Tool-sharing system model

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1 Introduction

This is a model of a neighborhood tool-sharing program made for the session 3 of the "Social Networks in Social Science Research" course at the University of Mannheim.

2 Model Design

We model a neighborhood tool-sharing network where participants can borrow and return tools. The network consists of nodes representing households connected by ties, signifying friendly relationships. Actors face a repeated decision over the number of rounds:

- **Cooperate (C)**: Return borrowed tools, maintain reputation, and keep access to the tool-sharing program.

- **Defect (D)**: Keep the tool without returning it, risking punishment and eventual exclusion.

Before the game starts, all actors contribute to acquiring the tools. By using the community tools, they receive benefit. Defection can result in punishment from direct neighbors (other actors in the network the defector is connected to). Repeated defection reduces the probability of future borrowing.

3 Model Parameters

- **Initial investment:** $C_{\text{init}} = 50$ (cost to join the program).

- **Benefit per round:** $B = 4$ (fixed gain from tool access).

- **Punishment per neighbor:** $P = 2$, applied with probability 0.6.

- **Future interaction penalty:** $F = 3$, applied when access is lost.

- **Borrowing probability:**

$$P_{\text{borrow},i} = \max(0, 1 - 0.1 \times \text{past defections})$$

Each defection reduces borrowing probability by 10%.

4   Utility Functions

4.1   Cooperation utility

If a participant always cooperates, their utility function is:

$$U_{\text{cooperate},i}(T) = -C_{\text{init}} + T \cdot B \tag{1}$$

$$U_{\text{cooperate},i}(T) = -50 + 4T$$

4.2   Defection utility

If a participant defects, their utility per round is:

$$U_{\text{defect},i}(T) = -C_{\text{init}} + \sum_{t=1}^{T} (B - E[P_i] - (1 - P_{\text{borrow},i})F) \tag{2}$$

where the expected punishment per round is:

$$E[P_i] = d_i \times (P \times 0.6) \tag{3}$$

and d_i is the degree (number of neighbors).

5 Main Strategies

5.1 Full cooperation

- Always return tools.

- No punishment is applied.

- Utility over T rounds:

$$U_{\text{cooperate},i}(T) = -50 + 4T$$

5.2 Full defection

- Always defect (keep tools).

- Faces punishment from neighbors.

- Borrowing probability decreases over time.

- Utility over T rounds:

$$U_{\text{defect},i}(T) = -50 + \sum_{t=1}^{T} (4 - E[P_i] - (1 - P_{\text{borrow},i})F)$$
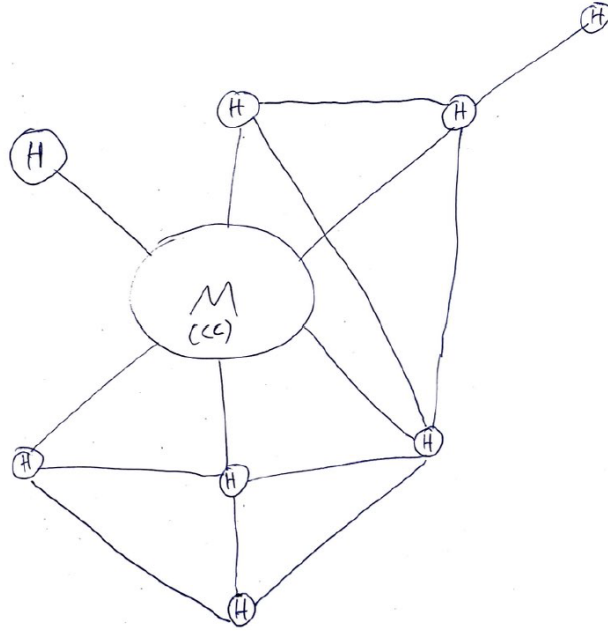
6 Network Structure

Figure 1: Network Structure for Tool-Sharing Program

7 Utility Calculations

Let's calculate some utilities for two different actors in the networks.

- **Actor A**: Rather isolated, only has one tie.

- **Actor B**: Connected to others, linked to fours neighbors.

We will calculate the utilities for the 3 above-mentioned strategies:

- **Full cooperation**: See 5.1 above.

- **Full defection**: See 5.2 above.

- **30% defection**: Defects randomly with 30% probability per round. Punishment and borrowing probability reductions are stochastic.
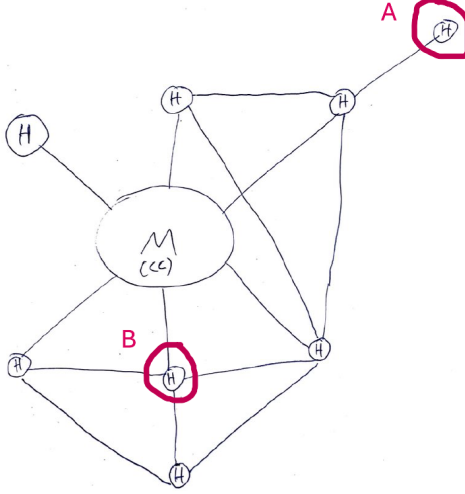
Figure 2: Actors in the Tool-Sharing Program

7.1 Utilities for selected actors

For each actor, we compute their total utility over 10, 50, and 100 rounds based on the three strategies:

Strategy	Actor A (1 tie)	Actor B (4 ties)
Full Cooperation - 10 rounds	-10.00	-10.00
Full Cooperation - 50 rounds	150.00	150.00
Full Cooperation - 100 rounds	350.00	350.00
Full Defection - 10 rounds	-35.50	-59.50
Full Defection - 50 rounds	-43.50	-163.50
Full Defection - 100 rounds	-112.90	-256.30
30% Defection - 10 rounds	-15.00	-25.50
30% Defection - 50 rounds	45.00	-50.30
30% Defection - 100 rounds	110.20	-90.10

Table 1: Utility Comparison for Selected Actors under Different Strategies