Pole Placement Control and Arbitrary Non-Sinusoidal Disturbance Cancellation for the Stabilization and Regulation of a Manipulator Arm

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***Abstract*— In this paper a joint angle regulation control task is achieved for a 3 degree of freedom robotic manipulator without a loss of generality. The applied control law consists of non-adaptive pole placement and adaptive high frequency periodic disturbance rejection. The disturbance is assumed to be of a known period and phase, and is modelled as a truncated Fourier series; thus, the controller may be applied to tasks involving non-sinusoidal or irregular disturbance of known period and phase.**

Keywords—robot manipulator, pole placement, control, disturbance rejection

I. Introduction

Manipulator robotics have been an emerging area of research in the recent years. Robotic manipulators have the ability of performing tasks with more accuracy, power, and speed in comparison to humans. These performance measurements are all a direct result of the controller being applied to the manipulator. Automation applications in the industry often involve repetitive operations. Periodic disturbances induced by the repetitive operations must be compensated to achieve precise functioning. Many repetitive tasks induce periodic disturbances of a known period (equal to the period of the repetitive task), however, these disturbances are non-sinusoidal and may be highly irregular in terms of their profile. In order to realize repetitive tasks requiring accuracy, the suppression of irregular periodic disturbances is a problem to be solved.

Robotic controllers can be classified into two general categories: fixed designs, and adaptive designs [1]. Fixed designs require the exact knowledge of the system and environment. However, due to the complexity of manipulator dynamics and the variety of environmental conditions, fixed controller designs can be less effective when applied to complex robot manipulators. Adaptive control has been recognised as an effective approach for robot manipulator controller design due to the presence of nonlinearities and uncertainties in robot dynamic models and environmental disturbances [1]. A few common adaptive control methods include model-reference control [2], self-tuning adaptive control [3], and linear perturbation adaptive control [1]. These methods are effective control schemes. However, these methods often assume the manipulator dynamics are completely unknown.

Periodic disturbance suppression of known period disturbances is a prevalent field of research in the realm of repetitive control. This includes the adaptive suppression of periodic disturbances that consist of a fundamental wave and harmonics [4], as well as the suppression of clogging torques in PMDC motors utilizing a disturbance observer where the harmonic frequencies which sum to approximate the disturbance torque are determined by the motor's build (slot and pole count) [5]. The latter is closely related to the approach described in this paper; however, the intended application concerns the control of manipulators with the disturbance being specific to the repetitive task at hand, and thus, concerns a larger scope for possible periodic disturbance profiles [6]. Disturbance observers are a common method for removing system disturbances and a future research area can potentially be to compare the performance with the disturbance rejection technique developed in this paper.

In the case of a 3 degree of freedom manipulator, the dynamics of the system are in fact known and can take the form of explicit equations. For this reason, a control scheme was developed such that the system dynamics are known; meanwhile, the environment contains uncertainties. Although in application, a repetitive periodic disturbance would arise from a repetitive task, for simplicity a regulation task is considered. Since the pole placement controller's derivation for a regulation task versus a tracking task (of some desired repetitive trajectory, for example) does not entail any added complexity, this simplification can be made without a loss of generality. Thus, the proposed control mechanism is intended to be used in conjunction with a wide variety of non-redundant manipulators for the task of regulation and adaptive disturbance rejection without a loss of generality.

The following section includes a summary of the relevant mathematical preliminaries including the manipulator dynamics and nature of the environmental disturbance. In section III the parameter estimation scheme to learn the unknown disturbance is described. In section IV the derivation of the pole placement control with disturbance rejection is tackled. The simulation and testing conditions used for the development of this system are summarized and a discussion of the results follows in section V. The simulation results are also compared with other control methods. Finally, in section VI the repetitive regulation task control scheme is concluded and potential future applications of this work are discussed.

II. MATHEMATICAL PRELIMINARIES

A. Manipulator Dynamics

The kinematics of the system is described by a 2-bar linkage with a total of three joints to form a 2-link planar system on a rotating base. The dynamics of the manipulator are strongly nonlinear, and can be written in the general form known as the Euler-Lagrange equation [7],

$$M\big(q(t)\big)\ddot{q}(t) + C\big(q(t),\dot{q}(t)\big)\dot{q}(t) + G\big(q(t)\big) = F(t) + \tau(t) \quad (1)$$

where M represents the generalized inertia matrix and C represents the Coriolis matrix of the robotic system. $F(t)$ represents the external force applied by the environment to the manipulator (which takes the form of a multi-frequency periodic disturbance torque applied to each joint), $\tau(t)$ represents the motor input joint torques and G represents the joint torques due to gravity. It is assumed that there is no friction or noise in the actuators and sensors as well as no gravity acting on the manipulator. The gravity joint torques are included the rest of the paper to further generalize the derived control scheme. The function M and thus C are assumed to be known where M and C are related by [7],

$$\frac{d}{dq}M\big(q(t)\big) = 2C\big(q(t),\dot{q}(t)\big) \quad (2)$$

It is assumed that the function $M\big(q(t)\big)$ and thus $C\big(q(t),\dot{q}(t)\big)$ are known. The manipulator system is described by it's modified DH (Denavit-Hartenberg) parameters and rigid body properties that are shown in Table 1. For more details on the Denavit-Hartenberg convention refer to [7]. A schematic of the manipulator is shown in Fig. 1.

B. Periodic Disturbance

The externally applied disturbance torques acting on the manipulator, $F(t)$, is assumed to be of a known period and phase, and is constructed as a summation of multiple sinusoids with varying frequencies and unknown amplitudes. In this case, the disturbance at each joint, *i*, can be characterized as follows,

$$\mathrm{F(t)}_i = A_{1i}\sin(\omega_{1i}t + \varphi_{1i}) + A_{2i}\sin(\omega_{2i}t + \varphi_{2i}) \quad (3)$$

where A is the amplitude in Nm, t is the time in seconds, ω is the frequency in rad/s, and ϕ is the phase in radians of each sinusoidal component of the disturbance. It is assumed that the amplitudes are the only unknowns in (3). These amplitudes are the values to be estimated by a parameter estimator algorithm. Some knowledge of the periodic disturbance term to be estimated is required prior to learning; thus, the sinusoids that are summed together to return an estimate of the disturbance term are chosen beforehand such that the summation of sinusoids can closely approximate the disturbance.

The disturbance rejection task to be achieved is a simplification of a known period disturbance cancellation problem. The summation of sinusoids that make up the disturbance estimate takes the form of a truncated Fourier series. A Fourier series is a periodic function of related sinusoids. With

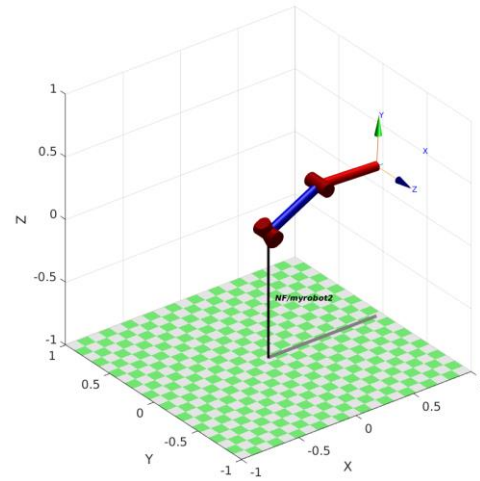

Fig. 1. Robotic manipulator designed for simulations

appropriate weights a truncated Fourier series can be used to approximate any periodic disturbance (with a known period). In a full problem learning the amplitudes of a truncated Fourier series will allow the algorithm to approximately learn any known period disturbance. This is true for any periodic disturbance even if it is of an irregular (non-sinusoidal) profile if the sinusoidal components of unknown amplitudes can closely recreate the disturbance. Thus, the approximation of a general periodic disturbance (of known period) takes the form of a full Fourier series.

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\left(\frac{2\pi kx}{T}\right) + \sum_{k=1}^{\infty} b_k \sin\left(\frac{2\pi kx}{T}\right) \quad (4)$$

where T is the known period. For some set of Fourier coefficients a_k and b_k defined by the integrals:

$$a_k = \frac{2}{T}\int_0^T f(x)\cos\left(\frac{2\pi kx}{T}\right)dx \quad (5)$$

$$b_k = \frac{2}{T}\int_0^T f(x)\sin\left(\frac{2\pi kx}{T}\right)dx \quad (6)$$

Here, a_k and b_k are the amplitudes of the kth torque harmonic. A total of 2k harmonics are considered and assumed to be of known frequencies.

As it is not possible to learn an infinite number of unknown amplitudes a truncation of the full series is necessary. The minimum number of sinusoids should be chosen such that the series is still able to closely recreate the disturbance. The need to learn the phase of the constituents of the external disturbance is eliminated by the use of the Fourier series.

III. PARAMETER ADAPTATION ALGORITHM

The dynamics equation presented in (1) can be rearranged to form a static parametric model (SPM) to facilitate learning of the unknown amplitudes [8]. This is done in a 2-step process. First the parametric model is formed as to isolate the unknown parameters and then a modified least squares algorithm is used to facilitate the online learning of the amplitudes of each sinusoid that forms the unknown disturbance (as a summation).

TABLE I. RIGID BODY PROPERTIES AND DENAVIT-HARTENBERG (DH) PARAMETERS DEFINED FOR THE SELECTED MANIPULATOR

	DH Parameters				Rigid body properties		
	d (m)	θ (rad)	a (m)	α (rad)	mass (kg)	COM (m)	Inertia ($kg \cdot m^2$)
Joint 1 || Base	0	π/2	0	0	0.5	0	eye(3)
Joint 2 || Link 1	0	π/2	0	0	1	0.325	eye(3)
Joint 3 || Link 2	0.5	-π/2	0	0	1	0.2175	eye(3)
Tool	0.5	0	0	0	~	0	~

A. Static Parametric Model

Equation (1) of the system is reformulated into the form of an SPM [8]. Each joint, *i*, of the system was considered separately, resulting in the following form:

$$z_i = \theta_i^{*T} \phi_i \tag{7}$$

where $\theta^* \epsilon R^3$ is the parameter vector with all unknown parameters, $z \epsilon R$ and $\phi \epsilon R^3$ are the regressor signals available for measurement. The signal z corresponds to the filtered disturbance applied to each joint. In common applications, measurement of the joint angles is done with motor encoders. However, joint velocities are not measured and only approximated using a differential. Thus, a stable filter of the order of the highest derivative of 'q' (joint angles) is required, which is 2. Thus, a second order stable filter with poles located at -λ is applied.

The system is a second order system with the first and second derivatives of the state q unknown, a filter is applied of the same order. Each of the components in (**Error! Reference source not found.**7) are defined as,

$$\hat{z}_i = \frac{[M(q)s^2q]_i}{(s+\lambda)^2} + \frac{[C(q,\dot{q})sq]_i}{(s+\lambda)^2} + \frac{G_i}{(s+\lambda)^2} - \frac{\tau_i}{(s+\lambda)^2} \tag{8}$$

$$\theta_i^* = [A_{1i} \quad A_{2i}]^T \tag{9}$$

$$\phi_i = \left[\frac{\sin(\omega_{1i}t+\varphi_{1i})}{(s+\lambda)^2} \quad \frac{\sin(\omega_{1i}t+\varphi_{1i})}{(s+\lambda)^2}\right]^T \tag{10}$$

The notation $[\ .\ ]_i$ denotes the i^{th} row of the enclosed matrix or vector.

B. Least Squares Adaptation Algorithm

The identification of the parameters is achieved through the application of a least squares algorithm with forgetting factor [9]. The past estimation errors are penalized; however, the forgetting factor β ensures that the new measurements are given a higher weight. A normalization factor, m_s, is applied to ensure that the signals remain bounded. The recursive form of this algorithm is

$$\dot{\theta}_i(t) = P_i(t)\varepsilon_i(t)\phi(t) \tag{11}$$

$$\dot{P}_i(t) = \beta_i P_i(t) - P_i(t)\frac{\phi^T(t)\phi(t)}{m_s^2}P_i(t) \tag{12}$$

$$\varepsilon = \frac{z-\hat{z}}{m_s^2} \tag{13}$$

$$m_s^2 = 1 + \alpha\phi\phi^T \tag{14}$$

where P is the covariance matrix of the system, and ε is the normalized estimation error. The design constants include the forgetting factor β, and the normalization constant α.

The convergence of parameter estimation relies on the richness of the regressor signals in the estimation algorithm. Convergence is ensured in this system as the regressor signals are guaranteed to contain multiple sinusoidal frequencies for each joint.

IV. POLE PLACEMENT CONTROL

A pole placement control scheme is proposed to incorporate with nonlinear and time varying dynamic behaviours of the manipulator system. The scheme employs a reference set of poles to provide for a specified closed-loop performance and asymptotically regulate the output of the system. However, the pole placement scheme requires a controllable state-space representation, this is problem is tackled by modifying the control input torque.

A. Ficticious State-Space Representation

The system can be represented in a modified state space form as follows:

$$\begin{bmatrix}\dot{q}\\ \ddot{q}\end{bmatrix} = \begin{bmatrix}[0]_n & I_n\\ [0]_n & -M^{-1}C\end{bmatrix}\begin{bmatrix}q\\ \dot{q}\end{bmatrix} + \begin{bmatrix}[0]_n\\ M^{-1}\end{bmatrix}(\tau - G) + \begin{bmatrix}[0]_n\\ M^{-1}\end{bmatrix}F \tag{15}$$

Where $[0]_n$ refers to a nxn square matrix containing all zeros and I_n refers to the identity matrix. The joint angles, q, and their derivatives in this equation are vectors containing one term for each joint in the manipulator. Note that $[0]_n, I_n, M, C$ ϵR^n where n represents the number of joints in the manipulator. However, with respect to each joint torque input the above state space system is not controllable, thus, a modified representation of the system is necessary in which the controllability matrix is full rank. The solution proposed includes a compensation term in the controller generated input torque that results in an augmented system including an extraneous addition/subtraction operation. This results in the following manipulator dynamics equation:

$$M(q(t))\ddot{q}(t) + C(q(t),\dot{q}(t))\dot{q}(t) + G(q(t)) = F(t) + \tau(t) + M(q(t))q(t) - M(q(t))q(t) \tag{16}$$

The resulting modified (and controllable) state-space representation takes the following nonlinear time varying system form.

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + \eta(t) \quad (17)$$

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} [0]_n & I_n \\ I_n & -M^{-1}C \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} [0]_n \\ M^{-1} \end{bmatrix} (\tau - G - Mq) + \begin{bmatrix} [0]_n \\ M^{-1} \end{bmatrix} F \quad (18)$$

As can be seen from the single output (corresponding to a single joint torque) controllability matrix in (19), the modified state space system avoids columns of zeros in both $A(t)$ and thus in the controllability matrix [8], ensuring that it is full rank. This is the case for the controllability of all joints.

$$P_{ci} = [B_i \quad AB_i \quad A^2B_i \quad A^3B_i \quad A^4B_i \quad A^5B_i] \quad (19)$$

It must be noted that although modifying the control input torques (τ) to include a dummy torque (Mq) allows for a controllable state-space representation, this is not guarenteed to be optimal in terms of minimizing the controller's power input.

B. *Desired Response*

The criteria of choosing a reference characteristic polynomial for the closed-loop system is not only based on stability (i.e. with stable poles) but is also required to satisfy some prespecified performance criteria such as rise time, settling time and damping. As this state space system is a 6th order system (characteristic polynomial contains 6 poles), the response can be approximated with the dominant pole pair [10]. This is a reasonable approximation provided the non-dominant poles have a much quicker response than the dominant pole pair. The stability of the system is ensured when all poles are stable, meaning that they are located on the left-hand s-plane. Consequently, the four non-dominant poles are located at -6 and -5 as pairs (chosen arbitrarily). In order to avoid pole-zero cancellations, the poles are not to be placed in the vicinity of the pair of zeros located at -1. In this solution the dominant pole pair is placed at -2. This results in the pole placements in the complex plane shown in Fig. 2.

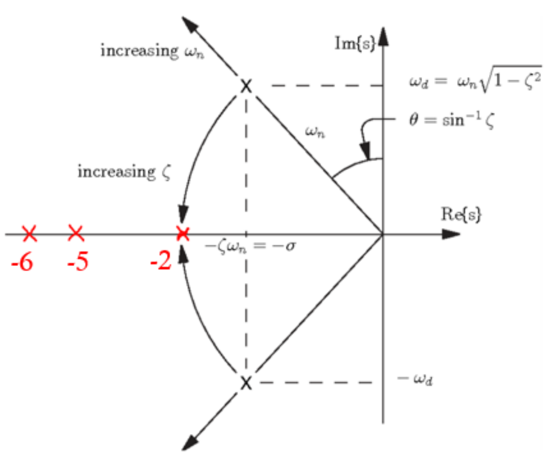

Fig. 2. Illustration of the pole selection of the desired system dynamics

TABLE II. SYSTEM PERFORMANCE CRITERIA DETERMINED BY THE POLE PLACEMENT DESIGN

Rise Time(s)	Settling Time (s)	Settling Min. (s)	Settling Max. (s)	Peak	Peak Time(s)
1.6790	2.9170	0.2252	0.2498	0.2498	4.69

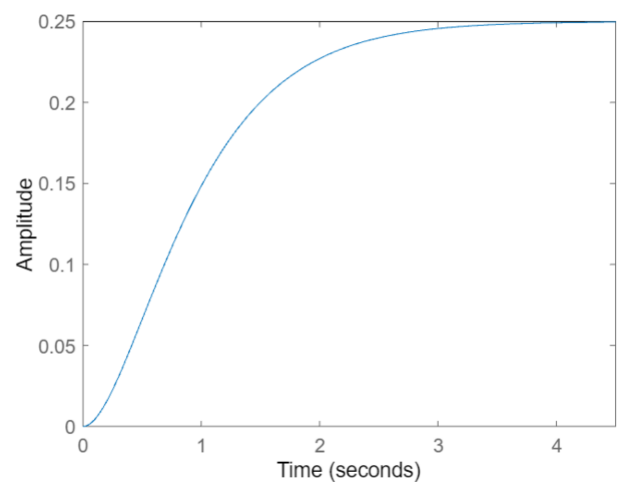

Fig. 3. Critically damped step response desired for the system response

The dominant pole placements are such that the response is approximately second order with the following performance criteria as well as the desired step response is shown in Fig. 3.

The dominant transfer function, $G_{dominant}(s)$, can be written as follows:

$$G_{dominant}(s) = \frac{1}{(s+2)^2} = \frac{1}{s^2+4s+4} \quad (20)$$

The desired response is critically damped; thus, the damping factor is equal to one. The natural frequency is calculated to be 2 rad/s and the time constant of decay of each pole is 5 s.

C. *Pole Placement Scheme*

By first cancelling the torques due to gravity, the dummy torques, and the disturbance torques, the closed-loop poles can then be placed using a feedback gain K_c. This will only achieve the desired closed loop pole locations if the periodic disturbance is well estimated. This results in the following input into the plant and closed loop system:

$$u(t) = \tau_{ppc}(t) = -K_c(t) \cdot x(t) \quad (21)$$

$$\dot{x}(t) = \big(A(t) - B(t) \cdot K_c(t)\big)x(t) \quad (22)$$

K_c is designed by matching the characteristic polynomial of the state space system with a desired characteristic polynomial (refer to section *IV.B*) and then solving for the unknown gain matrix, K_c. Note that the K_c matrix is time varying and must be solved for at every timestep.

$$\det(sI - A^*) = \det(sI - A(t) + B(t) \cdot K_c(t)) \quad (23)$$

Where A^* corresponds to a system with the previously outlined desired response of the modified state space system representation. The matrix A^* is a fictitious system matrix and does not correspond to real manipulator system due to the inclusion of the dummy term. Nonetheless, the desired response is achieved when the control input joint torques compensate for the dummy term. The estimated disturbance ($\hat{F}(t)$) is injected into the plant to cancel its effects on the system response, thus, if estimation of the sinusoidal disturbance is perfect, it removes oscillations of the manipulator's joints due to the disturbance. Consequently, the final controller input is calculated to be:

$$\tau(t) = -K_c(t) \cdot x(t) + G + M(q)q(t) - \hat{F}(t) \quad (24)$$

V. SIMULATION

The derived controller scheme is tested in a simulation setting. The simulation is constructed using Simulink and MATLAB tools. A description of the set up and results follow in the next sections.

A. Simulation Description

A high-level block diagram of the overall flow of information is shown in Fig. 4.

The properties of the manipulator are determined using the Peter Corke's Robotics Toolbox in MATLAB [11]. This toolbox is used to produce the dynamics equation of the manipulator (1) in terms of the joint positions and velocities. Code generated symbolic expressions from the Seriallink Manipulator class return the M and G matrices as functions of the manipulator's current configuration and the C matrix as a function of the manipulator's current configuration and joint velocities. These functions make up the block labelled as 'calculate dynamics' in Fig, 5.

The equations used for the parameter estimation, controller, plant, and filter were derived in Section III and IV. The periodic disturbance is composed of a summation of sine waves with differing constant frequencies and zero phase (refer to section *II.B*) where the true amplitudes of the sine waves at each joint as well as their frequencies are shown in Table III. Notice how the implementation involves a disturbance being applied to joint 1 only. The initial position of the joint angles was arbitrarily chosen such that they are all at 0.1 radians. The joints are all static at the beginning of the simulation. The applied stable filter is chosen such that $\lambda = 1$. Design parameters for the least squares implementation for each joint are shown in Table IV.

B. Simulation Results

As can be seen from Fig. 5, the parameter estimates converge to the true parameter values. This is due to the controller generated input being persistently exciting which requires the generated input signal to be sufficiently rich of order of 6. As can be seen from Fig. 6, the estimated parameter values return an accurate approximate of the disturbance torque applied to each joint. These two graphs demonstrate the accurate estimation of the external disturbance on each joint.

The estimated periodic disturbance is actively injected into the controller in order to eliminate the effects of the external disturbance on the manipulator system. The available control parameter estimates are treated as if they are the true ideal control parameters. This approach is known as the certainty equivalence approach [12]. As the parameter estimate errors can potentially be large at the beginning of the simulation, this can

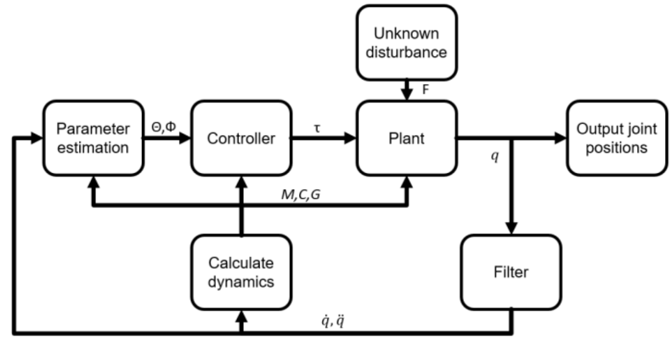

Fig. 4. High level block diagram of the system architecture

TABLE III. TRUE VALUES OF THE DISTURBANCE AMPLITUDES AND FREQUENCIES

Joint	Variable	Value
1	A_1	0.05 Nm
	A_2	0.01 Nm
	ω_1	10 rad/s
	ω_2	20 rad/s
2	A_3	0 Nm
	A_4	0 Nm
	ω_3	10 rad/s
	ω_4	20 rad/s
3	A_5	0 Nm
	A_6	0 Nm
	ω_5	10 rad/s
	ω_6	20 rad/s

TABLE IV. LEAST SQUARES ALGORITHM DESIGN PARAMETERS

Design parameter	Value
α	0.1
β	0.5
P(0)	I_2
$\theta(0)$	$[\ 0.001, 0.001\ ]^T$

result in large parameter estimate edits and direct injection of a varying approximated disturbance torque that may be far from the true disturbance. This can prove to be problematic as it can lead to large and oscillating control inputs and undesirable motion of the manipulator. A possible solution could be to place a condition on the disturbance estimation error such that the disturbance cancellation will not be applied until the errors are below a threshold.

In the implementation, the least squares estimator eases large parameter value edits due to initially large disturbance torque estimation errors. This is because the covariance matrix P is initialized as a small value and thus early parameter updates are slower in comparison to, for example, the gradient estimator. Additionally, the least squares estimator is less prone to oscillatory behaviour than the gradient estimator due to its integral action.

To demonstrate the effectiveness of the pole placement algorithm with disturbance rejection, different variations of the system are compared in simulation. These variations include monitoring the system response without applying any control input, injecting a disturbance rejection term by using the estimated joint disturbances, applying non-adaptive pole

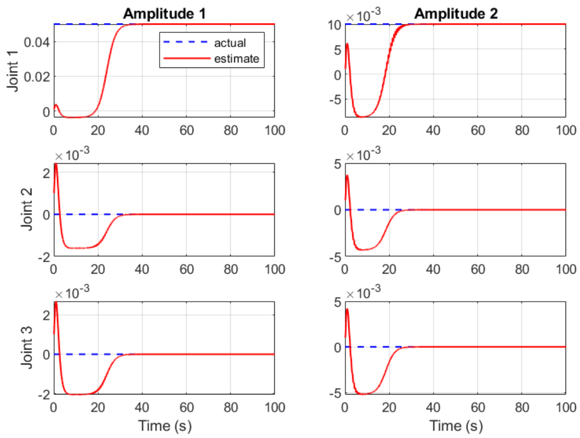

Fig. 5. Online parameter estimation of the disturbance amplitudes using a least squares algorithm with forgetting factor

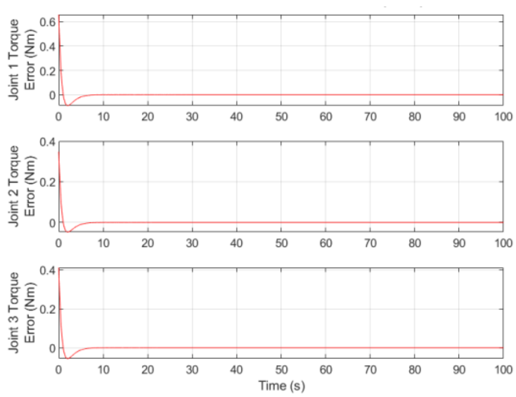

Fig. 6. Error of the online estimation of the disturbance applied to each joint

placement control for regulation, and finally, applying pole placement control with adaptive disturbance rejection. The same disturbance and initial conditions were applied in all four cases, and the resulting joint positions are plotted in Fig. 7 and Fig. 8.

The resulting joint angles produced by the system with no controller input versus with only disturbance rejection are not regulated (refer to Fig. 7). Also, as can be seen from Fig. 7, the joint angles show reduced oscillation when disturbance rejection is included. This confirms that the disturbance estimation is able act independently to approximate the disturbance torques.

The asymmetrical nature of the disturbance torque results in continuous motion in the joints when subject to the disturbance alone. In Fig. 7, the velocity of joint 1 and joint 3 can be seen to reduce when the time reaches 25 s. This is due to the fact that the parameter estimates have converged to their correct values, thus, the disturbance rejection cancels out the effects of the disturbance which in turn results in the disturbance no longer contributing to the velocity of the joints. Any continued motion of the joints following the successful cancellation of the disturbance torques is due to the momentum of the links. Although there is no disturbance torque applied to joint 3, since joint 1 and 3 are highly coupled (forming the planar section of the manipulator), their plots are similar such that when the disturbance being applied to joint 1 is learned the oscillatory behaviour of joint 3 is also suppressed.

The plots of the resulting joint angles from applied pole placement with and without disturbance rejection demonstrate proper regulation as the joint angles converge to zero (Fig. 8). As the pole placement algorithm was designed such that the system response determined by the dominant poles is critically damped, there is no overshoot in the regulation of the joint angles. A magnified view of the data following convergence of the estimated disturbance to the actual one demonstrates that the standard pole placement implementation is unable to rid the system of the oscillations produced by the disturbance. As can be seen from Fig. 8 the scale of the joint angle oscillation for joint 2 is much smaller than the other joints. This is due to the fact that joint 2 is far less coupled to joint 1 and 3 than joint 1 is to joint 3. The irregular profile of the disturbance affecting joint 2 is also a result of the low coupling between joint 2 and joint 1 (where the disturbance torque is applied). The addition of the disturbance cancellation term to the pole placement only control input is able to fully eliminate these oscillations. Thus, the control task of regulation and disturbance cancellation is only complete with this final proposed controller (including pole placement and disturbance rejection).

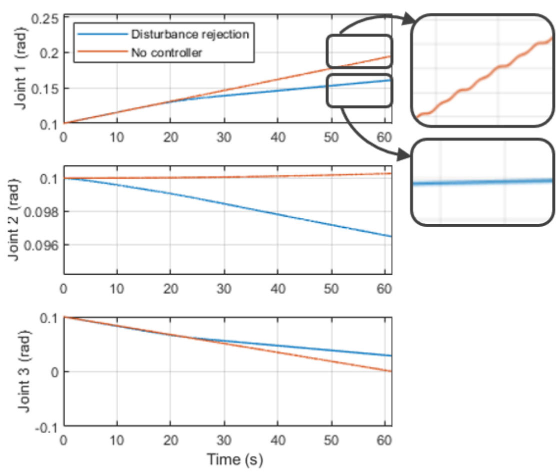

Fig. 7. Comparison of resulting joint positions with no controller versus disturbance rejetion

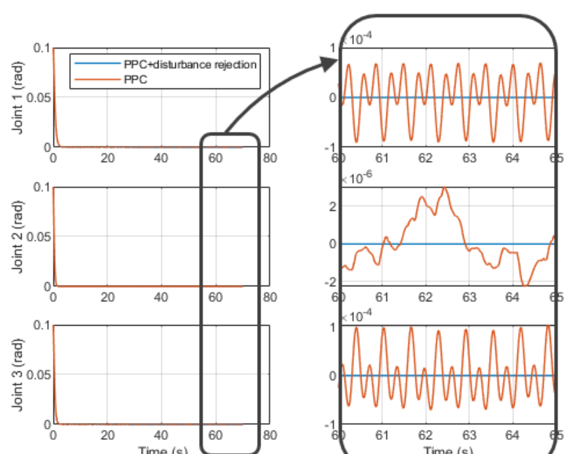

Fig. 8. Comparison of resulting joint positions with pole placement control (PPC) versus pole placement control with disturbance rejection

VI. CONCLUSION

In this paper a joint angle regulation control task was achieved in simulation for a 2 link, 3 joint (3 degree of freedom) robotic manipulator using a control law consisting of non-adaptive pole placement with adaptive high frequency periodic disturbance rejection. The controller consists of a pole placement control scheme that achieves a critically damped and optimal response with a learnt injected disturbance rejection term. The disturbance is represented as a truncated Fourier series that allows for the realization of an arbitrary known period disturbance with a possibly irregular profile by an online least squares learning algorithm. The simulation results confirm that this control objective is achieved.

As was previously mentioned in the disturbance modelling section, this simulation acts as a proof of concept for arbitrary disturbance rejection without a loss of generality. Thus, in a real-world application the harmonics of a reduced Fourier series can be used as a vector regressor to approximate the amplitudes for a range of frequencies for an arbitrary externally applied periodic disturbance (of a known period). The reduced Fourier series would consist of a collection of possible disturbance signal frequencies, where the frequencies chosen are determined by the irregular profile of the disturbance to be estimated.

The simplified robot model neglects friction, noise, and gravity. Apart from this ideal model, in a real world application the dynamics of the robotic system are often not exactly known. This would mean that an indirect adaptive parameter estimation model would have to be applied to accurately approximate the inertia, Coriolis, and gravitational effects, as well as any damping matrix concerning joint friction before pole placement and disturbance rejection can be effectively applied for the regulation task.

The applications of the proposed control scheme involve any disturbance that may be highly non-sinusoidal (irregular) and is of a known period. In an industrial manufacturing setting, for example, repetitive tasks conducted by mechanical manipulators are often timed and thus of a known period. Examples of such a task are pick and place or robotic deburring. However, straightforward non-adaptive control is not enough to ensure high accuracy in repetitive tasks over the lifetime of the manipulator as unforeseen configuration and joint velocity related issues often arise. This includes ‘sticking’ of joint at certain configurations, vibration experienced at a state of the robot that develops due to wear, and many other disturbances that match the period at which the task is executed. This is the type of problem the proposed scheme attempts to solve as these disturbances are most often non-sinusoidal. A specific example of an application of the proposed regulation scheme with known period non-sinusoidal disturbance cancellation is an industrial deburring manipulator where the parts are consecutively, at a given frequency, pushed against a ‘soft’ acting manipulator.

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