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EC 551 - Homework #1

Questions:-

Show intermediate steps and not just final solution

✓1. Find the canonical SOP form of the following Boolean function:

$$F(a, b, c) = \Sigma m(1, 2, 3, 5, 7)$$

✓2. Express the function $F(a,b,c,d) = (a'bcd + a'bc'd + ab'c'd' + ab'cd')'$ as a sum of minterms.

✓3. Find all the prime implicants in the complement of the following expressions.

(i) $ab + a'b'$

(ii) $b' + (cd' + e)a$

(iii) $(a+b+c')(b'+c)(a+c)$

✓4. Minimize the following Boolean expression using K-map. Include the following steps:

A. Identify the prime implicants (and write them as Boolean expressions)

B. Identify and select the essential prime implicants

C. Add from the remaining prime implicants to create a minimal cover

$$f(a,b,c,d) = a'b(c'd + c'd') + a'bcd + a(c+d) + ac'd$$

✓ 5. Design a two-level circuit that implements a 4-bit minority function, that is, the output is a 1 if less than two inputs are asserted.

✓ 6. Tseitin Transformation : Use a set of clauses (POS) to represent the functionality of a three-input NAND gate. You can use a, b, c to represent the three inputs and f to represent the output.

✓ 7. Identify all the possible glitches that could occur when the input makes a transition in a circuit that uses the minimal cover you identified in Problem 4C to implement f(a, b, c, d) in Problem 4. Show how you can eliminate these glitches using additional prime implicants.

✓ 8. Draw a Mealy FSM that recognizes the input patterns (i.e, outputs 1) "101" and "010".
Make sure your FSM has the minimal no. of states.
Choose a minimal ~~bit~~ bitwidth encoding for the states of this FSM, what are the unreachable states on the corresponding circuit implementation of this FSM?

A1) $F(a,b,c) = \Sigma m(1,2,3,5,7)$

	a	b	c	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

$\therefore F(a,b,c)$

$= \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c + abc$

The canonical SOP form is :-

$F(a,b,c) = \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c + abc$ Ans

A2) $F(a,b,c,d) = \overline{(\bar{a}bcd + \bar{a}b\bar{c}d + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d})}$

$= M_5 \cdot M_7 \cdot M_8 \cdot M_{10}$

$= m_0 + m_1 + m_2 + m_3 + m_4 + m_6 + m_9 + m_{11} + m_{12} + m_{13} + m_{14} + m_{15}$

(P.T.O)

	a	b	c	d	F
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

$F(a, b, c, d)$

$= \Sigma m(0,1,2,3,4,6,9,11,12,13,14,15)$

A3) To find prime implicants in the COMPLIMENT of :-

(i) $\overline{ab + a'b'} = (\overline{ab})\,(\overline{\bar{a}\bar{b}})$

$= (\bar{a}+\bar{b})\,(\bar{\bar{a}}+\bar{\bar{b}}) = (\bar{a}+\bar{b})\,(a+b)$

$= \cancel{a\bar{a}}^{0} + a\bar{b} + \bar{a}b + \cancel{b\bar{b}}^{0}$

$= a\bar{b} + \bar{a}b \leftrightarrow$ prime implicants.

a \ b	0	1
0		1
1	1	

(ii) $b' + (cd' + e)a$

$\overline{\bar{b} + [(c\bar{d}+e)a]} = (\bar{\bar{b}})\,[\overline{(c\bar{d}+e)a}]$

$= b \cdot [\overline{(c\bar{d}+e)} + \bar{a}] = b\,[(\overline{c\bar{d}} \cdot \bar{e}) + \bar{a}]$

$= b\,[(\bar{c}+d)\bar{e} + \bar{a}] = b\,[\bar{c}\bar{e} + d\bar{e} + \bar{a}]$

$= b\bar{c}\bar{e} + bd\bar{e} + b\bar{a}$

$= b\bar{c}\bar{e} + bd\bar{e} + b\bar{a}$ ~~→ prime implicants~~.

↳ prime implicant Ans

The 5 i/p kmap results in same ~~so~~ as above.

(iii) $(a+b+\bar{c})(\bar{b}+c)(a+c)$

$= \overline{(a+b+\bar{c})} + \overline{(\bar{b}+c)} + \overline{(a+c)}$

$= (\bar{a}.\bar{b}.\bar{\bar{c}}) + (\bar{\bar{b}}.\bar{c}) + (\bar{a}.\bar{c})$

$= (\bar{a}\bar{b}c) + (b\bar{c}) + (\bar{a}\bar{c})$

$= (\bar{a}\bar{b}c + b\bar{c} + \bar{a}\bar{c})$ ~~prime implicant~~.

↳ ~~prime implicant~~

c \ ab	00	01	11	10
0	1 (0)	1 (2)	1 (6)	(4)
1	1 (1)	(3)	(7)	(5)

$\bar{a}\bar{b}c$
001 → 1

$b\bar{c}$
X 1 0
1 1 0 → 6
0 1 0 → 2

$\bar{a}\bar{c}$
0 X 0
~~0 0 0 → 0~~
0 1 0 → 2

$\bar{a}\bar{b}$, $\bar{a}\bar{c}$, $b\bar{c}$ are the prime implicants.

A4) A) $F = \Sigma m(4, 5, 7, 9, 10, 11, 13, 14, 15)$

ab \ cd	$\bar{c}\bar{d}$ 00	$\bar{c}d$ 01	cd 11	$c\bar{d}$ 10
00 $\bar{a}\bar{b}$	(0)	(1)	(3)	(2)
01 $\bar{a}b$	1 (4)	1 (5)	1 (7)	(6)
11 ab	(12)	1 (13)	1 (15)	1 (14)
10 $a\bar{b}$	(8)	1 (9)	1 (11)	1 (10)

(i) $\bar{a}b\bar{c}$

(ii) bd

(iii) ad

(iv) ac

$F = \bar{a}b\bar{c} + bd + ad + ac$

(A) m_4 is in $\bar{a}b\bar{c}$

m_5 is in $\bar{a}b\bar{c}$ and bd

m_7 is in bd

m_{13}, m_{15} are in bd and ad

m_9 is in ad

m_{10} is in ac

m_{11} is in ad and ac

m_{14} is in ac

Prime Implicants : $\bar{a}b\bar{c}$, bd, ad, ac

(B) EPI list :

m_4 is covered only by $\bar{a}b\bar{c}$

m_7 is covered only by bd

m_9 is covered only by ad

m_{14} and m_{10} are covered only by ac

$\therefore$ EPI = $\bar{a}b\bar{c}$, bd, ad, ac

(C) Since in this case the EPI are the PI the minimal cover is given as:-

$F = \bar{a}b\bar{c} + bd + ad + ac$

A5)

a	b	c	d	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

→ Truth table based on the condition stated.

$F = \sum m(0, 1, 2, 4, 8)$

ab \ cd	00	01	11	10
00	1 0	1 1	3	1 2
01	1 4	5	7	6
11	12	13	15	14
10	1 8	9	11	10

(i) $\bar{a}\bar{b}\bar{c}$

(ii) $\bar{a}\bar{b}\bar{d}$

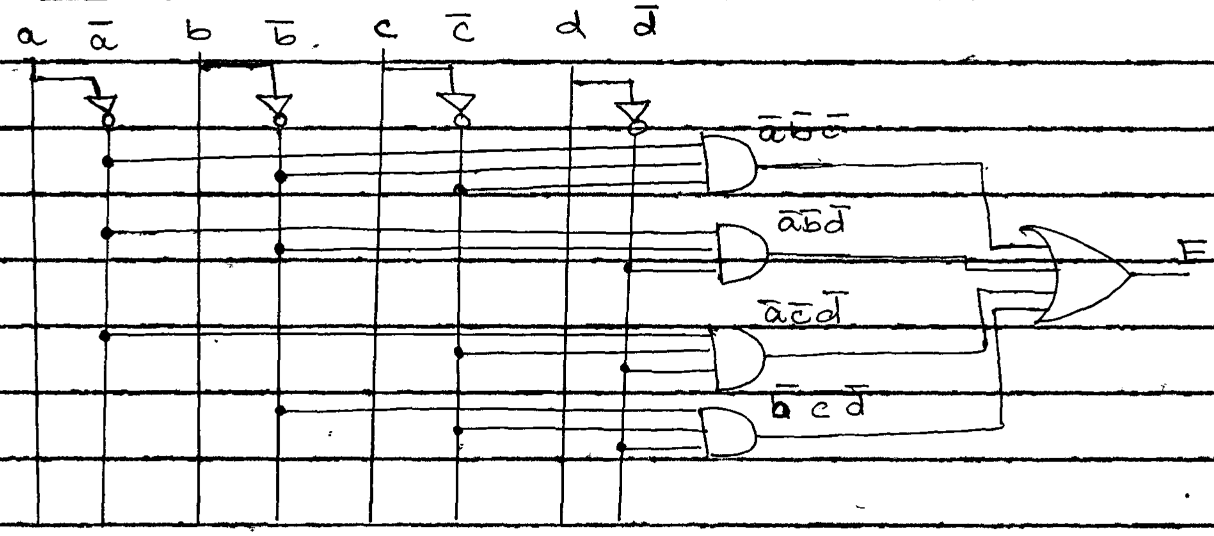
(ii) $\bar{a}\bar{c}\bar{d}$

(iv) $\bar{b}\bar{c}\bar{d}$

$F = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{d} + \bar{a}\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d}$

$F = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{d} + \bar{a}\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d}$

Two-level circuit :-



A6) 3 input NAND gate

a	b	c	$\bar{f}_{AND}$	f_{NAND}
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

The NAND gate operates properly when the following conditions hold:

1. Output f is true when atleast a or b or c is false
2. Output f is false when all a, b, and c are true

$(f \rightarrow \bar{a} \vee \bar{b} \vee \bar{c}) \wedge (\bar{f} \rightarrow (a \wedge b \wedge c))$

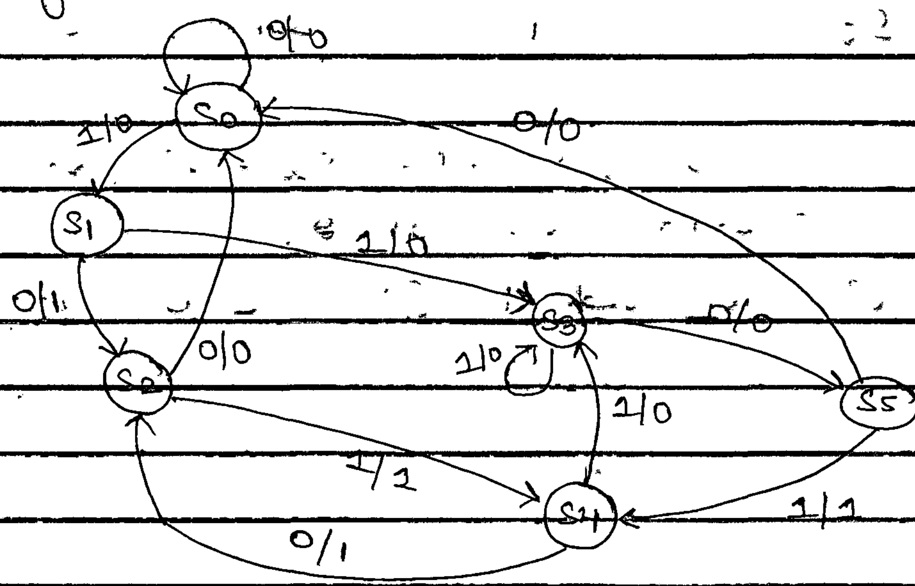
$(\bar{f} \vee (\bar{a} \vee \bar{b} \vee \bar{c}) \wedge (f \vee (a \wedge b \wedge c)))$

$(\bar{f} \vee (\bar{a} \vee \bar{b} \vee \bar{c})) \wedge (f \vee (a \wedge b \wedge c))$

$(\bar{f} \vee \bar{a} \vee \bar{b} \vee \bar{c}) \wedge (f \vee a) \wedge (f \vee b) \wedge (f \vee c)$

A7) In problem 4c) -
The minimal cover includes the essential prime implicants which are also the prime implicants in this case. Therefore there should not be ~~any~~ any glitches present.

A8) Mealy FSM



There are 6 states S0, S1, S2, S3, S4, S5
requires 3 bits for encoding.

Current state	i/p	Next state	o/p
S_0	0	S_0	0
S_0	1	S_1	0
S_1	0	S_2	1
S_1	1	S_3	0
S_2	0	S_0	0
S_2	1	S_4	1
S_3	0	S_5	0
S_3	1	S_3	0
S_4	0	S_2	1
S_4	1	S_3	0
S_5	0	S_0	0
S_5	1	S_4	1

Taking S_0 as 000 $S_1 \rightarrow 001$ $S_2 \rightarrow 010$ $S_3 \rightarrow 011$

$S_4 \rightarrow 100$ $S_5 \rightarrow 101$

(S_6) 110 and 111 (S_7) are unreachable.