Tutorial - 2

1)

i	j
0	1
1	2
3	3
6	4
10	5

No. of times loop is running be k

$S_k = 1 + 3 + 6 + 10 + \cdots T_k$

$S_{k-1} = 1 + 3 + 6 + \cdots T_{k-1}$

Subtracting both

$S_k - S_{k-1} = 1 + 2 + 3 + 4 + \cdots + (k-1)$

$T_k = \frac{(k-1)k}{2}$

Given that k^{th} term is n

$T_k = n$

$\frac{k(k-1)}{2} = n \Rightarrow \frac{k^2}{2} - \frac{k}{2} = n$

Ignoring lower order terms & constants

$\Rightarrow k^2 = n$

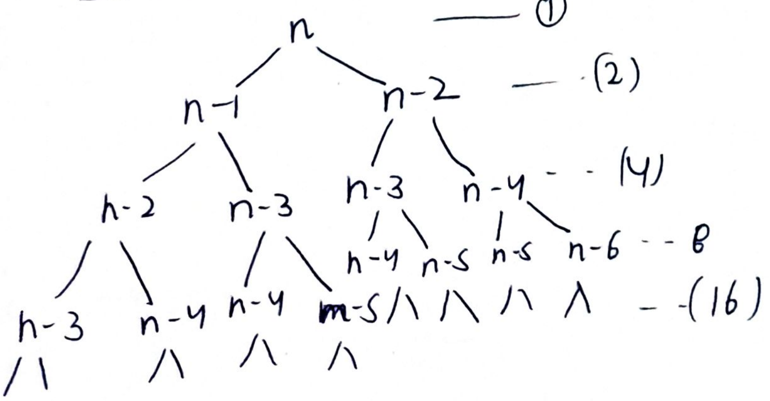
$\Rightarrow k = \sqrt{n}$

$\Rightarrow \boxed{T(n) = O(\sqrt{n})}$

2) $T(n) = T(n-1) + T(n-2) + O(1)$

For recursive fibonacci solution

Recursion Tree :



No. of tims function is running will be sum of the series :

$S = 1 + 2 + 4 + \cdots + 2^n$

$= \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1$

Time Complexity:

$\boxed{T(n) = O(2^n)}$

From recursion tree height of tree $= n$ } After removing constants

$\Rightarrow \boxed{\text{Space Complexity} = O(n)}$

3) Code having time complexity:

$O(n \log n)$:
```
for (int i=1 ; i<=n ; i++)
{
    for (int j=1 ; j<n ; j=j*2)
        printf("Hello");
}
```

$O(n^3)$:
```
for (int i=0 ; i<n ; i++)
{ for (int j=0; j<n ; j++)
  {
      for (int k=0 ; k<n ; k++)
          printf("Hello");
  }
}
```

$O(\log(\log n))$:
```
for (int i=2 ; i<=n ; i= pow (i,3))
{ printf("Hello");
}
```

Here n can be any positive integer

$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + cn^2$

Ignoring lower order terms:

$T(n) = T\left(\frac{n}{2}\right) + cn^2$

using master theorem

$a=1 \quad b=2 \quad f(n)=n^2$

$c = \log_b a = \log_2 1 = 0$

$\boxed{0 < n^2}$ True

$\Rightarrow \boxed{T(n) = \theta(n^2)}$

)

i	j
1	n
2	n/2
3	n/3
4	n/4

Time complexity will be sum of series

$S = \frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \cdots$

$= \sum_{i=1}^{n} \left(\frac{n}{i}\right)$

complexity $= n \times \sum_{i=1}^{n} \left(\frac{1}{i}\right)$

$\boxed{T(n) = n\log n}$

6) Sequencing:

$2, 2^k, (2^k)^k, ((2^k)^k)^k \ldots$

Generalising: $2^{k^0}, 2^{k^1}, 2^{k^2}, \ldots 2^{k^{\lambda-1}}$

<u>Assumption</u>: let no. of terms be λ

<u>Given</u>: last term is n

$\Rightarrow 2^{k^{\lambda-1}} = n$

$k^{\lambda-1} \log 2 = \log n$

$k^{\lambda-1} = \log n$ [Ignoring constant ($\log 2$)]

$(\lambda-1)\log k = \log n$

$\boxed{\lambda = \log(\log n)}$ [Ignoring constant terms]

Time complexity: $\boxed{T(n) = O(\log(\log n))}$

8) a) $100 < \log(\log n) < \log n < (\log n)^2 < \sqrt{n} < n < n(\log n)$
$< \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

b) $1 < \log(\log n) < \sqrt{\log n} < \log n < \log 2n < 2(\log n) < n <$
$n \log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

c) $96 < \log_8 n < \log 2n < 5n < n(\log_6 n) < n(\log_2 n) <$
$\log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$