,rms}^* = V_{s,rms} \left(\frac{V_{s,rms}}{Z_{in}} \right)^* = \frac{V_{s,rms}V_{s,rms}^*}{Z_{in}^*} = \frac{\left(\frac{208}{\sqrt{2}}\right)^2}{|Z_{in}|} \angle Z_{in}$$

where

$$|Z_{in}| = \frac{\sqrt{(RX_2^2)^2 + (X_1X_2^2 + R^2X_1 + R^2X_2)^2}}{R^2 + X_2^2}$$

and

$$\angle Z_{in} = \tan^{-1} \left(\frac{X_1X_2^2 + R^2X_1 + R^2X_2}{RX_2^2} \right)$$

(b) For $\omega = 40$ rad/s, 14 mH $\rightarrow jX_1 = j0.56 \Omega$ and 28 mH $\rightarrow jX_2 = j1.12 \Omega$

Adding capacitor with impedance $Z_c = -jX_c = -j/\omega C$, in parallel to 28 mH inductor:

$$Z_{in} = jX_1 + (R//jX_2//-jX_c)$$

$$Z_{in} = jX_1 + \left(\frac{1}{R} + \frac{1}{jX_2} + \frac{1}{-jX_c}\right)^{-1} = jX_1 + \frac{RX_2X_c}{X_2X_c - jRX_c + jRX_2}$$

$$Z_{in} = \frac{RX_2X_c + jX_1(X_2X_c - jRX_c + jRX_2)}{X_2X_c - jRX_c + jRX_2}$$

$$Z_{in} = \frac{((RX_2X_c) - RX_1(X_2 - X_c) + jX_1X_2X_c)(X_2X_c - jR(X_2 - X_c))}{(X_2X_c)^2 + (R(X_2 - X_c))^2}$$

Z_{in} must be real for unity PF. Setting its imaginary part to zero we obtain a quadratic equation with X_c as the unknown:

$$(X_1X_2^2 + R^2X_2 + R^2X_1)X_c^2 - R^2(X_2 + 2X_1X_2)X_c + R^2X_1X_2^2 = 0$$

Positive valued solutions for X_c satisfy the requirements and the necessary capacitance is:

$$C = \frac{1}{\omega X_c}$$

For example, for $R = 2 \Omega$, we obtain $X_c = \{0.956, 0.396\} \Omega$ or two capacitance values

$$C = \frac{1}{\omega X_c} = \{26.1, 63.1\} mF, \text{ which provide unity PF.}$$

Chapter 12

Ebrahim Forati
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1)

a)

$$V_{cb} = -V_{ec} + V_{eb} = 8.35 \text{ V}$$

b)

$$\begin{aligned}V_{be} &= 0.65 \text{ V} \\I_b &= 1 \mu\text{A} \\P &= 0.65 \mu\text{W}\end{aligned}$$

2)

a)

$$\begin{aligned}V_{gs} &= -0.2 \text{ V} \\V_{dg} &= 3.2 \text{ V}\end{aligned}$$

b)

$$P_{gs} = V_{gs} I_g = -20 \text{ pW}$$

If current is going into the gate, 20 pW is generating in the junction.

3)

a)

$$V_{cn} = 400 \angle 273^\circ$$

b)

$$V_{an} - V_{bn} = 400 \angle 33^\circ - 400 \angle 153^\circ = 692.8 \angle 3^\circ \text{ V}$$

c)

$$V_{ax} = V_{an} + V_{nc} + V_{cx} = 639.4 \angle 31.12 \text{ V}$$

d)

$$V_{bx} = V_{ba} + V_{ax} = 327.7 \angle 116.15 \text{ V}$$

4)

A power source having more than two terminals with different phases(amplitude or phase) between terminals is called a polyphase source.

Economically they are more efficient for long distance power transmission.

For balanced sources, amplitudes of voltages between any two terminals are the same and they have 120 degrees phase difference.

5)

$$V_{21} = -V_{12} = -9 \angle 30$$

$$V_{13} = V_{12} - V_{32} = 9.97 \angle 12.76$$

$$V_{34} = V_{31} + V_{14} = -9.97 \angle 12.76 + 2 \angle 10 = 7.97 \angle -166$$

$$V_{24} = V_{21} + V_{14} = -9 \angle 30 + 2 \angle 10 = 7.15 \angle -144$$

6)

$$V_{12} = V_{14} + V_{24} = 9 - j - (3 + 3j) = 6 - 4j = 7.2\angle - 33.7^\circ \text{ V}$$

$$V_{32} = V_{34} - V_{24} = 8 - (3 + 3j) = 5 - 3j = 5.8\angle - 30.9^\circ \text{ V}$$

$$V_{13} = V_{14} - V_{34} = 9 - j - (8) = 1 - j = 1.4\angle - 45^\circ \text{ V}$$

7)

a)

$$\begin{aligned} I_{ab} &= 9 \text{ A} \\ I_{cd} &= 8 \text{ A} \\ I_{de} &= 9 \text{ A} \\ I_{fe} &= -10 \text{ A} \\ I_{be} &= 1 \text{ A} \end{aligned}$$

b)

$$R = \frac{125}{9}$$

8)

a)

$$\begin{aligned} I_{gh} &= -5 \text{ A} \\ I_{cd} &= 5 \text{ A} \\ I_{dh} &= 1 \text{ A} \end{aligned}$$

$$\begin{aligned} I_{ed} &= -4 \text{ A} \\ I_{ei} &= 3 \text{ A} \\ I_{jf} &= -1 \text{ A} \end{aligned}$$

c)

Left mesh: 5 A

Middle mesh: 4 A

Right mesh: 1 A

9)

a) All of voltages

b) all except de and f

10) they need higher power. If they don't increase voltage, they will need higher current and therefore thicker wires which is not economic.

11)

a)

$$V_{an} = 110 \angle 0^\circ$$

$$V_{bn} = -110 \angle 0^\circ$$

b)

$$V_{z3} = 220 \angle 0^\circ$$

c)

$$P_{z_1} = \frac{V_{an}^2}{\text{Re}(z_1)} = \frac{110^2}{50} = 242 \text{ W}$$

$$P_{z_2} = \frac{V_{an}^2}{\text{Re}(z_2)} = \frac{110^2}{100} = 121 \text{ W}$$

$$P_{z_3} = \frac{V_{an}^2}{\text{Re}(z_3)} = \frac{220^2}{100} = 484 \text{ W}$$

$$P_{V_{an}} = -[242 + 484 / 2] = -484 \text{ W}$$

$$P_{V_{an}} = -[121 + 484 / 2] = -363 \text{ W}$$

d)

resistor and capacitor

R=100 ohm

$$\frac{1}{\omega C} = 90 \Rightarrow C = \frac{1}{2\pi \times 60 \times 90} = 29.47 \mu\text{F}$$

12)

$$I_1 = 1.8 \angle -128.4 \text{ A}$$

$$I_2 = 0.58 \angle 179.2 \text{ A}$$

$$I_1 = 1.68 \angle -0.24 \text{ A}$$

a)

$$P_1 = 3.47 \text{ Watts}$$

$$P_2 = 0.33 \text{ Watts}$$

b)

$$P_{200\Omega} = 488.55 \text{ Watts}$$

$$P_{50\Omega} = 256.8 \text{ Watts}$$

$$P_{10\Omega} = 3.37 \text{ Watts}$$

c)

$$P_{total} = 748.72 \text{ Watts}$$

$$Q = 500 \times I_2^2 = 168.68$$

$$\cos \varphi = \frac{748.72}{\sqrt{748.72^2 + 168.68^2}} = 0.9755$$

13)

a)

$$\cos(\varphi) = 0.9806$$

b)

$$C = 3.2 \text{ mF}$$

14)

a) $I=0 \text{ A}$

b)

$$z = 155 + 25j \Omega$$

or

$$z = 250 - 67j \Omega$$

15)

a)

$$400\angle 33^\circ + 400\angle -87^\circ + 400\angle -207^\circ = 335 + 217j + 21 - 399j - 356 + 181j = 0$$

b)

positive sequence. Because,

$$\angle V_{bn} = \angle V_{an} - 120^\circ$$

16)

$$I_a = \frac{240}{50 + 2\pi \times 50 \times 0.5i} = 1.45\angle -72.3^\circ \text{ A}$$

$$I_b = \frac{240\angle -120^\circ}{50 + 2\pi \times 50 \times 0.5i} = 1.45\angle -192.3^\circ \text{ A}$$

$$I_c = \frac{240\angle 120^\circ}{50 + 2\pi \times 50 \times 0.5i} = 1.45\angle 47.7^\circ \text{ A}$$

b)

$$\cos(\varphi) = 0.3$$

c)

$$P = 3|V||I|\cos\varphi = 3 \times 240 \times 1.45 \times 0.3 = 313.2 \text{ Watts}$$

17)

a)

$$\begin{aligned} V_{an} &= 208\angle 0 \text{ V} \\ V_{bn} &= 208\angle -120 \text{ V} \\ V_{cn} &= 208\angle 120 \text{ V} \\ V_{ab} &= \sqrt{3} \times 208\angle 30 \text{ V} \\ V_{bc} &= \sqrt{3} \times 208\angle -90 \text{ V} \\ V_{ca} &= \sqrt{3} \times 208\angle -210 \text{ V} \end{aligned}$$

$$\begin{aligned} I_{aA} &= 208\angle 0 \text{ mA} \\ I_{bB} &= 208\angle -120 \text{ mA} \\ I_{cC} &= 208\angle 120 \text{ mA} \end{aligned}$$

b)

$$\begin{aligned} V_{an} &= 208\angle 0 \text{ V} \\ V_{bn} &= 208\angle -120 \text{ V} \\ V_{cn} &= 208\angle 120 \text{ V} \\ V_{ab} &= \sqrt{3} \times 208\angle 30 \text{ V} \\ V_{bc} &= \sqrt{3} \times 208\angle -90 \text{ V} \\ V_{ca} &= \sqrt{3} \times 208\angle -210 \text{ V} \end{aligned}$$

$$\begin{aligned} I_{aA} &= 1.87\angle -25.8 \text{ A} \\ I_{bB} &= 1.87\angle -145.8 \text{ A} \\ I_{cC} &= 1.87\angle 94.2 \text{ A} \end{aligned}$$

C)

$$\begin{aligned}
 V_{an} &= 208\angle 0^\circ \text{ V} \\
 V_{bn} &= 208\angle -120^\circ \text{ V} \\
 V_{cn} &= 208\angle 120^\circ \text{ V} \\
 V_{ab} &= \sqrt{3} \times 208\angle 30^\circ \text{ V} \\
 V_{bc} &= \sqrt{3} \times 208\angle -90^\circ \text{ V} \\
 V_{ca} &= \sqrt{3} \times 208\angle -210^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 I_{aA} &= 1.87\angle 25.8^\circ \text{ A} \\
 I_{bB} &= 1.87\angle -94.2^\circ \text{ A} \\
 I_{cC} &= 1.87\angle 145.8^\circ \text{ A}
 \end{aligned}$$

18)

a)

$$\begin{aligned}
 V_{an} &= 208\angle 0^\circ \text{ V} \\
 V_{bn} &= 208\angle -120^\circ \text{ V} \\
 V_{cn} &= 208\angle 120^\circ \text{ V} \\
 V_{ab} &= \sqrt{3} \times 208\angle 30^\circ \text{ V} \\
 V_{bc} &= \sqrt{3} \times 208\angle -90^\circ \text{ V} \\
 V_{ca} &= \sqrt{3} \times 208\angle -210^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 I_{aA} &= 205.9\angle 0^\circ \text{ mA} \\
 I_{bB} &= 205.9\angle -120^\circ \text{ mA} \\
 I_{cC} &= 205.9\angle 120^\circ \text{ mA}
 \end{aligned}$$

b)

$$\begin{aligned}
 V_{an} &= 208\angle 0^\circ \text{ V} \\
 V_{bn} &= 208\angle -120^\circ \text{ V} \\
 V_{cn} &= 208\angle 120^\circ \text{ V} \\
 V_{ab} &= \sqrt{3} \times 208\angle 30^\circ \text{ V} \\
 V_{bc} &= \sqrt{3} \times 208\angle -90^\circ \text{ V} \\
 V_{ca} &= \sqrt{3} \times 208\angle -210^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 I_{aA} &= 1.73\angle -23.57^\circ \text{ A} \\
 I_{bB} &= 1.73\angle -143.57^\circ \text{ A} \\
 I_{cC} &= 1.73\angle 96.43^\circ \text{ A}
 \end{aligned}$$

c)

$$\begin{aligned}
 V_{an} &= 208\angle 0^\circ \text{ V} \\
 V_{bn} &= 208\angle -120^\circ \text{ V} \\
 V_{cn} &= 208\angle 120^\circ \text{ V} \\
 V_{ab} &= \sqrt{3} \times 208\angle 30^\circ \text{ V} \\
 V_{bc} &= \sqrt{3} \times 208\angle -90^\circ \text{ V} \\
 V_{ca} &= \sqrt{3} \times 208\angle -210^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 I_{aA} &= 1.73\angle 23.57^\circ \text{ A} \\
 I_{bB} &= 1.73\angle -96.43^\circ \text{ A} \\
 I_{cC} &= 1.73\angle 143.57^\circ \text{ A}
 \end{aligned}$$

19)

a)

$$\begin{aligned}
 Z_p &= 0.926 - 0.261j \\
 V_{an} &= \frac{208}{\sqrt{3}}\angle -30^\circ \text{ V} \\
 V_{bn} &= \frac{208}{\sqrt{3}}\angle -150^\circ \text{ V} \\
 V_{cn} &= \frac{208}{\sqrt{3}}\angle 90^\circ \text{ V}
 \end{aligned}$$

b)

$$\begin{aligned}V_{ab} &= 208\angle 0 \text{ V} \\V_{an} &= 208\angle -120 \text{ V} \\V_{an} &= 208\angle 120 \text{ V}\end{aligned}$$

c)

$$\begin{aligned}I_{aA} &= \frac{208}{0.926 - 0.261j} = 216.19\angle 15.7 \text{ A} \\I_{bB} &= 216.19\angle -104.3 \text{ A} \\I_{cC} &= 216.19\angle 135.7 \text{ A}\end{aligned}$$

d)

$$P = 3R|I|^2 = 3 \times 0.926 \times 216.19^2 = 129.84 \text{ kW}$$

20)

a)

$$\begin{aligned}I &= 6.8 \text{ A} \\z_p &= 8.5\angle 31.8 \text{ } \Omega\end{aligned}$$

b)

$$\begin{aligned}I &= 5.64 \text{ A} \\z_p &= 10.22\angle -23.07 \text{ } \Omega\end{aligned}$$

21)

a)

$$\cos(\varphi) = 0.9688$$

b)

$$P = 22.76 \text{ kW}$$

c)

$$\cos \varphi = 0.9803$$

$$P = 18.36 \text{ kW}$$

22)

a)

$$V_L = 184.4 \text{ V}$$

b)

$$V_p = 106.4 \text{ V}$$

23)

a)

$$P = \frac{210}{\sqrt{3}} |I| \times 0.75 \Rightarrow |I| = 1.43 \text{ A}$$

$$P = 390 \text{ W}$$

b)

$$z = \frac{\frac{210}{\sqrt{3}}}{1.43} \angle -41.4^\circ = 63.6 - 56j$$

$$z_{new} = \frac{1}{\frac{1}{63.6 - 56j} + 1} \approx 1$$

$$I \approx V = \frac{210}{\sqrt{3}}$$

$$P = \left(\frac{210}{\sqrt{3}} \right)^2 \times 3 = 44.1 \text{ kW}$$

24)

a)

$$P_c = 3VI = 3 \frac{300}{\sqrt{3}} \times \frac{300}{\sqrt{3}z_{total}}$$

$$P_c = 3VI = 3 \frac{300}{\sqrt{3}} \times \frac{300}{\sqrt{3}(3.7 - 0.95j)} = 22.76 \text{ kW}$$

b)

$$P_c = 3VI = 3 \frac{300}{\sqrt{3}} \times \frac{300}{\sqrt{3}(4.7 - 0.95j)} = 18.36 \text{ kW}$$

25)

$$z = 0.774 - 27.8j \Omega$$

$$I = \frac{115}{\sqrt{3}} / z = 0.0664 + 2.3856j \text{ A}$$

$$|I| = 2.38 \text{ A}$$

$$P = \text{Re} \left(3 \times \frac{V^2}{z} \right) = 13.22 \text{ W}$$

26)

$$\cos(\varphi) = 0.7$$

$$V_L = 208\sqrt{3}$$

a)

$$I_p = 13.22 \text{ A}$$

$$I_L = 22.9 \text{ A}$$

b)

$$\begin{aligned}
 z_p &= \frac{208\sqrt{3}}{13.22} \angle -45.6^\circ = 27.25 \angle -45.6^\circ \\
 z_{new} &= 9.44 - 9.44j \quad \Omega \\
 \cos(\varphi_{new}) &= 0.707 \\
 I_L &= 15.58 \text{ A} \\
 P &= \sqrt{3} \times 208 \times I_L \times 0.707 = 3.97 \text{ kW}
 \end{aligned}$$

27)

a)

$$\begin{aligned}
 z_p &= 0.0002 - 0.318j \quad \Omega \\
 I_p &= \frac{400\sqrt{3}}{|z_p|} = 2.17 \text{ KA}
 \end{aligned}$$

b)

$$I_L = \sqrt{3}I_p = \frac{400\sqrt{3}}{|z_p|} = 3.77 \text{ KA}$$

c)

$$V_L = 400\sqrt{3}$$

d)

$$\cos(\varphi) = 6.7 \times 10^{-4}$$

e)

$$P = \sqrt{3}V_L I_L \cos(\varphi) = 3 \text{ kW}$$

28)

$$z_p = 94.2 + 195.5j$$

$$I_L = \frac{400}{|z_p|} = 1.82 \text{ A}$$

$$V_L = 400\sqrt{3} \text{ V}$$

$$I_p = I_L = 1.84 \text{ A}$$

$$\cos(\varphi) = 0.4341$$

b)

$$V_L = V_p = 400\sqrt{3}$$

$$I_p = \frac{400\sqrt{3}}{|z_p|} = 3.19 \text{ A}$$

$$I_L = \sqrt{3}I_p = 5.53 \text{ A}$$

$$\cos(\varphi) = 0.4341$$

29)

Y connection:

$$P = 948 \text{ W}$$

Delta connection:

$$P = 2.88 \text{ kW}$$

30)

a)

$$\begin{aligned}
 3I_{p1} &= \frac{10 \text{ k}}{400} = 25 \text{ A} \\
 3I_{p2} &= \frac{25 \text{ k}}{400} = 62.5 \text{ A} \\
 |z_1| &= 48 \Omega \\
 |z_2| &= 19.2 \Omega \\
 z_1 &= 36 + 31.75j \Omega \\
 z_2 &= 15.36 - 11.52 \Omega \\
 z_1 \parallel z_2 &= 15.97 - 4.87j \Omega \\
 \cos(\varphi) &= 0.956 \text{ lead}
 \end{aligned}$$

b)

$$\sqrt{3} \times 400 \times \sqrt{3} \times \frac{400}{|z_1 \parallel z_2|} \times 0.956 = 27.5 \text{ kW}$$

c)

$$\begin{aligned}
 I_{p1} &= \frac{400}{|z_1|} = 8.33 \text{ A} \\
 I_{p2} &= \frac{400}{|z_2|} = 20.83 \text{ A}
 \end{aligned}$$

31)

$$\begin{aligned}
 V_L I_L &= 8.34 \text{ KVA} \\
 V_L &= 400 - RI_L \\
 RI_L^2 &= 100 \\
 \Rightarrow I &= 21.12 \text{ A} \\
 R &= 224 \text{ m}\Omega
 \end{aligned}$$

32)

$$V_{an} = \frac{160}{\sqrt{3}} \angle 240^\circ \text{ V}$$
$$I_{bB} = 36.08 \angle 95.54^\circ \text{ A}$$

33)

34)

35)

36)

a) 1.54 KW up scaled

b) 2.154 kW up scaled

c) 614 W up scaled

37) P=170.6 Watts

38) P=-184.54 Watts

39)

$$P_A = 850 \text{ Watts}$$

$$P_B = -225 \text{ Watts}$$

Yes, sum of them is the total power.

40)

$$P_1 = P_2 = P_3 = 861.26 \text{ Watts}$$

41)

There is no need to neutral line. If one of loads fails to open, phases remain correct without any need to neutral ground.

42)

a)

$$V_{AB} = 208\sqrt{3}\angle 30^\circ = 360\angle 30^\circ \text{ V}$$

$$V_{BC} = 360\angle -90^\circ \text{ V}$$

$$V_{CA} = 360\angle 150^\circ \text{ V}$$

$$I_A = 2.8\angle -90^\circ \text{ A}$$

$$I_B = 2.8\angle -210^\circ \text{ A}$$

$$I_C = 2.8\angle 30^\circ \text{ A}$$

b)

$$V_{AB} = 208\sqrt{3}\angle 30^\circ = 360\angle 30^\circ \text{ V}$$

$$V_{BC} = 360\angle -90^\circ \text{ V}$$

$$V_{CA} = 360\angle 150^\circ \text{ V}$$

$$I_A = 0.95\angle -90^\circ \text{ A}$$

$$I_B = 0.95\angle -210^\circ \text{ A}$$

$$I_C = 0.95\angle 30^\circ \text{ A}$$

43)

a) Yes, it is a delta connected load.

b)

$$\begin{aligned}V_{an} &= 120 \angle 0^\circ \text{ V} \\V_{bn} &= 120 \angle 180^\circ \text{ V} \\V_{ab} &= 240 \angle 0^\circ \text{ V}\end{aligned}$$

$$\begin{aligned}I_{Z_{AB}} &= 96 - 48j \text{ A} \\I_{Z_{AN}} &= 2.4 + 16.8j \text{ A} \\I_{Z_{BN}} &= -37 + 15j \text{ A} \\I_a &= 98.4 - 31.2j \text{ A} \\I_b &= -133 + 63j \text{ A} \\I_n &= 34.6 - 31.8j \text{ A}\end{aligned}$$

c) if $I_{AN} \neq I_{NB}$.

44)

We can connect computer equipments as a Y load to the available three phase power.

$$V_p = \frac{208}{\sqrt{3}} = 120 \text{ V}$$

Chapter 13

1)

a) $V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = -0.04 \sin(8t) \text{ V}$

b) $V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = 0.3 \cos(100t) \text{ V}$

c) $V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = 0.1 \cos(8t) - 0.04 \sin(8t - 40^\circ) \text{ V}$

2)

$$\begin{cases} V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ V_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \end{cases}$$

a)

$$V_1 = 0.8 \sin(40t) \text{ V}$$

b)

$$V_2 = 2 \sin(40t + 15^\circ) \text{ V}$$

c)

$$\begin{cases} V_1 = 24 \sin(40t) \text{ V} \\ V_2 = 60 \sin(40t + 15^\circ) \text{ V} \end{cases}$$

3)

a) $V_1 = 10.7 \sin(70t) \text{ nV}$

b)

$$V_2 = 41.25 \sin(5t - 30^\circ) \mu\text{V}$$

c)

$$V_2 = 25.5 \cos(5t) \mu\text{V}$$

4)

$$\begin{cases} V_1 = -L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ V_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{cases}$$

a)

$$V_2 = -4.25\sqrt{2}e^{-t} \mu\text{V}$$

b)

$$V_2 = 40 \sin(10t) \mu\text{V}$$

c)

$$V_2 = 489.63 \sin(70t) \mu\text{V}$$

5)

a) 1 and 4, 2 and 3

b) 3 and 1, 4 and 2

c) 1 and 3, 2 and 4

6)

$$M = 0.5 \text{ H}$$

7)

$$i_1 = -\frac{800}{1.5} \sin(5t) \text{ A}$$

8)

a)

$$\begin{aligned} V_1 &= -200 \cos(40t) - 100 \sin(40t) \text{ } \mu\text{V} \\ V_2 &= -600 \cos(40t) - 100 \cos(40t) \text{ } \mu\text{V} \end{aligned}$$

b)

$$\begin{aligned} V_1 &= -200 \cos(40t) - 100 \sin(40t) \text{ } \mu\text{V} \\ V_2 &= 600 \cos(40t) + 100 \cos(40t) \text{ } \mu\text{V} \end{aligned}$$

9)

a)

$$\begin{aligned} V_1 &= -6 \sin(2000t + 13^\circ) + 0.4 \cos(400t) \text{ } \mu\text{V} \\ V_2 &= 6 \cos(400t) - 1.2 \sin(2000t + 13^\circ) \text{ } \mu\text{V} \end{aligned}$$

b)

$$\begin{aligned} V_1 &= -6 \sin(2000t + 13^\circ) - 0.4 \cos(400t) \text{ } \mu\text{V} \\ V_2 &= 6 \cos(400t) + 1.2 \sin(2000t + 13^\circ) \text{ } \mu\text{V} \end{aligned}$$

10)

$$I_1 = 7.24 - 2.69i \text{ mA}$$

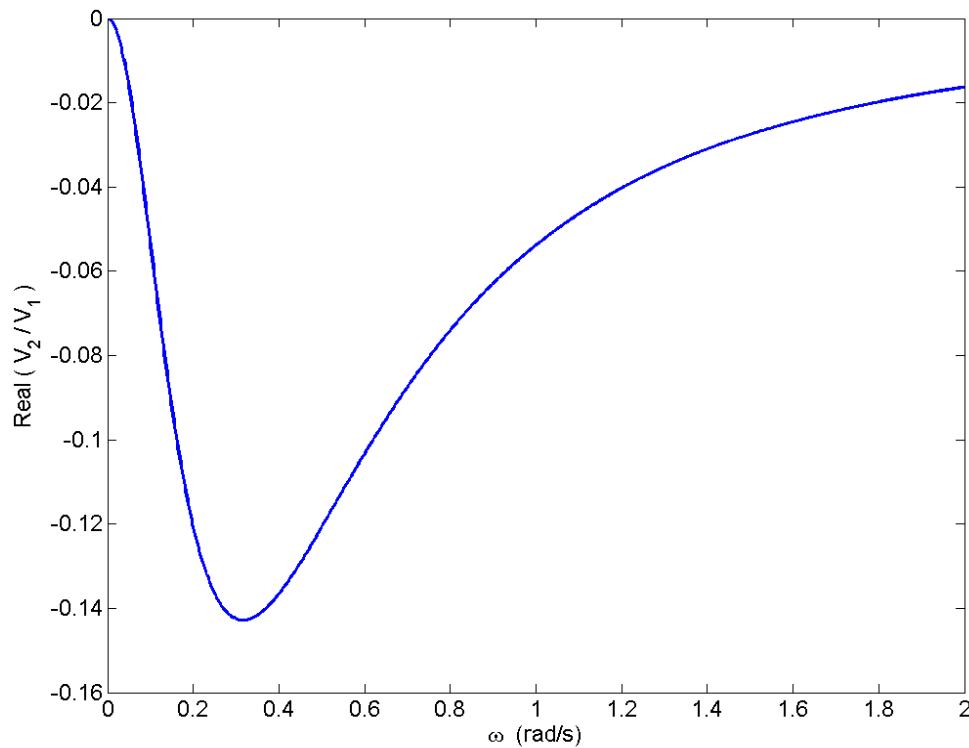
$$I_2 = -2.45 - 0.74i \text{ mA}$$

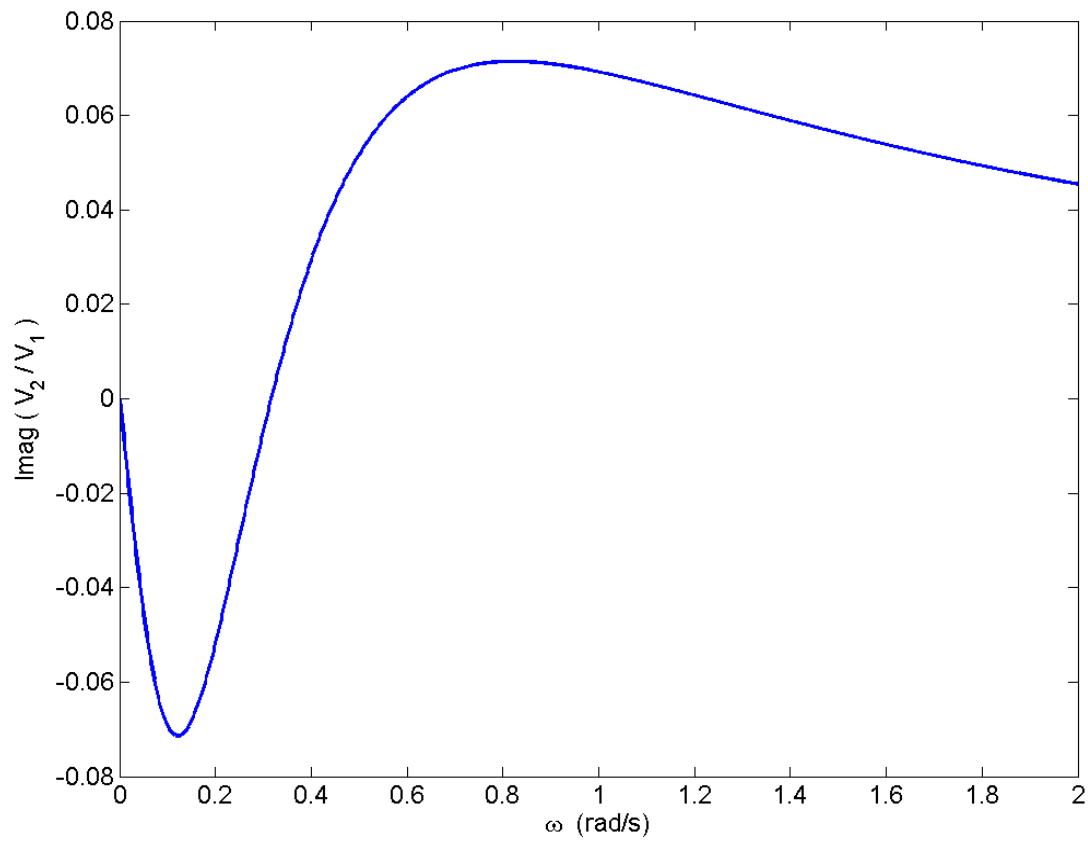
$$\frac{V_2}{V_1} = \frac{0.870(-2.45 - 0.74i)}{40} = (-53.3 - 16.2i) \times 10^{-3}$$

$$\frac{I_2}{I_1} = -0.26 + 0.2i$$

11)

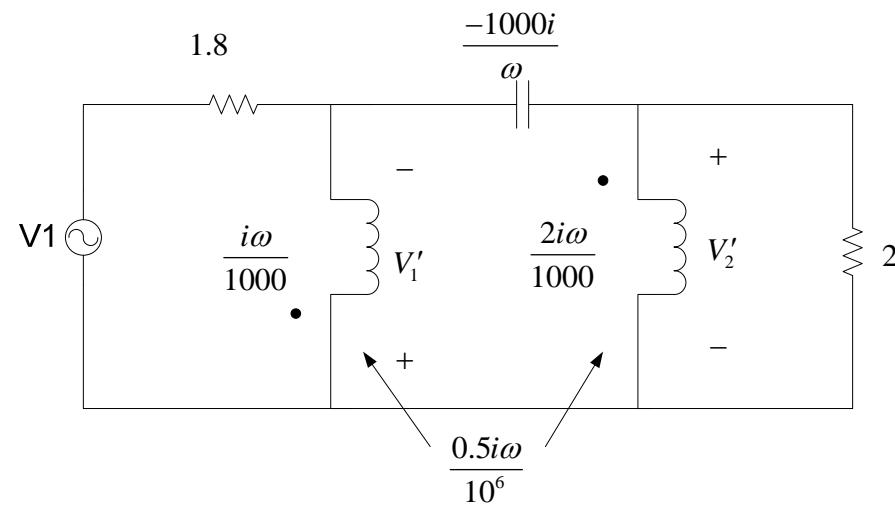
$$\frac{V_2}{V_1} = \frac{-2i\omega}{4\omega^2 + (2 + 6i\omega)(1 + 4i\omega)}$$





12)

a)



b)

$$\begin{cases} -V_1 + 1.8I_1 - V'_1 = 0 \\ -V'_2 + 2I_3 = 0 \\ V'_1 + V'_2 - \frac{1000i}{\omega} I_2 = 0 \\ V'_2 = \frac{2i\omega}{1000} (I_2 - I_3) + \frac{0.5i\omega}{10^6} (I_2 - I_1) \\ V'_1 = \frac{i\omega}{1000} (I_2 - I_1) + \frac{0.5i\omega}{10^6} (I_2 - I_3) \end{cases}$$

c)

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V'_1 \\ V'_2 \end{bmatrix} = \begin{bmatrix} 3.15 - 0.58i \\ 1.69 + 2.63i \\ -0.67 + 1.70i \\ -2.31 - 1.05i \\ -1.34 + 3.40i \end{bmatrix}$$

13)

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V'_1 \\ V'_2 \end{bmatrix} = \begin{bmatrix} 3.60 - 2.08i \\ 3.18 + 1.55i \\ 0.48 + 2.15i \\ -2.90 - 0.33i \\ 0.97 + 4.31i \end{bmatrix}$$

14)

a)

$$P_s = -0.5 \times 2 \times 9.2 = -9.2 \text{ mW}$$

b)

$$P_{10\Omega} = 0.5 \times 10 \times |9.2 - 40.7i \text{ mA}|^2 = 8.7 \text{ mW}$$

$$P_{5\Omega} = 0.5 \times 5 \times |-5.2 + 12.7i \text{ mA}|^2 = 47 \text{ } \mu\text{W}$$

c)

$$P = 0$$

d)

$$P = 0$$

15)

$$M = 4 \text{ mH}$$

16) I think the question is wrong. Three inductors cannot be coupled with those mutual inductances. (spice simulation confirms that). One of the mutual inductances is the maximum possible while the other one is not zero. But, with the given values, the answer is:

$$V_{AG} = 20i \times I_{s1} = 40i \Rightarrow$$

$$V_{AG}(t) = 40 \cos(t + 90^\circ)$$

17)

$$V_{GC} = 14 + 32i$$

$$V_{BC} = 42 + 24i$$

$$V_{AG} = 21 + 320i$$

$$V_{AB} = V_{AG} + V_{GC} + V_{CB} = -7 + 328i$$

18)

inductors don't have dot!

19)

$$i_c = \frac{30 \times 10^{-6} t u(t)}{(t^2 + 0.01)^2} \text{ A} \quad \text{for } t > 0$$

20)

a)

$$\begin{cases} v_A = 0.5 \cos 10t \\ v_B = 2 \cos 10t \end{cases}$$

b)

$$\begin{cases} v_A = -2 \sin(20t) - 0.4 \sin(20t - 100^\circ) \\ v_B = -1.6 \sin(20t - 100^\circ) - 3 \sin(20t) \end{cases}$$

c)

$$\begin{cases} V_1 = (L_1 + M) \frac{dI_A}{dt} + M \frac{dI_B}{dt} \\ V_2 = (L_2 + M) \frac{dI_A}{dt} + L_2 \frac{dI_B}{dt} \end{cases}$$

21)

a)

$$\begin{cases} (4 - 5i\omega)I_1 - 4i\omega I_3 = 0 \\ (6 + 5i\omega)I_1 - 2i\omega I_2 - 6I_3 = 0 \\ -6I_1 - 4i\omega I_2 + (11 + 6i\omega)I_3 = 0 \end{cases}$$

b)

$$I_3 = \frac{\begin{vmatrix} 4-5i\omega & 0 & 0 \\ 6+5i\omega & -2i\omega & 100 \\ -6 & -4i\omega & 0 \end{vmatrix}}{\begin{vmatrix} 4-5i\omega & 0 & -4i\omega \\ 6+5i\omega & -2i\omega & -6 \\ -6 & -4i\omega & 11+6i\omega \end{vmatrix}} = \frac{400i\omega(4-5i\omega)}{(4-5i\omega)\{(-2i\omega)(11+6i\omega)-24i\omega\}-4i\omega\{-4i\omega(6+5i\omega)-12i\omega\}} = -3.88 + 1.97i$$

22)

a)

$$\begin{cases} V_1 = i\omega L_1 I_1 - i\omega M I_2 + R_1 I_1 \\ V_2 = i\omega L_2 I_2 - i\omega M I_1 + R_2 I_2 \end{cases}$$

b)

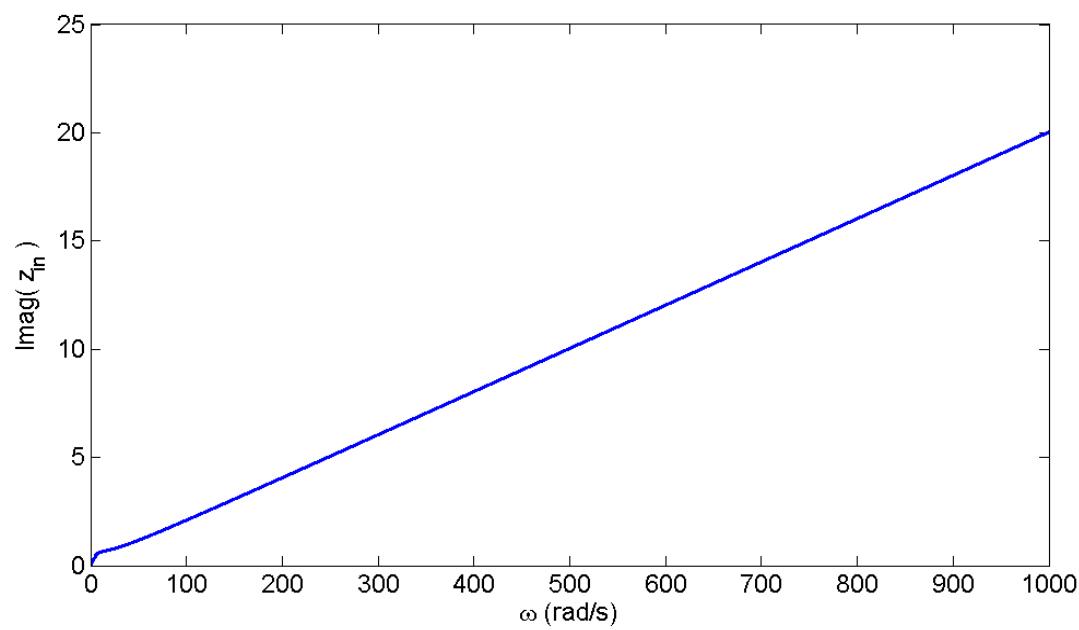
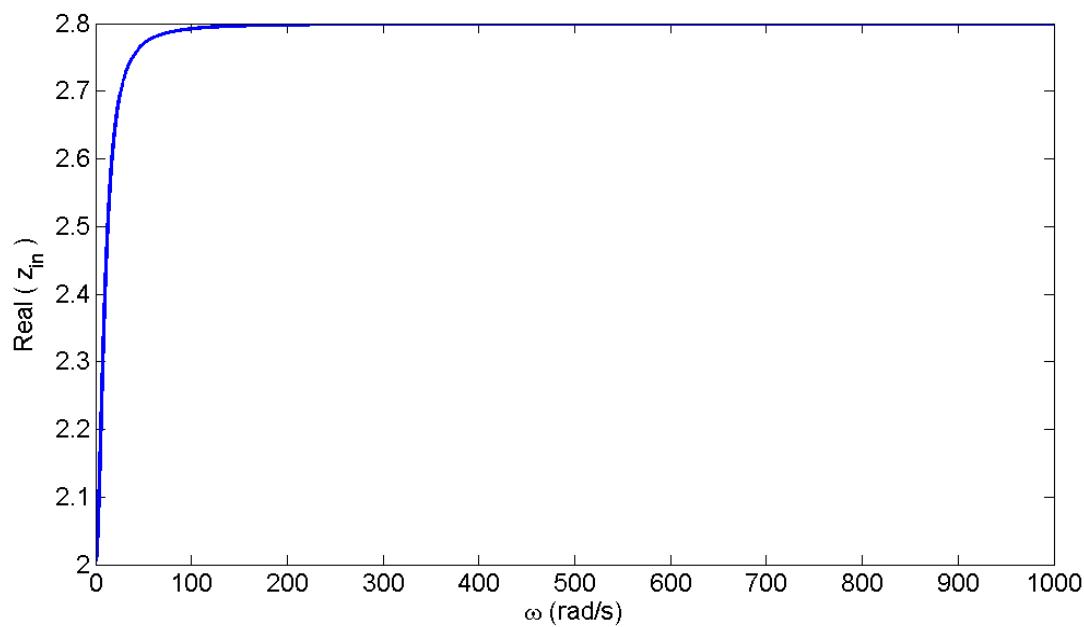
$$\begin{cases} V_1 = i\omega L_1 I_1 + i\omega M I_2 + R_1 I_1 \\ V_2 = -i\omega L_2 I_2 - i\omega M I_1 - R_2 I_2 \end{cases}$$

23)

a)

$$z_{in} = \frac{10 - 0.01\omega^2 + 1.5i\omega}{5 + 0.5i\omega}$$

b)



c)

$$z_{in}(\omega = 50) = 2.76 + 1.15i$$

24)

a) $k=1$

b) $w=349.78 \text{ [rad/s]}$

25)

a) $M=5.3 \text{ [mH]}$

b) $w= 0.32 \text{ [rad/s]}$

c) $w= 17.64 \text{ [rad/s]}$

26)

In order to solve this question, we need to have the value of the resistance, which is in series with the voltage source (v_1).

27) $Z=0.057+j0.879.$

28)

a) $w=0.0525 \text{ [rad/s]} (t=0); w=0.0153 \text{ [rad/s]} (t=5\text{ms});$

b) $w=0.044 \text{ [rad/s]} (t=0); w=0.0128 \text{ [rad/s]} (t=5\text{ms});$

29)

$$V_1 = 0.46 \angle 118^\circ \text{ V}$$

$$V_2 = 1.12 \angle 128^\circ \text{ V}$$

The average powers [Watt]:

$P_{v1} = 0.1058$

$P_{v2} = 0.3136$

30)

$$A=10+j*2e-5$$

$$B=10+j2e-5;$$

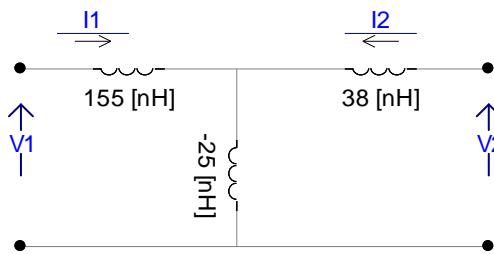
$$C=10+j1.99e-5;$$

$$D=10+j2e-5$$

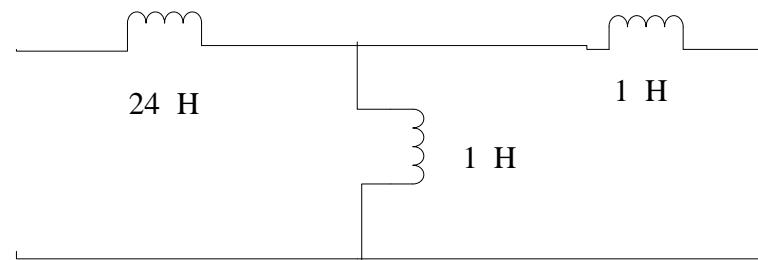
**In this question because the value of the mutual inductance is too small, the impedance seen from the left-hand-side of the circuit will be approximately equal to the R_1+jwL_1 .

This is because two winding are loosely connected, and also the value of the w (frequency) is too small.

31)



32)

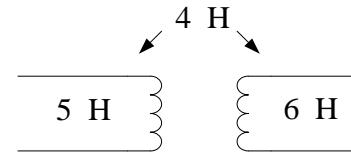


b)

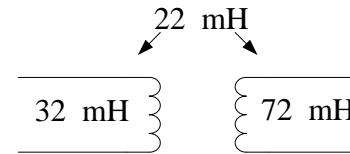
$$V_{CD} = \frac{1}{5} \sin(45t)$$

33)

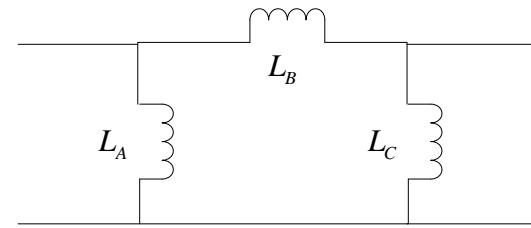
a)



b)



34)



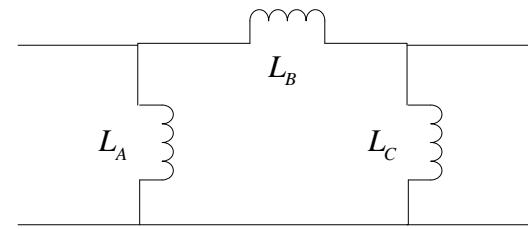
$$L_A = \frac{1065}{38} \text{ nH}$$

$$L_B = \frac{1065}{-25} \text{ nH}$$

$$L_C = \frac{1065}{155} \text{ nH}$$

35)

a)



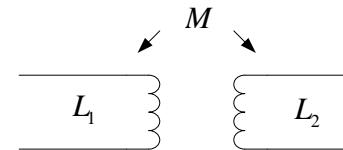
$$L_A = 49 \text{ nH}$$

$$L_B = 49 \text{ nH}$$

$$L_C = \frac{49}{24} \text{ nH}$$

36)

a)



$$M = \frac{4}{7}$$

$$L_1 = \frac{6}{7}$$

$$L_2 = \frac{12}{7}$$

b)

$$M = \frac{110}{41} \text{ mH}$$

$$L_1 = \frac{360}{41} \text{ mH}$$

$$L_2 = \frac{660}{41} \text{ mH}$$

37)

$$\frac{i_l}{V_s} = \frac{2i\omega}{28\omega^2 - 3 - 20i\omega}$$

$$\frac{V_l}{V_s} = \frac{4i\omega}{28\omega^2 - 3 - 20i\omega}$$

38)

$$V_0 = -4.1 \cos 1000t - 4.5 \sin 1000t$$

39)

$$Z_{in} = \frac{7.85\omega^2 - 2.618i\omega}{1 + 0.384i\omega}$$

40)

a)

$$\begin{cases} V_2 = 6V_1 \\ I_2 = \frac{1}{6}I_1 \end{cases} \Rightarrow V_2 = 24 \angle 32^\circ$$

$$I_2 = \frac{V_2}{Z_L} = \frac{24}{\sqrt{2}} \angle 77^\circ$$

$$V_2 = 24 \angle 32^\circ$$

b)

$$I_2 = \frac{V_2}{Z_L} = \infty$$

$$V_2 = 12 \angle 118^\circ$$

c)

$$I_2 = \frac{V_2}{Z_L} = \frac{12 \angle 118^\circ}{1.5 \angle 10^\circ} = 8 \angle 98^\circ$$

41)

a)

$$\begin{cases} V_2 = 6V_1 \\ I_2 = \frac{1}{6} I_1 \end{cases} \Rightarrow I_2 = \frac{244}{6} = \frac{122}{3} \angle 0^\circ \text{ mA}$$

$$V_2 = Z_L I_2 = (5 - 2i) \frac{122}{3} \angle 0^\circ = 203.3 - 81.3i \text{ mV}$$

b)

$$\begin{cases} V_2 = 6V_1 \\ I_2 = \frac{1}{6} I_1 \end{cases} \Rightarrow I_2 = \frac{100}{6} \angle 0^\circ \text{ mA}$$

$$V_2 = Z_L I_2 = (2i) \frac{100}{6} \angle 0^\circ = 33.3 \angle 90^\circ \text{ mV}$$

42)

$$i_1 = \frac{2\angle 0^\circ}{3 + 21.4 \times 10^{-4}} \approx \frac{2}{3} \text{ A}$$

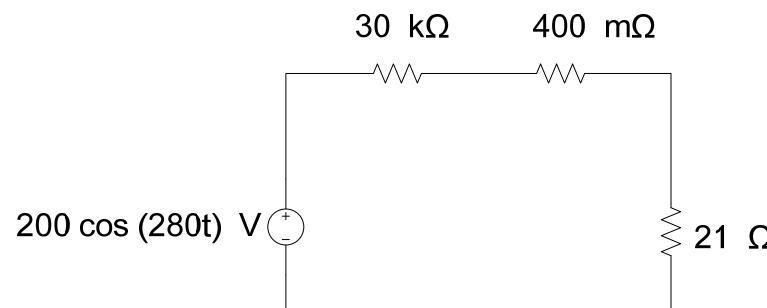
$$i_2 = 0.01i_1 = \frac{2}{300} \text{ A}$$

$$P_{21\Omega} = 0.5 \times 21 \times \left(\frac{2}{300} \right)^2 = 467 \text{ } \mu\text{W}$$

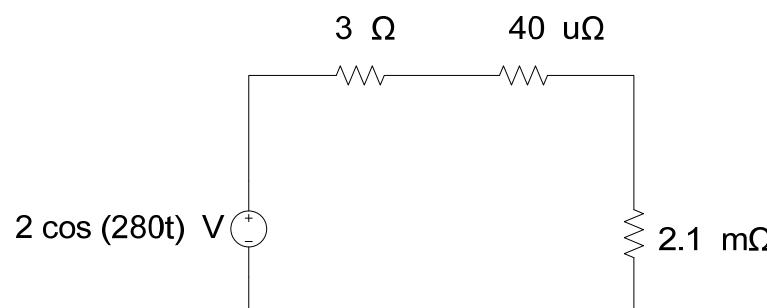
$$P_{0.4\Omega} = 0.5 \times 0.4 \times \left(\frac{2}{300} \right)^2 = 8.88 \text{ } \mu\text{W}$$

43)

a)



b)



44)

$$I_{1.5\Omega} = 5 \frac{50 + \frac{1}{81} \times \left(38 + \frac{1}{16} \times (9) \right)}{1.5 + 50 + \frac{1}{81} \times \left(38 + \frac{1}{16} \times (9) \right)} = 4.85 \text{ A}$$

$$I_{50\Omega} = 5 - 4.85 = 0.15 \text{ A}$$

$$I_{38\Omega} = \frac{1}{9} I_{50\Omega} = \frac{0.15}{9} \text{ A}$$

$$I_{9\Omega} = \frac{1}{4} I_{38\Omega} = \frac{0.15}{36} \text{ A}$$

$$P_{1.5\Omega} = 0.5 \times 1.5 \times (4.85)^2 = 17.64 \text{ W}$$

$$P_{50\Omega} = 0.5 \times 1.5 \times (0.15)^2 = 169 \text{ mW}$$

$$P_{38\Omega} = 0.5 \times 1.5 \times \left(\frac{0.15}{9} \right)^2 = 208 \text{ } \mu\text{W}$$

$$P_{9\Omega} = 0.5 \times 1.5 \times \left(\frac{0.15}{36} \right)^2 = 13 \text{ } \mu\text{W}$$

45)

a)

$$V_1 = -0.75 \text{ V}$$

$$V_2 = \frac{15}{2} \left(\frac{100 \times \frac{4}{225}}{4 + \frac{400}{225}} \right) V_1 = -\frac{45}{26} \text{ V}$$

b)

$$P_{100\Omega} = 0.5 \times \frac{\left(\frac{45}{26}\right)^2}{100} = 15 \text{ mW}$$

$$P_{4\Omega} = 0.5 \times \frac{\left(V_1 - V_2 \frac{2}{15}\right)^2}{4} = 0.5 \times \frac{\left(0.75 - \frac{3}{13}\right)^2}{4} = 16.9 \text{ mW}$$

$$I_{2\Omega} = 25 \times \frac{27000}{2700 + 2 + 25 \times \left(4 + \frac{400}{225}\right)} = 23.71 \text{ mA}$$

$$P_{2\Omega} = 0.5 \times 2 \times \left(23.71 \times 10^{-3}\right)^2 = 0.562 \text{ mW}$$

$$P_{2.7k\Omega} = 0.5 \times 2.7 \times 10^3 \times \left(1.29 \times 10^{-3}\right)^2 = 2.24 \text{ mW}$$

46)

$$i_x = 5.96 - 3.44i \text{ A}$$

$$V_2 = -65.41 + 37.76i \text{ V}$$

47)

a)

$$i_x = -10.42 + 6.02i \text{ A}$$

$$V_2 = 22.56 - 13.02i \text{ V}$$

b)

$$i_x = 8.42 - 4.86i \text{ A}$$

$$V_2 = 27.11 - 15.65i \text{ V}$$

48)

a)

$$V_2 = \frac{4}{4 + \frac{1}{900}} \frac{V_s}{30} = 3.89 \text{ V}$$

b)

$$V_2 = V_s = 117 \angle 0^\circ \text{ V}$$

c)

$$V_2 = 3.96 \angle 0^\circ \text{ V}$$

49)

a)

$$V_2 = V_s = 720 \angle 0^\circ \text{ V}$$

b)

$$V_2 = 676.73 \angle 0^\circ \text{ V}$$

c)

$$V_2 = 664.61 \angle 0^\circ \text{ V}$$

50) It is not possible. Maximum power that the source can deliver to such a circuit is less than 2 watts while 200 volts across the load means 20kw!

If we change the question to have 200 mv across the load, the answer is:

$$6.85\left(\frac{a}{b}\right)^2 - 18.5\left(\frac{a}{b}\right) + 9.25 = 0 \Rightarrow \left(\frac{a}{b}\right) = 2, \frac{2}{3}$$

51)

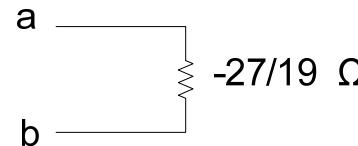
$$V_x = \frac{1.85 \parallel 0.1}{1.85 \parallel 0.1 + 314} 2 \angle 0^\circ = 0.63 \text{ mV}$$

52)

$$V_x = \frac{1.85 \parallel (200 \times 16)}{1.85 \parallel (200 \times 16) + 5.3} 2 \angle 0^\circ = 0.5175$$

$$I_l = -\frac{1}{4} \frac{V_x}{200} = -0.64 \text{ mA}$$

53)



54)

a and b (turn ratios) are not given

55)

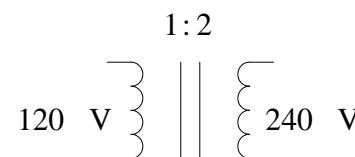
question is wrong. maximum deliverable power is about 5W. to have an answer, we should increase "c". For example, if we consider c=25 s, the answer is b/a=20.3

56)

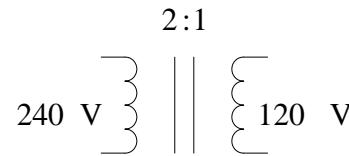
a) **I1 = 9.534 < -24.29**

b) P=1.9 [kW]

57)



58)



Because razors do not consume a lot of energy. The transformer size was probably small and did not let a lot of energy to be converted.

59)

a)

$$\frac{V_2}{V_s} = \frac{-i\omega 4\sqrt{5}}{200 + 6.5i\omega}$$

b)

$$\frac{V_2}{V_s} = \frac{-48.72i\omega}{200\sqrt{5} - 0.182\omega^2 + 57.4\sqrt{5}i\omega}$$

c)

$$\frac{V_2}{V_s} = \frac{-80}{204}$$

60)

- a) To couple some energy to his coil without paying for that.
- b) somehow. He can use energy only when we are using. The more energy we use, the more energy couples to his coil.
- c) False, magnetic field can be propagated even through vaccum.

1. (a) $8 + j$
 (b) $8e^{-9t}$
 (c) 22.5
 (d) $4e^{-j9}$
 (e) $-2je^{j11}$

2. (a) $(-1)^* = -1$
 (b) $\left(\frac{-j}{5\angle 20^\circ}\right)^* = \left(\frac{1\angle -90^\circ}{5\angle 20^\circ}\right)^* = (0.2\angle -110^\circ)^* = 0.2\angle 110^\circ$
 (c) $(5e^{-j5} + 2e^{j3})^* = ((1.4183 + j4.7946) + (-1.98 + j0.2822))^* = (-0.5617 + j5.0769)^* = -0.5617 - j5.0769$
 (d) $((2 + j)(8\angle 30^\circ)e^{j2t})^* = (2.23e^{j26.56^\circ} \times 8e^{j30^\circ} \times e^{j2t})^* = (17.84e^{j(56.56^\circ+2t)})^* = 17.84e^{-j(56.26^\circ+2t)}$

3. (a) $5e^{-j50t} e^{j50t}$
 (b) $(2 + j)e^{j9t}(2 - j)e^{-j9t}$
 (c) $(1 - j)e^{j78t}(1 + j)e^{-j78t}$
 (d) $-je^{-5jt}$

4. (a) $s = \pm j100 \text{ s}^{-1}$
 (b) $s = 0$
 (c) $s = -7 \pm j80 \text{ s}^{-1}$
 (d) $s = 8\text{s}^{-1}$
 (e) $s = -2 \pm j4, -1 \pm j4 \text{ s}^{-1}$

5. (a) $f = 7e^{-9t} \sin(100t + 9^\circ) = 3.5[e^{j81}e^{(-j100-9)t} + e^{-j81}e^{(j100-9)t}]$
 so $s = -9 - j100, s^* = -9 - j100$
 (b) $f = \cos 9t$
 so $s = 9j, s^* = -9j$
 (c) $f = 2\cos(90^\circ - 45t)$
 so $s = -j45, s^* = j45$

6. (a) $A e^{10t} \cos(3t + \phi_1)$
 (b) $Ae^{0.25t}$
 (c) $A + Be^t + C \cos(t + \phi_1) + De^t \cos(t + \phi_2)$

7. (a) $U = 0.71e^{0.2t}$
 for $t=0$, $U=0.71 \text{ V}$, $I=0.71/280=2.57 \text{ mA}$
 for $t=0.1 \text{ s}$, $U=0.72 \text{ V}$, $I=2.57 \text{ mA}$
 for $t=0.5 \text{ s}$, $U=0.78 \text{ V}$, $I=2.8 \text{ mA}$

- (b) $U = 0.285e^{-t} \cos(2t - 45^\circ)$
 for $t=0$, $U=0.202 \text{ V}$, $I=0.72 \text{ mA}$
 for $t=0.1 \text{ s}$, $U=0.21 \text{ V}$, $I=0.77 \text{ mA}$

for t=0.5 s., U=0.196 V, I=0.6 mA

8. (a) $s = -1 + j200\pi$

The neighbor is approaching nearer to you as the signal is rapidly changing with time.

(b) As the imaginary part of the frequency starts to decrease suddenly, the distance tends to increase.

9. (a) $V_{(t)} = 9(\cos 4t + j \sin 4t)$

so, $\text{Re}[V_{(t)}] = 9\cos 4t$;

(b) $V_{(t)} = 12 - 9j$

so $\text{Re}[V_{(t)}] = 12$;

(c) $V_{(t)} = 5\cos 100t - 43j \sin 100t$

so $\text{Re}[V_{(t)}] = 5\cos 100t$;

(d) $V_{(t)} = (2\cos 3t - \sin 3t) + j(2\sin 3t + \cos 3t)$

so $\text{Re}[V_{(t)}] = 2\cos 3t - \sin 3t$

10. (a) The real part is missing.

(b) The complex frequency of the signal is : $s = -2 + j60 \text{ s}^{-1}$

(c) The fact that $\text{Im}\{V_x\} > \text{Re}\{V_x\}$ implies that the phase angle is large.

(d) The $\text{Re}(s)$ denotes the damping factor and the $\text{Im}(s)$ denotes sinusoidal variation. So if $\text{Im}(s)$ is larger than the real part, the signal is changing rapidly with time.

11. (a) $s=5, V_{(t)} = \text{Re}[19e^{5t+j84}]$

(b) $s=0, V_{(t)} = \text{Re}[19e^{j84}]$

(c) $s=-4+j, s=5, V_{(t)} = \text{Re}[19e^{j84}e^{(-4+j)t}]$

12. (a) $s = -1 + j100; s^* = -1 - j100$

(b) $\mathbf{V}=2.5\angle -20^\circ \text{ V}$;

$$v(t) = \text{Re}[2.5e^{-j20} e^{(-1+j100)t}] = [2.5e^{-t} \cos(100t - 20^\circ)]$$

(c) Applying KVL, we obtain,

$$v(t) = Ri + L \frac{di}{dt} + \frac{1}{C} \int idt = 1.5i + 1.8 \frac{di}{dt} + 0.25 \int idt$$

We construct an s-domain voltage $\mathbf{V}(s) = 2.5\angle -20^\circ$ with s given above and assume the current of the form Ie^{st}

On substituting in the above equation, we get,

$$2.5\angle -20^\circ e^{st} = 1.5Ie^{st} + 1.8sIe^{st} + \frac{0.25}{s} Ie^{st}$$

Suppressing the exponential factor, we may write

$$I = \frac{2.5\angle -20^\circ}{1.5 + 1.8s + \frac{0.25}{s}} = \frac{2.5\angle -20^\circ}{180\angle -90^\circ} = 0.0138\angle -110^\circ \text{ A}$$

In time domain, we can write this as

$$i(t) = 0.0138 e^{-t} \cos(100t - 110^\circ) A$$

13. $V(t) = 1.5I + 1.8sI + \frac{0.25}{s}I$

(a) $s = -2 + j2$

we can get: $V(t) = 20.75 \angle 151.5^\circ = 20.75e^{-2t} \cos(2t + 151.5^\circ)$

(b) $s = -3 + j$

we can get: $V(t) = 21.75 \angle 186^\circ = 21.75e^{-3t} \cos(t + 186^\circ)$

14. $\mathbf{s} = -200 + j150$

$$v_1(t) = i(t) \cdot 21; v_2(t) = 0.1 \frac{di(t)}{dt}$$

In frequency domain,

$$\frac{V_2}{V_1} = \frac{0.1s}{21} = \frac{s}{210} = \frac{250 \angle 143.130^\circ}{210} = 1.19 \angle 143.130^\circ$$

15. $s = -150 + j100, V_2 = 5 \angle -25^\circ$,

so $I = \frac{V_2}{Ls} = 0.278 \angle -171^\circ, V_1 = IR = 5.84 \angle -171^\circ$

we can get: $V = V_1 + V_2 = 6.12 \angle -171^\circ$,

and $V(t) = 6.12e^{-150t} \cos(100t - 171^\circ)$

16. $\mathbf{s} = -1 + j2; I_s = 2.3 \angle 5^\circ A$

Applying KCL in the upper node of the given circuit, we obtain,

$$i_s = i_R + i_C = \frac{v}{R} + C \frac{dv}{dt}$$

We construct an s-domain voltage $\mathbf{I}_s(s) = 2.3 \angle 5^\circ A$ with s given above and assume the voltage of the form Ve^{st}

On substituting in the above equation, we get,

$$2.3 \angle 5^\circ e^{st} = 0.5Ve^{st} + 0.25sVe^{st}$$

Suppressing the exponential factor, we may write

$$V = \frac{2.3 \angle 5^\circ}{0.5 + 0.25s} = \frac{2.3 \angle 5^\circ}{0.559 \angle 63.435^\circ} = 4.114 \angle -58.435^\circ V$$

In time domain, we can write this as

$$v(t) = 4.114 e^{-t} \cos(2t - 58.435^\circ) V$$

17. $I_1 = \frac{V(t)}{R} = 0.9 \angle 75^\circ, I_2 = \frac{V(t)}{0.25/s} = 0.72 \angle 218.5^\circ$

so $I = I_1 + I_2 = 0.52 \angle 129.4^\circ$,

and $i(t) = 0.52e^{-2t} \cos(1.5t + 129.4^\circ)$

18. $v_s = 10 \cos 5t$

(a) $\mathbf{s} = j5$

(b) $\mathbf{V}_s = 10 V$

(c) $Z_{R1} = 1; Z_c = \frac{1}{sC} = \frac{10}{0.4 * j5} = -j0.5;$

$Z_L = sL = (j5) * 1.5 = j7.5;$

$Z_{R2} = 2.2;$

Total impedance of the circuit is given by,

$$\begin{aligned} Z_{total} &= (Z_{R1} \parallel Z_c) + Z_L + Z_{R2} \\ &= -\frac{j0.5}{1-j0.5} + 2.2 + j7.5 \\ &= \frac{5.95+j5.9}{1-j0.5} = 2.4 + j7.1 \end{aligned}$$

$$\therefore I_x = \frac{V_s}{Z_{total}} = \frac{10}{2.4+j7.1} = 1.334 \angle -71.322^\circ A$$

$$(d) i_x(t) = 1.334 \cos(5t - 71.322^\circ) A$$

$$19. Z_{eq} = 2.2 + 1.5s + \frac{0.4/s}{1+0.4/s} = 0.3 + 0.42j, U = I_x Z_{eq} = 1.032 \angle 64.5^\circ$$

so, $U_x = 1.032e^{-t} \cos(0.5 + 64.5^\circ)$

$$20. i_{s1} = 20e^{-3t} \cos 4t A; i_{s2} = 30e^{-3t} \sin 4t A$$

$$(a) Is_1 = 20; Is_2 = 30 \angle -90^\circ = -j30; s = -3 + j4$$

$$Z_R = 5;$$

$$Z_c = \frac{1}{sC} = \frac{10}{-3+j4} = -1.2 - j1.6;$$

$$Z_L = sL = (-3 + j4) * 2 = -6 + j8;$$

Applying principle of superposition in the given circuit, we obtain,

$$\begin{aligned} V_x &= Is_1 \frac{(Z_c + Z_L) \parallel Z_R}{Z_c + Z_L} Z_L + Is_2 \frac{(Z_c + Z_R) \parallel Z_L}{Z_L} Z_L \\ &= 20 \frac{5(-7.2+j6.4)}{-2.2+j6.4} \frac{-6+j8}{-7.2+j6.4} - j30 \frac{(-6+j8)(3.8-j1.6)}{-2.2+j6.4} \\ &= \frac{-600+j800-j30(-22.8+12.8+j30.4+j9.6)}{-2.2+j6.4} \\ &= \frac{600+j1100}{-2.2+j6.4} = \frac{1253 \angle 61.389}{6.76 \angle 108.97} = 185.35 \angle -47.58^\circ \end{aligned}$$

(b) In time domain, we can write this as

$$v_x(t) = 185.35 e^{-3t} \cos(4t - 47.58^\circ) V$$

$$21. (a) F(x) = \int_0^\infty 2.1U(t)dt = \boxed{\frac{2.1}{s}}$$

$$(b) F(x) = \int_0^\infty e^{-st} 2U(t-1)dt = \boxed{\frac{2}{s} e^{-2s}}$$

$$(c) F(x) = \int_0^\infty e^{-st} [5U(t-2) - 2U(t)]dt = \boxed{\frac{5}{s} e^{-2s} - \frac{2}{s}}$$

$$(c) F(x) = \int_0^\infty e^{-st} 3U(t-b)dt = \boxed{\frac{3}{s} e^{-bs}}$$

$$22. (a) F(s) = \int_{0-}^\infty 5u(t-6)e^{-st}dt = 5 \int_6^\infty e^{-st} dt = 5 \frac{e^{-st}}{-s} \Big|_6^\infty = \boxed{\frac{5}{s} e^{-6s}}$$

$$(b) F(s) = \int_{0-}^\infty 2e^{-t} u(t)e^{-st}dt = 2 \int_0^\infty e^{-(s+1)t} dt = 2 \frac{e^{-(s+1)t}}{-(s+1)} \Big|_0^\infty = \boxed{\frac{2}{s+1}}$$

$$(c) F(s) = \int_{0-}^\infty 2e^{-t} u(t-1)e^{-st}dt = 2 \int_1^\infty e^{-(s+1)t} dt = 2 \frac{e^{-(s+1)t}}{-(s+1)} \Big|_1^\infty = \boxed{\frac{2}{s+1} e^{-(s+1)}}$$

$$(d) F(s) = \int_{0^-}^{\infty} e^{-2t} \sin 5t u(t) e^{-st} dt = \int_0^{\infty} e^{-2t} \frac{1}{j2} (e^{j5t} - e^{-j5t}) e^{-st} dt = \frac{1}{j2} \left[\frac{e^{-(s+2-j5)t}}{-(s+2-j5)} \right]_0^{\infty} - \left[\frac{e^{-(s+2+j5)t}}{-(s+2+j5)} \right]_0^{\infty} = \frac{1}{j2} \left(\frac{1}{s+2-j5} - \frac{1}{s+2+j5} \right) = \boxed{\frac{5}{(s+2)^2 + 25}}$$

23. (a) $F(s) = \frac{1}{s^2} e^{-s}$
 (b) $F(s) = \frac{2}{s^2}$
 (c) $F(s) = \frac{12}{s^2 + 4}$
 (d) $F(s) = \frac{s}{s^2 + 10000}$

24. (a) On employing Eq.[14] and evaluating by integration by parts, we obtain,

$$F(s) = \int_{0^-}^{\infty} t u(t) e^{-st} dt = \int_0^{\infty} t e^{-st} dt = \left[\frac{te^{-st}}{-s} \right]_0^{\infty} + \frac{1}{s} \int_0^{\infty} e^{-st} dt = \frac{1}{s^2}$$

Also it is given that,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} F(s), \text{ assuming } \mathcal{L}\{f(t)\} = F(s)$$

Here, let $f(t) = u(t)$, $F(s) = \frac{1}{s}$

Therefore,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} \left(\frac{1}{s} \right) = \frac{1}{s^2} \text{ which is the same as what we found by employing Eq.[14].}$$

(b) On employing Eq.[14] and evaluating by integration by parts, we obtain,

$$F(s) = \int_{0^-}^{\infty} t^2 u(t) e^{-st} dt = \int_0^{\infty} t^2 e^{-st} dt = \left[\frac{t^2 e^{-st}}{-s} \right]_0^{\infty} + \frac{2}{s} \left[\left[\frac{te^{-st}}{-s} \right]_0^{\infty} + \frac{1}{s} \int_0^{\infty} e^{-st} dt \right] = \frac{2}{s^3}$$

Also it is given that,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} F(s), \text{ assuming } \mathcal{L}\{f(t)\} = F(s)$$

Here, let $f(t) = tu(t)$, $F(s) = \frac{1}{s^2}$ using the results from part (a)

Therefore,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} \left(\frac{1}{s^2} \right) = \frac{2}{s^3} \text{ which is the same as what we found by employing Eq.[14].}$$

(c) On employing Eq.[14] and evaluating by integration by parts, we obtain,

$$F(s) = \int_{0^-}^{\infty} t^3 u(t) e^{-st} dt = \int_0^{\infty} t^3 e^{-st} dt = \left[\frac{t^3 e^{-st}}{-s} \right]_0^{\infty} + \frac{3}{s} \int_0^{\infty} t^2 e^{-st} dt = \frac{6}{s^4}$$

Also it is given that,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} \mathbf{F}(s), \text{ assuming } \mathcal{L}\{f(t)\} = \mathbf{F}(s)$$

$$\text{Here, let } f(t) = t^2 u(t), F(s) = \frac{2}{s^3}$$

Therefore,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} \left(\frac{2}{s^3} \right) = \frac{6}{s^4} \text{ which is the same as what we found by employing Eq.[14].}$$

(d) On employing Eq.[14] and evaluating by integration by parts, we obtain,

$$F(s) =$$

$$\int_{0^-}^{\infty} te^{-t} u(t) e^{-st} dt = \int_0^{\infty} te^{-(s+1)t} dt = \left[\frac{te^{-(s+1)t}}{-(s+1)} \right]_0^{\infty} + \frac{1}{s+1} \int_0^{\infty} e^{-(s+1)t} dt = \frac{1}{(s+1)^2}$$

Also it is given that,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} \mathbf{F}(s), \text{ assuming } \mathcal{L}\{f(t)\} = \mathbf{F}(s)$$

$$\text{Here, let } f(t) = e^{-t} u(t), F(s) = \frac{1}{s+1}$$

Therefore,

$$\mathcal{L}(tf(t)) = -\frac{d}{ds} \left(\frac{1}{s+1} \right) = \frac{1}{(s+1)^2} \text{ which is the same as what we found by employing Eq.[14].}$$

25. (a) $t + 4, s > 0$
 (b) $(t + 1)(t + 2), s > 0$
 (c) $e^{-0.5t} U(t), s > -0.5$
 (d) $\sin 10t \cdot U(t + 1), s > -1$

$$\begin{aligned} 26. \mathcal{L}\{f(t) + g(t) + h(t)\} &= \int_{0^-}^{\infty} (f(t) + g(t) + h(t)) e^{-st} dt \\ &= \int_{0^-}^{\infty} f(t) e^{-st} dt + \int_{0^-}^{\infty} g(t) e^{-st} dt + \int_{0^-}^{\infty} h(t) e^{-st} dt \\ &= \boxed{\mathcal{L}\{f(t)\} + \mathcal{L}\{g(t)\} + \mathcal{L}\{h(t)\}} \end{aligned}$$

27. (a) $F(s) = \frac{3}{s} e^{-2s}$
 (b) $F(s) = \frac{3}{s+2} + \frac{5}{s}$
 (c) $F(s) = 1 + \frac{1}{s} - \frac{1}{s^2}$
 (d) $F(s) = 5$

$$\begin{aligned} 28. (a) g(t) &= [5u(t)]^2 - u(t) \\ \therefore G(s) &= \int_0^{\infty} [5]^2 e^{-st} dt - \int_0^{\infty} u(t) e^{-st} dt = 25 \frac{e^{-st}}{-s} \Big|_0^{\infty} - \frac{e^{-st}}{-s} \Big|_0^{\infty} = \boxed{\frac{24}{s}} \\ (b) g(t) &= 2u(t) - 2u(t-2) \end{aligned}$$

$$\therefore G(s) = 2 \int_0^\infty e^{-st} dt - 2 \int_2^\infty e^{-st} dt = 2 \frac{e^{-st}}{-s} \Big|_0^\infty - 2 \frac{e^{-st}}{-s} \Big|_2^\infty =$$

$$\frac{2}{s}(1 - e^{-2s})$$

(c) $g(t) = tu(2t)$

$$\therefore G(s) = \int_0^\infty te^{-st} dt = \left[\frac{te^{-st}}{-s} \right]_0^\infty + \frac{1}{s} \int_0^\infty e^{-st} dt = \frac{1}{s^2}$$

(d) $g(t) = 2e^{-t}u(t) + 3u(t)$

$$\therefore G(s) = 2 \int_0^\infty e^{-(s+1)t} dt + 3 \int_0^\infty e^{-st} dt = 2 \frac{e^{-(s+1)t}}{-(s+1)} \Big|_0^\infty + 3 \frac{e^{-st}}{-s} \Big|_0^\infty =$$

$$\frac{2}{s+1} + \frac{3}{s}$$

29. (a) $f(t) = U(t)$

(b) $f(t) = 1.55\delta(t) - 2U(t)$

(c) $f(t) = e^{-1.5t}U(t)$

(d) $f(t) = 5[tU(t) + U(t) + \delta(t)]$

30. (a) $G(s) = \frac{1.5}{(s+9)^2}$

$$\therefore g(t) = 1.5te^{-9t}u(t)$$

(b) $G(s) = \frac{2}{s} - 0$

$$\therefore g(t) = 2u(t)$$

(c) $G(s) = \pi$

$$\therefore g(t) = \pi\delta(t)$$

(d) $G(s) = \frac{a}{(s+1)^2} - a, a > 0$

$$\therefore g(t) = ae^{-t}tu(t) - a\delta(t)$$

31. (a) $\delta(t)|_{t=1} = 0$

(b) $5\delta(t+1) + u(t+1)|_{t=1} = 1$

(c) $\int_{-1}^2 \delta(t) dt = 1$

(d) $3 - 2 \int_{-1}^2 \delta(t) dt = 1$

32. (a) $[\delta(2t)]^2|_{t=1} = 0$

(b) $2\delta(t-1) + u(-t+1)|_{t=0} = 1$

(c) $\frac{1}{3} \int_{-0.001}^{0.003} \delta(t) dt = 0$

(d) $\frac{1}{\left[\frac{1}{2} \int_0^2 \delta(t-1) dt \right]^2} = 4$

33. (a) $\int_{-\infty}^{\infty} 2\delta(t-1) dt = 2$

(b) $\frac{1}{1} = 1$

(c) $\frac{\sqrt{3}}{1} - 0 = \sqrt{3}$

(d) 1

34. (a) $\int_{-\infty}^{+\infty} e^{-100} \delta\left(t - \frac{1}{5}\right) dt = e^{-100}$

(b) $\int_{-\infty}^{+\infty} 4t \delta(t - 2) dt = 8$

(c) $\int_{-\infty}^{+\infty} 4t^2 \delta(t - 1.5) dt = 0$

(d) $\frac{\int_{-\infty}^{+\infty} (4-t) \delta(t-1) dt}{\int_{-\infty}^{+\infty} (4-t) \delta(t+1) dt} = \frac{3}{5}$

35. (a) $f(t) = 5\delta(t) + 5tU(t) - 5e^{-t}U(t)$

(b) $f(t) = U(t) + 50e^{-40t}U(t)$

(c) $f(t) = 0.5u(t) + 4tu(t) + 4te^{-5t}u(t) + 2\delta(t)$

(d) $f(t) = 4te^{-5t}u(t) + 2e^{-t}u(t) + e^{-3t}u(t)$

36. (a) $G(s) = \frac{3(s+1)}{(s+1)^2} + \frac{2s}{s^2} - \frac{1}{(s+2)^2} = \frac{3}{(s+1)} + \frac{2}{s} - \frac{1}{(s+2)^2}$

$$\therefore g(t) = 3e^{-t}u(t) + 2u(t) - te^{-2t}u(t)$$

(b) $G(s) = -\frac{10}{(s+3)^3}$

$$\therefore g(t) = -10 \frac{1}{2} t^2 e^{-3t} u(t) = -5t^2 e^{-3t} u(t)$$

(c) $G(s) = 19 - \frac{8}{(s+3)^2} + \frac{18}{(s^2+6s+9)} = 19 + \frac{10}{(s+3)^2}$

$$\therefore g(t) = 19\delta(t) + 10te^{-3t}u(t)$$

37. (a) $F(s) = \frac{1}{s+2}, f(t) = 2e^{-2t}u(t)$

(b) $F(s) = 1, f(t) = \delta(t)$

(c) $F(s) = \frac{3}{s+2}, f(t) = 3e^{-2t}u(t)$

(d) $= \frac{2s}{s+1.5}, f(t) = 2\delta(t) - 3e^{-0.5t}U(t)$

38. (a) $V(s) = \frac{s^2+2}{s} + 1$

$$\therefore v(t) = \frac{d\delta(t)}{dt} + 2u(t) + \delta(t)$$

(b) $V(s) = \frac{s+8}{s} + \frac{2}{s^2} = 1 + \frac{8}{s} + \frac{2}{s^2}$

$$\therefore v(t) = \delta(t) + 8u(t) + 2tu(t)$$

(c) $V(s) = \frac{s+1}{s(s+2)} + \frac{2s^2-1}{s^2} = \frac{1}{s+2} + \frac{1}{s(s+2)} + 2 - \frac{1}{s^2}$

$$\therefore v(t) = e^{-2t}u(t) + 0.5(1 - e^{-2t})u(t) + 2\delta(t) - tu(t)$$

(d) $V(s) = \frac{s^2+4s+4}{s} = s + 4 + \frac{4}{s}$

$$\therefore v(t) = \frac{d\delta(t)}{dt} + 4\delta(t) + 4u(t)$$

39. (a) $F(s) = \frac{17}{3} \frac{1}{s-3} + \frac{1}{3s}, f(t) = \frac{(17e^{-3t}+1)}{3} u(t)$

(b) $f(t) = 7\delta(t) + \left[\frac{6\sqrt{5}}{5} \sinh\left(\frac{\sqrt{5}t}{2}\right) e^{-1.5t} - 1 \right] U(t)$

(c) $f(t) = 2t + \left[\frac{4}{10} \cosh(2\sqrt{10}t) - \frac{3\sqrt{10}}{10} \sinh(2\sqrt{10}t) \right] e^{-8t} U(t) + U(t)$

(d) $f(t) = 2te^{-t}u(t)$

(e) $f(t) = (\frac{14}{9}e^{-4t} - \frac{49}{72}e^{-t} - \frac{7}{8}e^{-5t} + \frac{7}{6}te^{-t})u(t)$

40. (a) $G(s) = \frac{1}{s^2+9s+20} = \frac{1}{(s+4)(s+5)}$

Applying method of residues

$$\left. \frac{1}{s+4} \right|_{s=-5} = -1; \left. \frac{1}{s+5} \right|_{s=-4} = 1 \text{ so that,}$$

$$G(s) = \frac{1}{s+4} - \frac{1}{s+5}$$

$$\therefore g(t) = e^{-4t}u(t) - e^{-5t}u(t)$$

(b) $G(s) = \frac{4}{s^3+18s^2+17s} + \frac{1}{s} = \frac{4}{s(s+1)(s+17)} + \frac{1}{s}$

Applying method of residues to the first term,

$$\left. \frac{4}{(s+1)(s+17)} \right|_{s=0} = \frac{4}{17}; \left. \frac{4}{s(s+17)} \right|_{s=-1} = -\frac{1}{4}; \left. \frac{4}{s(s+1)} \right|_{s=-17} = \frac{1}{68}$$

so that,

$$G(s) = \frac{4}{17s} - \frac{1}{4(s+1)} + \frac{1}{68(s+17)} + \frac{1}{s}$$

$$\therefore g(t) = \frac{1}{17}u(t) + \frac{1}{68}e^{-17t}u(t) - \frac{1}{4}e^{-t}u(t)$$

(c) $G(s) = 0.25 \frac{1}{(\frac{s}{2})^2 + 1.75s + 2.5} = \frac{1}{(s+2)(s+5)}$

Applying method of residues

$$\left. \frac{1}{s+5} \right|_{s=-2} = \frac{1}{3}; \left. \frac{1}{s+2} \right|_{s=-5} = -\frac{1}{3} \text{ so that,}$$

$$G(s) = \frac{1}{3(s+2)} - \frac{1}{3(s+5)}$$

$$\therefore g(t) = \frac{1}{3}e^{-2t}u(t) - \frac{1}{3}e^{-5t}u(t)$$

(d) $G(s) = \frac{3}{s(s+1)(s+4)(s+5)(s+2)}$

Applying method of residues,

$$\left. \frac{3}{(s+1)(s+4)(s+5)(s+2)} \right|_{s=0} = \frac{3}{40}; \left. \frac{3}{s(s+4)(s+5)(s+2)} \right|_{s=-1} = -\frac{1}{4};$$

$$\left. \frac{3}{s(s+1)(s+4)(s+5)} \right|_{s=-2} = \frac{1}{4}; \left. \frac{3}{s(s+1)(s+5)(s+2)} \right|_{s=-4} = -\frac{1}{8};$$

$$\left. \frac{3}{s(s+1)(s+4)(s+2)} \right|_{s=-5} = \frac{1}{20}$$

so that,

$$G(s) = \frac{3}{40s} - \frac{1}{4(s+1)} + \frac{1}{4(s+2)} - \frac{1}{8(s+4)} + \frac{1}{20(s+5)}$$

$$\therefore g(t) = \frac{3}{40}u(t) - \frac{1}{4}e^{-t}u(t) + \frac{1}{4}e^{-2t}u(t) - \frac{1}{8}e^{-4t}u(t) + \frac{1}{20}e^{-5t}u(t)$$

(e) MATLAB Verifications:

```

>> syms t s
A = 1/(s^2+9*s+20);
B = 4/(s^3+18*s^2+17*s)+1/s;
C = 0.25* 1/((s/2)^2+1.75*s+2.5);
D = 3/(s*(s+1)*(s+4)*(s+5)*(s+2));
a = ilaplace(A)
b = ilaplace(B)
c = ilaplace(C)
d = ilaplace(D)
a = 1/exp(4*t) - 1/exp(5*t)
b = 1/(68*exp(17*t)) - 1/(4*exp(t)) + 21/17
c = 1/(3*exp(2*t)) - 1/(3*exp(5*t))
d = 1/(4*exp(2*t)) - 1/(4*exp(t)) - 1/(8*exp(4*t)) + 1/(20*exp(5*t))
+ 3/40

```

41. (a) $F(s) = -\frac{1}{(s+2)^2} - \frac{1}{s+2} + \frac{1}{s+1}$, $f(t) = (-te^{2t} - e^{-2t} + e^t)u(t)$

(b) $F(s) = -\frac{2}{(s+2)^3} + \frac{1}{(s+2)^2}$, $f(t) = (t^2 + t)e^{-2t}u(t)$

(c) $F(s) = \frac{\frac{2}{7}}{s+2} - \frac{\frac{3}{7}}{s+6} + \frac{\frac{8}{21}}{s+9}$, $f(t) = (\frac{2}{7}e^{-2t} - \frac{3}{7}e^{-6t} + \frac{8}{21}e^{-9t})u(t)$

42. (a) $G(s) = \frac{1}{3s} - \frac{1}{2s+1} + \frac{3}{s^3+8s^2+16s} - 1 = \frac{1}{3s} - \frac{1}{2(s+0.5)} + \frac{3}{s(s+4)^2} - 1$

Applying method of residues to the third term,

$$\left. \frac{3}{(s+4)^2} \right|_{s=0} = \frac{3}{16}; \left. \frac{d}{ds} \left[\frac{3}{s} \right] \right|_{s=-4} = -\frac{3}{16}; \left. \frac{3}{s} \right|_{s=-4} = -\frac{3}{4} \text{ so that}$$

$$G(s) = \frac{1}{3s} - \frac{1}{2(s+0.5)} + \frac{3}{16s} - \frac{3}{16(s+4)} - \frac{3}{4(s+4)^2} - 1$$

$$\therefore g(t) = \frac{25}{48}u(t) - \frac{1}{2}e^{-0.5t}u(t) - \frac{3}{16}e^{-4t}u(t) - \frac{3}{4}te^{-4t}u(t) - \delta(t)$$

(b) $G(s) = \frac{1}{3s+5} + \frac{3}{\frac{s^3}{8}+0.25s^2} = \frac{1}{3s+5} + \frac{24}{s^2(s+2)}$

Applying method of residues to the second term,

$$\left. \frac{d}{ds} \left[\frac{24}{s+2} \right] \right|_{s=0} = -6; \left. \frac{24}{s+2} \right|_{s=0} = 12; \left. \frac{24}{s^2} \right|_{s=-2} = 6 \text{ so that}$$

$$G(s) = \frac{1}{3(s+1.667)} - \frac{6}{s} + \frac{12}{s^2} + \frac{6}{s+2}$$

$$\therefore g(t) = e^{-1.667t}u(t) - 6u(t) + 12tu(t) + 6e^{-2t}u(t)$$

(c) $G(s) = \frac{2s}{(s+a)^2} = 2 \left[\frac{1}{s+a} - \frac{a}{(s+a)^2} \right]$

$$\therefore g(t) = 2e^{-at}u(t) - 2ate^{-at}u(t)$$

43. (a) $F(s) = \frac{24}{(s+4)^2} + \frac{6}{s+4} - \frac{6}{s+2}$, $f(t) = (24te^{-4t} + 6e^{-4t} - 6e^{-2t})u(t)$

(b) $F(s) = 3 + \frac{15}{4} \frac{1}{(s+5)^2} - \frac{9}{8} \frac{1}{s+5} - \frac{21}{4} \frac{1}{s+7}$,

$$f(t) = 3\delta(t) + \left(\frac{15}{4}te^{-5t} - \frac{9}{8}e^{-5t} - \frac{21}{4}e^{-7t}\right)u(t)$$

$$(c) F(s) = 2 - \frac{1}{s+100} + \frac{s}{s^2+100}, f(t) = 2\delta(t) - (e^{100t} + \cos(10t))u(t)$$

$$(d) f(t) = \frac{1}{2}tu(t)$$

$$44. (a) G(s) = \frac{s}{(s+2)^3} = \frac{1}{(s+2)^2} - \frac{2}{(s+2)^3}$$

$$\therefore g(t) = te^{-2t}u(t) - t^2e^{-2t}u(t)$$

$$(b) G(s) = \frac{4}{(s+1)^4(s+1)^2} = \frac{4}{(s+1)^6}$$

$$\therefore g(t) = \frac{1}{120}t^5e^{-t}u(t)$$

$$(c) G(s) = \frac{1}{s^2(s+4)^2(s+6)^3} - \frac{2s^2}{s} + 9$$

Using Matlab to perform the partial fraction expansion of

$$\frac{1}{s^2(s+4)^2(s+6)^3}$$

```
>> N = [1];
```

```
>> D = poly([0 0 -4 -4 -6 -6 -6]);
```

```
>> [r, p, k] = residue(N, D)
```

r =

0.0081

0.0093

0.0069

-0.0078

0.0078

-0.0003

0.0003

p =

-6.0000

-6.0000

-6.0000

-4.0000

-4.0000

0

0

k =

[]

```
>> rats(r)
```

ans =

7/864

1/108

1/144

-1/128

1/128

-1/3456

1/3456

$$G(s) = \frac{1}{s^2(s+4)^2(s+6)^3} - 2s + 9 = \frac{1}{3456s^2} - \frac{1}{3456s} + \frac{1}{128(s+4)^2} - \frac{1}{128(s+4)} + \frac{1}{144(s+6)^3} + \frac{1}{108(s+6)^2} + \frac{7}{864(s+6)} - 2s + 9$$

$$\therefore g(t) = \boxed{\frac{1}{3456}tu(t) - \frac{1}{3456}u(t) + \frac{1}{128}te^{-4t}u(t) - \frac{1}{128}e^{-4t}u(t) + \frac{1}{288}t^2e^{-6t}u(t) + \frac{1}{108}te^{-6t}u(t) + \frac{7}{864}e^{-6t}u(t) - 2\frac{d\delta(t)}{dt} + 9\delta(t)}$$

(d) MATLAB Verifications

```
>> syms t s
A = s/(s+2)^3;
B = 4/((s+1)^4*(s+1)^2);
C = 1/(s^2*(s+4)^2*(s+6)^3);
a = ilaplace(A)
b = ilaplace(B)
c = ilaplace(C)

a = t/exp(2*t) - t^2/exp(2*t)
b = t^5/(30*exp(t))
c = t/3456 - 1/(128*exp(4*t)) + 7/(864*exp(6*t)) + t/(128*exp(4*t))
+ t/(108*exp(6*t)) + t^2/(288*exp(6*t)) - 1/3456
```

45. (a) $5(sI(s) - I(0)) - 7(s^2I(s) - sI(0) - I'(0)) + 9I(s) = \frac{4}{s}$

(b)

$$m(s^2P(s) - sP(0) - p'(0)) + \mu_f(sP(s) - P(0)) + k \int_{-\infty}^{\infty} e^{-st}P(t)dt = 0$$

(c) $s\Delta nP(s) - \Delta nP(0) = \frac{1}{\tau} \int_{-\infty}^{\infty} \Delta npe^{-st} dt + \frac{GL}{s}$

46. $v(0^-) = 1.5 \text{ V}; i_s = 700 u(t) \text{ mA}$

(a) Writing the differential equation from KCL, we obtain,

$$i_s = i_R + i_C = \frac{v}{R} + C \frac{dv}{dt}$$

$$0.7u(t) = 0.5v + \frac{0.5dv}{dt}$$

(b) Taking Laplace transform, we obtain,

$$\frac{0.7}{s} = 0.5V(s) + 0.5[sV(s) - v(0^-)]$$

$$\frac{0.7}{s} = 0.5V(s) + 0.5sV(s) - 0.75$$

(c) $V(s) = \frac{7}{5s(s+1)} + \frac{3}{2(s+1)}$

Applying method of residues to the first term,

$$\left. \frac{7}{5(s+1)} \right|_{s=0} = \frac{7}{5}; \left. \frac{7}{5s} \right|_{s=-1} = -\frac{7}{5} \text{ so that,}$$

$$V(s) = \frac{7}{5s} - \frac{7}{5(s+1)} + \frac{3}{2(s+1)}$$

$$\therefore v(t) = \boxed{\frac{7}{5}u(t) + \frac{1}{10}e^{-t}u(t) V}$$

$$47. Z_{eq} = \frac{1}{4s+1},$$

$$V(s) = I(s) * Z_{eq} = \frac{2}{(s+1)^2},$$

$$v(t) = 2te^{-t}u(t)$$

$$48. i(0^-) = 5 \text{ A}; \mathbf{V(s)} = \left(\frac{2}{s} - \frac{1}{s+1}\right) V$$

(a) Writing the differential equation applying KVL, we obtain,

$$v(t) = iR + \frac{Ldi}{dt}$$

$$v(t) = 4i + \frac{0.2di}{dt}$$

Taking Laplace transform, we obtain,

$$\frac{2}{s} - \frac{1}{s+1} = 4\mathbf{I(s)} + 0.2[\mathbf{sI(s)} - i(0^-)]$$

$$\mathbf{I(s)} = \frac{10}{s(s+20)} - \frac{5}{(s+1)(s+20)} + \frac{5}{s+20}$$

Applying method of residues to the first and the second term,

$$\frac{10}{s+20} \Big|_{s=0} = \frac{1}{2}; \frac{10}{s} \Big|_{s=-20} = -\frac{1}{2}$$

$$\frac{5}{s+1} \Big|_{s=-20} = -\frac{5}{19}; \frac{5}{s+20} \Big|_{s=-1} = \frac{5}{19}$$

so that,

$$\mathbf{I(s)} = \frac{1}{2s} - \frac{1}{2(s+20)} + \frac{5}{19(s+20)} - \frac{5}{19(s+1)} + \frac{5}{s+20} \text{ A}$$

$$(b) \therefore i(t) = \frac{1}{2}u(t) + \frac{181}{38}e^{-20t}u(t) - \frac{5}{19}e^{-t}u(t) \text{ A}$$

$$49. V(t) = 5i(t) + 5 \int i(t) dt,$$

$$I(s) = \frac{\frac{2}{5}}{s+1},$$

$$i(t) = \frac{2}{5}e^{-t}u(t)$$

$$50. v(0^-) = 4.5 \text{ V}; \mathbf{Vs(s)} = \frac{2}{s+1} \text{ V}$$

(a) Writing the differential equation applying KVL, we obtain,

$$v_s(t) = \frac{1}{C} \int_{-\infty}^t i(t') dt' + i(t)R$$

$$v_s(t) = 5 \int_{-\infty}^t i(t') dt' + 5i(t)$$

$$\text{We can write, } 5 \int_{-\infty}^t i(t') dt' = 5 \int_{-\infty}^{0^-} i(t') dt' + 5 \int_{0^-}^t i(t') dt'$$

$$= v(0^-) + 5 \int_{0^-}^t i(t') dt'$$

$$2e^{-t}u(t) = v(0-) + 5 \int_{0-}^t i(t')dt' + 5i(t)$$

(b) Taking Laplace transform, we obtain,

$$\frac{2}{s+1} = 5I(s) + \frac{4.5}{s} + \frac{5}{s} I(s)$$

$$I(s) = 0.4s - \frac{0.9}{s+1}$$

$$\therefore i(t) = 0.4 \frac{d\delta(t)}{dt} - 0.9 e^{-t}u(t) \text{ A}$$

$$51. Zeq = \frac{1.5s}{1+1.5s}$$

$$V(s) = \frac{450}{s+0.67},$$

$$v(t) = 450e^{-0.67t}u(t)$$

52. The function for the plot in Fig. 14.18 is given by

$$f(t) = 6u(t - 1.6) + 6u(t - 3) - 12u(t - 4.6)$$

$$\therefore F(s) = \frac{6}{s}(e^{-1.6s} + e^{-3s} - 2e^{-4.6s})$$

53. (a) $D(s) = s^3 + 13s^2 + 47s + 35$

s^3	1	47	0
s^2	13	35	0
s	44.3	0	
1	35		

Stable

(b) $D(s) = s^3 + 13s^2 + s + 35$

s^3	1	1	0
s^2	13	35	0
s	-x	0	
1	35		

Unstable

54. (a) $D(s) = s^2 + 3s + 8$

s^2	1	8	0
s	3	0	
1	8		

There is no sign change. Therefore, the system is Stable.

$$D(s) = s^2 + 3s + 8 = \left(s - 1.5 + \frac{j\sqrt{23}}{2}\right)\left(s - 1.5 - \frac{j\sqrt{23}}{2}\right)$$

Since the poles lie on the left hand plane, the system is stable.

(b) $D(s) = s^2 + 2s + 1$

s^2	1	1	0
s	2	0	
1	1		

There is no sign change. Therefore, the system is **Stable.**

$$D(s) = s^2 + 2s + 1 = (s + 1)^2$$

Since the poles lie on the left hand plane, the system is stable.

55. $f(0) = \lim_{s \rightarrow \infty} [sF(s)]$

- (a) $f(0) = \boxed{2}$
- (b) $f(0) = \boxed{0}$
- (c) $f(0) = \boxed{1}$
- (d) $f(0) = \boxed{-1}$

56. (a) $f(t) = u(t - 3); F(s) = \frac{1}{s} e^{-3s}$

$$f(0) = \lim_{s \rightarrow \infty} [sF(s)] = \boxed{0}$$

(b) $f(t) = 2e^{-(t-2)}u(t - 2); F(s) = \frac{2}{s+1} e^{-2s}$

$$f(0) = \lim_{s \rightarrow \infty} [sF(s)] = \boxed{0}$$

(c) $f(t) = \frac{u(t-2)+[u(t)]^2}{2}; F(s) = \frac{1}{2s} e^{-2s} + \frac{1}{2s}$

$$f(0) = \lim_{s \rightarrow \infty} [sF(s)] = \boxed{0.5}$$

(d) $f(t) = \sin 5t e^{-t}u(t); F(s) = \frac{5}{(s+2)^2+25}$

$$f(0) = \lim_{s \rightarrow \infty} [sF(s)] = \boxed{5}$$

57. (a) $f(\infty) = \boxed{-2}$

(b) $f(\infty) = \boxed{0}$

(c) $f(\infty) = \boxed{2}$

(d) $f(\infty) = \boxed{0}$

58. (a) $\frac{1}{s+18}$

$$f(0+) = \lim_{s \rightarrow \infty} [sF(s)] = \lim_{s \rightarrow \infty} \left[\frac{s}{s+18} \right] = \boxed{0}$$

$$f(\infty) = \lim_{s \rightarrow 0} [sF(s)] = \lim_{s \rightarrow 0} \left[\frac{s}{s+18} \right] = \boxed{0}$$

(b) $10 \left(\frac{1}{s^2} + \frac{3}{s} \right)$

$$f(0+) = \lim_{s \rightarrow \infty} [sF(s)] = \lim_{s \rightarrow \infty} \left[10s \left(\frac{1}{s^2} + \frac{3}{s} \right) \right] = \boxed{30}$$

$$f(\infty) = \lim_{s \rightarrow 0} [sF(s)] = \lim_{s \rightarrow 0} \left[10s \left(\frac{1}{s^2} + \frac{3}{s} \right) \right] = \boxed{\infty}$$

(c) $\frac{s^2-4}{s^3+8s^2+4s}$

$$f(0+) = \lim_{s \rightarrow \infty} [sF(s)] = \lim_{s \rightarrow \infty} \left[s \left(\frac{s^2-4}{s^3+8s^2+4s} \right) \right] = \boxed{1}$$

$$f(\infty) = \lim_{s \rightarrow 0} [sF(s)] = \lim_{s \rightarrow 0} \left[s \left(\frac{s^2-4}{s^3+8s^2+4s} \right) \right] = \boxed{-1}$$

(d) $\frac{s^2+2}{s^3+3s^2+5s}$

$$f(0+) = \lim_{s \rightarrow \infty} [sF(s)] = \lim_{s \rightarrow \infty} \left[s \left(\frac{s^2+2}{s^3+3s^2+5s} \right) \right] = \boxed{1}$$

$$f(\infty) = \lim_{s \rightarrow 0} [sF(s)] = \lim_{s \rightarrow 0} \left[s \left(\frac{s^2+2}{s^3+3s^2+5s} \right) \right] = \boxed{\frac{2}{5}}$$

59. (a) $f(0) = \boxed{1}, f(\infty) = \boxed{0}$
 (b) $f(0) = \boxed{\infty}, f(\infty) = \boxed{NaN}$
 (c) $f(0) = \boxed{\infty}, f(\infty) = \boxed{0}$

60. (a) $\frac{1}{s-1}$

Here, the pole lies on the right hand plane, so the final value theorem is not applicable.

(b) $\frac{10}{s^2-4s+4} = \frac{10}{(s-2)^2}$

Here, the poles lie on the right hand plane, so the final value theorem is not applicable.

(c) $\frac{13}{s^3-5s^2+8s-6} = \frac{13}{(s-3)(s^2-2s+2)}$

Here, the poles lie on the right hand plane, so the final value theorem is not applicable.

(d) $\frac{3}{2s^3-10s^2+16s-12} = \frac{3}{2(s-3)(s^2-2s+2)}$

Here, the poles lie on the right hand plane, so the final value theorem is not applicable.

61. $V = \frac{8}{s+2}$

(a) $I = \frac{1}{s+2}, R = \frac{V}{I} = 8, \text{resister: } 8 \Omega$

(b) $I = \frac{4}{s(s+2)}, R = \frac{V}{I} = 2s, \text{inductance: } 2H$

62. (a) $F(s) = \frac{16}{s-1}$

$$f(0+) = \lim_{s \rightarrow \infty} [sF(s)] = \lim_{s \rightarrow \infty} \left[\frac{16s}{s-1} \right] = 16$$

Here, the pole lies on the right hand plane, so function has an indeterminate final value.

(b) $\therefore f(t) = \boxed{16e^{-t}u(t)}$

(c) $i_c(t) = C \frac{dv(t)}{dt} = 2(-16)e^{-t} = \boxed{-32e^{-t} A}$

63. if only $i(s)$ applies to the circuit, $Zeq = \frac{2s+10}{3s+10}, Ic(s)' = \frac{5}{3} \frac{1}{(s+3.33)}$

if only $V(s)$ applies to the circuit, $Zeq = \frac{10}{s} + 3, Ic(s)'' = \frac{3s+10}{s} \frac{1}{s+4}$

so $Ic(s) = Ic(s)' - Ic(s)''$,

then $\boxed{i_c(t) = (\frac{5}{3}e^{-3.33} - 3e^{-4} - 2.5 + 25e^{-4t})u(t)}$

64. $i_s(t) = 2u(t+2) A; \mathbf{I}_s(s) = \frac{2}{s}$

$$(a) v_s(t) = 2u(t) \text{ V}; V_s(s) = \frac{2}{s}$$

$$Z_{R1} = 1;$$

$$Z_c = \frac{1}{sC} = \frac{10}{s};$$

$$Z_{R2} = 2;$$

Applying principle of superposition in the given circuit, we obtain,

$$\begin{aligned} I_C(s) &= I_s \frac{\left((Z_{R2} + Z_c) \parallel Z_{R1} \right)}{Z_{R2} + Z_c} - \frac{V_s(s)}{Z_{R1} + Z_c + Z_{R2}} \\ &= \frac{2}{s} \frac{\left(2 + \frac{10}{s} \right)}{\left(2 + \frac{10}{s} + 1 \right)} - \frac{\frac{2}{s}}{2 + \frac{10}{s} + 1} \\ &= 0 \end{aligned}$$

$$\therefore i_c(t) = 0 \text{ A};$$

$$(b) v_s(t) = te^{-t}u(t) \text{ V}; V_s = \frac{1}{(s+1)^2}$$

Applying principle of superposition in the given circuit, we obtain,

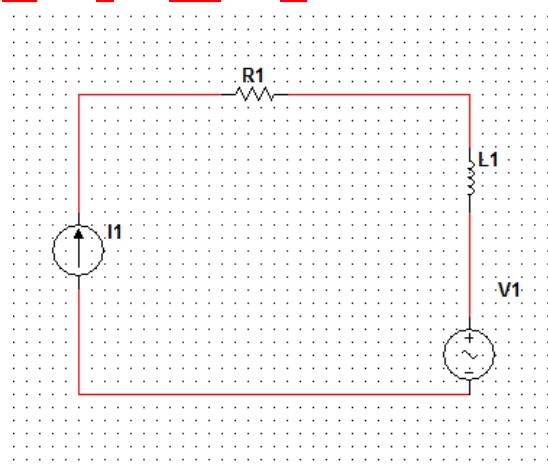
$$\begin{aligned} I_C(s) &= I_s(s) \frac{\left((Z_{R2} + Z_c) \parallel Z_{R1} \right)}{Z_{R2} + Z_c} - \frac{V_s(s)}{Z_{R1} + Z_c + Z_{R2}} \\ &= \frac{2}{s} \frac{\left(2 + \frac{10}{s} \right)}{\left(2 + \frac{10}{s} + 1 \right)} - \frac{\frac{1}{(s+1)^2}}{2 + \frac{10}{s} + 1} \\ &= \frac{2}{3s+10} - \frac{s}{(s+1)^2(3s+10)} \\ &= \frac{2}{3s+10} - \frac{1}{(s+1)(3s+10)} + \frac{1}{(s+1)^2(3s+10)} \\ &= \frac{2}{3s+10} + \frac{3}{7(3s+10)} - \frac{1}{7(s+1)} - \frac{3}{49(s+1)} + \frac{1}{7(s+1)^2} + \frac{9}{49(3s+10)} \\ &= \frac{128}{147(s+\frac{10}{3})} - \frac{10}{49(s+1)} + \frac{1}{7(s+1)^2} \end{aligned}$$

$$\therefore i_c(t) = \left(\frac{128}{147} e^{-\frac{10t}{3}} - \frac{10}{49} e^{-t} + \frac{1}{7} t e^{-t} \right) u(t) \text{ A}$$

$$65. (a) i(0) = \lim_{s \rightarrow \infty} sI(s) = 5 \text{ A}$$

$$(b) v(\infty) = \lim_{s \rightarrow 0} s^2 I(s) = 0$$

1. $I_1 = \boxed{2A}$, $R_1 = \boxed{4}$, $L_1 = \boxed{1.5s}$, $V_1 = \boxed{-3}$

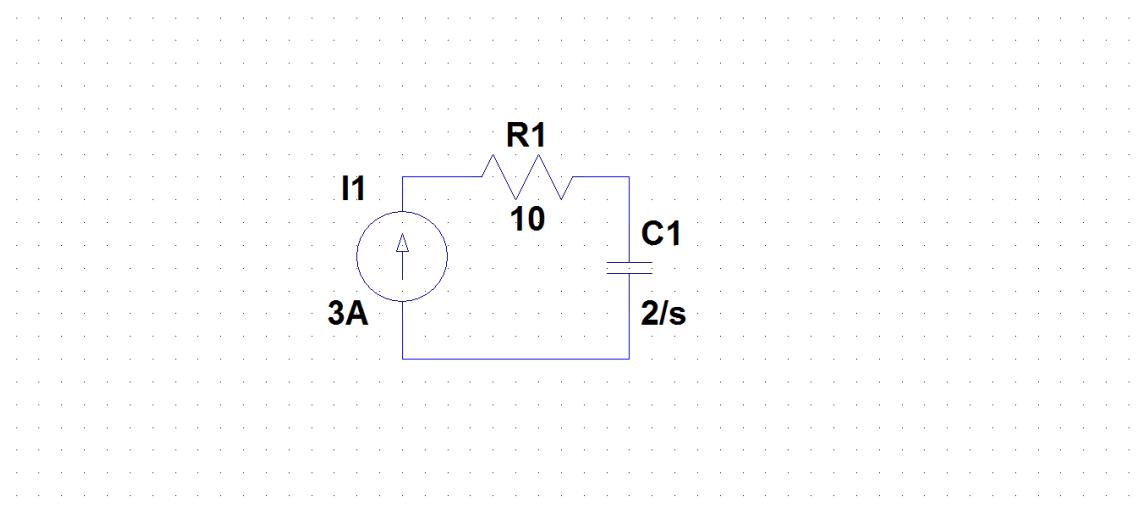


2. $R = 10\Omega, C = 500mF, I_1 = 3A$

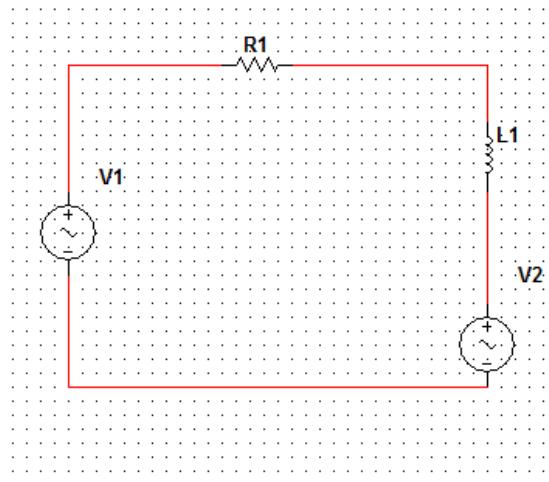
$$t = 0^-$$

$$v_C(0^-) = v_C(0^+) = v_C(0) = 0$$

$$R = 10\Omega, Z_C = 1/(500 \times 10^{-3}) s$$



3. $V_1 = 3 \frac{1.5}{s+1} + \frac{2}{s}, R_1 = 2.7, L_1 = 1.1s, V_2 = -1.1i(0)$



(a) $I(s) = \frac{V_1}{2.7 + 1.1s}, i(t) = (0.94e^{-t} - 1.68e^{-2.46t} + 0.74)u(t)$

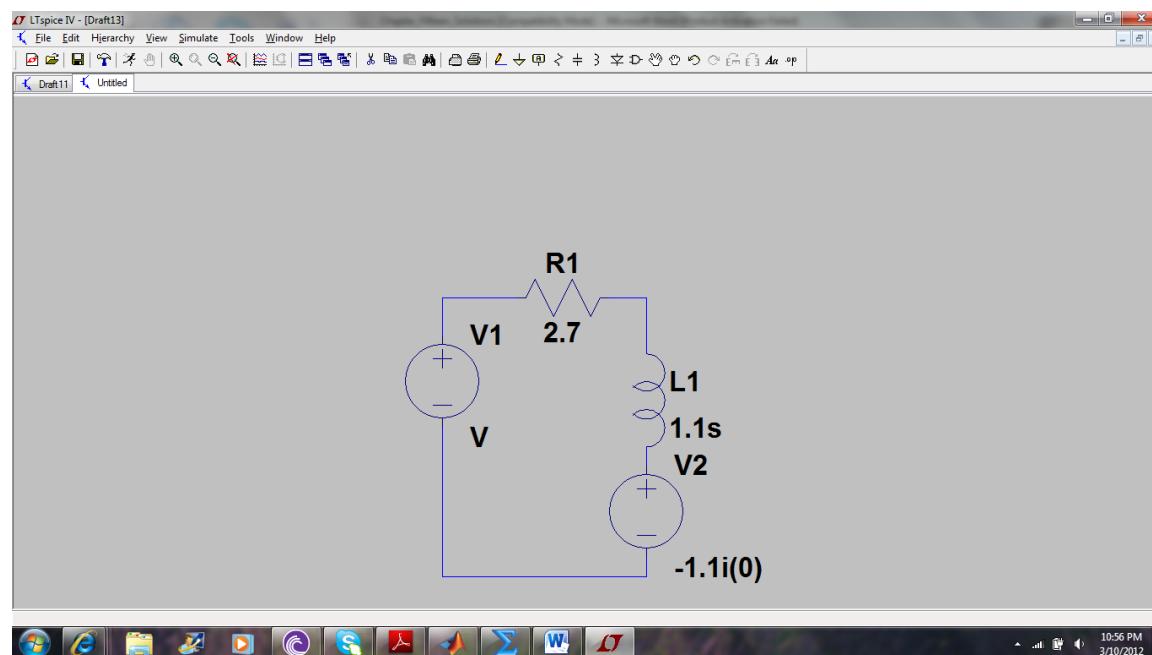
(b) $I(s) = \frac{V_1 - V_2}{2.7 + 1.1s}, i(t) = (0.94e^{-t} - 3.68e^{-2.45t} + 0.74)u(t)$

$$4. R = 2.7\Omega, L = 1.1H, v_s = 1.5e^{-t}u(t) + 2u(t)V$$

$$(a) i(0) = 0$$

$$V_s(s) = \frac{1.5}{s+1} + \frac{2}{s}$$

$$R_1 = 2.7\Omega, L_1 = 1.1s, V_2 = -1.1i(0^-)$$



$$I(s) = \frac{V_1}{R_1 + sL} = \frac{\frac{1.5}{s+1} + \frac{2}{s}}{2.7 + 1.1s} = \frac{3.5(s+0.57)}{1.1s(s+1)(s+2.45)}$$

$$V(s) = 1.1sI(s) = \frac{3.5(s+0.57)}{(s+1)(s+2.45)} = \frac{A}{(s+1)} + \frac{B}{s+2.45}$$

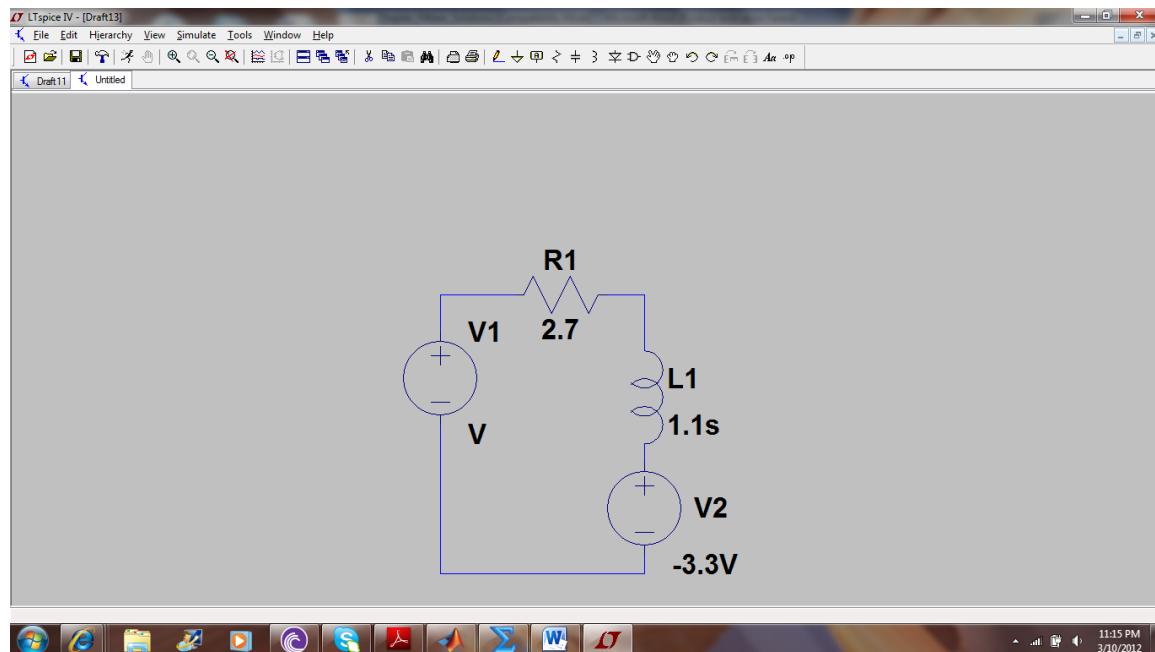
$$A = \frac{3.5(s+0.57)}{(s+2.45)} \Big|_{s=-1} = -1.0345$$

$$B = \frac{3.5(s+0.57)}{(s+1)} \Big|_{s=-2.45} = 345$$

$$V(s) = \frac{-1.0345}{(s+1)} + \frac{4.5345}{s+2.45}$$

$$v(t) = \left[-1.0345e^{-t} + 4.5345e^{-2.45t} \right] u(t)$$

(b) $i(0) = 3A$



$$I(s) = \frac{V_1 + 3.3}{R_L + sL} = \frac{\frac{1.5}{s+1} + \frac{2}{s}}{2.7 + 1.1s} = \frac{3.3s^2 + 6.8s + 2}{1.1s(s+1)(s+2.45)}$$

$$V(s) = 1.1sI(s) - 3.3 = \frac{-4.585s - 6.085}{(s+1)(s+2.45)} = \frac{A}{(s+1)} + \frac{B}{s+2.45}$$

$$A = \left. \frac{-4.585s - 6.085}{(s+2.45)} \right|_{s=-1} = -3.551$$

$$B = \left. \frac{-4.585s - 6.085}{(s+1)} \right|_{s=-2.45} = -1.0345$$

$$V(s) = \frac{-3.551}{(s+1)} + \frac{-1.0345}{s+2.45}$$

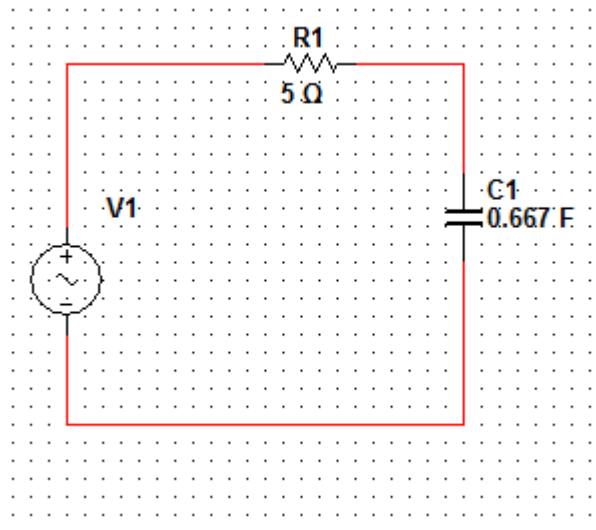
$$v(t) = \boxed{[-3.551e^{-t} - 1.0345e^{-2.45t}]u(t)}$$

$$5. -\frac{sVc}{1.5} + 2 + \frac{\frac{2}{s}-Vc}{5} = 0$$

(a) $Vc(s) = \frac{10s+2}{s+3}$

(b) $v_c(t) = 31.2e^{-31.3t}u(t) + 10\delta(t)$

(c) $V1=2u(t)$

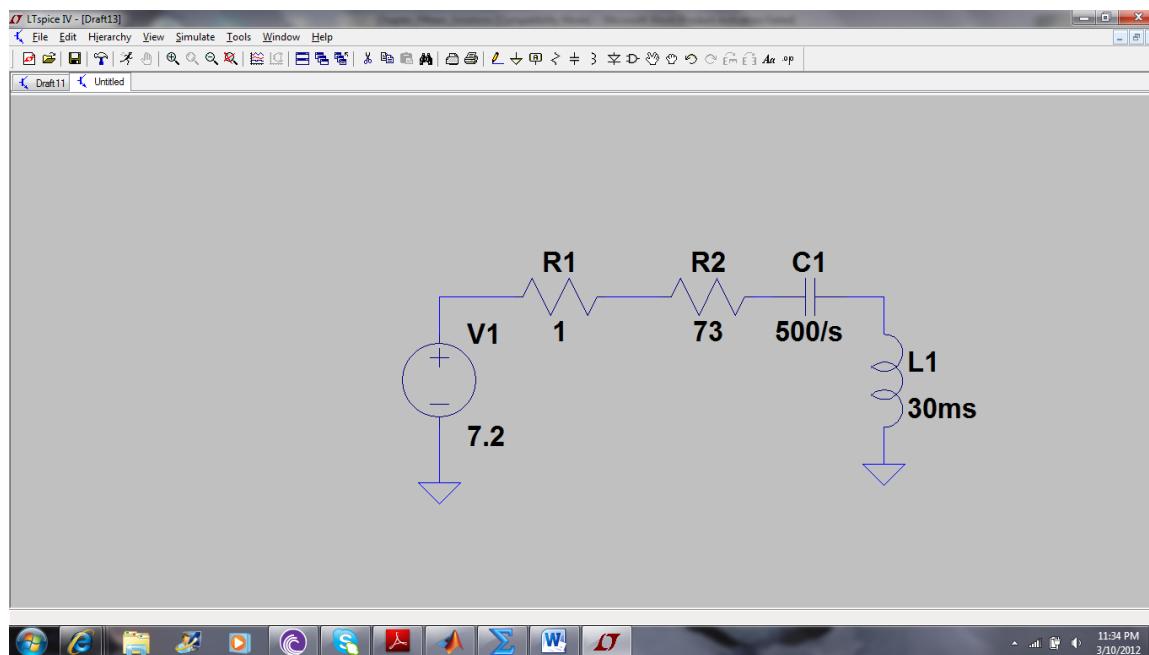


$$6. i_L(0^-) = 0, v_C(0^-) = 0$$

R_1, R_2 Will remain the same in the s-domain

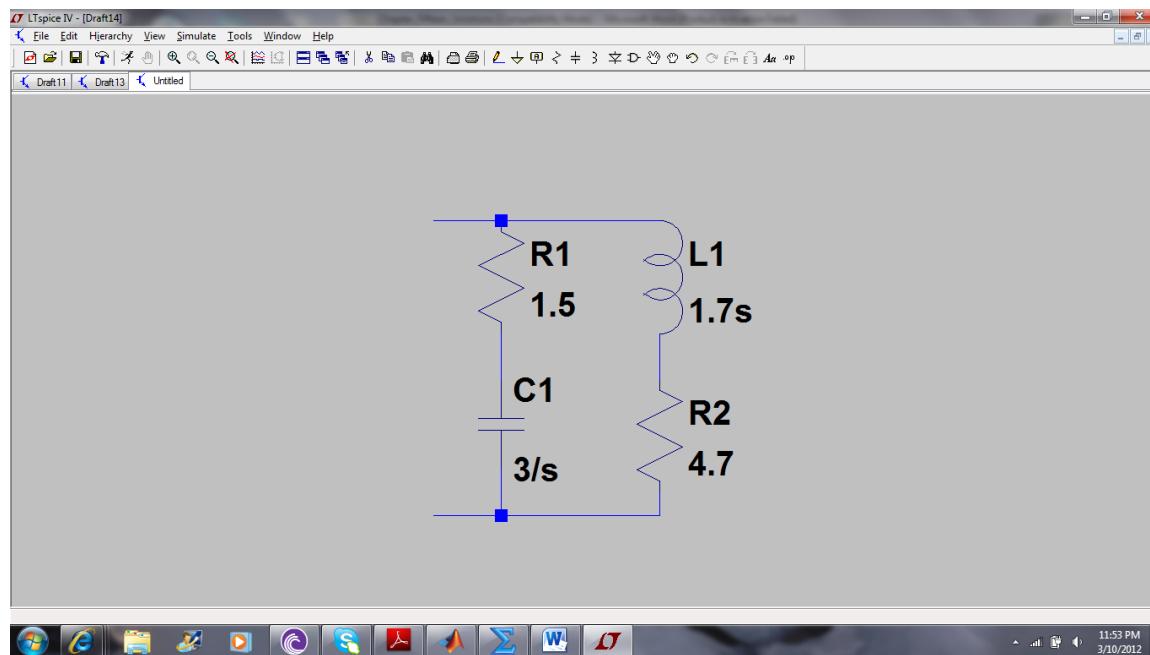
$$C \rightarrow 1/Cs = 500/s$$

$$L \rightarrow sL = 30 \times 10^{-3} s$$



$$7. Z_s = 0.5s + 3.3 \left| \frac{4}{s} \right| = 0.5s + \frac{13.2s}{3.3s+4}$$

8. Find input admittance $Y(s)$



$$Z_{eq} = Z_1 // Z_2$$

$$Z_1 = 4.7 + 1.7s$$

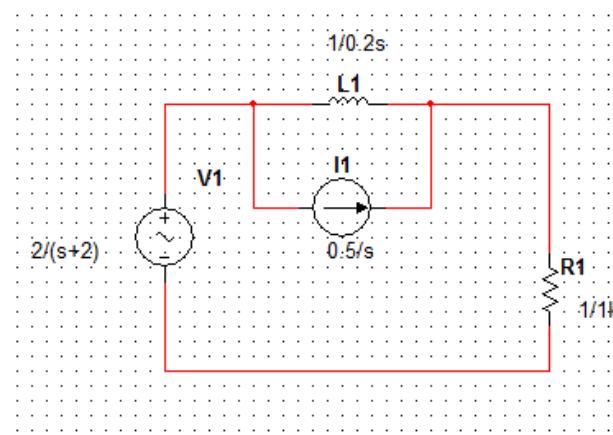
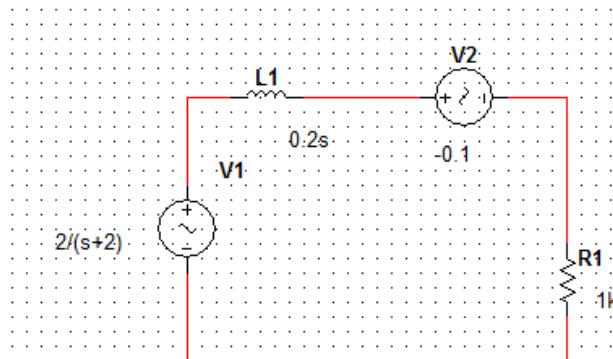
$$Z_2 = 1.5 + 3/s$$

$$Z_{eq} = Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(4.7 + 1.7s)(1.5 + 3/s)}{(4.7 + 1.7s) + (1.5 + 3/s)}$$

$$Z_{eq} = \frac{2.55s^2 + 7.05s + 19.2}{1.7s^2 + 6.2s + 3}$$

$$Y_{eq} = 1/Z_{eq} = \frac{1.7s^2 + 6.2s + 3}{2.55s^2 + 7.05s + 19.2}$$

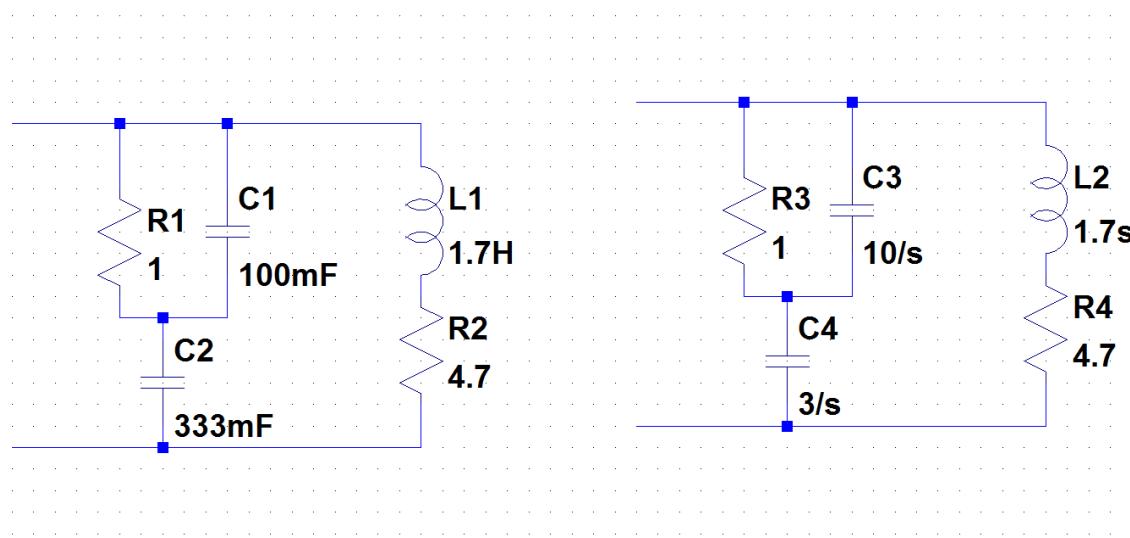
9. (a)



$$(b) I_s = \frac{\frac{2}{s+2} + 0.1}{0.2s + 1000}, V_s = I_s * 0.2s = \boxed{\frac{\frac{2}{s+2} + 0.1}{0.2s + 1000} * 0.2s}$$

$$(c) \boxed{v(t) = 0.1\delta(t) - (498e^{-5000t} - 0.0008e^{-2t})u(t)}$$

10.



$$Z_1 = 1\Omega // (10/S) = \frac{10}{S+10}$$

$$Z_2 = Z_1 + \frac{3}{s} = \frac{13s+3}{s(s+1)}$$

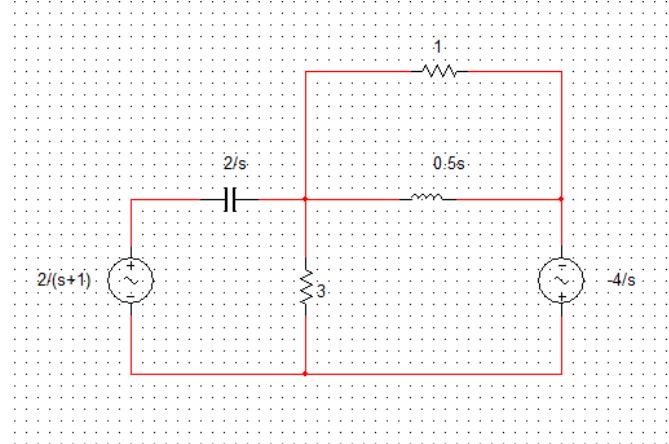
$$Z_3 = 1.7s + 4.7$$

$$Z_{in} = Z_2 // Z_3$$

$$Z_{in} = \frac{13(s+3)(1.7s+4.7)}{(13s+3) + s(1.7s+4.7)(s+1)}$$

11.

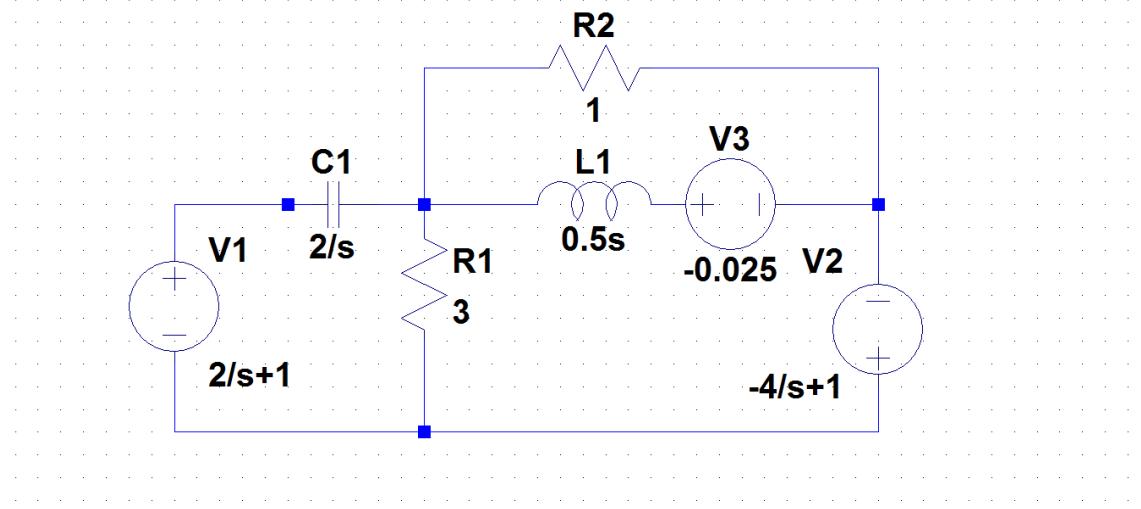
(a)



$$(c) \begin{cases} \frac{2}{s+1} + \frac{2}{s}I_1 + 3(I_1 - I_3) = 0 \\ I_2 + 0.5s(i_2 - i_3) = 0 \\ -\frac{4}{3} + 3I_3 + 0.5s(I_3 - I_2) = 0 \end{cases}$$

$$(d) \begin{cases} i_1 = (2.9e^{-0.67t} - 0.6e^{-1.5t} - 2e^t)u(t) \\ i_2 = e^{-1.5t}u(t) \\ i_3 = (1.33 - 0.33e^{-1.5t})u(t) \end{cases}$$

12.



Mesh 1:

$$\frac{-2}{s+1} + \frac{2}{s} I_1 + 3(I_1 - I_2) = 0$$

Mesh 2:

$$3(I_2 - I_1) + [0.5s(I_2 - I_3) + (-0.025)] - \frac{-4}{s+1} = 0$$

Mesh 3:

$$-(-0.025) + 0.5s(I_3 - I_2) + 1(I_3) = 0$$

$$I_1 = \frac{2(3s^2 + 4s)}{(s+1)(3s+2)}$$

$$I_2 = \frac{0.025(360s^3 + 963s^2 - 318)}{(s+1)(2s+3)(3s+2)}$$

$$I_3 = \frac{-0.025(-360s^3 - 231s^2 + 175s + 6)}{(s+1)(2s+3)(3s+2)}$$

$$i_1(t) = 2\delta(t) - 2.67e^{-0.6t} + 2e^{-t}$$

$$i_2(t) = 1.5\delta(t) + 1.4625e^{-1.5t} - 3.2e^{-0.67t} + e^{-t}$$

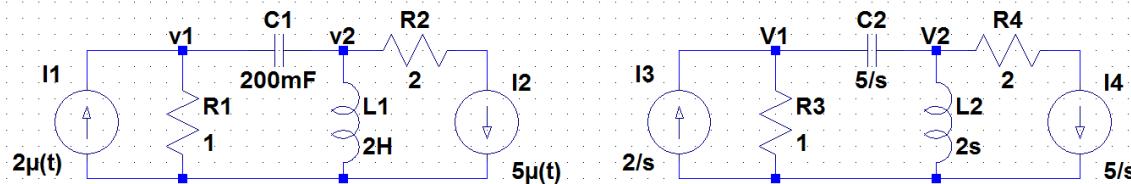
$$i_3(t) = 1.5\delta(t) - 4.3875e^{-1.5t} + 1.6e^{-0.67t} - e^{-t}$$

$$13. (a) \frac{Vx}{5} + \frac{Vx+\frac{2}{s}}{0.8s} + \frac{(Vx-\frac{3}{s})s}{4} = 0$$

$$(b) Vx = \frac{15s^2 - 50}{5s^3 + 4s^2 + 25s},$$

$$vx(t) = (5(\cos 2.27 - 0.036 \sin 2.2t)e^{-0.4t} - 2)u(t)$$

14.



At node 1:

$$\frac{2}{s} = \frac{V_1}{1} + \frac{(V_1 - V_2)}{5/s} \Rightarrow \frac{2}{s} = \frac{V_1}{1} + \frac{(V_1 - V_2)s}{5}$$

At node 2:

$$\frac{5}{s} = \frac{V_2}{2s} + \frac{(V_2 - V_1)}{5/s} \Rightarrow \frac{5}{s} = \frac{V_2}{2s} + \frac{(V_2 - V_1)s}{5}$$

```
>> eqn1='(V1/1)+(V1-V2)/(5/s)=2/s';
```

```
eqn2='(V2/2*s)+((V2-V1)*s)/5=5/s';
```

```
solution=solve(eqn1,eqn2,'V1','V2');
```

```
V1=solution.V1
```

```
V1 =
```

```
24/(s*(s + 7))
```

```
>> v1=ilaplace(V1)
```

```
v1 =
```

```
24/7 - 24/(7*exp(7*t))
```

```
>> V2=solution.V2
```

V2 =

$$(2*(7*s + 25))/(s^2*(s + 7))$$

>> v2=ilaplace(V2)

v2 =

$$(50*t)/7 - 48/(49*exp(7*t)) + 48/49$$

$$V_1 = \frac{24}{s(s+7)}$$

$$V_2 = \frac{2(7s+25)}{s^2(s+7)}$$

$$v_1(t) = \frac{24}{7} - \frac{24}{7}e^{-7t}$$

$$v_2(t) = \frac{50t}{7} - \frac{48}{49}e^{-7t} + \frac{48}{49}$$

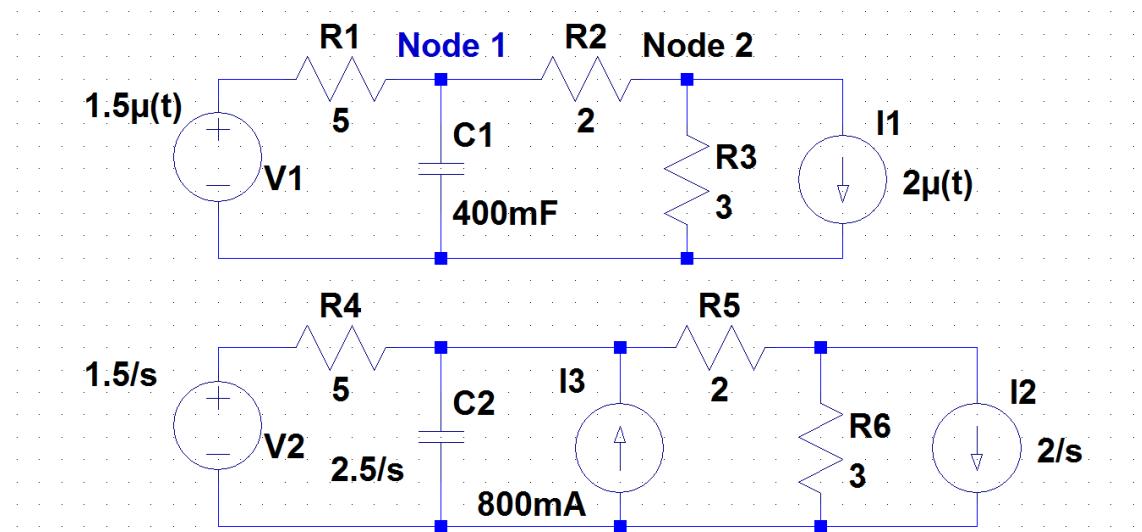
$$15. \begin{cases} \frac{4}{s+1} = \frac{V_1}{1} + \frac{(V_1-V_2)s}{5} \\ \frac{V_2}{2s} + \frac{(V_2-V_1)s}{5} = \frac{5}{s} \end{cases}$$

$$\text{so, } \begin{cases} V_1 = \frac{18s^2 + 10s + 20}{2s^3 + 3s^2 + 6s + 5} \\ V_2 = \frac{18s^2 + 60s + 50}{2s^3 + 3s^2 + 6s + 5} \end{cases}$$

$$v_1(t) = \left(\frac{14}{3}e^{-t} + \left(\frac{13}{169} \cos\left(\frac{\sqrt{39}}{4}t\right) - \frac{11\sqrt{39}}{169} \sin\left(\frac{\sqrt{39}}{4}t\right) \right) \frac{1}{3}e^{-0.25t} \right) u(t)$$

$$P = \frac{v_1(t)^2}{R} = v_1(t)^2$$

16.



At node 1:

$$\frac{V_1 - 1.5/s}{5} + 800 \times 10^{-3} + \frac{V_2 - V_1}{2} + \frac{V_1}{s \cdot 5/s} = 0$$

At node 2:

$$\frac{V_2}{3} + \frac{V_2 - V_1}{2} = \frac{2}{s}$$

$$V_1 = \frac{-0.25(8s - 15)}{s(s+1)}$$

$$V_2 = \frac{0.15(8s + 31)}{s(s+1)}$$

$$v_2(t) = 4.65 - 3.45e^{-t}$$

$$P = \frac{v_2^2(t)}{R} = \frac{(4.65 - 3.45e^{-t})^2}{3}$$

Solving in Matlab:

```
>> eqn1='(V1/(2.5/s))+((V1-(1.5/s))/5)+((V1-V2)/2)=-800e-3'
```

```
eqn1 =
```

```
(V1/(2.5/s))+((V1-(1.5/s))/5)+((V1-V2)/2)=-800e-3
```

```
>> eqn2='(V2/3)+((V2-V1)/2)=(2/s)'
```

```
eqn2 =
```

```
(V2/3)+((V2-V1)/2)=(2/s)
```

```
>> solution=solve(eqn1,eqn2,'V1','V2');
```

```
>> V1=solution.V1
```

```
V1 =
```

```
-(0.25*(8.0*s - 15.0))/(s*(s + 1.0))
```

```
>> V2=solution.V2
```

```
V2 =
```

```
(0.15*(8.0*s + 31.0))/(s*(s + 1.0))
```

```
>> v2=ilaplace(V2)
```

```
v2 =
```

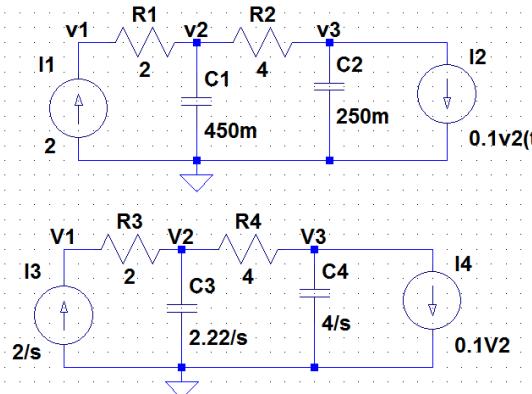
```
4.65 - 3.45/exp(1.0*t)
```

$$17. \left\{ \begin{array}{l} \frac{3}{s} = \frac{V_1}{5} + \frac{(V_1 - V_2)s}{10} \\ \frac{10}{s^2 + 4} = \frac{V_2}{2s} + \frac{(V_2 - V_1)s}{10} \end{array} \right. \text{ SO } \left\{ \begin{array}{l} V_1 = \frac{30s^4 + 100s^3 + 270s^2 + 600}{2s^5 + 5s^4 + 18s^3 + 20s^2 + 40s} \\ V_2 = \frac{30s^3 + 100s^2 + 320s}{2s^4 + 5s^3 + 18s^2 + 20s + 40} \end{array} \right.$$

then

$$vx(t) = v_2(t) = \left(\frac{300}{13} \cos 2t + \frac{289}{13} \sin 2t - \left(\frac{105}{231} \cos \left(\frac{\sqrt{55}t}{4} \right) + \frac{59\sqrt{55}}{231} \sin \left(\frac{\sqrt{55}t}{4} \right) \right) \frac{1}{13} e^{-1.25t} \right) u(t)$$

18. (a)



(b)

At node 1:

$$\frac{2}{s} = \frac{V_1 - V_2}{2} \Rightarrow V_1 - V_2 = \frac{4}{s} \quad (1)$$

At node 2:

$$0 = \frac{V_2 - V_1}{2} + \frac{V_2}{2.22/s} + \frac{V_2 - V_3}{4} \quad (2)$$

$$0 = \frac{-V_1}{2} + \left(\frac{3}{4} + \frac{s}{2.22} \right) V_2 - \frac{V_3}{4}$$

At node 3:

$$-0.1V_2 = \frac{V_3}{4/s} + \frac{V_3 - V_2}{4} \quad (3)$$

From eq. (1)

$$V_1 = \frac{4}{s} + V_2$$

$$\text{from}(3) \Rightarrow V_3 = \frac{0.6}{s+1} V_2$$

$$V_2 = \frac{\frac{2}{s}}{\frac{1}{2} + \frac{s}{2.22} - \frac{0.15}{s+1}}$$

Solving in Matlab:

```
>> syms s;
```

```
>> V2=(2/s)/(0.5+(s/2.22)-(0.15/s+1));
```

```
>> v2=ilaplace(V2)
```

```
v2 =
```

$$(8*2849^{(1/2)}*\exp((111*t)/200)*\sinh((3*2849^{(1/2)}*t)/200))/77$$

```
>> V1=(4/s)+V2
```

```
V1 =
```

$$4/s - 2/(s*(3/(20*s) - (50*s)/111 + 1/2))$$

```
>> v1=ilaplace(V1)
```

```
v1 =
```

$$(8*2849^{(1/2)}*\exp((111*t)/200)*\sinh((3*2849^{(1/2)}*t)/200))/77 + 4$$

```
>> V3=(0.6/(s+1))*V2
```

```
V3 =
```

$$-6/(5*s*(s + 1)*(3/(20*s) - (50*s)/111 + 1/2))$$

```
>> v3=ilaplace(V3)
```

```
v3 =
```

$$\frac{2664/(1777*\exp(t)) - (2664*\exp((111*t)/200)*(cosh((3*2849^{(1/2)}*t)/200) - (311*2849^{(1/2)}*\sinh((3*2849^{(1/2)}*t)/200))/8547))}{1777}$$

19.
$$\begin{cases} -\frac{6(0.45+0.22s)}{s^2+4} + \frac{1333}{s}(I_1 - I_2) + 2(I_1 - I_3) = 0 \\ 0.005I_1 + 0.001s(I_2 - I_4) - 0.001 + \frac{1333}{s}(I_2 - I_1) = 0 \\ \frac{1000}{s}(I_4 - I_3) + 0.001 + 0.001s(I_4 - I_2) = 0 \\ 2(I_3 - I_1) + \frac{1000}{s}(I_3 - I_4) + \frac{6s}{s^2+4} = 0 \end{cases}$$

so,
$$I_2 = \frac{I_1 = -\frac{936s-540}{s^2+4}}{\frac{234s^4-10^8s^3-3.1\times10^{10}s^2-6.2\times10^{13}s-3.6\times10^{13}}{1.1\times10^5s^4+3.3\times10^7s^3+6.7\times10^{10}s^2+1.3\times10^8s+2.7\times10^{11}}}$$

so, $i_1(t) = 936\cos 2t - 270\sin 2t$

20.

$$5V_2 + 3V_2 + \frac{V_1 - 14}{s^2} + \frac{V_1 - V_2}{1/500 \times 10^{-6} s} = 0$$

$$\frac{V_2 - V_1}{1/500 \times 10^{-6} s} + \frac{V_2}{2 \times 10^{-3} s} = 5V_2$$

$$V_2 = \frac{140}{s(5 \times 10^5 s^2 + 18.34 \times 10^6 s - 4.99 \times 10^4)}$$

$$v_2(t) = 0.0027998 e^{0.002726t} + 0.208 \times 10^{-6} e^{-36.676t} - 0.0028$$

(a)

$$v_2(1ms) = 8.14 \times 10^{-9}$$

(b)

$$v_2(100ms) = 5.686 \times 10^{-7}$$

(c)

$$v_2(10s) = 7.717 \times 10^{-5}$$

$$21. U_{eq} = \frac{12s^2}{(s+1)(s+2)(34s^2+47s+30)}$$

$$Z_{eq} = \frac{12(34s + 35)}{34s^2 + 47s + 30}$$

22. (a) $Z=2\Omega$

Transform the voltage source and 20Ω into current source

$$I_1 = \frac{s}{20(s+1)(s+2)}$$

$$Z_1 = 20\Omega / 8s = \frac{160s}{20+8s}$$

$$V_2(s) = I_1(s)Z_1 = \frac{8s^2}{(s+1)(s+2)(8s+20)}$$

$$Z_2 = Z_1 + 14 = \frac{160s}{20+8s} + 14$$

$$I_2 = \frac{V_2(s)}{Z_2} = \frac{8s^2}{(272s+280)(s+1)(s+2)}$$

$$Z_3 = Z_2 // (12/s) = \frac{12(272s+280)}{12(8s+20)+s(272s+280)}$$

$$V_3 = I_2 Z_3 = \frac{96s^2}{(s+1)(s+2)[12(8s+20)+s(272s+280)]}$$

$$I(s) = \frac{V_3}{Z + Z_3}, Z = 2\Omega$$

$$I(s) = \frac{96s^2}{(s+1)(s+2)[12(8s+20)+s(272s+280)]} \times \frac{1}{2 + \frac{12(272s+280)}{12(8s+20)+s(272s+280)}}$$

(b)

$$I(s) = \frac{V_3}{Z + Z_3}, Z = 1/2s\Omega$$

$$I(s) = \frac{96s^2}{(s+1)(s+2)[12(8s+20)+s(272s+280)]} \times \frac{1}{\frac{1}{2s} + \frac{12(272s+280)}{12(8s+20)+s(272s+280)}}$$

(c)

$$I(s) = \frac{V_3}{Z + Z_3}, Z = s + \frac{1}{2s} + 3\Omega$$

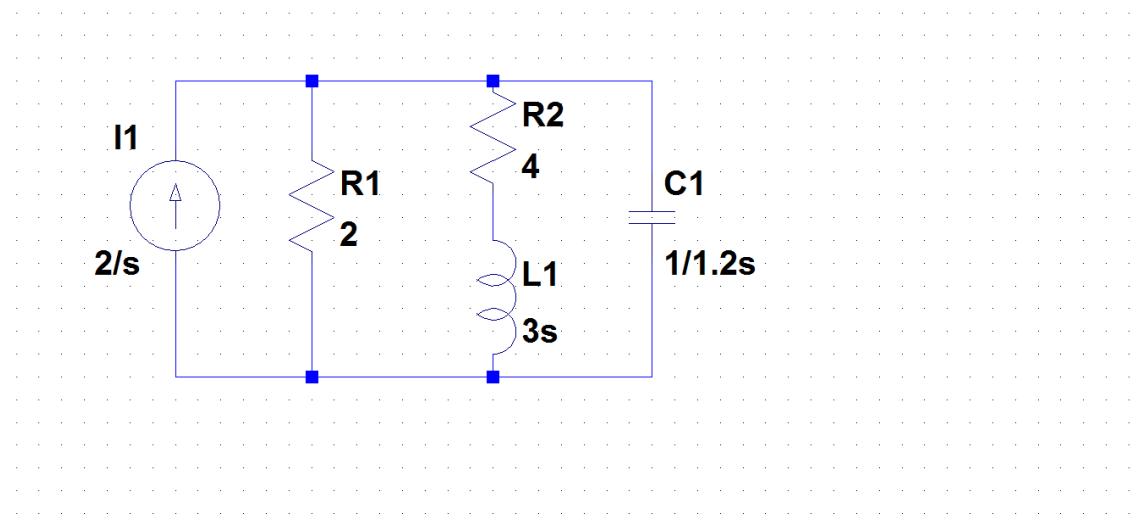
$$I(s) = \frac{96s^2}{(s+1)(s+2)[12(8s+20)+s(272s+280)]} \times \frac{1}{\left(s + \frac{1}{2s} + 3\right) + \frac{12(272s+280)}{12(8s+20)+s(272s+280)}}$$

23. (a) $U_{eq} = \frac{10(3s+4)}{s(18s^2+24s+5)}$; $Z_{eq} = \frac{5(3s+4)}{18s^2+24s+5}$;

(b) $U_{eq} = \frac{2(15s+46)}{5s+12}$; $Z_{eq} = \frac{s(15s+46)}{5s+12}$

(c) $U_{eq} = \frac{2(6s+8)}{s(3s+6)}$; $Z_{eq} = \frac{6s+8}{3s+6}$

24.



$$\frac{-4}{s} + 2(I + I_C) + (4 + 3s)(I - I_C) = 0$$

$$(4 + \frac{3}{s})(I_C - I) + I_C \times \frac{1}{1.2s} = 0$$

$$I = \frac{4(4 \times 10^{33}s + 3.83 \times 10^{33})}{s(1.85 \times 10^{34}s + 1.7 \times 10^{34})}$$

$$I = \frac{4(4 \times 10^{33}s + 3 \times 10^{33})}{s(1.85 \times 10^{34}s + 1.7 \times 10^{34})}$$

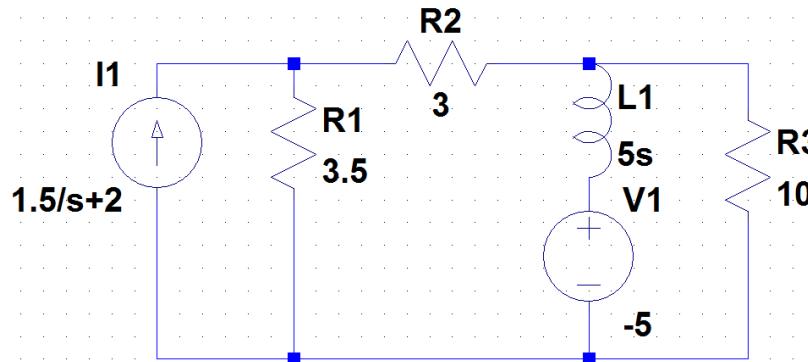
$$i_C(t) = 0.15898e^{-0.9189t} + 0.706$$

$$i(t) = 0.902 - 0.037e^{-0.9189t}$$

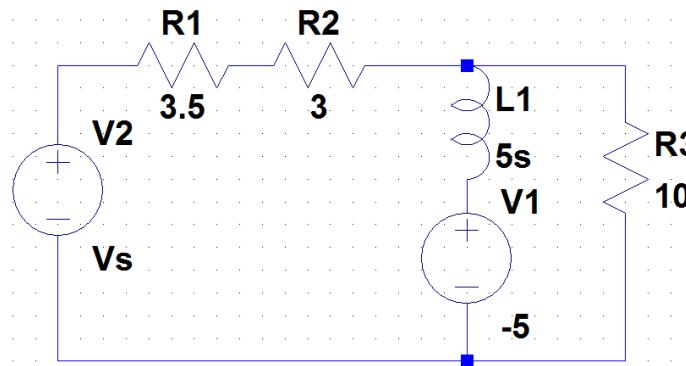
25. (a) $U_{eq} = \frac{17.5}{1.3+s}; Z_{eq} = \frac{6.5s}{1.3+s}$

(b) $I_s = \frac{350}{33s(5s + \frac{13}{9})}, i(t) = \frac{1050}{143} u(t)(1 - e^{\frac{13t}{45}})$

26.



$$V_s = 3.5I_s = \frac{5.25}{s+2}$$



$$-V_s + 6.5(I_L + I_x) + 5sI_L - 5 = 0$$

$$-5 + 5(I_x - I_L) + 10I_x = 0$$

$$I_x = \frac{0.25(20s^2 + 86s + 113)}{(15s + 26)(s + 2)}$$

$$i_x(t) = 1.5e^{-1.73t} + 0.33\delta(t) - 1.3125e^{-2t}$$

31.

	Zero	Pole
(a)	0	-12.5
(b)	0, -1	-5, -3
(c)	-4	-1, -7
(d)	2, -1	0,-7,-1

32.

(a) $s+4$

Zero at $s = -4$

Pole at ∞

(b)

$$\frac{2s}{s^2 - 8s + 16} = \frac{2s}{(s-4)(s+4)}$$

$zero = 0$

$poles = 4, -4$

(c)

$$\frac{4}{s^3 + 8s + 7}$$

$zero = \infty$

$poles$

$$s = 0.4044 + j2.9139$$

$$s = -0.8089$$

$$s = 0.4044 - j2.9139$$

(d)

$$\frac{s-5}{s^3 - 7s + 6} = \frac{s-5}{(s+3)(s-2)(s-1)}$$

$zero$

$$s = 5$$

$poles:$

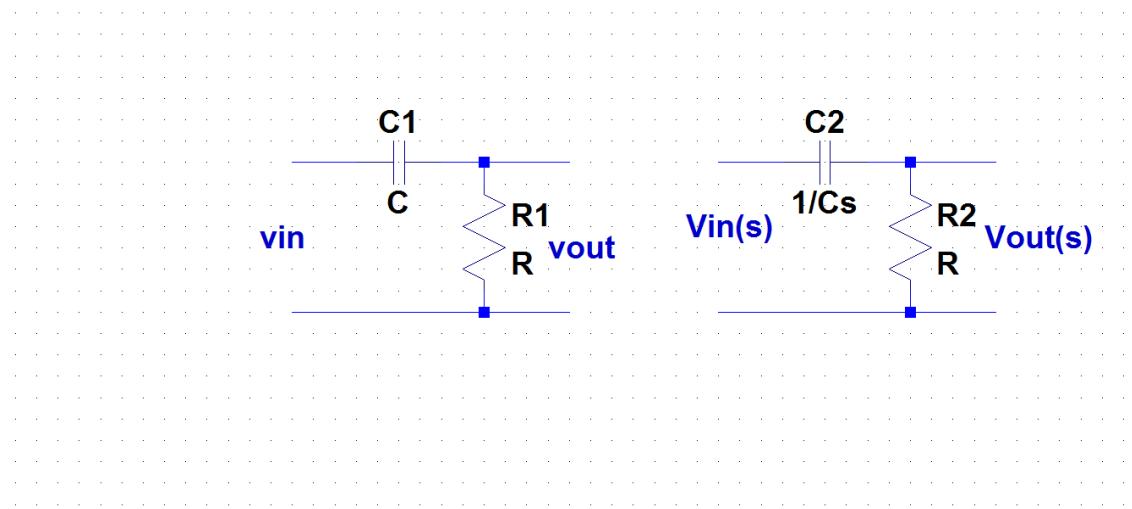
$$s = -3$$

$$s = 2$$

$$s = 1$$

33. (a) $s=0, s=-0.2;$
(b) $s=0, s=-4, s=-1, s=-3, s=-5;$
(c) $s=2, s=-2;$
(d) $s=1, s=-1, s=6, s=-6$

34.



(a)

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

$$0 = \frac{V_{out}(s)}{R} + \frac{V_{out} - V_{in}}{1/sC}$$

$$0 = \frac{V_{out}(s)}{R} + sCV_{out} - sCV_{in}$$

$$sCV_{in} = V_{out} \left(\frac{1}{R} + sC \right)$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{RCs}{1+RCs}}$$

(b)

Zero

$$RCs = 0$$

$$\boxed{s = 0}$$

Poles:

$$\boxed{1+RCs = 0}$$

$$\boxed{s = -1/RC}$$

35. (a) for circuit a:

$$\frac{V_{out}}{R} = \frac{V_{in} - V_{out}}{sL}, H = \frac{V_{out}}{V_{in}} = \frac{1}{\frac{sL}{R} + 1}$$

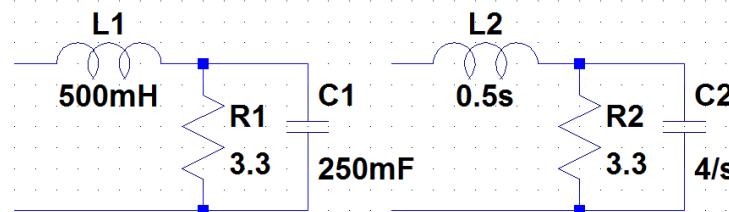
for circuit b:

$$\frac{V_{out}}{Ls} = \frac{V_{in} - V_{out}}{R}, H = \frac{1}{1 + \frac{R}{sL}}$$

(b)

	Zero	Pole
(a)	∞	$-R/L$
(b)	0,	$-R/L$

36.



$$Z_{in}(s) = 0.5s + 3.3 / (4/s)$$

$$Z_{in}(s) = 0.5s + \frac{3.3 \times (4/s)}{3.3 + (4/s)}$$

$$Z_{in}(s) = \frac{1.65s^2 + 2s + 13.2}{3.3s + 4}$$

Critical frequencies are the poles and the zeros

poles

$$s = -4/3.3$$

Zeros :

$$s_1 = -0.6061 + j2.7627$$

$$s_2 = -0.6061 - j2.7627$$

$$37. Y(s) = \frac{2650s+120}{(17s+47)(333s+200)} + \frac{1}{5}$$

zeros: s=-0.3285, -5.3774

poles: s=-47/17, -200/333

38. $H(s) = \frac{s}{s^2 + 8s + 7}$

(a) $v_{in}(t) = 3u(t)V$

$$V_{out}(s) = H(s)V_{in}(s)$$

$$V_{in}(s) = \frac{3}{s} \Rightarrow V_{out}(s) = \frac{s}{s^2 + 8s + 7} \times \frac{3}{s}$$

$$\therefore V_{out}(s) = \frac{3}{s^2 + 8s + 7}$$

(b) $v_{in}(t) = 25e^{-2t}u(t)$

$$V_{in}(s) = \frac{25}{s+2}$$

$$V_{out}(s) = H(s)V_{in}(s)$$

$$V_{out}(s) = \frac{25s}{(s+2)(s^2 + 8s + 7)}$$

(c) $v_{in}(t) = 4u(t+1)$

$$V_{in}(s) = 4 \times \frac{1}{s} \times e^s = \frac{4}{s} e^s$$

$$V_{out}(s) = H(s)V_{in}(s)$$

$$V_{out}(s) = \frac{4e^s}{(s^2 + 8s + 7)}$$

(d) $v_{in}(t) = 2\sin(5t)u(t)$

$$V_{in}(s) = 2 \times \frac{5}{s^2 + 25} = \frac{10}{s^2 + 25}$$

$$V_{out}(s) = H(s)V_{in}(s)$$

$$V_{out}(s) = \frac{10s}{(s^2 + 25)(s^2 + 8s + 7)}$$

$$39. H(s) = s + \frac{1}{s^2 + 23s + 60}$$

$$(a) Vin = \frac{2}{s} + 4, Vout = H(s) * Vin = \frac{2(s^3 + 23s^2 + 60s + 1)(2s + 1)}{s(s + 20)(s + 3)}$$

s=0, s=-20, s=-3; s=-0.5

$$(b) Vin = -\frac{5}{s+1}, Vout = \frac{5(s^3 + 23s^2 + 60s + 1)}{(s+20)(s+3)(s+1)}$$

s=-20, s=-3, s=-1

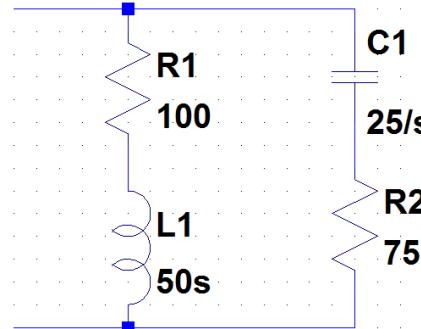
$$(c) Vin = \frac{4}{(s+2)^2}, Vout = H(s) * Vin = \frac{4(s^3 + 23s^2 + 60s + 1)}{(s+20)(s+3)(s+2)^2}$$

s=-20, s=-3, s=-2

$$(d) Vin = 5\sqrt{10} \frac{s+1}{(s+10)^2 + 25},$$

s=-20, s=-3, s=-5, s=-15, s=-10

40.



$$Z_1 = 75 + \frac{25}{s}$$

$$Z_2 = 100 + 50s$$

$$Z_{in} = Z_1 // Z_2 = \frac{(100 + 50s)(75 + \frac{25}{s})}{100 + 50s + 75 + \frac{25}{s}}$$

$$Z_{in} = \frac{(100 + 50s)(75s + 25)}{225s + 25}$$

Zeros :

$$s_1 = -100 / 50 = -2$$

$$s_2 = -25 / 75 = -0.333$$

Poles :

$$s = -25 / 225 = -0.111$$

41. $x(t)=u(t)$, $y(t)=u(t-1)$;

$$\text{So, } x(t) * y(t) = \int_{-\infty}^t u(t)u(t-z-1) dz$$

When $t < 1$, $x(t)*y(t)=0$; when $t > 1$, $x(t)*y(t)=t-1$

42. (a) $x(t) * x(t)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^t x(z)h(t-z)dz \quad \text{Equation (11)}$$

$$t < 0$$

$$x(t) * x(t) = 0$$

$$t > 0$$

$$x(t) * x(t) = \int_0^t (1 \times 1) dz = t$$

To check in frequency domain:

$$x(t) * x(t) = X(s) \times X(s) = \frac{1}{s^2}$$

$$\mathcal{L}^{-1}\{X^2(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$$

(b) $y(t) * \delta(t)$

$$y(t) = 3u(t-1)$$

$$y(t) * \delta(t) = \int_{-\infty}^t \delta(z)y(t-z)dz$$

$$43. f(t) = \begin{cases} 0, & t < 0 \\ 10t, & 0 \leq t < 2 \\ 20, & 2 \leq t < 4 \\ 10t - 20, & 4 \leq t < 6 \\ 40, & t \geq 6 \end{cases}$$

44.

$$h(t) = 2e^{-3t}u(t)$$

$$x(t) = u(t) - \delta(t)$$

$$y(t) = h(t) * x(t)$$

(a) convolution in the time domain

$$h(t) * x(t) = \int_{-\infty}^t x(z)h(t-z)dz$$

$$t < 0 \Rightarrow y(t) = 0$$

$$t > 0$$

$$y(t) = \int_{-\infty}^t [u(z) - \delta(z)](2e^{-3(t-z)})dz$$

$$y(t) = \left[\frac{2}{3} - \frac{8}{3}e^{-3t} \right] u(t)$$

$$(b) y(t) = \ell^{-1}\{X(s)H(s)\}$$

$$H(s) = \frac{2}{s+3}$$

$$X(s) = \ell\{u(t) - \delta(t)\} = \frac{1}{s} - 1 = \frac{1-s}{s}$$

$$Y(s) = X(s)H(s) = \frac{1-s}{s} \times \frac{2}{s+3} = \frac{2(1-s)}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$A = \frac{2}{3}, B = \frac{-8}{3}$$

$$Y(s) = \frac{2/3}{s} + \frac{-8/3}{s+3}$$

$$y(t) = \ell^{-1}\{Y(s)\} = \left[\frac{2}{3} - \frac{8}{3}e^{-3t} \right] u(t)$$

$$45. Z = \frac{5s+20}{s+9}, V_{in} = V_{out}$$

46.



$$Z_{in} = R + \frac{1}{Cs} = \frac{RCs + 1}{RCs}$$

$$Z_{in} = 2 + \frac{4}{s} = \frac{2s + 4}{s}$$

$$s = \sigma + j\omega$$

$$Z_{in} = \frac{2(\sigma + j\omega) + 4}{\sigma + j\omega}$$

$$|Z_{in}| = \frac{\sqrt{(2\sigma + 4)^2 + 4\omega^2}}{\sqrt{\sigma^2 + \omega^2}}$$

$$s = \sigma \Rightarrow |Z_{in}| = \frac{(2\sigma + 4)}{\sigma}$$

$$s = j\omega \Rightarrow |Z_{in}| = \frac{\sqrt{16 + 4\omega^2}}{\omega}$$

47. (a) $Z(\sigma) = \sigma^2 + \sigma$

(b) $Z(\omega) = -\omega^2 + j\omega, |Z| = \omega\sqrt{\omega^2 + 1}$

(c) $Z = (\sigma^2 + \sigma - \omega^2) + 2j\sigma\omega, |Z| = \sqrt{(\sigma^2 + \sigma - \omega^2)^2 + (2\sigma\omega)^2}$

48.

$$(a) \frac{s(s+4)}{(s+5)(s+2)}$$

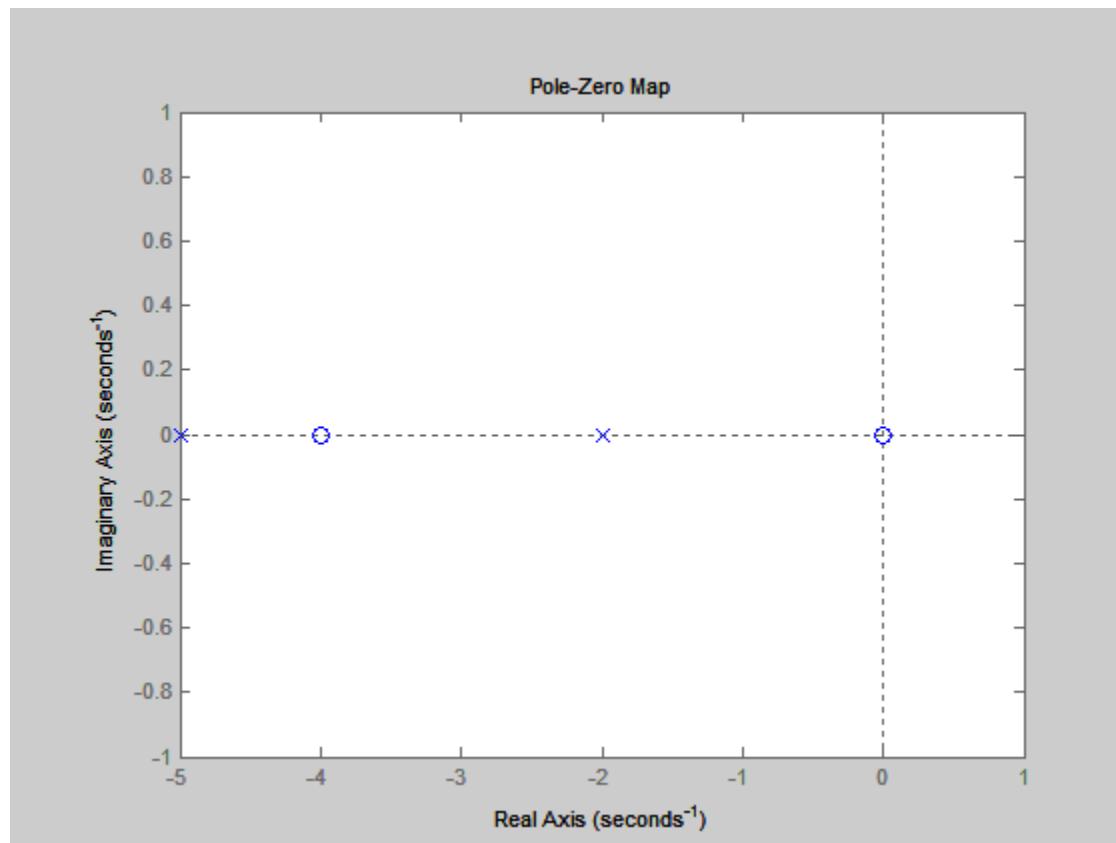
Zeros : $s = 0, s = -4$ *Poles :* $s = -5, s = -2$

```
H = tf([1 4 0], [1 7 10])
sgrid
pzmap(H)
```

Transfer function:

$$s^2 + 4s$$

$$s^2 + 7s + 10$$



$$(b) \frac{s-1}{s^2+8s+7}$$

Zeros: $s = 1$

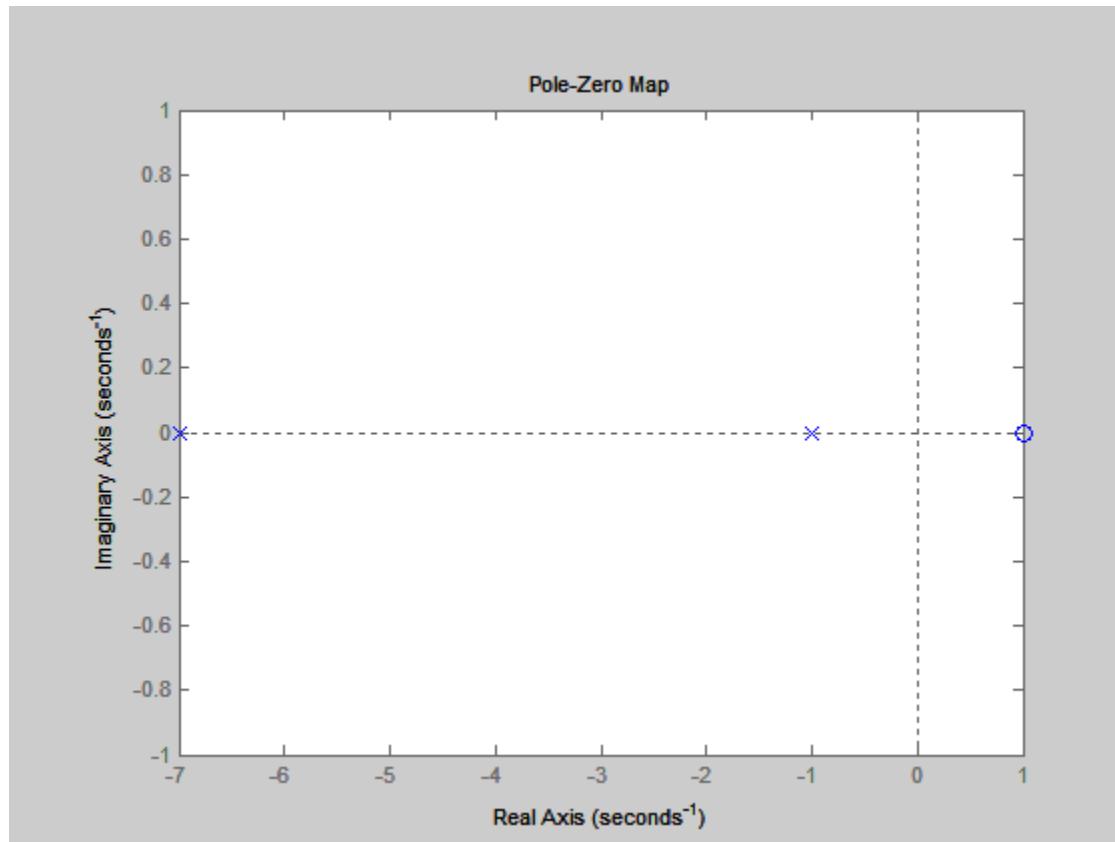
Poles: $s = -1, s = -7$

```
H = tf([1 -1], [1 8 7])
sgrid
pzmap(H)
```

Transfer function:

$$s - 1$$

$$s^2 + 8s + 7$$



$$(c) \frac{s^2 + 1}{s(s^2 + 10s + 16)}$$

Zeros: $s = \pm\sqrt{-1} = \pm j$

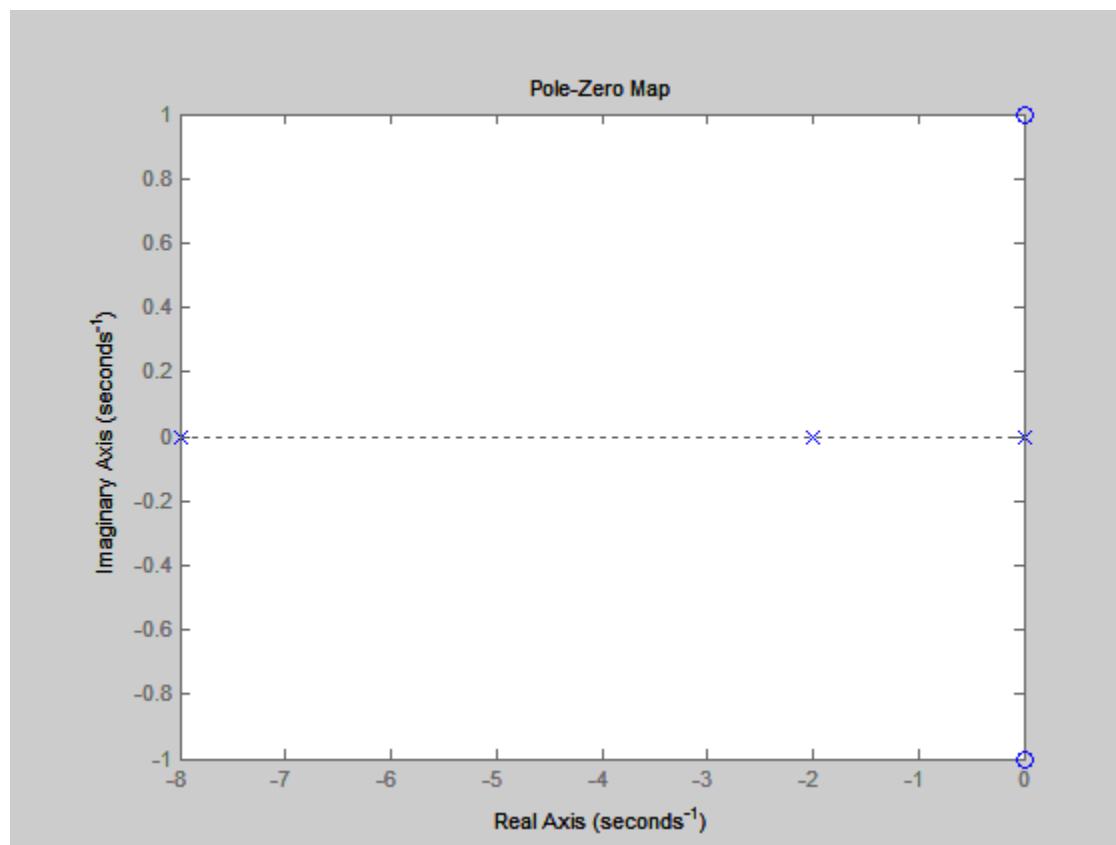
Poles: $s = 0, s = -2, s = -8$

```
H = tf([1 0 1], [1 10 16 0])
sgrid
pzmap(H)
```

Transfer function:

$$s^2 + 1$$

$$s^3 + 10s^2 + 16s$$



$$(d) \frac{5}{s^2 + 2s + 5}$$

Zeros: $s = \infty$

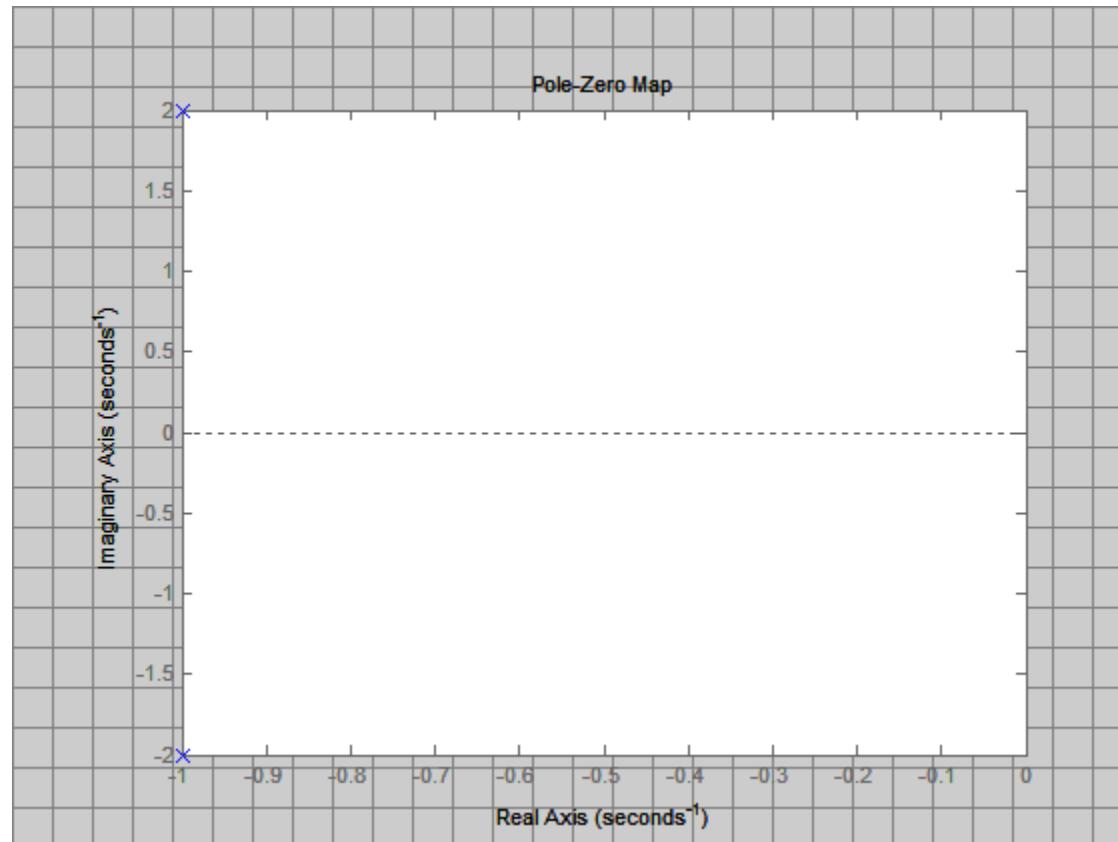
Poles: $s = -1 + 2j, s = -1 - 2j$

```
H = tf([5], [1 2 5])
sgrid
pzmap(H)
```

Transfer function:

5

$s^2 + 2s + 5$

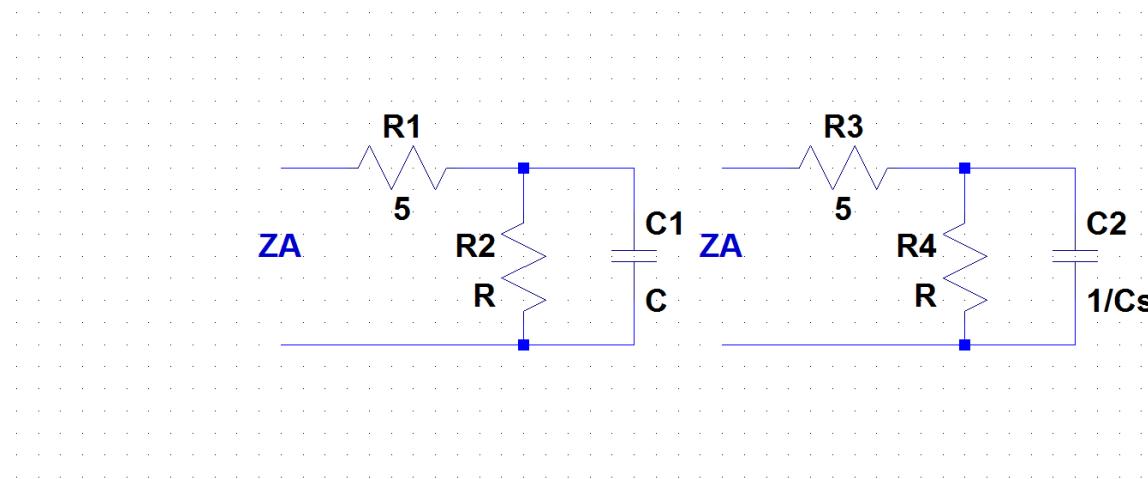


$$49. H(s) = \frac{k(s+2)}{s^2 + 2s + 5}$$

(a) $H(0) = 1, H(s) = \frac{5}{2} \frac{s+2}{s^2 + 2s + 5}$

(b) $H(0) = -5, H(s) = -\frac{25}{2} \frac{s+2}{s^2 + 2s + 5}$

50.



$$Z_A = \left(R // \frac{1}{C_s} \right) + 5 = \frac{(R+5) + 5RCs}{RCs + 1}$$

$$\text{Zero : } (R+5) + 5RCs = 0$$

$$s = \frac{-(R+5)}{5RC} = -10$$

$$20 + Z_A = 20 + \frac{(R+5) + 5RCs}{RCs + 1}$$

$$= \frac{25RCs + R + 25}{RCs + 1}$$

$$s_{\text{zero}} = \frac{-(R+25)}{25RC}$$

R = 20Ω

C = 0.25F

51.

(a)
$$H(s) = 100 \frac{s+2}{s^2+2s+5}$$

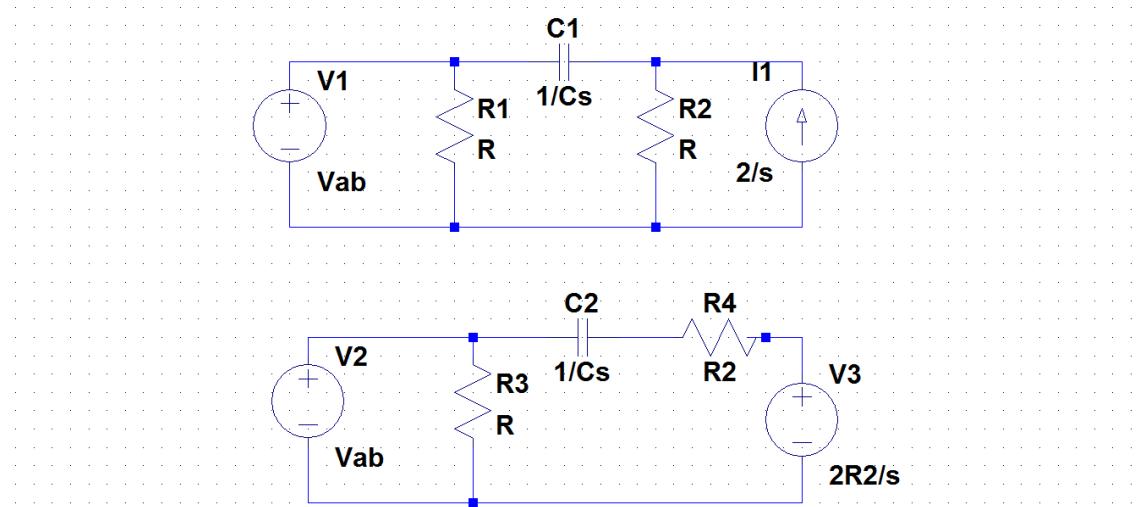
(b)
$$H(j\omega) = 100 \frac{j\omega+2}{-\omega^2+2j\omega+5}, |H(j\omega)| = 100 \sqrt{\frac{\omega^2+4}{\omega^4+14\omega^2+25}}$$

(e)
$$\omega_{max} = -1.67$$

$$53. \begin{cases} \left(4 + \frac{2}{s}\right)I_1 + 2s(I_1 - I_2) - 2 = 0 \\ 2I_2 + 2s(I_2 - I_1) + 2 = 0 \end{cases}$$

$$\text{So, } \begin{cases} I_1 = \frac{s}{3s^2+3s+1} \\ I_2 = -\frac{2s+1}{3s^2+3s+1} \end{cases}, \text{ and } i_2(t) = -\frac{2}{3} \cos\left(\frac{\sqrt{3}t}{6}\right) e^{-0.5t}$$

54.



$$Z_1 = R_2 + 1/C_s$$

$$V_{ab} = \frac{2R_2}{s} \times \frac{R_1}{R_1 + Z_1}$$

$$V_{ab} = \frac{2R_1 R_2 C}{1 + (CR_1 + CR_2)s}$$

 Natural frequency in V_{ab} :

$$s = \frac{-1}{(CR_1 + CR_2)}$$

$$55. I_{in} = \frac{2s^2 + 16s + 16}{2s^3 + 13s^2 + 12s}$$

$$(a) \begin{cases} I_2(s) = \frac{1}{2s^2 + 13s + 12} \\ I_1(s) = I_{in} - I_2(s) = \frac{2s^2 + 15s + 16}{2s^3 + 13s^2 + 12s} \end{cases}$$

$$\text{So } i_2(t) = 0.12e^{-1.11t} - 0.12e^{-5.39t}$$

$$56. H(s) = \frac{V_{out}}{V_{in}}$$

(a) $5(s+1)$

Zero at $s=-1$

Fig. 15.43 (b)

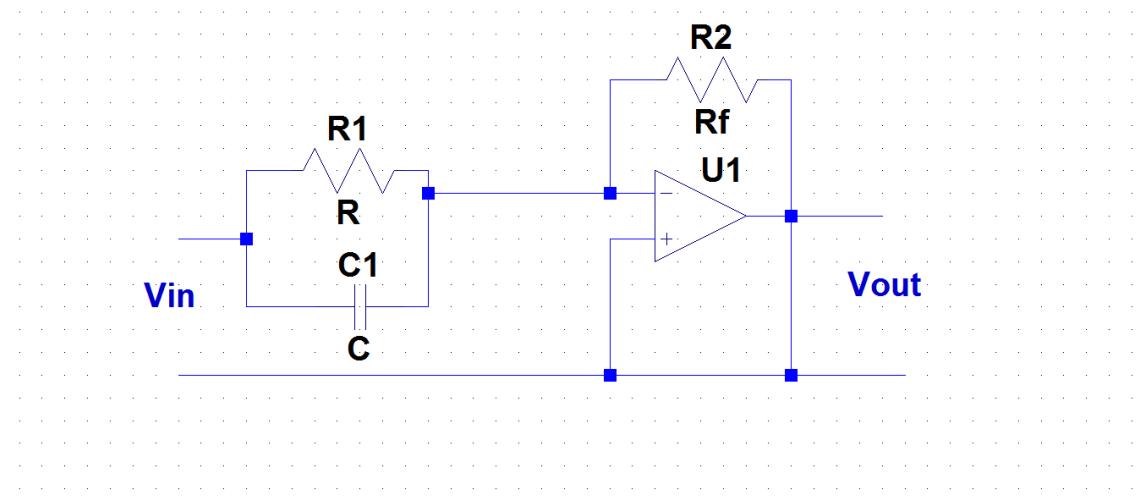
$$H(s) = -R_f C_1 \left(s + \frac{1}{R_i C_1} \right)$$

$$\text{zero : } s = \frac{-1}{R_i C_1} = -1$$

$$R_i = 100k\Omega \Rightarrow C_1 = 10\mu F$$

$$H(s) = -R_f C_1 (s + 1) = -R_f 10 \times 10^{-6} (s + 1) = 5(s + 1)$$

$$-R_f 10 \times 10^{-6} = 5 \Rightarrow R_f = 50k\Omega$$



56. (b)

$$H(s) = \frac{5}{s+1}$$

 pole : $s = -1$

Fig 15.43(a)

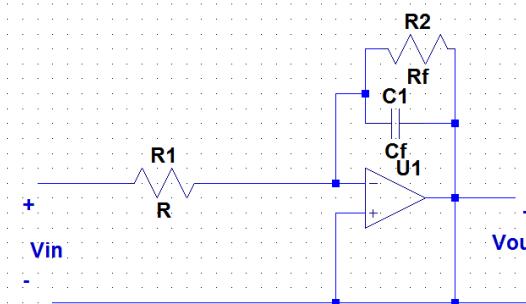
$$H(s) = -\frac{\frac{1}{R_1 C_f}}{s + \left(\frac{1}{R_f C_f}\right)}$$

$$\text{pole} = -\frac{1}{R_f C_f} = -1$$

$$R_f = 100k\Omega$$

$$C_f = \frac{1}{R_f} \Rightarrow C_f = 10\mu F$$

$$\frac{1}{R_1 C_f} = 5 \Rightarrow R_1 = 20k\Omega$$



56. (c)

$$H(s) = 5 \frac{s+1}{s+2}$$

 Pole : $s = -2$

$$\frac{1}{R_{fA} C_{fA}} = 2$$

$$R_{fA} = 100k\Omega$$

$$C_{fA} = \frac{1}{2 \times 100 \times 10^3} \Rightarrow C_{fA} = 5\mu F$$

$$H_A(s) = -\frac{\frac{1}{R_{1A} C_{fA}}}{s + \left(\frac{1}{R_{fA} C_{fA}}\right)} = -\frac{2 \times 10^5 / R_{1A}}{s+2}$$

 Zero : $s = -1$

$$\frac{1}{R_{1B} C_{1B}} = 1 \Rightarrow R_{1B} = 100k\Omega$$

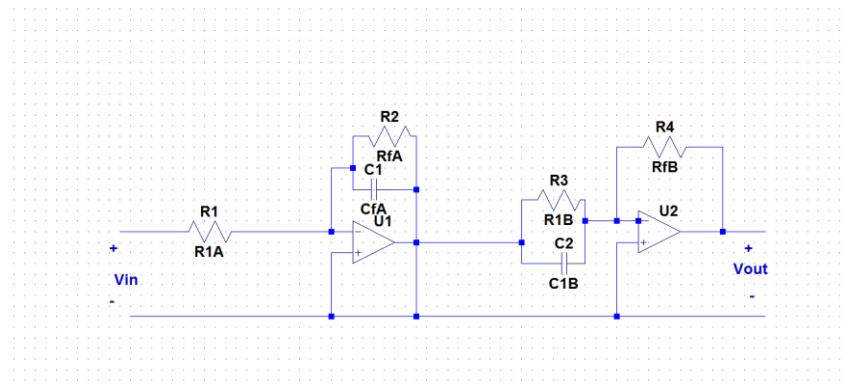
$$C_{1B} = \frac{1}{R_{1B}} \Rightarrow C_{1B} = 10\mu F$$

$$H_B(s) = -R_{fB} C_{1B} \left(s + \frac{1}{R_{1B} C_{1B}} \right) = -10 \times 10^{-6} R_{fB} (s+1)$$

$$H(s) = H_A(s)H_B(s) = 2 \cdot \frac{R_{fB}}{R_{1A}} \cdot \frac{(s+1)}{(s+2)}$$

$$2 \cdot \frac{R_{fB}}{R_{1A}} = 5 \Rightarrow R_{fB} = 100k\Omega$$

$$R_{1A} = \frac{2R_{fB}}{5} = 40k\Omega$$



57. (a), cascade 2 stage of fig/ 15.43 (b).

$$\text{We get: } H1 = H2 = -\sqrt{2}(s + 1) = -Rf * C1 \left(s + \frac{1}{R1*C1} \right)$$

So, $R1=1 \text{ k}\Omega$, $C1=1 \mu\text{F}$, $Rf=1.41 \text{ k}\Omega$

(b) two stage of Fig. 15.43 (a)

$$\text{Stage 1: } H1 = -\frac{\frac{1}{R1*Cf1}}{s+\frac{1}{Rf1*Cf1}} = -\frac{3}{s+500},$$

Then, $Cf1=1 \mu\text{F}$, $R1=333 \Omega$, $Rf1=2 \Omega$

$$\text{Stage 2: } H1 = -\frac{\frac{1}{R2*Cf2}}{s+\frac{1}{Rf2*Cf2}} = -\frac{1}{s+100}$$

Then, $Cf2=1 \mu\text{F}$, $R2=2 \text{ k}\Omega$, $Rf1=100 \Omega$

$$58. H(s) = \frac{V_{out}}{V_{in}} = 5 \frac{s + 10^4}{s + 2 \times 10^5}$$

Pole: $s = -2 \times 10^5$

$$\frac{1}{R_{fA}C_{fA}} = 2 \times 10^5$$

$$R_{fA} = 100k\Omega$$

$$C_{fA} = \frac{1}{2 \times 10^5 \times 100 \times 10^3} \Rightarrow C_{fA} = 5 \times 10^{-11} F$$

$$H_A(s) = -\frac{\frac{1}{R_{1A}C_{fA}}}{s + \left(\frac{1}{R_{fA}C_{fA}}\right)} = -\frac{2 \times 10^{10} / R_{1A}}{s + 2 \times 10^5}$$

Zero: $s = -10^4$

$$\frac{1}{R_{1B}C_{1B}} = 1 \Rightarrow R_{1B} = 100k\Omega$$

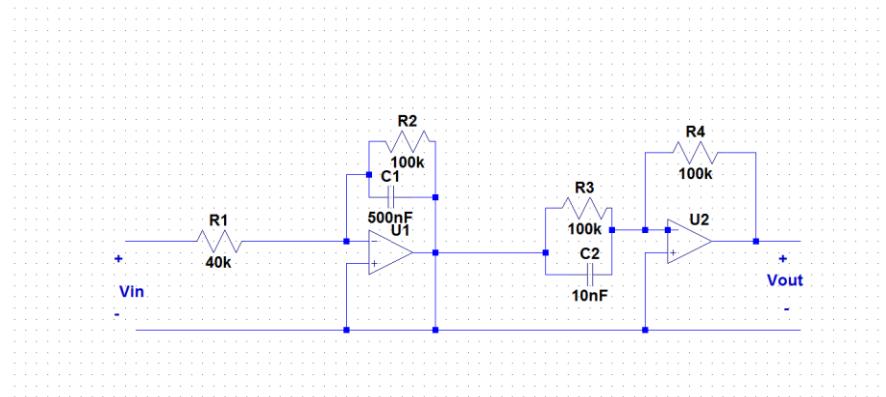
$$C_{1B} = \frac{1}{10^4 R_{1B}} \Rightarrow C_{1B} = 10nF$$

$$H_B(s) = -R_{fB}C_{1B} \left(s + \frac{1}{R_{1B}C_{1B}} \right) = -1 \times 10^{-10} R_{fB} (s + 10^4)$$

$$H(s) = H_A(s)H_B(s) = 2 \cdot \frac{R_{fB}}{R_{1A}} \cdot \frac{(s + 10^4)}{(s + 2 \times 10^5)}$$

$$2 \cdot \frac{R_{fB}}{R_{1A}} = 5 \Rightarrow R_{fB} = 100k\Omega$$

$$R_{1A} = \frac{2R_{fB}}{5} = 40k\Omega$$



59. 3 stages:

Stage 1: Fig. 15.43(b); stage 2&3: Fig. 15.43(a)

Then, $\begin{cases} C1 = 1 \mu F \\ Rf1 = 3 k\Omega \\ R1 = 20 \Omega \end{cases}$ $\begin{cases} Cf2 = Cf3 = 1 \mu F \\ R2 = R3 = 1 k\Omega \\ Rf2 = 13.3 \Omega \end{cases}$

$$60. H(s) = \frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_1}$$

(a)

$$Z_1(s) = 10^3 + (10^8 / s), Z_f(s) = 500$$

$$H(s) = \frac{-500}{10^3 + (10^8 / s)} = \frac{-500s}{10^8 + 10^3 s}$$

(b)

$$Z_1(s) = 5000, Z_f(s) = 10^3 + (10^8 / s)$$

$$H(s) = \frac{-\left(10^3 + (10^8 / s)\right)}{5000} = -2 \times 10^{-4} \left(\frac{10^8 + 10^3 s}{s} \right)$$

(c)

$$Z_1(s) = 10^3 + (10^8 / s), Z_f(s) = 10^4 + (10^8 / s)$$

$$H(s) = \frac{-Z_f}{Z_1} = -\frac{10^4 + (10^8 / s)}{10^3 + (10^8 / s)}$$

$$H(s) = -\frac{10^8 + 10^4 s}{10^8 + 10^3 s}$$

1. (a) $Q_0=100; \xi=0.005;$

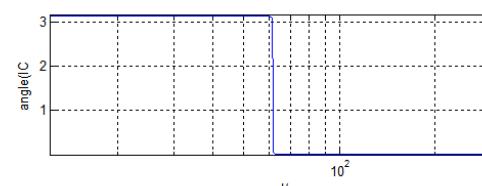
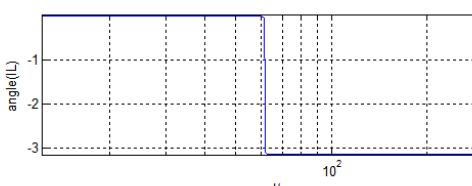
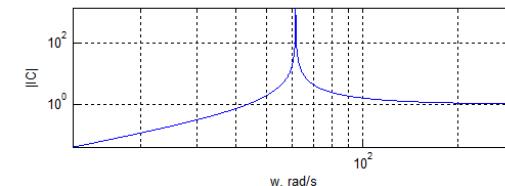
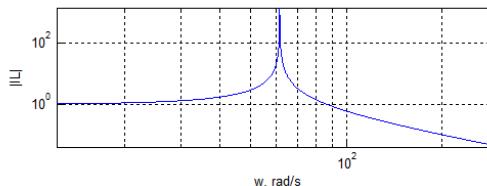
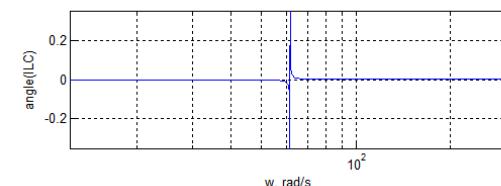
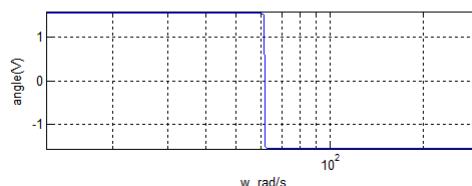
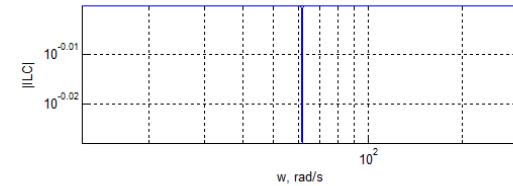
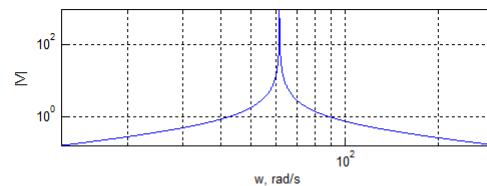
(b) $Q_0=0.1, \xi=5;$

(c) $q_0=1000, \xi=5e-4;$

(d) $Q_0=1, \xi=0.5$

2. (a) $\alpha = 22.73e-3$; $\omega_0 = 61.54 \text{ rad/s}$; $\zeta = 369e-6$; $f_0 = 9.8 \text{ Hz}$; $\omega_d = 61.54 \text{ rad/s}$

(b)



Note $\angle V = 0^\circ$ at ω_0 .

(c) $|I_L| = -|I_C|$ at ω_0 .

$$3. |Y| = \sqrt{\frac{1}{R^2} + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

So $1/12.3 < |Y| < 9.93$,

Then $0.1 < |Z| < 12.3$

4. (a) $Q_0 = 0.5$

(b) Require $|Z(\omega)| = 0.9$ $Z_{\max} = 0.9$ $R = 0.9(5) = 4.5$ or $|Y(\omega)| = 1/4.5$ or

$$|Y(\omega)| = \left| \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} = \frac{1}{4.5}$$

Solve quadratic equation to obtain: $\omega_1 = 62.7 \text{ rad/s}$; $\omega_2 = 159.5 \text{ rad/s}$

5. $Z(\omega) = (R_1 + j\omega L) // (R_2 + (R_3 // (1/j\omega C)))$, where $R_1 = 1 \Omega$, $R_2 = 2 \Omega$ and $R_3 = 100k\Omega$ then

$$Z(\omega) \approx \frac{100e3 - 0.4\omega^2 + j(200e3)\omega}{100e3 - 0.2\omega^2 + j2.2\omega}$$

At resonance $Z(\omega)$ is purely resistive. Set $\text{Imag}\{Z(\omega)\} = 0$ to obtain ω_0 , or

$$40\omega_0^2 - 20e6 = 0 \Rightarrow \omega_0 = 707.1 \text{ rad/s}$$

6.

$$Z(\omega) = \frac{100e3 - 0.4\omega^2 + j(200e3)\omega}{100e3 - 0.2\omega^2 + j2.2\omega}$$

$$7. Y = sC + 10^{-5} + \frac{1}{1+0.2s}$$

$$|Y(\omega_0)| = 6e-5,$$

(b) When $Im(Y) = 0$, $\omega_0 = 707$ rad/s

(c) $Q_{max} = 707$

8. (b) At resonance, $Z(\omega)$ is real and $Z(\omega_0) = 8054.4 \Omega$

(a) When $\text{Im}(Z) = 0$, $\omega_0 = 707.1 \text{ rad/s}$

(c) Let source voltage $V_S = 1\angle 0^\circ \text{ Volt}$ be connected at input terminals.

$$\text{Then, } V_C = \frac{V_S}{1+j0.014} \approx V_S = V_L$$

Maximum energy stored at ω_0 is:

$$w_C(t) + w_L(t) = \frac{1}{2}Cv_C^2(t) + \frac{1}{2}Li_C^2(t) = \frac{1}{2}Cv_C^2(t) + \frac{1}{2}L(\int v_L dt)^2 = \frac{\cos^2 \omega_0 t}{200e3} + \frac{\sin^2 \omega_0 t}{200e3} = 10^{-5}$$

Total energy lost per period at ω_0 is:

$$(P_{2\Omega} + P_{100K\Omega})T_0 = 44.4 \times 10^{-9}$$

$$\text{Therefore, } Q_0 = 2\pi \frac{\text{Maximum energy stored at } \omega_0}{\text{Total energy lost per period at } \omega_0}$$

$$Q_0 = 1414$$

$$9. Q = \frac{5.06e-3}{\frac{1}{\omega} + (5.97e-14) \omega},$$

$$\omega_0 = 4e6,$$

$$Q_{max} = 10368.6$$

10. $\omega_0 = 3162 \text{ rad/s}$, $R = \frac{Q_0}{\sqrt{\frac{C}{L}}} = 47.43 \Omega$ and $B = \omega_0/Q_0 = 210.8 \text{ rad/s}$

(a) $Z(3162) = Z(\omega_0) = 47.43 \Omega$

(b) $|Z(\omega)| \approx \frac{R}{\sqrt{1+N^2}}$ and $\arg Z(\omega) = -\tan^{-1}N$. So, $Z(3000) = 25.8 \angle 57^\circ \Omega$

(c) $Z(3200) = 44.7 \angle -20^\circ \Omega$

(d) $Z(2000) = 4.3 \angle 85^\circ \Omega$

(d) $Z(3000) = 25.3 \angle 58^\circ \Omega; Z(3200) = 45 \angle -20^\circ \Omega; Z(2000) = 3.3 \angle 86^\circ \Omega$

$$11. |Y| = \frac{1}{R} \sqrt{1 + Q^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2}$$

- (a) $|Y|=0.3016, |Z|=3.3156;$
- (b) $|Y|=0.0551, |Z|=18.15;$
- (c) $|Y|=0.0349, |Z|=28.65;$
- (d) $|Y|=1/32.5, |Z|=32.5$

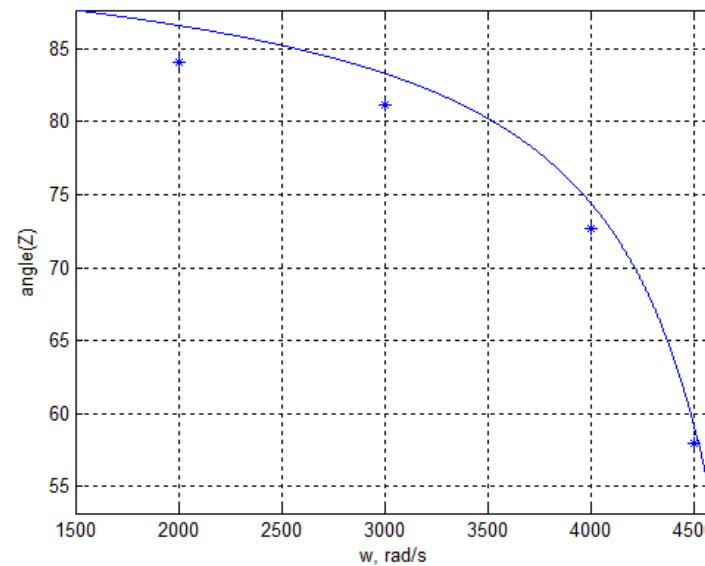
12. $C = 200 \mu F$; $R = 8 \Omega$

(a) $\arg Z(2000) = \angle 84^\circ$

(b) $\arg Z(3000) = \angle 82^\circ$

(c) $\arg Z(4000) = \angle 73^\circ$

(d) $\arg Z(4500) = \angle 58^\circ$



(d)

13. (a) B~1 kHz;

(b) B~10 MHz

14.(a) $C = 1 \text{ F}$ (b) $R = 2.5 \Omega$ (c) $Q = 2.5; w_1 = 0.82 \text{ rad/s}; w_2 = 1.22 \text{ rad/s}; B = 0.4 \text{ rad/s}$

$$15. Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} = 7, \text{ so } C = 3 \text{ nF}, Z = R + j(\omega L - \frac{1}{\omega C})$$

(a) $|Z| = 500 \text{ k}\Omega$

(b) $I_s = 10^{-20} \cos(4.25 \times 10^8 t) + 1.7 \times 10^{-6} \sin(4.25 \times 10^8 * t) - 8 \times 10^{-11} * e^{-3 \times 10^{-7} * t}$

16. $Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} = 7$; so for $Q_0 = 5$, $R = 141.4 \Omega$

(b) $\arg Z(90K) = \angle -82.9^\circ, \arg Z(100K) = \angle -82.8^\circ, \arg Z(110K) = \angle -82.6^\circ$

(c) $Z = R + j(\omega L - \frac{1}{\omega C})$

$\arg Z(90K) = \angle -87.7^\circ, \arg Z(100K) = \angle -87.5^\circ, \arg Z(110K) = \angle -87.2^\circ$

$\text{error}(90K) = 5.4\%, \text{error}(100K) = 5.4\%, \text{error}(110K) = 5.3\%$

$$17. \text{ (a)} Q_0 = R \sqrt{\frac{C}{L}} = 1.11, Y = \frac{1}{R} [1 + jQ_0 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)]$$

So, $|Y| = 0.201$, $|Z| = 4.97 \Omega$

$$\text{(b)} Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} = 0.894, Z = R + j(\omega L - \frac{1}{\omega C})$$

So, $|Z| = 4.95 \Omega$

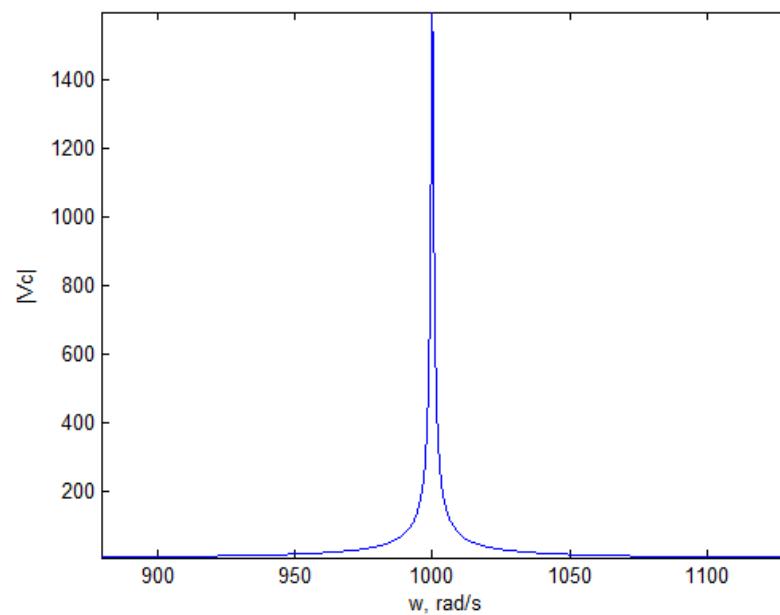
18. Loop current is $I = \frac{1.5\angle 0^\circ V}{Z_{in}}$, where $Z_{in} = 3.75 + j\left(4\omega - \frac{4 \times 10^3}{\omega}\right) \Omega$

Z_{in} is real at resonance, so $\omega_0 = 1000 \text{ rad/s}$

At resonance, $I(\omega_0) = 0.4 \text{ A}$.

V_c is at maximum at resonance: $V_c(\omega_0) = -j \frac{0.4(4 \times 10^3)}{\omega_0} \Rightarrow |V_c(\omega_0)| = 1.6 \text{ kV}!!$

This is a dangerous voltage!



$$19. Z_{in} = R + j(\omega L - \frac{1}{\omega C}),$$

$\omega_0=1.4e5$; $Q_0=14.1$

20. $C = 2.85 \mu F$

(a) When $R_1 = 0$ then $\omega_0 = 3417 \text{ rad/s}$ and $|Z(\omega_0)| = 150 \Omega$

(b) $|Z(700)| = 21.7 \Omega$

(c) $|Z(800)| = 25 \Omega$

21. (a) $X_p = -3.3e3$, $X_s = -75.6$,

So $Q = 0.15$, $R_s = 488.9 \Omega$, $C_s = 69 \mu F$

(b) $X_p = 40$, $X_s = 39.74$

So $Q = 12.5$, $R_s = 3.2 \Omega$, $L_s = 200 \text{ mH}$

22. (a) $X_s(40) = -0.25$, $Q(40) = 0.125$, $X_s(80) = -0.125$, $Q(80) = 0.0625$

For $\omega = 40 \text{ rad/s}$, $X_p(40) = -16.25$, $R_p = 2.03125 \Omega$, $C_p = 1.54 \text{ mF}$

For $\omega = 80 \text{ rad/s}$, $X_p(80) = -16.25$, $R_p = 2.0078 \Omega$, $C_p = 389 \mu\text{F}$

(b) $X_s(40) = .12$, $Q(40) = 0.06$, $X_s(80) = .24$, $Q(80) = 0.12$

For $\omega = 40 \text{ rad/s}$, $X_p(40) = 33.45$, $R_p = 2.0072 \Omega$, $L_p = 0.836 \text{ H}$

For $\omega = 80 \text{ rad/s}$, $X_p(80) = 16.91$, $R_p = 2.029 \Omega$, $L_p = 0.211 \text{ H}$

$$23. . Z = \frac{8(12\omega^4 - 13985\omega^3 i - 3999000\omega^2 + 890500000\omega i + 15000000000)}{3\omega^4 - 1150\omega^3 i - 394800\omega^2 + 33600000\omega i + 8000000000}$$

when $\text{im}(Z)=0$, $\omega=\omega_0$

24. At ω_0 set imaginary part of input impedance, $Z(\omega)$, to zero to obtain $\omega_0 = 41 \text{ rad/s}$

Then, $Z(0.95\omega_0) = Z(39) = 1.845 \angle -2.2^\circ \Omega$, $I_s(39) = 0.542 \angle 2.2^\circ A$ and

$$V_x(39) = 0.98 \angle 2.2^\circ V$$

25. $\omega_0=1 \text{ rad/s}$

(a) $kf=63700$,

So $R=1 \Omega$, $C=47 \mu\text{F}$, $L=2.2 \mu\text{H}$

(b) $km=500000$

So $R=500 \text{ k}\Omega$, $C=2 \mu\text{F}$, $L=166.7 \text{ kH}$

(c) $km=25$, $kf=238900$

So $R=25 \Omega$, $L=34.9 \text{ H}$, $C=50 \mu\text{F}$

26. $\omega_0=1 \text{ rad/s}$

(a) $k_f=2700$,

So $R=1 \Omega$, $C=1.85 \text{ mF}$, $L=74 \mu\text{H}$

(b) $k_m=100$

So $R=100 \Omega$, $C=50 \text{ mF}$, $L=20 \text{ H}$

(c) $k_m=25$, $k_f=4.71 \times 10^6$

So $R=25 \Omega$, $L=6.37 \text{ mH}$, $C=708 \text{ pF}$

27. $R' = 1 \Omega k$, $C' = 3.5 \mu F$, $L' = 285 \text{ mH}$

$$\text{So } I_{in} = \frac{Vin}{1000} + \frac{Vin}{\frac{3.5}{s} + 0.285s} + \frac{0.2Vin}{1000}$$

$$\text{Then } Z_{in} = \frac{Vin}{I_{in}} = \frac{200s}{57s^2 + 700} + \frac{3}{25000}$$

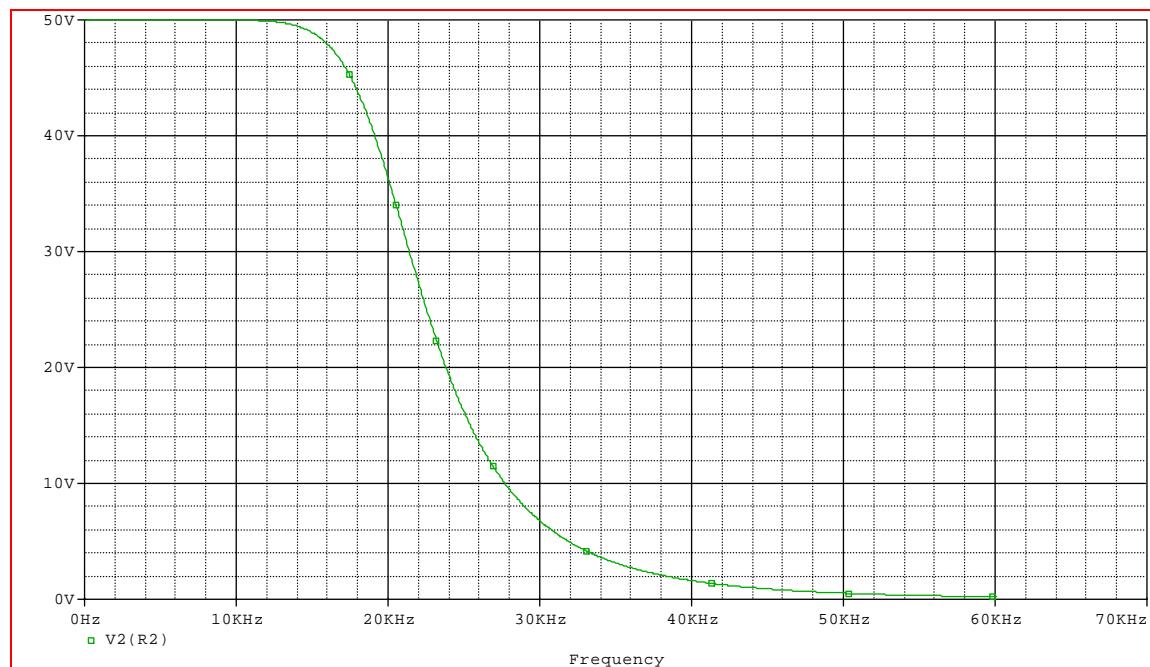
28. $km = 0.5$; $kf = 20e-3$

$100 \Omega \rightarrow 50 \Omega$

$9.82\mu\text{H} \rightarrow 245.5 \mu\text{H}$

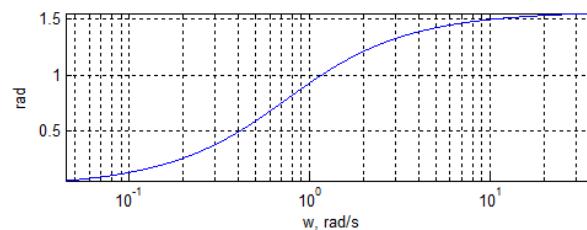
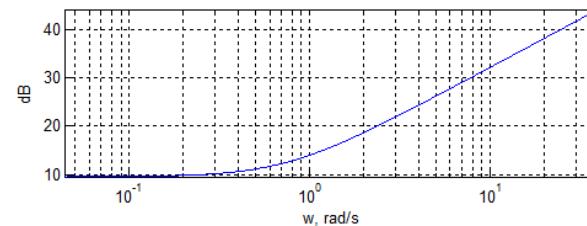
$31.8 \mu\text{H} \rightarrow 780 \mu\text{H}$

$2.57 \text{ nF} \rightarrow 257 \text{ nF}$

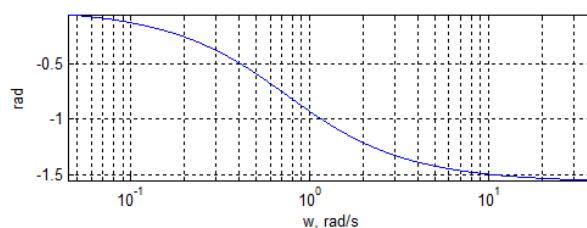
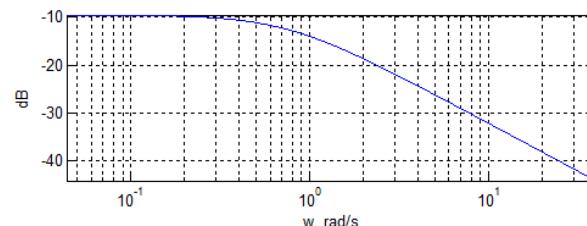


29. all elements are in parallel: $C=-j1e6$, $R=1250$, $L=1.25j$, source: $3.5j*I_x$

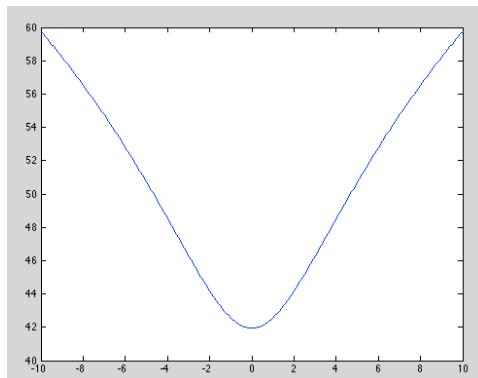
30. (a) $HdB = 20\log\sqrt{9 + 16\omega^2}$; $\text{ang}H(j\omega) = \tan^{-1}\left(\frac{4}{3}\omega\right) \text{ rad}$



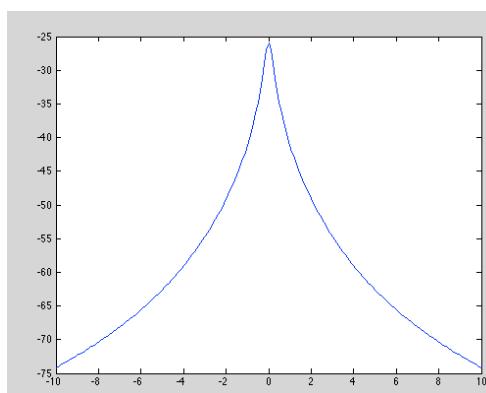
(b) $HdB = -20\log\sqrt{9 + 16\omega^2}$; $\text{ang}H(j\omega) = -\tan^{-1}\left(\frac{4}{3}\omega\right) \text{ rad}$



31. (a) $HdB = 20\log(\sqrt{9 + 16\omega^2})$

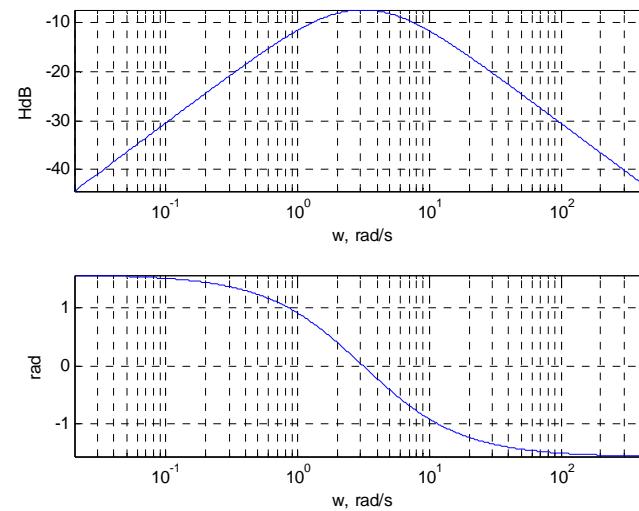


(b) $HdB = -20\log(\sqrt{9 + 16\omega^2})$



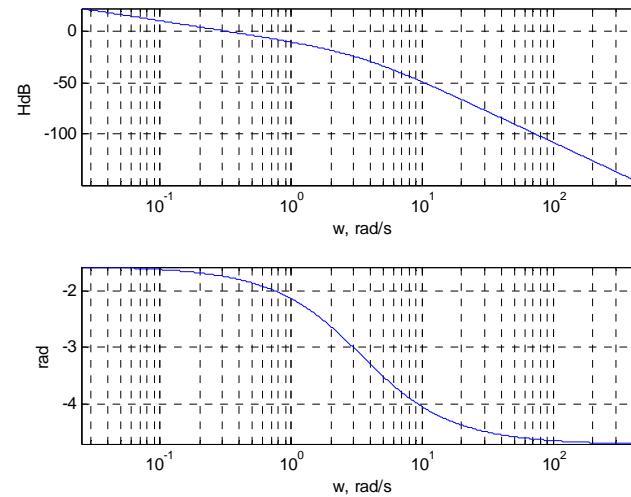
32. (a)

$$H(s) = \frac{0.3s}{(1 + \frac{s}{2})(1 + \frac{s}{5})}$$

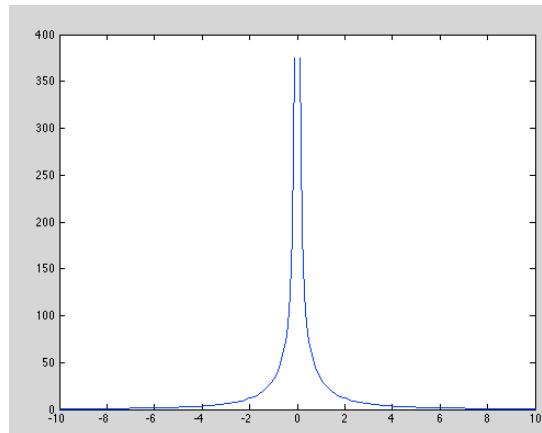


(b)

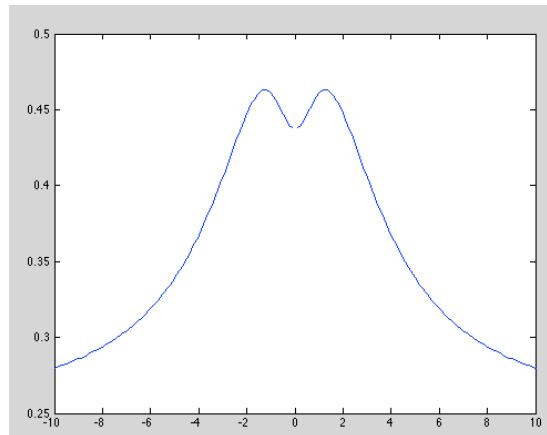
$$H(s) = \frac{1}{3s(1 + \frac{s}{3})(1 + \frac{s}{4})}$$



33. (a) $H(j\omega) = \frac{j\omega+300}{j8\omega-5\omega^2}$

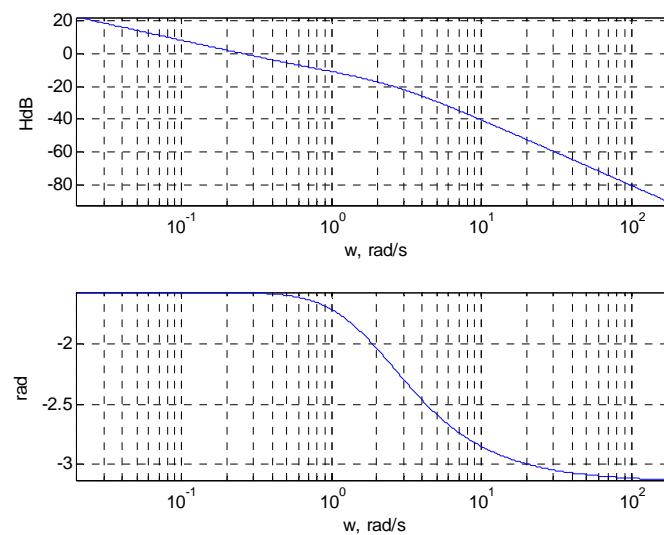


(b) $|H| = \frac{1}{-64\omega^2-25\omega^4} \sqrt{(1492\omega^2)^2 + (5\omega^2 + 2400\omega)^2}$



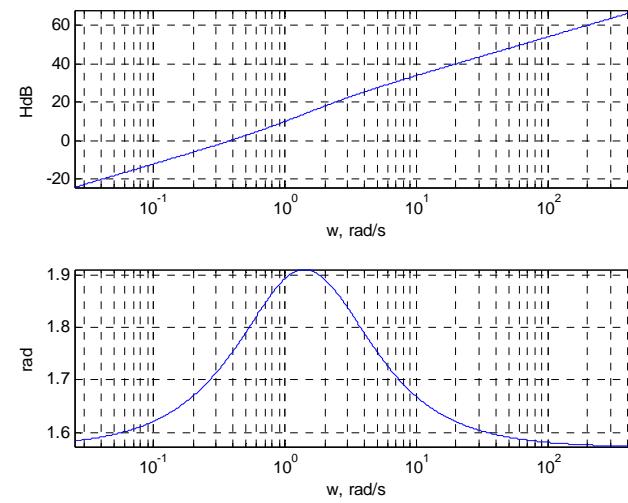
34. (a)

$$H(s) = \frac{0.25(1+s)}{s \left(1 + \frac{s}{2}\right)^2}$$



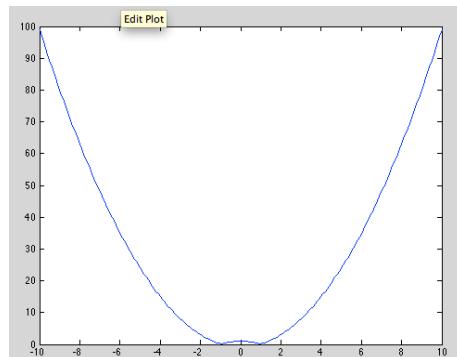
(b)

$$H(s) = \frac{2.5s(1+s)}{\left(1 + \frac{s}{2}\right)}$$

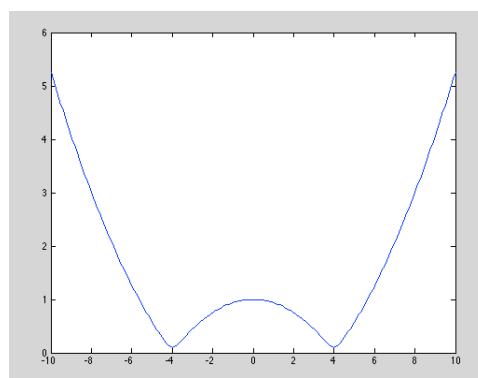


35.

(a)

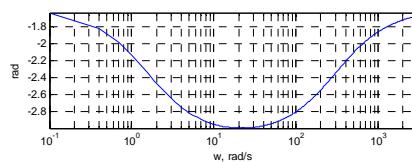
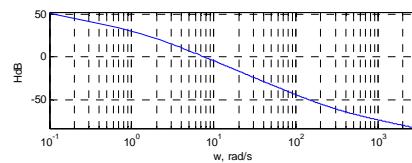


(b)

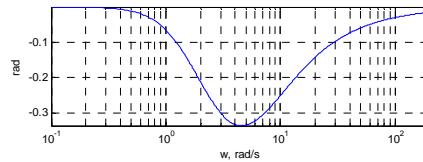
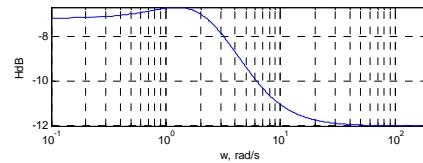


36.

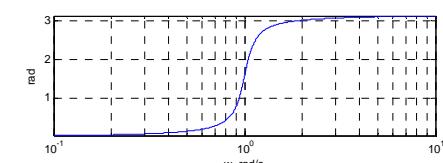
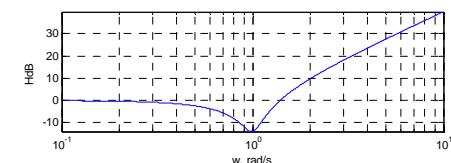
33 (a)
$$H(s) = \frac{300(1+\frac{s}{300})}{8s(1+\frac{5s}{8})}$$



33(b)
$$H(s) = \frac{0.43(1+\frac{s}{1.2})(1+\frac{s}{5.8})}{(1+\frac{s}{2})^2}$$



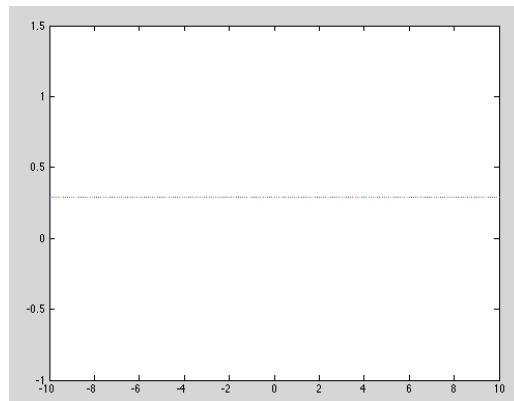
35(a)
$$H(s) = 1 + 0.2s + s^2$$



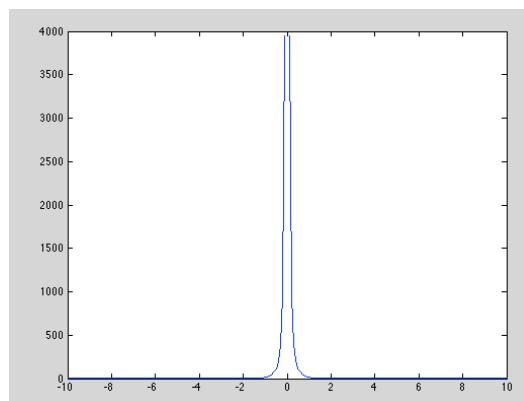
35(b)
$$H(s) = 1 + 0.1\left(\frac{s}{4}\right) + \left(\frac{s}{4}\right)^2$$

37.

(a)

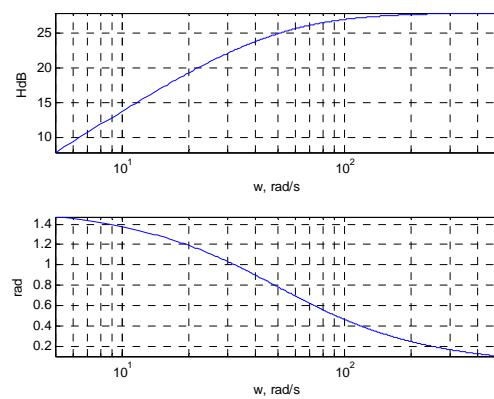


(b)



38. (a) $H(s) = \frac{0.5s}{1+\frac{s}{50}}$

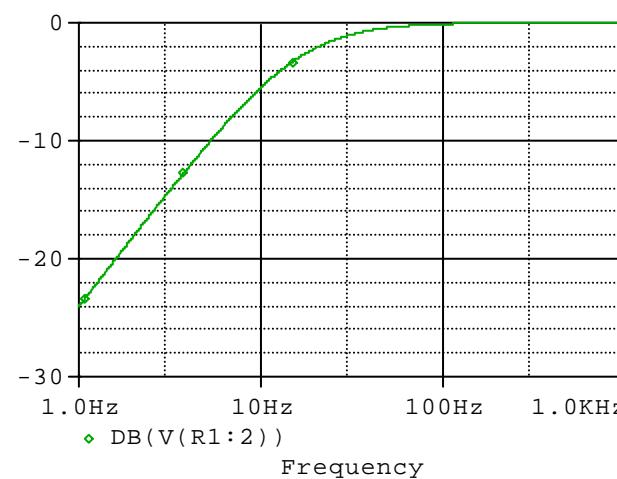
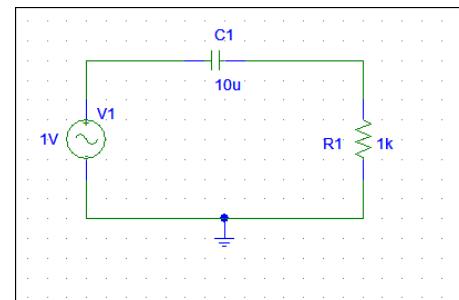
(b)



$$39. \frac{V_1}{50} = -V_{in} * 0.25s, \frac{Vi}{100} = -\frac{V_{out}}{200} \Big| \Big| \frac{1}{0.25s}$$

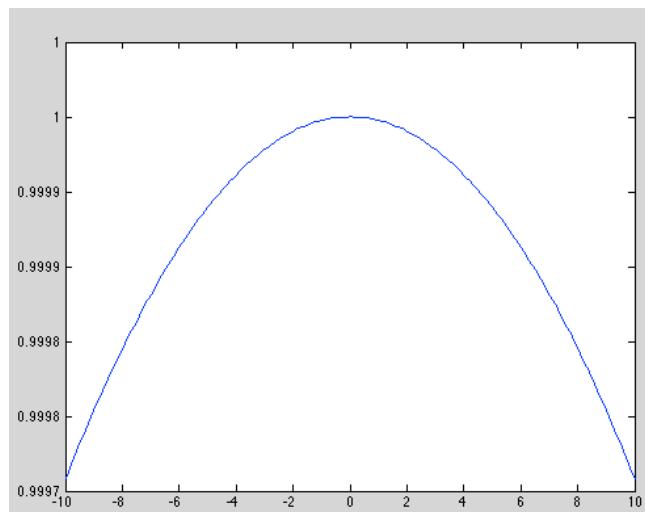
$$\text{so, } H(s) = \frac{25s}{50s+1}$$

$$40. H(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{s}{100}}{1 + \frac{s}{100}}$$

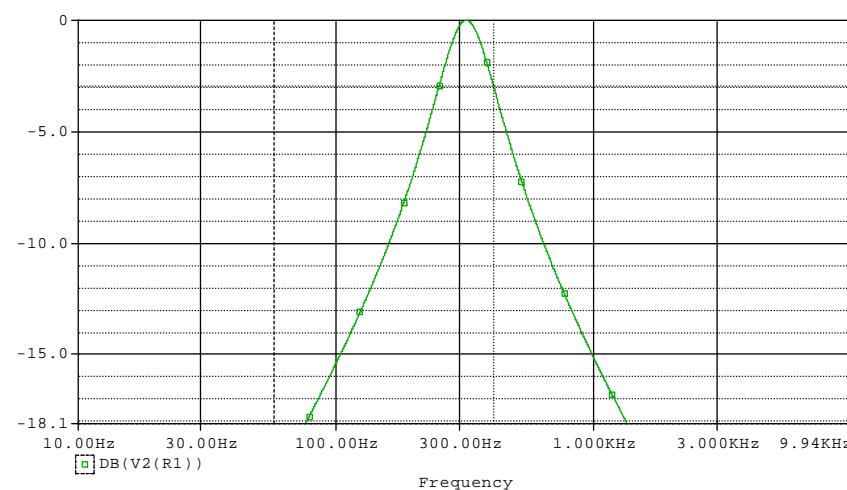
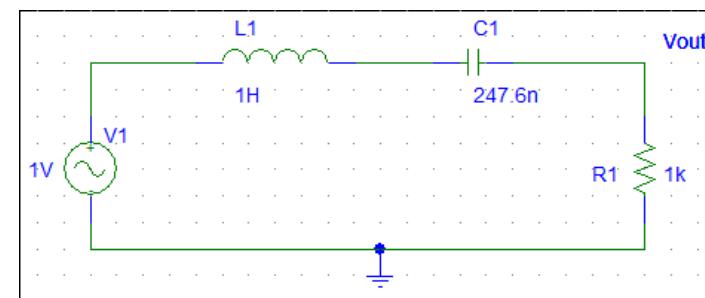


41. (a) $H(s) = \frac{1}{1+RCS}$, choose R=1 kΩ, C=2.2 μF,

(b) $H(j\omega) = \frac{1}{1+2.2 \times 10^{-3} j\omega}$,

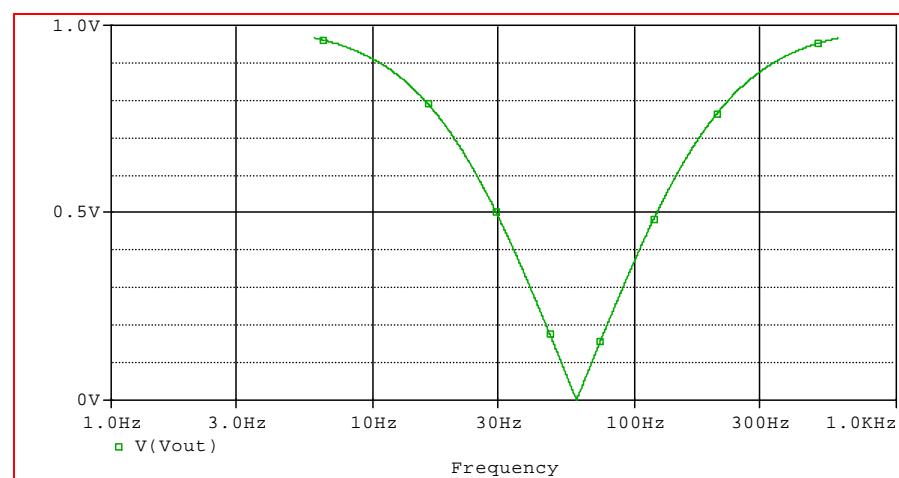
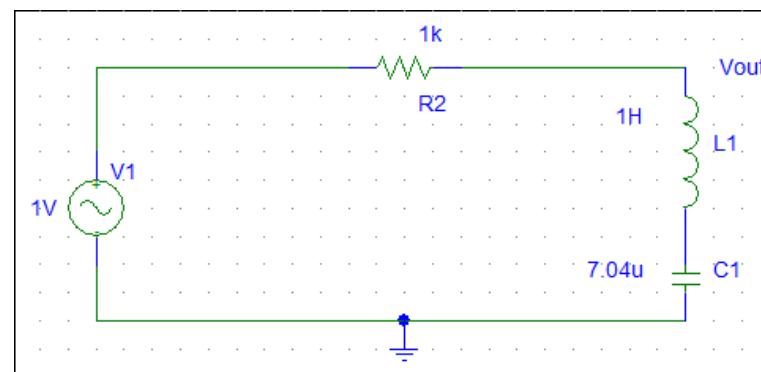


42. (a)



43. $\omega_L = \frac{\sqrt{(RC)^2 + 4LC} - RC}{2LC}$, $\omega_L = \frac{\sqrt{(RC)^2 + 4LC} + RC}{2LC}$

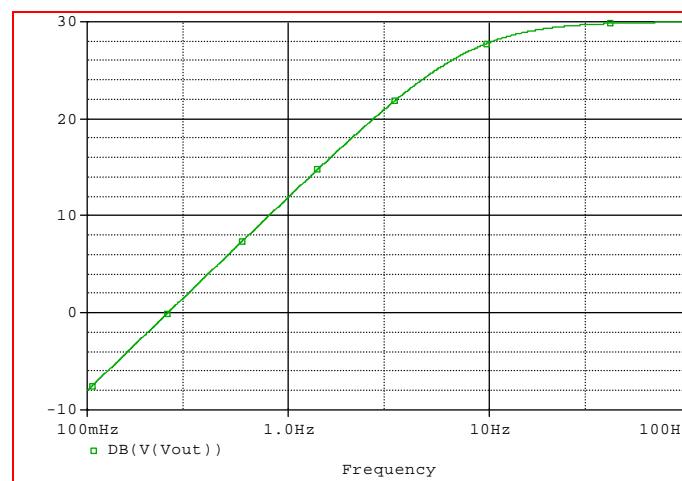
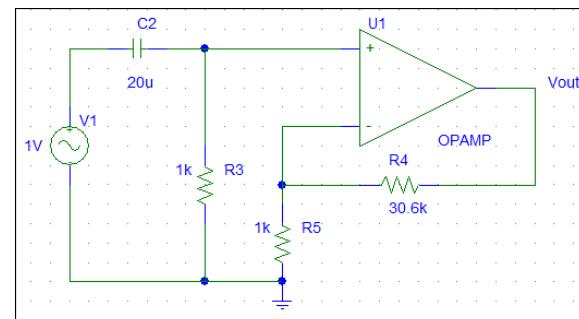
44.



$$45. \text{ Fig. 16.41(b), } V_{out} = V_{in} \left(\frac{1}{1+sR_2*C} \right) \left(1 + \frac{R_f}{R_1} \right)$$

assume R1=100 Ω, C=1 μF, so Rf=1.7 kΩ, R2=1 kΩ

46.



47. stage 1, LPF, $\omega_H=1000 \text{ rad/s}$, $A_v=10 \text{ dB}$

so, $R_1=100\Omega$, $R_f=216 \Omega$, $C=1 \mu\text{F}$, $R_2=909 \Omega$

stage 2: HPF, $\omega_L=100 \text{ rad/s}$, $A_v=10 \text{ dB}$

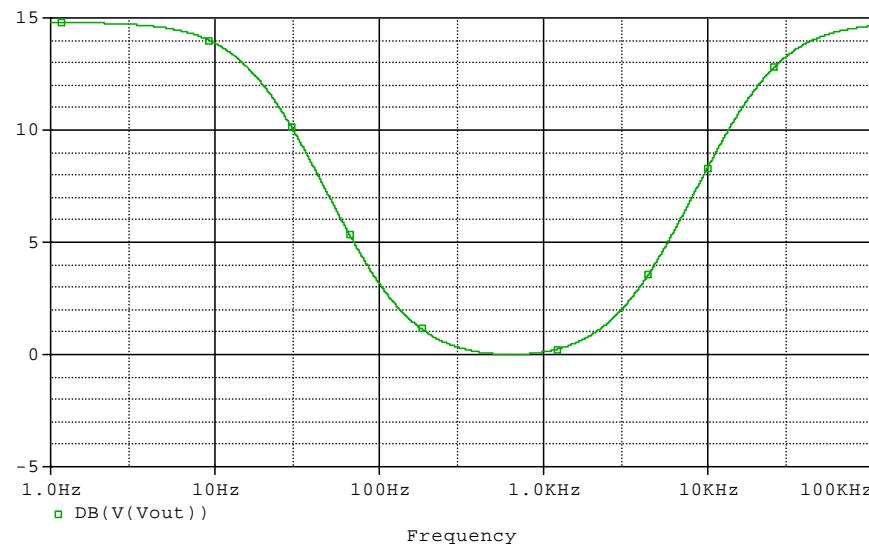
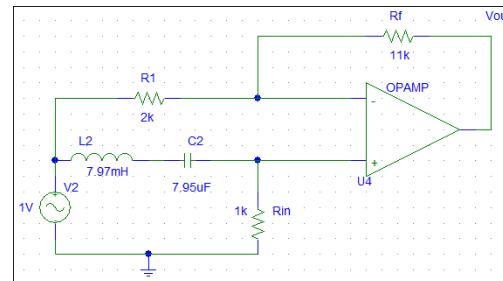
so, $R'_1=100 \Omega$, $R'_f=216 \Omega$, $C'=1 \mu\text{F}$, $R'_2=10 \text{ k}\Omega$

48. First design a RLC bandpass filter with the specified cutoff frequencies, taking the output across L and C, instead of instead R, to obtain the complementary bandstop response.

$$\frac{1}{LC} = \omega_H \omega_L = (2\pi 20000)(2\pi 20) = 15.79e6 \text{ and } B = \frac{R}{L}$$

Selecting $R = 1 \text{ k}\Omega$ results in $L = 7.97 \text{ mH}$ and $C = 7.95 \mu\text{F}$

For the required gain of 15, may use a non-inverting opamp circuit. A PSpice simulation confirms the design.



49. LPF, $f_L=500$ Hz,

assume $C=1 \mu\text{F}$, so $R_2=795 \Omega$, $R_1=100 \Omega$, R_f adjustable to meet the requirement of A_v

50. The coefficients of the given transfer function are $a_0 = \left(\frac{1}{RC}\right)^2$ and $a_1 = \frac{2}{RC}$. For the LP Butterworth function these are 1 and 1.4142, respectively, and so it cannot be used to implement the given transfer function. For the LP Chebychev function these are 0.708 and 0.6449, respectively, and they do not relate as the coefficients in the given function.

51. (a) Fig. 16.48, assume $Rq=R2=R$, $C1=C2=C$;

$$H(s) = \frac{k}{s^2 + 1.4142s + 1}, \frac{Vo}{Vi} = \frac{\frac{G}{R^2C^2}}{s^2 + \left(\frac{2}{RC} + \frac{1-G}{RC}\right)s + \frac{1}{R^2C^2}},$$

so, $RC=1$, $G=1.586$,

set $RB=1\text{ k}\Omega$, so $RA=586\text{ }\Omega$,

set $C=1\text{ nF}$, and $Km=187$, so $R=187\text{ }\Omega$.

$$(b) Av = \frac{1.00s^4}{s^2 + 0.6449s + 0.708}, RC = 1.189, G = 1.4158$$

set RB=1 kΩ, so, RA=416 Ω,

and C=1 nF, Km=0.187, so R=222 Ω

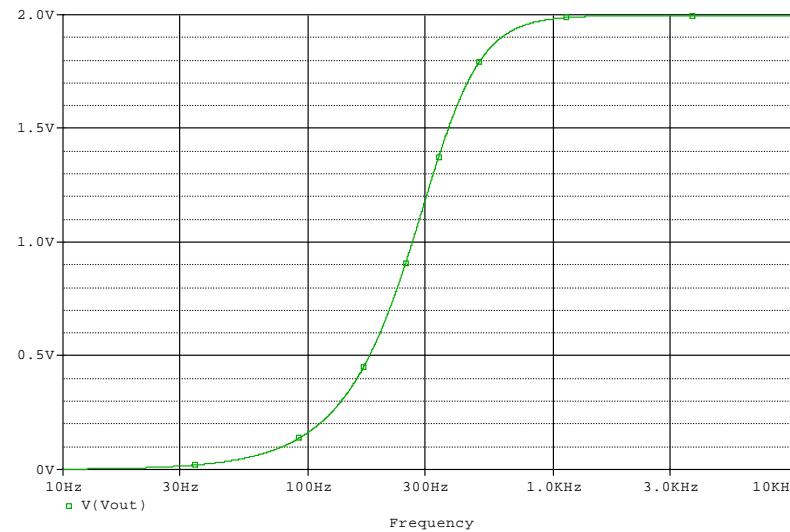
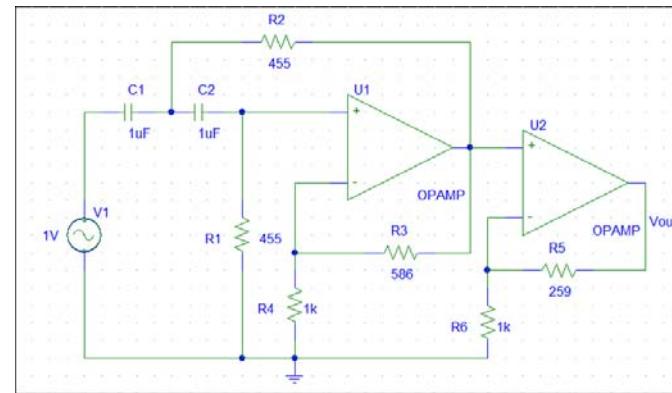
52. 2nd order HP filter implemented on the Sallen-Key amplifier with $C_1=C_2=C$ and $R_1=R_2=R$. yields relationships: $a_0 = \frac{1}{R^2 C^2} = 1$; $a_1 = \frac{3-G}{RC} = 1.4142$; $G = \frac{R_A+R_B}{R_B}$

Letting $RC = 1$ we obtain $G = 3-a_1 = 1.5858$. Selecting $R_B = 1 \text{ k}\Omega$ provides $R_A = 586 \Omega$.

Setting $R = 1 \Omega$ and $C = 1 \mu\text{F}$ and using $k_f = 2\pi 350/1 = 2199$ we obtain

$$1\mu\text{F} = \frac{1\text{F}}{k_f k_m} = \frac{1\text{F}}{2199 k_m} \Rightarrow k_m = \frac{1}{2199 e^{-6}} = 455 \text{ so that } R' = k_m R = 455 \Omega.$$

Provided gain is $20\log(G) = 4 \text{ dB}$. The extra 2 dB, or gain of 1.259 can be provided with a non-inverting op amp circuit with $R_1=1 \text{ k}\Omega$ and $R_f = 259 \Omega$.



53. $RC=1$, $G=1.586$,

set $RB=1 \text{ k}\Omega$, so, $RA=586 \Omega$,

$Km=1120$, so $R=1.12 \text{ k}\Omega$

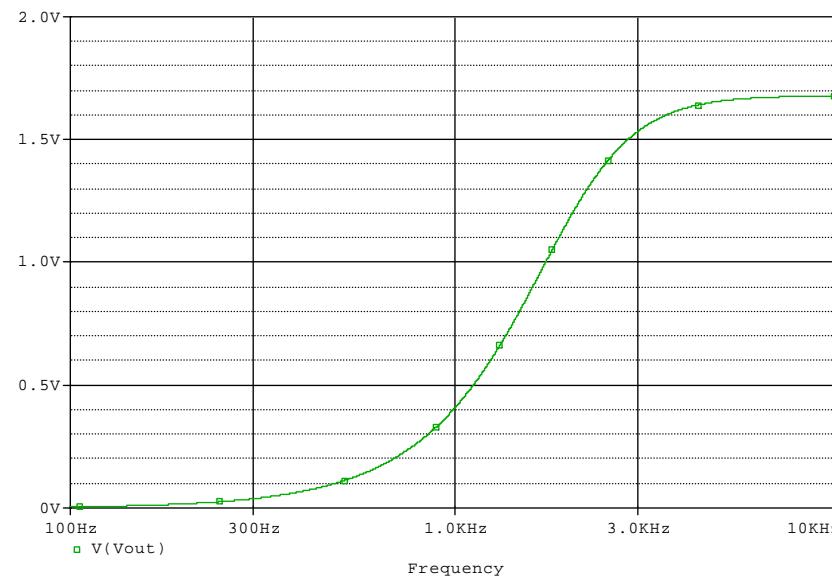
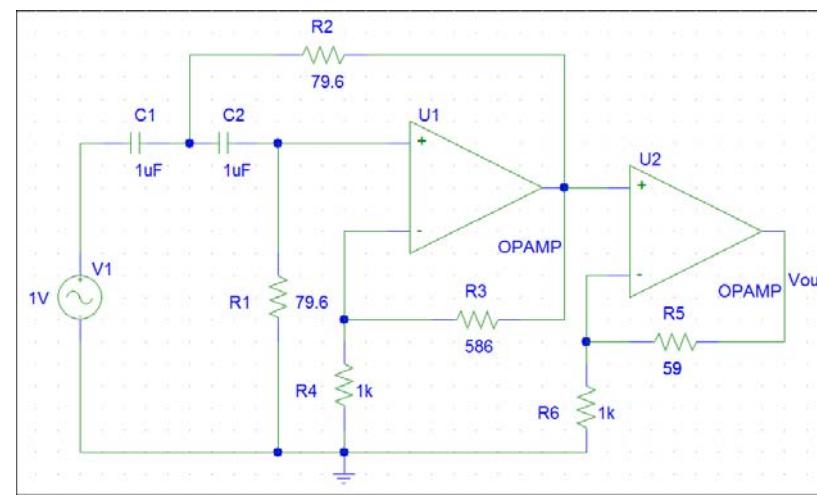
54. 2nd order HP filter implemented on the Sallen-Key amplifier with $C_1=C_2=C$ and $R_1=R_2=R$. yields relationships: $a_0 = \frac{1}{R^2 C^2} = 1$; $a_1 = \frac{3-G}{RC} = 1.4142$; $G = \frac{R_A+R_B}{R_B}$

Letting $RC = 1$ we obtain $G = 3-a_1 = 1.5858$. Selecting $R_B = 1 \text{ k}\Omega$ provides $R_A = 586 \Omega$.

Setting $R = 1 \Omega$ and $C = 1 \mu\text{F}$ and using $k_f = 2\pi 2000/1 = 12566$ we obtain

$$1\mu\text{F} = \frac{1\text{F}}{k_f k_m} = \frac{1\text{F}}{12566 k_m} \Rightarrow k_m = \frac{1}{12566e-6} = 79.6 \text{ so that } R' = k_m R = 79.6 \Omega.$$

Provided gain is $20\log(G) = 4 \text{ dB}$. The extra 0.5 dB, or gain of 1.059 can be provided with a non-inverting op amp circuit with $R_1=1 \text{ k}\Omega$ and $R_f = 59 \Omega$.



55. Fig.16.49, set $R_1=R_2=R$, $C_1=C_2=C$,

we get $RC=1$, $G=4$

set $R_A=3 \text{ k}\Omega$, $R_B=1 \text{ k}\Omega$

$K_f=3768$, $K_m=256$

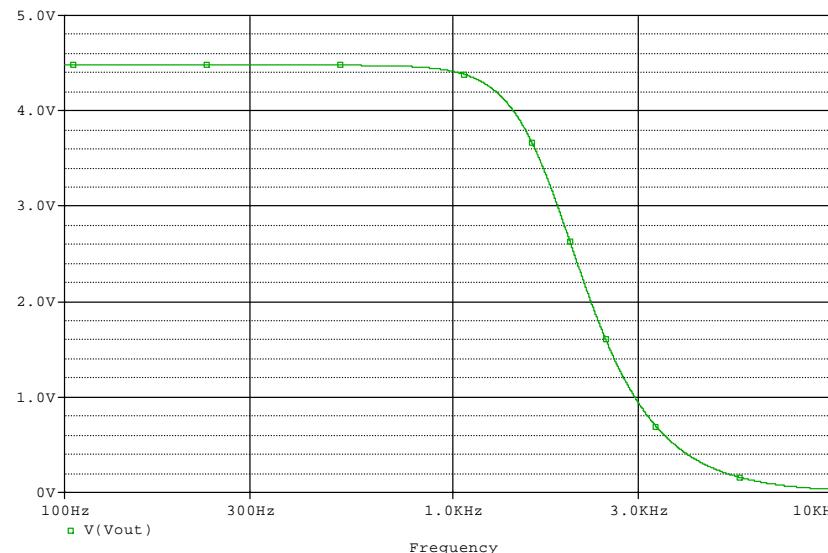
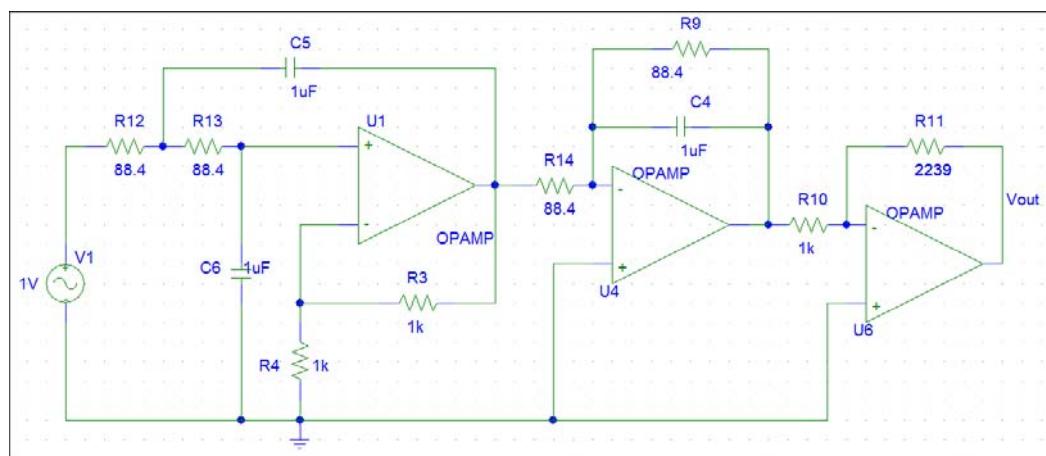
So $R=256 \Omega$, $C=1 \mu\text{F}$

56.

(a) 3rd order Butterworth LPF has TF: $H(s) = \frac{Ka_0}{s^3 + 2s^2 + 2s + 1} = Ka_0 \left(\frac{1}{s^2 + s + 1} \right) \left(\frac{1}{s + 1} \right) = H_1(s)H_2(s)$. $H_1(s)$ can be implemented by the Sallen-Key amplifier circuit. Setting $RC = 1$ yields $G = 2$, which is realized by selecting $R_A = R_B = 1 \text{ k}\Omega$. The gain of this stage is then $20\log(G) = 6 \text{ dB}$. Setting $R = 1 \Omega$ and $C = 1 \mu\text{F}$ and using $k_f = 2\pi 1800/1 = 11310$ we obtain

$$1\mu\text{F} = \frac{1\text{F}}{k_f k_m} = \frac{1\text{F}}{11310 \cdot k_m} \Rightarrow k_m = \frac{1}{11310 \cdot e^{-6}} = 88.4 \text{ so that } R' = k_m R = 88.4 \Omega.$$

$H_2(s)$ can be implemented by an op amp inverting circuit with resistor-coupled input and a parallel resistor-capacitor combination as feedback, which, setting capacitors as C and resistor as R yields TF: $H_2(s) = \frac{-1}{1+RCS}$. Setting $RC = 1$ results in $H_2(s) = \frac{-1}{1+s}$. With appropriate scaling, $C = 1 \mu\text{F}$ and $R = 88.4 \Omega$. To achieve the required 13 dB gain, we use an additional inverting op amp circuit to provide gain of 7 dB or 2.239, which is achieved with $R_1 = 1 \text{ k}\Omega$ and $R_f = 2239 \Omega$. (Phase inversion corrects for phase inversion of previous stage.)

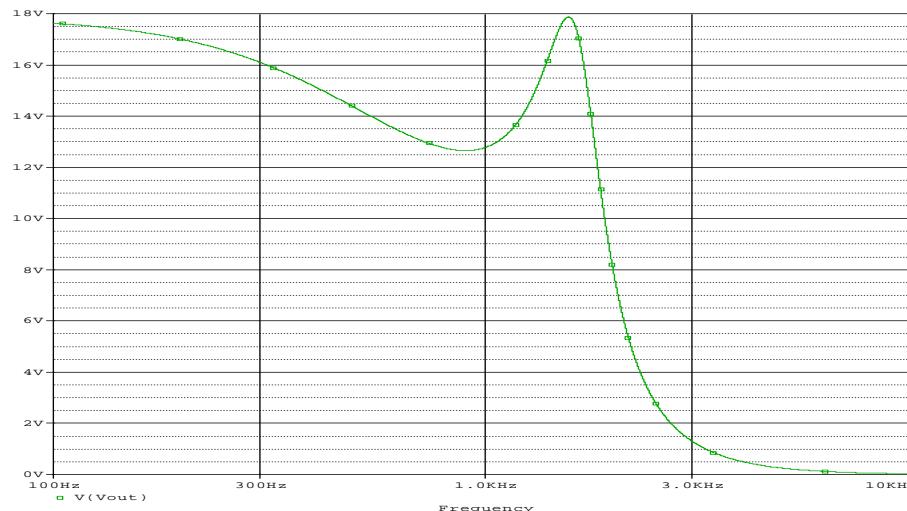
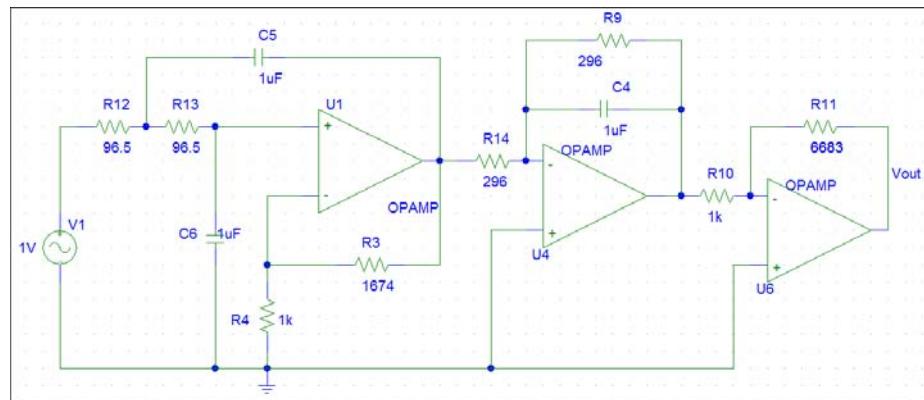


56. (b) 3rd order Chebychen LPF with TF $H(s) = \frac{Ka_0}{s^3 + 2s^2 + 2s + 1}$ has a real root at $s = -0.2986$. Long division of the denominator polynomial with $(s+0.2986)$ yields the factored form:

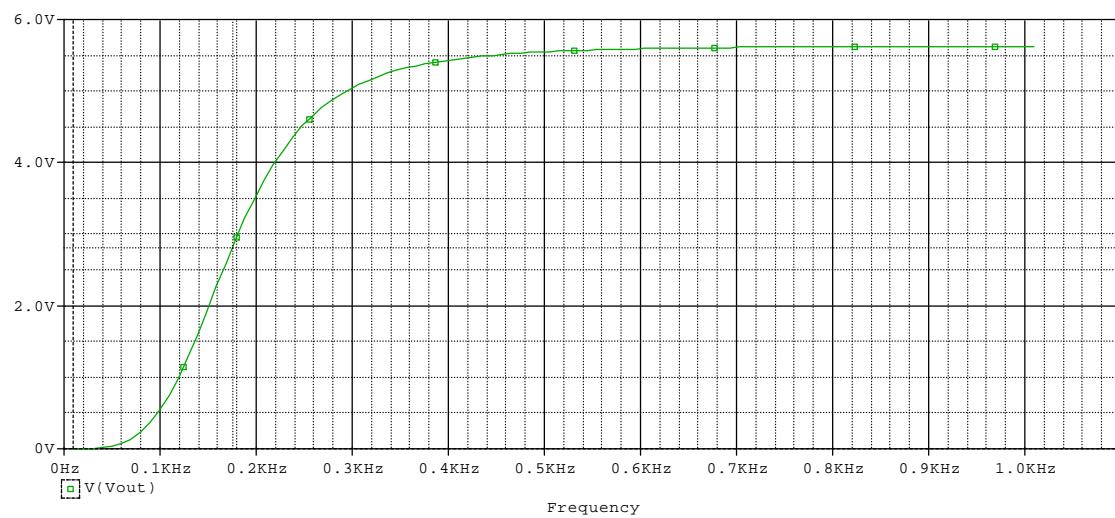
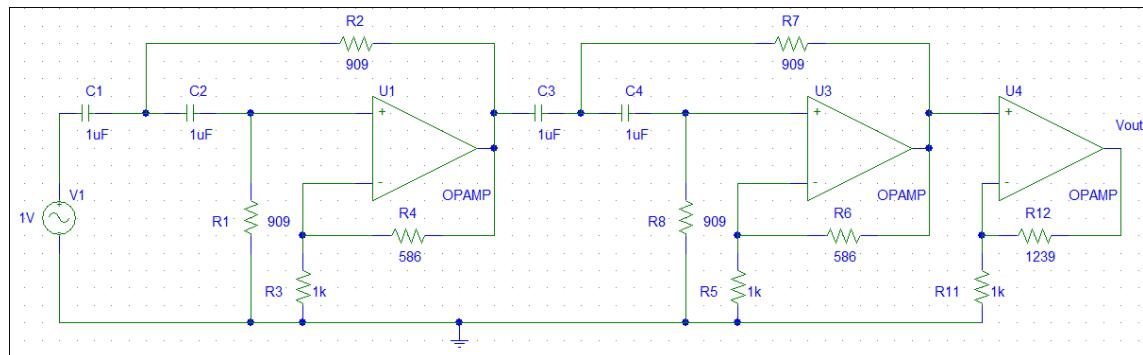
$H(s) = Ka_0 \left(\frac{1}{s^2 + 0.2986s + 0.8393} \right) \left(\frac{1}{s + 0.2986} \right) = H_1(s)H_2(s)$ $H_1(s)$ can be implemented by the reference Sallen-Key amplifier circuit with $RC = 1.0915$ yielding $G = 2.6741$, which is realized by selecting $R_B = 1 \text{ k}\Omega$ and $R_A = 1674 \Omega$. The gain of this stage is then $20\log(G/(RC)^2) = 20\log(2.2443) = 7 \text{ dB}$. Setting $R = 1.0915 \Omega$ and $C = 1 \mu\text{F}$ and using $k_f = 2\pi 1800/1 = 11310$ we obtain $1\mu\text{F} =$

$\frac{1F}{k_f k_m} = \frac{1F}{11310 k_m} \Rightarrow k_m = \frac{1}{11310 e^{-6}} = 88.42$ so that $R' = k_m R = 96.5 \Omega$. $H_2(s)$ can be implemented by an op amp inverting circuit with resistor-coupled input and a parallel resistor-capacitor combination as feedback, which, setting capacitors as C and resistor as R yields TF:

$H_2(s) = \frac{-1}{1+RCS} = \frac{-\frac{1}{RC}}{s + \frac{1}{RC}} = \frac{0.2986}{s + 0.2986}$. Setting $1/RC = 0.2986$ or $RC = 3.349$ with appropriate scaling, $C = 1 \mu\text{F}$ and $R = 296 \Omega$. This stage has gain $20\log(0.2986) = -10.5$. To achieve the required 13 dB gain, we use an additional inverting op amp circuit to provide gain of $13 - 7 + 10.5 = 16.5 \text{ dB}$ or 6.683, which is achieved with $R_1 = 1 \text{ k}\Omega$ and $R_f = 6683 \Omega$. (Phase inversion corrects for phase inversion of previous stage.) The design meets the ripple and cutoff frequency specifications.

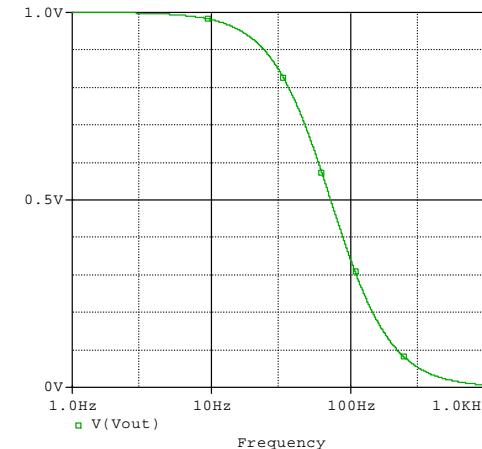
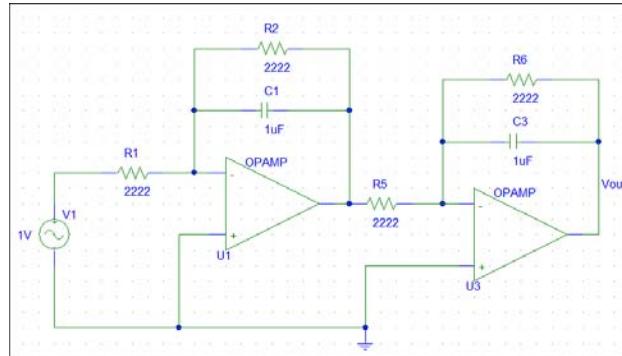


57. A 4th order Butterworth HP filter can be implemented by cascading two identical Sallen-Key amplifier circuits, each realizing a 2nd order HP TF. The achieved gain is $G^2 = 2.5154$ or 8 dB. The extra 7 dB gain, or 2.239, can be realized with a non-inverting op amp circuit with $R_B = 1 \text{ k}\Omega$ and $R_A = 1239 \Omega$. The specifications are realized, with pass band gain of 15 dB and cutoff frequency at 175 Hz = 1100 rad/s, but with 6 dB attenuation at ω_c due to the presence of the double poles for the cascaded identical stages.

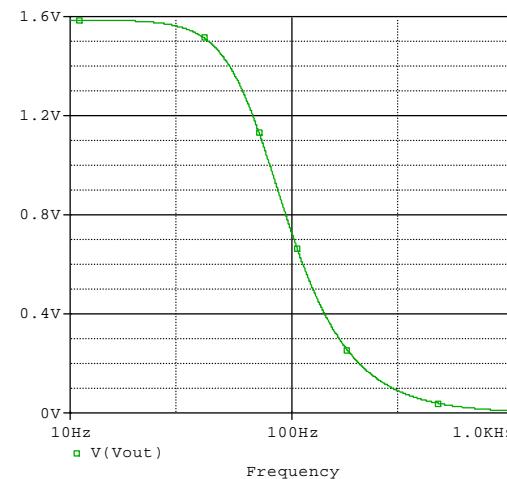
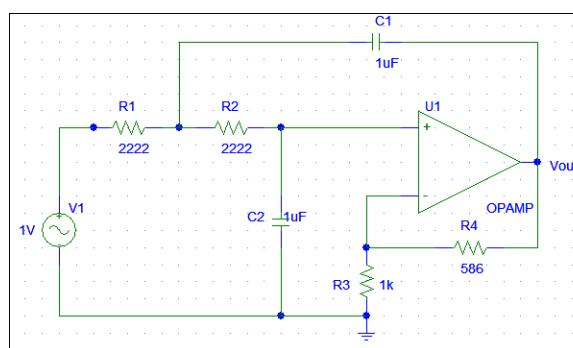


58. TF of equation [36] implemented by cascading two circuits of Figure 15.43(a) with $R_1 = R_f = R = 1 \Omega$ and $C_f = C = 1 \mu F$ has form $H(s) = \frac{1}{(s+1)^2}$. Setting $C = 1 \mu F$ and using $k_f = 450$ we obtain

$1\mu F = \frac{1F}{k_f k_m} = \frac{1F}{450 k_m} \Rightarrow k_m = 2222$ so that $R' = k_m R = 2222 \Omega$. Its simulation shows that at $\omega_c = 450 \text{ rad/s}$ the attenuation is at 6 dB due to the presence of the double pole there, while its 3-dB bandwidth is at 289 rad/s.



On the other hand, a 2nd order Butterworth LP has TF $H(s) = \frac{1}{s^2 + 1.4142s + 1}$ may conveniently be implemented using the Sallen-Key amplifier, with $G = 1.586$ achieved with $R_B = 1 K\Omega$ and $R_A = 586 \Omega$. Selecting $C=1 \mu F$ scaling provides $R = 2222 \Omega$. Its simulation shows a 4-dB gain, while its 3-dB bandwidth at 450 rad/s as required by the specifications.



59. $Q_0=45$, $\omega_0=1.68e6$, sp $C=0.45 \text{ nF}$, $L=0.27 \text{ mH}$,

$Q_0=45$, $\omega_0=5e6$, so, $C=0.45 \text{ nF}$, $L=89 \mu\text{H}$.

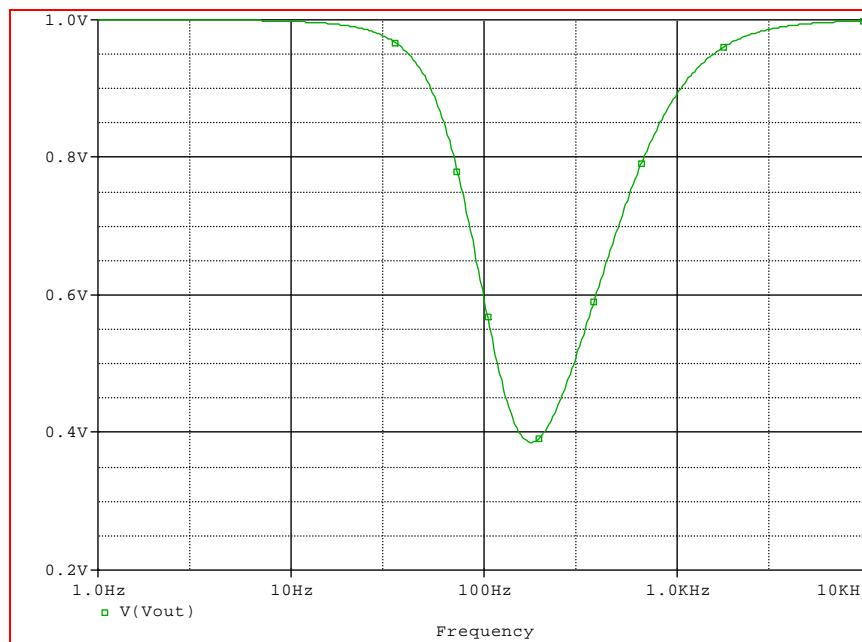
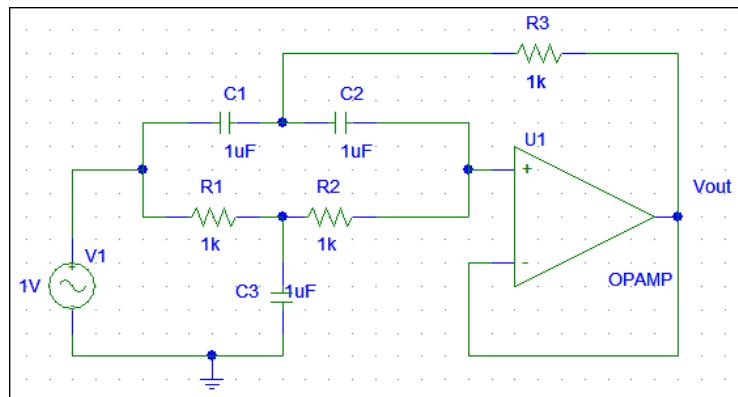
So, $C=0.45 \text{ nF}$, $L_{\max}=270 \mu\text{H}$, $L_{\min}=89 \mu\text{H}$

60.

$$H(s) =$$

$$\frac{sC_1R_1R_3(s^3C_1R_1R_2R_3C_2(C_1+C_2)+s^2(C_1R_1R_2C_3+C_1R_1R_3C_2+C_1^2R_1R_3+C_1R_1+C_2C_3R_2R_3+C_1C_3R_2R_3)+s(C_1R_1+C_3R_2+C_1R_3+R_3)+sC_1R_1R_3)}{s^3C_1C_2C_3R_1R_2R_3+s^2C_1R_1(3C_2R_3+C_3R_2)+s(C_1R_1+C_1R_3+R_2R_3)+1}$$

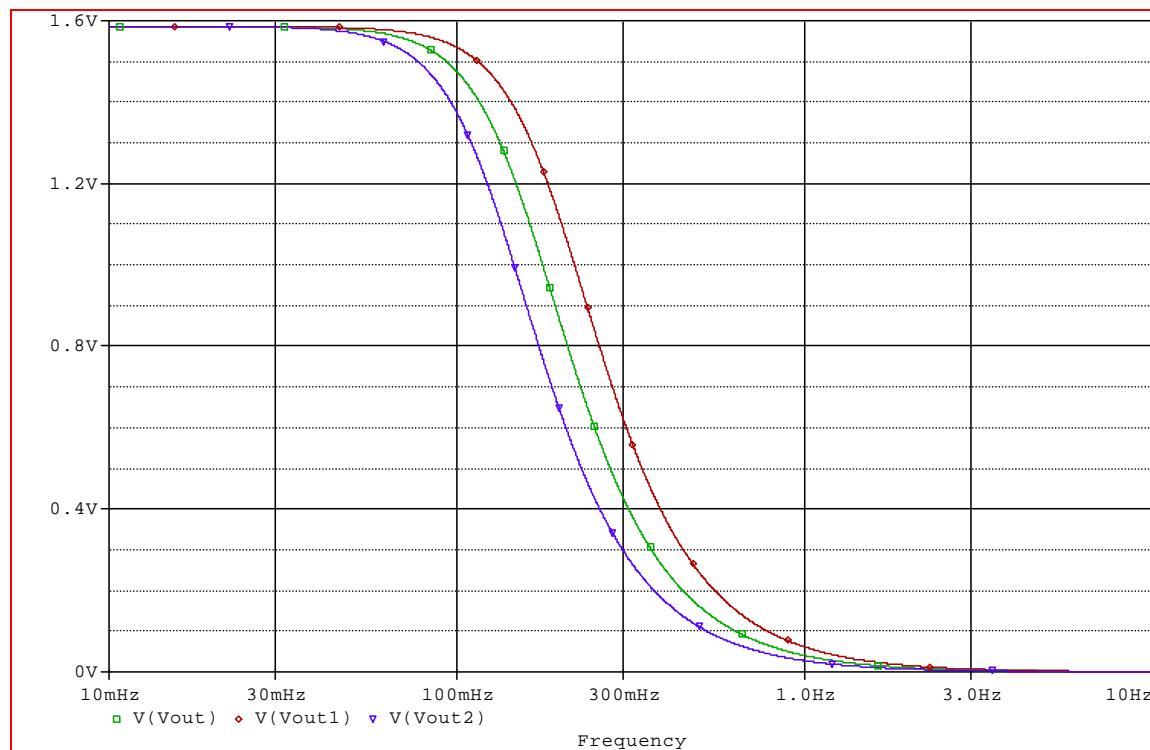
Which is a band stop filter. A simulation with cutoff at 1100 rad/s (175 Hz) is included:



61. $RC=8e-4$, set $C= 1 \mu\text{F}$, so $R=800 \Omega$,

and $1+R_f/R_1=1.26$, set $R_1= 1 \Omega$, so $R_f=260 \Omega$

62. Observing TF of circuit of Figure 16.48 in equation [40] the greatest deviation from the target design occurs when all components are simultaneously 10% above or 10% below their nominal values. PSpice simulation with component values altered accordingly can yield the extent of tolerance variations. Here nominal values are: $R = 1 \Omega$, $C = 1 \mu\text{F}$, $R_A = 586 \Omega$ and $R_B = 1 \text{k}\Omega$. The nominal cutoff frequency is 1 rad/s. For components 10% below their nominal values that frequency changes to 1.22 rad/s, or +22%. For components 10% above their nominal values that frequency changes to 0.82 rad/s or -18%.



63. only connected to "+" terminal: $V_o = \left(1 + \frac{R_A}{R_B}\right) V +$

only connected to "-" terminal: $V_1 = V_i \left(\frac{1}{s \cdot R_1 \cdot C_1}\right)$

so, $\frac{V_{out}}{V_{in}} = \frac{(R_A + R_B)(R_3 + 2sC_1 + sC_2)}{C_1 \cdot C_2 \cdot R_1 \cdot R_B \cdot s^2 + C_2 \cdot R_B \cdot s + R_3 \cdot R_A + R_3 \cdot R_B}$

64.

65. 3rd order LPF, and 2nd order HPF

LPF (fig. 16.49) $R_a=3 \text{ k}\Omega$, $R_b=1 \text{ k}\Omega$, $C=1 \mu\text{F}$, $R=159 \Omega$, $C_f=1 \mu\text{F}$, $R_f=R_1=159 \Omega$

HPF (fig. 16.48) $R_A=1 \text{ k}\Omega$, $R_B=568 \Omega$, $C=1 \mu\text{F}$, $R=1593 \Omega$

e7na E4U
kol mlfatna sr8a mn ElCoM

$$1. \Delta Z = \begin{vmatrix} -2 & 4 & 0 \\ 5 & 1 & -9 \\ 2 & -5 & 4 \end{vmatrix}, \Delta 11 = \begin{vmatrix} 1 & -9 \\ -5 & 4 \end{vmatrix}, I1 = 1.757 A$$

$$2. (a) \begin{bmatrix} 100 & -45 & 30 \\ 75 & 0 & 80 \\ 48 & 200 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ -0.1 \\ 0.5 \end{bmatrix}$$

$$(b) V_2 = \frac{A_2}{\Delta_Y} = \frac{-3013}{-1181050} = 2.6 \text{ mV}$$

3. (a)
$$\begin{cases} 470I_1 - 470I_2 = V \\ -470I_1 + 12670I_2 - 10000I_3 - 2200I_4 = 0 \\ -10000I_2 + 13200I_3 - 2200I_4 = 0 \\ -2200I_2 - 2200I_3 + 9100I_4 = 0 \end{cases}$$

(b) $\Delta Z = \begin{vmatrix} 470 & -470 & 0 & 0 \\ -470 & 12670 & -10000 & -2200 \\ 0 & -10000 & 12300 & -2200 \\ 0 & -2200 & -2200 & 9100 \end{vmatrix} = 1.13e14, \Delta_{11} = 2.9e11$

(c) $Z_{in} = 389 \Omega$

4.
$$\begin{cases} V_1 = 870I_1 - 870I_2 \\ 0 = -870I_1 + (870+870+1000)I_2 - 1000I_3 - 870I_4 \\ 0 = -1000I_2 + 1100I_3 - 100I_4 - 100I_5 \\ 0 = -870I_2 - 100I_3 + (870+220+100)I_4 \\ 0 = -100I_3 + 200I_5 \end{cases}$$

$$\Delta_Z = \begin{vmatrix} 870 & -870 & 0 & 0 & 0 \\ -870 & 2740 & -1000 & -870 & 0 \\ 0 & -1000 & 1100 & -100 & -100 \\ 0 & -870 & -100 & 1190 & 0 \\ 0 & 0 & -100 & 0 & 200 \end{vmatrix} = 2.7687e13 \Omega^5$$

$$\Delta_{11} = \begin{vmatrix} 2740 & -1000 & -870 & 0 \\ -1000 & 1100 & -100 & -100 \\ -870 & -100 & 1190 & 0 \\ 0 & -100 & 0 & 200 \end{vmatrix} = 2.4750e11 \Omega^4$$

$$Z_{in} = \frac{\Delta_Z}{\Delta_{11}} = 111.87 \Omega$$

$$5. \Delta Y = \begin{vmatrix} 3s & 0 & 0 \\ -3s & 33s & -10s \\ 0 & -10s & 17s \end{vmatrix} = 1383s^3, \Delta 11 = 461s^2, Y_{in} = \boxed{3s}$$

6. (a) $\omega = 1 \text{ rad/s}$

$$\Delta Z = \begin{vmatrix} j0.05 & -j0.05 & 0 \\ -j0.05 & 6 + j0.15 & -6 \\ 0 & -6 & 6 - j50e6 \end{vmatrix} = 1.5002e7 \angle 0.95^\circ \Omega^3$$

$$\Delta_{11} = \begin{vmatrix} 6 + j0.15 & -6 \\ -6 & 6 - j50e6 \end{vmatrix} = 3.0009e8 \angle -88.6^\circ \Omega^2$$

$$Z_{in} = \frac{\Delta_Z}{\Delta_{11}} = 50 \angle 89.5^\circ \text{ m}\Omega$$

(b) $\omega = 1 \text{ rad/s}$

$$\Delta Z = \begin{vmatrix} j16 & -j16 & 0 \\ -j16 & 6 + j48 & -6 \\ 0 & -6 & 6 - j156e3 \end{vmatrix} = 8.1263e7 \angle 79.4^\circ \Omega^3$$

$$\Delta_{11} = \begin{vmatrix} 6 + j48 & -6 \\ -6 & 6 - j156e3 \end{vmatrix} = 7.5462e6 \angle -7^\circ \Omega^2$$

$$Z_{in} = \frac{\Delta_Z}{\Delta_{11}} = 10.8 \angle 86.5^\circ \Omega$$

$$7. (a) \begin{cases} -0.063j * V1 + (V1 - V2)(-0.032j) = 0 \\ (V2 - V1)(-0.032j) + V2 * 0.067 + (V2 - V3)(-0.032j) = 0 \\ (V3 - V2)(-0.032j) + V3 * j * 6.3 \times 10^{-6} = 0 \end{cases}$$

$$\text{So, } Y_{in} = \frac{6(-6.5\omega^3 + 6.5j\omega^2 + 3.2 \times 10^{19}\omega - 1.2 \times 10^{21}j)}{\omega(-1.3j\omega^3 - 1600\omega^2 + j6.5 \times 10^{19}\omega + 3.9 \times 10^{21}j)}$$

$$(b) V = \frac{I_{in}}{Y_{in}} = 0.0514 \text{ V}$$

$$8. (a) \left. \begin{array}{l} V_1 = 10I_1 + 4(I_2 - I_3) \\ 0 = -4I_1 + 19I_3 \\ 0 = I_1 - I_2 + 2I_3 \end{array} \right\} \Rightarrow \Delta_Z = \begin{vmatrix} 10 & 4 & -4 \\ 0 & -4 & 19 \\ 1 & -1 & 2 \end{vmatrix} = 170 \Omega^3$$

$$(b) \Delta_{11} = \begin{vmatrix} -4 & 19 \\ -1 & 2 \end{vmatrix} = 11 \Omega^2. \quad Z_{in} = \frac{\Delta_Z}{\Delta_{11}} = 15.45 \Omega$$

9. (a)
$$\begin{cases} \frac{V_1}{R_{in}} + \frac{V_1 - V_2}{28} = 0 \\ \frac{V_1 - V_2}{20} + \frac{V_1}{R_x} = 0 \end{cases}$$

(b) $R_{in}=R_x$

10. (a) Let input voltage be 1 V. For deal op amp Type equation here. $V_i = 0$. Then, $V_{R4} = 1$ V.

$$I_4 = \frac{1}{R_4}, \quad V_4 = i_4 \left(R_4 + \frac{1}{j\omega C} \right) = 1 + \frac{1}{j\omega CR_4}, \quad I_{R_3} = \frac{1 - V_4}{R_3} = \frac{1}{R_3} \left(1 - 1 - \frac{1}{j\omega CR_4} \right) = -\frac{1}{j\omega CR_3 R_4}$$

$$I_{R_2} = I_{R_3}, \quad V_{12} = R_2 I_{R_2} + 1 = 1 - \frac{R_2}{j\omega CR_3 R_4}, \quad I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{1 - V_{12}}{R_1} = \frac{R_2}{j\omega CR_1 R_3 R_4} = I_{in}$$

$$\therefore Z_{in} = \frac{1}{I_{in}} = \frac{j\omega CR_1 R_3 R_4}{R_2}$$

(b) For $R_1 = 4 \text{ k}\Omega$, $R_2 = 10 \text{ k}\Omega$, $R_3 = 10 \text{ k}\Omega$, $R_4 = 1 \text{ k}\Omega$ and $C = 200 \text{ pF}$

$$Z_{in} = \frac{j\omega(200e-12)(4e3)(10e3)(1e3)}{10e3} = j\omega 8 \times 10^4 \Omega \text{ or } \text{equivalent to a } 0.8 \text{ mH inductor.}$$

$$11. \begin{cases} I_1 = \frac{V_1}{1000} + \frac{V_1 - V_2}{10000} \\ I_2 = \frac{V_2}{8000} + \frac{V_2 - V_1}{10000} \end{cases}$$

$$\text{So, } Y = \begin{bmatrix} 11 \times 10^{-4} & -10^{-3} \\ -10^{-4} & \frac{1}{8000} + \frac{1}{10000} \end{bmatrix}$$

$$12. \begin{cases} I_1 = \frac{V_1}{11} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_2}{10} \\ I_2 = \frac{V_2}{20} + \frac{V_2 - V_1}{8} + \frac{V_2 - V_1}{1} \end{cases}$$

So,
$$Y = \begin{bmatrix} 0.316 & -0.225 \\ -0.225 & 0.275 \end{bmatrix} S$$

$$13.(a) \begin{cases} I1 = 0.1s * V1 + 0.2s * (V1 - V2) \\ I2 = 0.25sV2 + 0.2s * (V2 - V1) \end{cases}$$

$$(b) Y = \begin{vmatrix} 0.3s & -0.2s \\ -0.2s & 0.15s \end{vmatrix}$$

$$(c) P = \frac{U^2}{R} = 10s$$

$$14. \begin{cases} I_1 = \frac{V_1 - V_2}{540} + \frac{V_1 - V_3}{200} & (1) \\ I_2 = \frac{V_2 - V_1}{540} + \frac{V_2 - V_3}{400} & (2) \end{cases} \text{ and } \frac{V_3 - V_1}{200} + \frac{V_3}{510} + \frac{V_3 - V_2}{400} = 0 \quad (3)$$

Substituting (3) into (1) and (2) we obtain: $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4.21e - 3 & -3.17e - 3 \\ -3.2e - 3 & 3.7e - 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

So $Y = \begin{bmatrix} 4.2 & -3.17 \\ -3.17 & 3.7 \end{bmatrix} mS$

$$15. (a) Y = \begin{vmatrix} 11e - 4 & -1e - 3 \\ -1e - 4 & 2.25e - 4 \end{vmatrix}, \begin{cases} I_1 = 11e - 4 * V_1 - 1e - 3 * V_2 \\ I_2 = -1e - 4 * V_1 + 2.25e - 4 * V_2 \end{cases}$$

(b) $p=9e-6 \text{ W}$

$$16. (a) \begin{cases} I1 = \frac{V1}{1k} + \frac{V1-V2}{10k} \\ I2 = \frac{V2}{8k} + \frac{V2-V1}{10k} \end{cases} \quad \text{---(1)} \quad \text{so, } Y = \begin{bmatrix} 11 & -1 \\ -1 & 2.25 \end{bmatrix} \times 10^{-4} S$$

(b) For the one-port network:

$I1 = \frac{9-V1}{1k}$ and $I2 = -\frac{V2}{4k}$ substituting into (1) we obtain $V1 = 4.329$ V and $V2 = 0.9114$ V.

Then $I1=4.67$ mA, and $P_{in} = 42$ mW

$$17. \begin{cases} I1 = \frac{V1 - 1*I1}{2} - 5I1 \\ I2 = -5I1 + \frac{V2}{5} + \frac{V2}{1} \end{cases}$$

$$\text{So, } Y = \begin{vmatrix} 6.25 & 0 \\ -31.25 & 1.2 \end{vmatrix}$$

18.
$$\begin{cases} I_1 = 0.6V_2 + \frac{V_1}{5k} + \frac{V_1 - V_a}{20k} & (1) \\ I_2 = \frac{V_2 - V_a}{10k} & (2) \\ \frac{V_a - V_1}{20k} + \frac{V_a - V_2}{10k} - 0.1I_1 = 0 & (3) \end{cases}$$

(3) can be written as: $V_a = \frac{V_1}{3} + \frac{V_2}{1.5} + 667I_1 \quad (4)$

Substitute (4) into (1): $I_1 = (241.4e - 6)V_1 + 0.6V_2 \quad (5)$

Substitute (4) and (5) into (2): $I_2 = (-17.23e - 6)V_1 + (41.52e - 3)V_2 \quad (6)$

$\therefore \mathbf{y} = \begin{bmatrix} 241.4e - 6 & 0.6 \\ -17.23e - 6 & 41.52e - 3 \end{bmatrix} S$

(a) $I_1 = y_{11}V_1 + y_{12}V_2 = (0.6)(1) = 0.6 A$
 $I_2 = y_{21}V_1 + y_{22}V_2 = (41.52e - 3)(1) = 41.52 mA$

(b) $I_1 = (241.4e - 6)(-8) + (0.6)(3) = 1.8 A$
 $I_2 = (-17.23e - 6)(-8) + (41.52e - 3)(3) = 125 mA$

(c) $I_1 = (241.4e - 6)(5) + (0.6)(5) = 3 A$
 $I_2 = (-17.23e - 6)(5) + (41.52e - 3)(5) = 207 mA$

$$19. \begin{cases} I1 = \frac{V1 - 5I1}{1} + 0.3I1 \\ I2 = \frac{V2}{2} - 0.3I1 + \frac{V2}{5} \end{cases}$$

So,
$$Y = \boxed{\begin{pmatrix} \frac{1}{5.7} & 0 \\ -\frac{1}{19} & 0.7 \end{pmatrix}}$$

20. (a) The input is applied at terminals G-S, while the output is taken from terminals D-S.

(b) $I_g = y_{is}V_{gs} + y_{rs}V_{ds}$

$$I_d = y_{fs}V_{gs} + y_{os}V_{ds}$$

$$y_{is} = \frac{I_g}{V_{gs}} \Big|_{V_{ds}=0} = j\omega(C_{gs} + C_{gd})$$

$$y_{rs} = \frac{I_g}{V_{ds}} \Big|_{V_{gs}=0} = -j\omega C_{gd}$$

$$y_{fs} = \frac{I_d}{V_{gs}} \Big|_{V_{ds}=0} = g_m - j\omega C_{gd}$$

$$y_{os} = \frac{I_d}{V_{ds}} \Big|_{V_{gs}=0} = \frac{1}{r_d} + j\omega(C_{gs} + C_{gd})$$

(c) $y_{is} = j\omega(3.4 + 1.4) \times 10^{-12} = j4.8\omega pS$

$$y_{rs} = -j\omega 1.4 \times 10^{-12} = -j1.4\omega pS$$

$$y_{fs} = 4.7 \times 10^{-13} - j\omega 1.4 \times 10^{-12} S$$

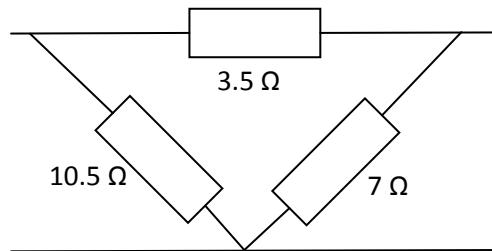
$$y_{os} = 10^{-4} + j\omega(0.4 + 1.4) \times 10^{-12} S$$

21. (a) simplify the circuit to a single resistor with value of 5.54 kΩ

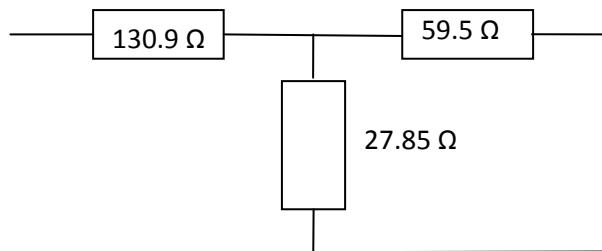
(b) $P = I^2R = \boxed{22.16 W}$

(c) $P = \frac{U^2}{R} = \boxed{14.62 W}$

22. (a)



(b)



23. (a) $\omega = 50, Z_{in} = 69.9j$

(b) $\omega = 100, Z_{in} = 141j$

24. (a) $\omega = 50, \quad Z_{in} = 5.875\angle 7.6^\circ \Omega$ (b) $\omega = 100, \quad Z_{in} = 6 \Omega$

25. $Z_{in}=1.86 \text{ M}\Omega$

26. $Z_{in} = 8.27 \Omega$

27.

(a) From solution of problem 14, $Y = \begin{bmatrix} 4.21 & -3.17 \\ -3.17 & 3.7 \end{bmatrix} mS$

So, $-y_{12} = 3.17 mS$

$$y_{11} + y_{12} = 1.04 mS$$

$$y_{22} + y_{12} = 0.53 mS$$

And the dependent source is turned off.

(b) For circuit of Figure 17.43: $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4.21e - 3 & -3.17e - 3 \\ -3.2e - 3 & 3.7e - 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$$\text{Since } I_2 = -V_2/2 \text{ we get } -0.5V_2 = (-3.17e - 3)V_1 + (3.7e - 3)V_2 \Rightarrow V_1 = 158.9V_2$$

$$\text{And since } I_1 = 1 \text{ A we get } 1 = (4.21e - 3)V_1 + (-3.17e - 3)V_2$$

$$V_2 = 1.502 \text{ volt.}$$

$$P_{out} = \frac{V_2^2}{2} = 1.128 W$$

Same result for circuit of Figure 17.13(a)

28. (a) From solution of problem 14, $Y = \begin{bmatrix} 4.21 & -3.17 \\ -3.17 & 3.7 \end{bmatrix} mS$

So, $-y_{12} = 3.17 mS$

$y_{11} + y_{12} = 1.04 mS$

$y_{22} + y_{12} = 0.53 mS$

And the dependent source is turned off.

(b) For circuit of Figure 17.43: $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4.21e - 3 & -3.17e - 3 \\ -3.2e - 3 & 3.7e - 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Since $I_2 = -V_2/1$ we get $-V_2 = (-3.17e - 3)V_1 + (3.7e - 3)V_2 \Rightarrow V_1 = 313.66V_2$

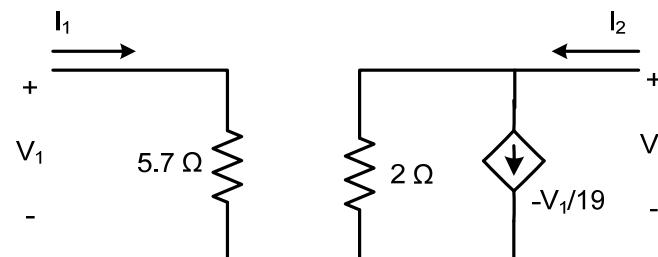
$I_1 = 10mA$, and $10e - 3 = (4.21e - 3)V_1 + (-3.17e - 3)V_2$

$V_2 = 7.6 mV, I_2 = -7.6 mA, \text{ and } V_1 = 2.38 V$

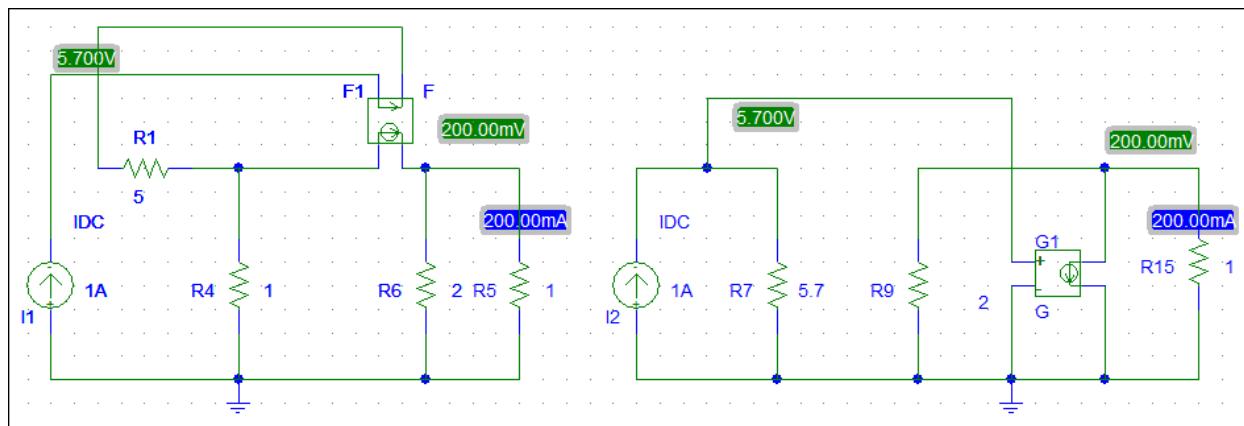
Same result for circuit of Figure 17.13(a)

29. $y_{11}=0.0041$, $y_{12}=-0.0031$, $y_{22}=0.0037$

$$30. (a) Y = \begin{bmatrix} \frac{1}{5.7} & 0 \\ -\frac{1}{19} & 0.5 \end{bmatrix} S$$



(b)



$$31. \begin{cases} I1 = 0.1V1 - 0.05V2 \\ I2 = -0.5V1 + 0.2V2 \\ V1 = 1 - 10I1 \\ V2 = -5I2 \end{cases}, \text{ so } \begin{cases} I1 = 0.027 \\ I2 = -0.182 \\ V1 = 0.727 \\ V2 = 0.901 \end{cases}$$

(a) $Gv = \frac{V2}{V1} = 1.239$

(b) $Gi = \frac{I2}{I1} = -0.741$

(c) $Gp = -\frac{V2*I2}{V1*I1} = 8.35$

(d) $Zin = \frac{V1}{I1} = 26.9 \Omega$

(e) $Zout = \frac{V2}{I2} = -4.9 \Omega$

32.

$$\mathbf{z} = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} \Omega \Rightarrow \mathbf{y} = \begin{bmatrix} -\frac{1}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{2}{13} \end{bmatrix} S$$

$$\mathbf{z} = \begin{bmatrix} 1000 & 470 \\ 2500 & 900 \end{bmatrix} \Omega \Rightarrow \mathbf{y} = \begin{bmatrix} -3.3 & 1.7 \\ 9.1 & -3.6 \end{bmatrix} mS$$

$$\mathbf{y} = \begin{bmatrix} 1 & 5 \\ 6 & 30 \end{bmatrix} mS \Rightarrow \mathbf{z} = \begin{bmatrix} -5.8 & 96 \\ 1.15 & -19 \end{bmatrix} \times 10^{17} \Omega$$

$$\mathbf{y} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} S \Rightarrow \mathbf{z} = \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} \\ -\frac{1}{5} & \frac{1}{5} \end{bmatrix} \Omega$$

33. $Z = \begin{bmatrix} 125 & 25 \\ 25 & 75 \end{bmatrix}$

$$34. Z = \begin{bmatrix} 125 & 25 \\ 25 & 75 \end{bmatrix}, V_2 = -10I_2, V_1 = (6\angle 0^\circ mA - I_1)50$$

$$\text{So, } \begin{cases} 0.3 - 50I_2 = 125I_1 + 25I_2 \\ -10I_2 = 25I_1 + 75I_2 \end{cases} \Rightarrow \begin{cases} 0.3 = 175I_1 + 25I_2 \\ -10I_2 = 25I_1 + 85I_2 \end{cases} \Rightarrow \begin{cases} I_1 = 1.8\angle 0^\circ mA \\ I_2 = -0.5\angle 0^\circ mA \end{cases} \Rightarrow \begin{cases} V_1 = 0.21\angle 0^\circ V \\ V_2 = 5.3\angle 0^\circ mV \end{cases}$$

$$\therefore G_V = 0.025, G_I = -0.294, G_P = 7 \times 10^{-3}$$

$$35. (a) Z1 = \begin{vmatrix} 9 & 6 \\ 6 & 8 \end{vmatrix}, Z2 = \begin{vmatrix} 158.7 & 27.8 \\ 27.8 & 87.3 \end{vmatrix},$$

$$\text{so } Z = Z1 + Z2 = \boxed{\begin{vmatrix} 167.7 & 33.8 \\ 33.8 & 95.3 \end{vmatrix}}$$

$$(b) Y1 = \begin{vmatrix} \frac{8}{36} & -\frac{6}{36} \\ -\frac{6}{36} & \frac{9}{36} \end{vmatrix}, Y2 = \begin{vmatrix} \frac{87.3}{1300} & -\frac{27.8}{1300} \\ -\frac{27.8}{1300} & \frac{158.7}{1300} \end{vmatrix}$$

$$\text{So } Y = Y1 + Y2 = \boxed{\begin{vmatrix} 0.289 & -0.188 \\ -0.188 & 0.372 \end{vmatrix}}$$

36. (a) $z = \begin{bmatrix} 16.54 & 15.66 \\ 15.8 & 17.55 \end{bmatrix} k\Omega$

(b) $G_I = -0.7, G_V = 0.64, G_P = 0.45$

$$37. \boxed{Z_{11} = \frac{V_1}{I_1} = \frac{5(I_1 - 0.8V_2)}{I_1}},$$

$$\boxed{Z_{22} = \frac{V_2}{I_2} = \frac{0.1V_1 + 2I_2}{I_2}}$$

$$\boxed{Z_{12} = \frac{V_1}{I_2} = 20 \Omega}$$

$$\boxed{Z_{21} = \frac{V_2}{I_1} = \frac{V_2}{0.8V_2} = 1.25 \Omega}$$

$$38. \quad y = \begin{bmatrix} 30 & 10 \\ 70 & 43.33 \end{bmatrix} mS, \quad z = \begin{bmatrix} 72.3 & -16.7 \\ -116.9 & 50 \end{bmatrix} \Omega$$

39. $V1 = Ia * 100000, V1 = Ib * (-2000j), V1 = 10000 * Ic + (Ic + 0.01V1)(-10000j)$

$$Z11 = -0.0049 + 0.0055j$$

$$V2 = Zc * 10000, V1 = (i2 - iC - 0.04v1)(385 - 1.9 \times 10^3)$$

$$\text{So, } z22 = 4.94 \times 10^3 + 2.88 \times 10^3j, Z12 = 48.5 - 30.9j$$

$$V1 = Ic * Z11, Ic = (-0.0047 + 0.0053j)I1$$

$$\text{So, } Z21 = -470 + 530j$$

40.
$$\mathbf{h} = \begin{bmatrix} 166.7 \Omega & 1/3 \\ -1/3 & \frac{1}{75} S \end{bmatrix}$$

$$41. h1 = \begin{vmatrix} 50 \Omega & 1 \\ -1 & 40 \text{ ms} \end{vmatrix}, h2 = \begin{vmatrix} 1.67 \Omega & 0.67 \\ -0.67 & 13.3 \text{ ms} \end{vmatrix}$$

42. (a)
$$\mathbf{z} = \begin{bmatrix} 3500 & -300 \\ -5 & 100 \end{bmatrix} \Omega$$

(b)
$$\mathbf{y} = \begin{bmatrix} 0.5 & 1.5 \\ 2.5 & 17.5 \end{bmatrix} mS$$

43. (a)
$$h = \begin{vmatrix} 20 \Omega & -2 \\ 5 & 0.1 s \end{vmatrix}$$

(b)
$$h = \begin{vmatrix} 100 \Omega & -2 \\ 5 & 0.14 s \end{vmatrix}$$

44. (a) $\boxed{\mathbf{h} = \begin{bmatrix} 5 \text{ k}\Omega & 0.55 \times 10^{-4} \\ 300 & 39 \mu\text{S} \end{bmatrix}}$

(b) $\boxed{G_I = \frac{I_2}{I_1} = h_{21} = 300}$

(c) $\boxed{Z_{out} = \frac{1}{h_{22}} = 25.64 \text{ k}\Omega}$

(d) KVL at input loop: $(100 + h_{11})|I_1| = 5 \text{ mV} \Rightarrow |I_1| = \frac{5 \text{ mV}}{100 + h_{11}}$

$$|V_1| = h_{11}|I_1| = h_{11} \frac{5 \text{ mV}}{100 + h_{11}}$$

$$|V_2| = \frac{|V_1|}{h_{12}} = \frac{h_{11} \frac{5 \text{ mV}}{100 + h_{11}}}{h_{12}} \Rightarrow \boxed{|V_2| = 89.13 \text{ Volt}}$$

$$45. \begin{cases} V1 = 1 - 5I1 \\ V2 = 2 * I2 \end{cases}$$

so

$$\begin{cases} V1 = \frac{1}{6}V \\ V2 = \frac{2}{3}V \\ I1 = \frac{1}{6}A \\ I2 = \frac{1}{3}A \end{cases}$$

46. Convert to t-parameters using Table 17.1, multiply resulting matrices and convert back to h.

$$\mathbf{h}_A = \begin{bmatrix} 50 \Omega & 1 \\ -1 & 40 \mu S \end{bmatrix} \text{ and } \mathbf{h}_B = \begin{bmatrix} 16.67 \Omega & 0.33 \\ -0.33 & 13.35 \mu S \end{bmatrix}$$

$$\mathbf{t}_A = \begin{bmatrix} 0.3 & 50 \Omega \\ 40 mS & 1 \end{bmatrix} \text{ and } \mathbf{t}_B = \begin{bmatrix} 1.0009 & 50.0051 \Omega \\ 40.1 mS & 3.0003 \end{bmatrix}$$

$$\mathbf{t} = \mathbf{t}_A \mathbf{t}_B = \begin{bmatrix} 5.0053 & 300.03 \Omega \\ 80.1 mS & 5.0005 \end{bmatrix}, \text{ and}$$

$$\boxed{\mathbf{h} = \begin{bmatrix} 60 \Omega & 0.2 \\ -0.2 & 16 \mu S \end{bmatrix}}$$

May confirm by combining two original networks and apply basic circuit laws.

47. Convert to y-parameters using Table 17.1, add resulting matrices and convert back to h.

$$\mathbf{h}_A = \begin{bmatrix} 50 \Omega & 1 \\ -1 & 40 \mu S \end{bmatrix} \text{ and } \mathbf{h}_B = \begin{bmatrix} 16.67 \Omega & 0.33 \\ -0.33 & 13.35 \mu S \end{bmatrix}$$

$$\mathbf{y}_A = \begin{bmatrix} 20 & -20 \\ -20 & 60 \end{bmatrix} mS \text{ and } \mathbf{y}_B = \begin{bmatrix} 60 & -20 \\ -20 & 20 \end{bmatrix} mS$$

$$\mathbf{y} = \mathbf{y}_A + \mathbf{y}_B = \begin{bmatrix} 80 & -40 \\ -40 & 80 \end{bmatrix} mS, \text{ and}$$

$$\boxed{\mathbf{h} = \begin{bmatrix} 12.5 \Omega & 0.5 \\ -0.5 & 60 \mu S \end{bmatrix}}$$

May confirm by combining two original networks and apply basic circuit laws.

$$48. (a) \boxed{\mathbf{y} = \begin{bmatrix} \frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R} \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} R & 1 \\ -1 & 0 \end{bmatrix}}$$

$$(b) \boxed{\mathbf{z} = \begin{bmatrix} R & R \\ R & R \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} 0 & 1 \\ -1 & \frac{1}{R} \end{bmatrix}}$$

$$49. I_1 = \frac{V_1}{1000} - 10^{-5}V_2, V_2 = I_2 * 10000 - 100V_1$$

So
$$h = \begin{vmatrix} 1000 \Omega & 0.01 \\ 10 & 5 \text{ k}\Omega \end{vmatrix}$$

$$50. (a) \begin{cases} I_1 = \frac{V_1}{5} + \frac{V_1 - V_2}{10} \\ I_2 = \frac{V_2}{20} + \frac{V_2 - V_1}{10} \end{cases} \Rightarrow \mathbf{t} = \begin{bmatrix} 1.5 & 10 \Omega \\ 350 \text{ mS} & 3 \end{bmatrix}$$

$$(b) \mathbf{h} = \begin{bmatrix} \frac{10}{3} \Omega & \frac{1}{3} \\ -\frac{1}{3} & 0.1167 \text{ S} \end{bmatrix}$$

$$51. \begin{cases} V1 = (631.6 + 12530)I1 + 0.2V2 \\ V2 = 12530I1 + 0.2V2 \end{cases}, \text{ so } t11 = 1.04$$

$$\text{and } \begin{cases} -I2 = \frac{V1}{631.6 + (1890||12530)} * \frac{12530}{12530 + 1890} \end{cases}, \text{ so } t12 = 0.803 \Omega$$

$$\text{and } \begin{cases} I1 = \frac{V1 - 0.2V2}{12530 + 631.6} \\ V2 = I1 * 12530 + 0.2V2 \end{cases}, \text{ so } t21 = 6.35 \times 10^{-5} \text{ s}$$

and $V2 = I1(1890 + 12530) + I1 * 12530$, so $t22 = 1.15$

then, $t = \begin{vmatrix} 1.04 & 0.803 \Omega \\ 6.35 \times 10^{-5} \text{ s} & 1.15 \end{vmatrix}$

$$52. \text{ (a) } \mathbf{ab} = \begin{bmatrix} 9.5 & 6 \\ 5.5 & 3 \end{bmatrix}, \text{ (b) } \mathbf{ba} = \begin{bmatrix} 8.5 & 7 \\ 5.5 & 4 \end{bmatrix}, \text{ (c) } \mathbf{ac} = \begin{bmatrix} -18 \\ 0 \end{bmatrix}$$

$$\text{(d) } \mathbf{bc} = \begin{bmatrix} -5 \\ -3.5 \end{bmatrix}, \text{ (e) } \mathbf{bac} = \begin{bmatrix} -27 \\ -18 \end{bmatrix}, \text{ (f) } \mathbf{aa} = \begin{bmatrix} 27 & 18 \\ 9 & 18 \end{bmatrix}$$

$$53 \text{ (a)} \quad t = t1 * t2 = \begin{vmatrix} 16.8 & 1.87 \times 10^4 \\ 0.004 & 4.2 \end{vmatrix}$$

$$(b) \quad t = t2 * t1 = \begin{vmatrix} 6.67 & 1.14 \times 10^4 \\ 8.26 \times 10^{-3} & 1.43 \end{vmatrix}$$

54. (a) $t_A = \begin{bmatrix} \frac{3}{2} & 1 \Omega \\ \frac{1}{2}S & 1 \end{bmatrix}, \quad t_B = \begin{bmatrix} \frac{7}{4} & 3 \Omega \\ \frac{1}{4}S & 1 \end{bmatrix}, \quad t_C = \begin{bmatrix} \frac{11}{6} & 5 \Omega \\ \frac{1}{6}S & 1 \end{bmatrix}$

(b) $t = t_A t_B t_C = \begin{bmatrix} 6.19 & 19.88 \Omega \\ 2.48 S & 8.12 \end{bmatrix}$

(c) $\left\{ \begin{array}{l} \frac{V_x - V_1}{1} + \frac{V_x}{2} + \frac{V_x - V_y}{3} = 0 \\ I_1 = \frac{V_1 - V_x}{1} \\ \frac{V_y - V_x}{3} + \frac{V_y}{4} + \frac{V_y - V_2}{5} = 0 \\ I_2 = \frac{V_2 - V_y}{5} + \frac{V_2}{6} \end{array} \right\} \Rightarrow y = \begin{bmatrix} 408.4 & -50.3 \\ -50.3 & 311.3 \end{bmatrix} mS$

Converting these y-parameters into t-parameters confirms (b).

55. (a) $t = t_{right} * t_{left} = \begin{vmatrix} 2.12 & 36.15 \\ 0.12 & 2.49 \end{vmatrix}$

(b) $t = t_{left} * t_{right} = \begin{vmatrix} 3.01 & 82.1 \\ 0.047 & 1.59 \end{vmatrix}$

56. (a) Using the t-parameter definitions we can get: $t = \begin{bmatrix} 0.1 & 20 \Omega \\ 2.8S & 4 \end{bmatrix}$

(b) t-parameter relationship equations: $\begin{aligned} V_1 &= t_{11}V_2 - t_{12}I_2 \\ I_1 &= t_{21}V_2 - t_{22}I_2 \end{aligned}$

KVL at input loop: $V_1 = V_s - 100I_1 = V_s - 100(t_{21}V_2 - t_{22}I_2) = V_s - 100t_{21}V_2 + 100t_{22}I_2$

But $V_1 = t_{11}V_2 - t_{12}I_2 = V_s - 100t_{21}V_2 + 100t_{22}I_2$

Turning Vs source off: $Z_{out} = \frac{V_2}{I_2} = \frac{t_{12} + 100t_{22}}{t_{11} + 100t_{21}} = \boxed{Z_{out} = 1.5 \Omega}$

57. $t = t * t * t = \begin{vmatrix} 307 & 2.2 \times 10^4 \\ 0.252 & 181 \end{vmatrix}$

$$58. (a) \boxed{\mathbf{t}_a = \begin{bmatrix} 1 & 0 \\ \frac{1}{a} & 1 \end{bmatrix}, \quad \mathbf{t}_b = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}, \quad \mathbf{t}_c = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & a \end{bmatrix}}$$

$$(b) \boxed{\mathbf{t} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{10} & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{50} & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.58 & 14 \Omega \\ 0.115S & 4.5 \end{bmatrix}}$$

$$59. \quad Z = \begin{vmatrix} 15 & 5 \\ 5 & 15 \end{vmatrix}, h = \begin{vmatrix} 13.3 & 0.33 \\ -0.33 & 0.667 \end{vmatrix}, Y = \begin{vmatrix} 0.075 & -0.025 \\ -0.025 & 0.075 \end{vmatrix}, t = \begin{vmatrix} 3 & 40 \\ 0.5 & 3 \end{vmatrix}$$

$$60. \quad y = \begin{bmatrix} 0.075 & -0.025 \\ -0.025 & 0.075 \end{bmatrix}^4 = \begin{bmatrix} 53.13 & -46.88 \\ -46.88 & 53.13 \end{bmatrix} \mu S$$

Use Table 17.1 to convert to the following:

$$z = \begin{bmatrix} 15 & 5 \\ 5 & 15 \end{bmatrix} \Omega$$

$$h = \begin{bmatrix} \frac{1000}{75} \Omega & 1/3 \\ -1/3 & \frac{1}{15} S \end{bmatrix}$$

$$t = \begin{bmatrix} 3 & 40 \Omega \\ 0.2 S & 3 \end{bmatrix}$$

61. $t = t * t * t * t = \begin{vmatrix} 1561 & 13920 \\ 174 & 1561 \end{vmatrix},$

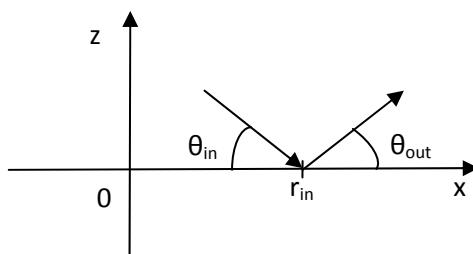
$$Y = \begin{vmatrix} 0.112 & -1.05 \\ -7.2 \times 10^{-5} & 0.112 \end{vmatrix},$$

$$Z = \begin{vmatrix} 8.97 & 84.1 \\ 0.0057 & 8.97 \end{vmatrix},$$

$$h = \begin{vmatrix} 0.892 & 9.38 \\ -6.4 & 0.11 \end{vmatrix}$$

$$62. (a) A = \frac{r_{out}}{r_{in}} \Big|_{\theta_{in}=0}, \quad B = \frac{r_{out}}{\theta_{in}} \Big|_{r_{in}=0}, \quad C = \frac{\theta_{out}}{r_{in}} \Big|_{\theta_{in}=0}, \quad D = \frac{\theta_{out}}{\theta_{in}} \Big|_{r_{in}=0}$$

(b) A perfectly reflecting flat mirror:



$$63. (a) t1 = \begin{vmatrix} 1 & 0.5d \\ 0 & 1 \end{vmatrix}, t = t1 * t1 = \begin{vmatrix} 1 & d \\ 0 & 1 \end{vmatrix}$$

$$(b) r_{out} = A * r_{in} + B * \theta_{in}, \theta_{out} = C * r_{in} + D * \theta_{in}$$

Unit: A: 1; B: m/rad; C: rad/m; D: 1