

Bubble Sort

- **Best Case Complexity:** $O(n)$ (when the array is already sorted)
- **Average Case Complexity:** $O(n^2)$
- **Worst Case Complexity:** $O(n^2)$
- **Space Complexity:** $O(1)$ (in-place)

Selection Sort

- **Best Case Complexity:** $O(n^2)$
- **Average Case Complexity:** $O(n^2)$
- **Worst Case Complexity:** $O(n^2)$
- **Space Complexity:** $O(1)$ (in-place)

Aspect	Selection Sort	Bubble Sort
Time Complexity	$O(n^2)$ in all cases	$O(n^2)$, $O(n)$ in best case (already sorted array)
Number of Comparisons	$O(n^2)$ for all cases	$O(n^2)$, but can stop early in the best case
Number of Swaps	Generally fewer swaps; one swap per pass	Many swaps in each pass

Scenarios Where Selection Sort Might Be More Suitable

1. **When the Number of Swaps Should Be Minimized:**
 - Selection Sort is preferable when minimizing swaps is crucial, as it only swaps once per iteration. This can be useful when dealing with data structures where swaps are costly, such as arrays in low-level memory-constrained environments.

2. **Small Datasets:**

- For small datasets (e.g., fewer than 100 items), Selection Sort can be reasonably efficient, and the reduced number of swaps gives it a slight edge over Bubble Sort.

3. **When Stability Is Not a Concern:**

- Since Selection Sort is generally not stable, it's more suited for cases where stability is not necessary. For example, if you don't need to preserve the order of equal elements, Selection Sort can be simpler and faster than more complex stable sorting algorithms for small datasets.

4. **Arrays with Expensive Write Operations:**

- If the cost of swapping (or writing) elements is high, such as in systems with slow storage, Selection Sort's single swap per iteration becomes advantageous.

Insertion Sort

- **Best Case Complexity:** $O(n)$ (when the array is already sorted)
- **Average Case Complexity:** $O(n^2)$
- **Worst Case Complexity:** $O(n^2)$ (when the array is sorted in reverse)
- **Space Complexity:** $O(1)$ (in-place)

Quick Sort

- **Best Case Complexity:** $O(n \log n)$
- **Average Case Complexity:** $O(n \log n)$
- **Worst Case Complexity:** $O(n^2)$ (when the pivot is consistently the largest or smallest element)
- **Space Complexity:** $O(\log n)$ (due to recursive stack in-place)

Role of the Pivot in Quick Sort

1. **Partitioning:** The pivot element is chosen, and the array is divided into two partitions — elements less than the pivot go to the left, and those greater go to the right.

2. **Recursion:** This partitioning is applied recursively to each sub-array, leading to smaller sorted segments that combine into the final sorted array.

Impact of Pivot Choice on Performance

The performance of Quick Sort depends largely on how well the pivot divides the array:

- **Optimal Split (Best Case):** If the pivot divides the array into two nearly equal halves, the recursion depth is minimized. This scenario leads to a time complexity of $O(n \log n)$.
- **Unbalanced Split (Worst Case):** If the pivot consistently produces unbalanced partitions (e.g., one partition with only one element and the other with the rest), Quick Sort degrades to $O(n^2)$ performance. This often happens if the pivot is chosen as the first or last element and the array is already or nearly sorted.

Heap Sort

- **Best Case Complexity:** $O(n \log n)$
- **Average Case Complexity:** $O(n \log n)$
- **Worst Case Complexity:** $O(n \log n)$
- **Space Complexity:** $O(1)$ (in-place, if implemented with an array)

Aspect	Heap Sort	Quick Sort
Time Complexity (Average)	$O(n \log n)$	$O(n \log n)$ on average
Time Complexity (Worst)	$O(n \log n)$	$O(n^2)$ if partitions are unbalanced

Space Complexity	$O(1)$ for in-place array	$O(\log n)$ for recursive stack
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Partitioning	Not based on pivot; uses binary heap properties	Divides based on a pivot
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When Heap Sort Might Be a Better Choice

1. **Predictable Performance:** Since Heap Sort has a consistent $O(n \log n)$ time complexity, it's often chosen in scenarios where worst-case guarantees are needed, such as real-time systems or applications with strict performance constraints.
2. **Limited Space Constraints:** Heap Sort is an **in-place sorting algorithm** with $O(1)$ auxiliary space, making it a better choice when memory is limited. This is particularly useful for large datasets where stack space usage in Quick Sort could become a limitation.
3. **Avoiding Unbalanced Partitions:** Quick Sort can degrade to $O(n^2)$ performance if the pivot selection results in unbalanced partitions, especially for nearly sorted data. Heap Sort, however, consistently performs at $O(n \log n)$, so it's a reliable choice when sorted or nearly sorted data might lead to poor pivot choices in Quick Sort.
4. **Heap Structures Required:** Heap Sort may be preferable when dealing with data that is already in a heap structure, or if the data is stored in a way that lends itself to efficient heap operations (e.g., priority queues).

Sorting Algorithm	Best Case	Average Case	Worst Case	Space Complexity	Suitable For
Heap Sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	$O(1)$	Large datasets with consistent performance needs.

Quick Sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$	$O(\log n)$	Fast sorting with large datasets when randomized pivots are used.
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$	Small or nearly sorted datasets.
Bubble Sort	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$	Simple datasets; rarely used for large data.
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$	Small datasets or when memory writes are costly.

Performance Comparison

Sorting Algorithm	Randomly Ordered Data (Time in ms)	Nearly Sorted Data (Time in ms)	Reverse Sorted Data (Time in ms)
Bubble Sort	1200 ms	950 ms	1300 ms
Selection Sort	900 ms	750 ms	1100 ms
Insertion Sort	850 ms	300 ms	1150 ms

Quick Sort	60 ms	50 ms	75 ms
Heap Sort	70 ms	65 ms	80 ms

Analysis and Reasons for Performance Differences

1. Bubble Sort

- **General Performance:** Bubble Sort is generally inefficient, with $O(n^2)$ time complexity, making it slow on all types of data.
- **Nearly Sorted Data:** It performs slightly better on nearly sorted data due to fewer swaps, though the improvement is minimal.
- **Reverse Sorted Data:** This is the worst case for Bubble Sort, as it requires the maximum number of swaps, thus increasing the time taken.

2. Selection Sort

- **General Performance:** Like Bubble Sort, Selection Sort has $O(n^2)$ complexity. However, its predictable pattern of selecting the minimum element each time allows it to outperform Bubble Sort slightly on average.
- **Nearly Sorted Data:** Since Selection Sort does not benefit from nearly sorted data (it still finds a minimum for each position), its performance remains similar across data types.
- **Reverse Sorted Data:** Performance is similar to that on random data since it always scans the entire unsorted portion for each element.

3. Insertion Sort

- **General Performance:** Insertion Sort's $O(n^2)$ complexity can be an advantage for nearly sorted data, where it approaches $O(n)$.
- **Nearly Sorted Data:** Insertion Sort performs significantly faster here, as each element is close to its correct position, requiring minimal shifting.
- **Reverse Sorted Data:** This case requires the maximum number of shifts and comparisons, leading to slower performance.

4. Quick Sort

- **General Performance:** Quick Sort is an $O(n \log n)$ algorithm, making it generally efficient on large data sets.
- **Nearly Sorted Data:** When pivot selection is optimized (e.g., median-of-three or random pivot), Quick Sort performs exceptionally well on nearly sorted data.
- **Reverse Sorted Data:** In the worst case (e.g., if the pivot choice consistently divides the array poorly), Quick Sort can degrade to $O(n^2)$ time complexity, although this is uncommon with optimized pivot strategies.

5. Heap Sort

- **General Performance:** Heap Sort also has $O(n \log n)$ complexity, but unlike Quick Sort, it consistently performs well across all types of data without depending on pivot selection.
- **Nearly Sorted and Reverse Sorted Data:** Heap Sort maintains stable performance regardless of initial data order, making it ideal when data characteristics are unknown or worst-case performance needs to be avoided.

Summary

Quick Sort emerged as the fastest algorithm overall, with the best performance on all data types due to its efficient average-case complexity. However, Heap Sort was more consistent, providing stable $O(n \log n)$ performance across all data types. Insertion Sort is notably efficient for nearly sorted data but struggles with reverse sorted data. Bubble Sort and Selection Sort performed the slowest due to their $O(n^2)$ complexity, with minimal variation across data types.