Logical Reasoning and Inequalities
(18B12MA312)

PBL Project Report

Project Title:

Binary Logic and its Applications

Project Members:

Pranav Bhargava (9920103112, F4)
Sarthak Pant (9920103126, F5)
Tanisha Jaiswal (9920103094, F4)
Jahanvi Punia (9920103117, F4)
Manav Seedhar(9920103102,F4)

Submitted to:

Dr. Lakhveer Kaur

Department of Mathematics

Jaypee Institute of Information Technology,Noida

November 2022

INTRODUCTION

In digital computers, everything is based on either a truth value or a false one, i.e, 1 or 0. The fundamental basis of all digital communication relies on the two values of 1's and 0's and forms the foundation of Boolean algebra and logic, which in turn gives rise to Logic Gates and the various practical applications associated with it. Since the time of computer/digital evolution, Binary logic and its applications in various fields have been accepted worldwide and forms the fundamental basic approach to every aspect of the digital world.

Binary logic is also the basis of electronic systems, such as cell phones and computers. It works on 1's and 0's and includes logical addition, subtraction, multiplication and division. It also includes logic gates AND, OR and NOT which translates the input signals to specific output forms. This in turn gives rise to various applications of digital circuits including flip flops, computer architecture, etc. It facilitates computing, robotics and other electronic applications.

METHODOLOGY

Any information is stored in data "words", representing data or instructions, made up from strings of individual "bits" with binary values of 1 or 0. These values are analogous to Boolean logic propositions and the statements or conclusions derived from them which were defined as "true" or "false". The binary logic may be divided into some sub-categories including:

- Boolean logic(including boolean algebra)
- Logic Gates
- Bivalent Logic or two-valued logic, logic satisfying the principle of bivalence

Boolean Algebra :

Boolean algebra is based upon the two values of Binary Logic. The two values may have many representations such as true or false, 1 or 0, and "on" or "off". It is this property which was recognised and developed by Claude Shannon in 1937 which makes it so useful for implementing logic functions by means of electronic circuits. This gives rise to specific laws concerning the boolean logic, based on the basic operations of AND, OR and NOT, described and follows:

NOT A = Q

A	Q
0	1
1	0

A OR B = Q

A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

A AND B = Q

A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

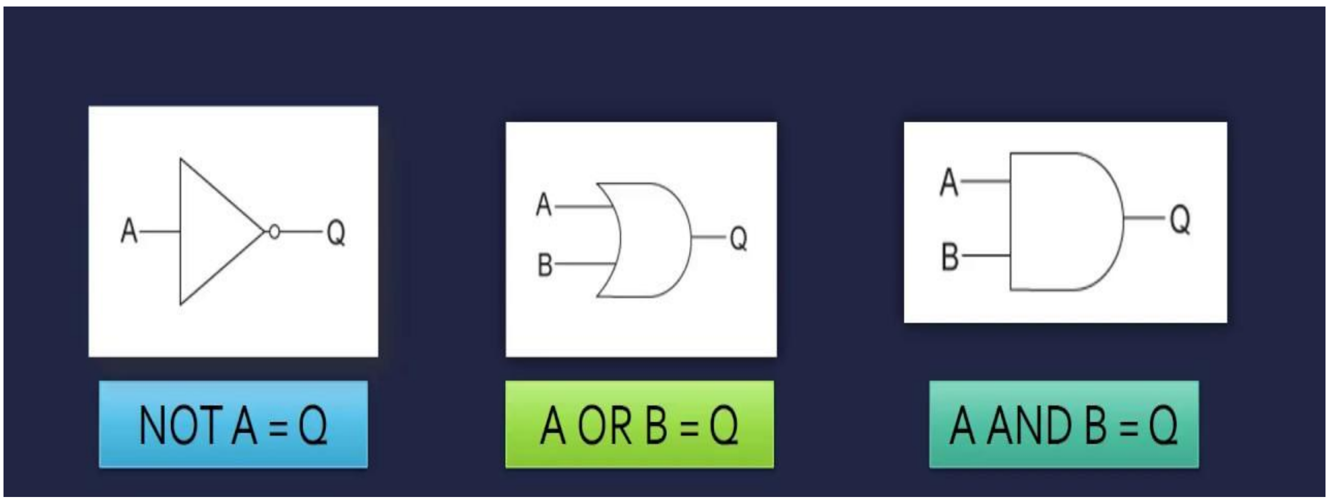
The identities associated with the basic logical operations are termed as ***Boolean identities*** or ***boolean laws*** which are described a below:

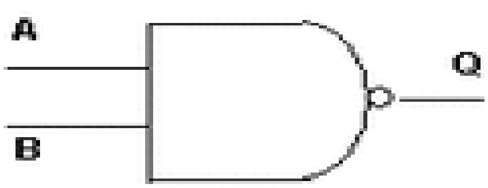
Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0 + x = x$
Null (or Dominance) Law	$0x = 0$	$1 + x = 1$
Idempotent Law	$xx = x$	$x + x = x$
Inverse Law	$x\bar{x} = 0$	$x + \bar{x} = 1$
Commutative Law	$xy = yx$	$x + y = y + x$
Associative Law	$(xy)z = x(yz)$	$(x+y) + z = x + (y+z)$
Distributive Law	$x + yz = (x+y)(x+z)$	$x(y+z) = xy + xz$
Absorption Law	$x(x+y) = x$	$x + xy = x$
DeMorgan's Law	$(\overline{xy}) = \bar{x} + \bar{y}$	$(\overline{x+y}) = \bar{x}\bar{y}$
Double Complement Law	$\bar{\bar{x}} = x$	

The boolean functions are implemented with the combination of the above laws and operations and hence finds its place in the implementation of the digital world.

Logic gates:

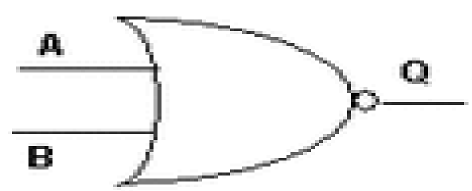
We see that Boolean functions are implemented in digital computer circuits called gates. A gate is an electronic device that produces a result based on two or more input values. In reality, gates consist of one to six transistors, but digital designers think of them as a single unit. Integrated circuits contain collections of gates suited to a particular purpose. Following are the basic logic gates, which in turn gives rise to "Universal Gate" (as all other gates and operations can be performed with these two gates or the combination of them) of NAND and NOR as shown below:





NAND

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0



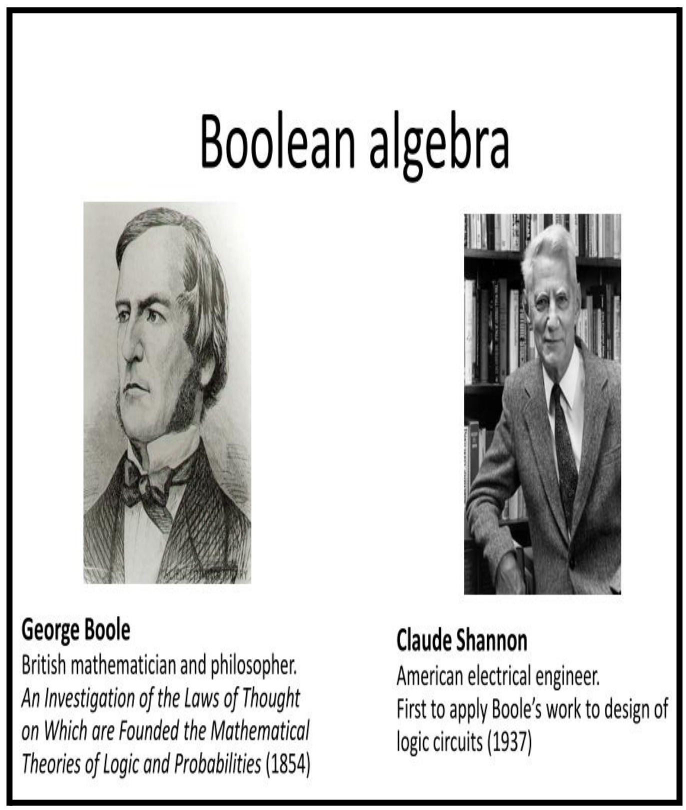
NOR

A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

HISTORY & RESEARCH

The forerunner of Boolean algebra was the conceptual algebra of Gottfried Wilhelm Leibniz. The Leibniz conceptual algebra is a priori equivalent to the Boolean set algebra. Boolean algebra predates the modern development of abstract algebra and mathematical logic. However, both are associated with realm origins. In an abstract setting, Boolean algebra was perfected by Jevons, Schröder, Huntington, and others in the late nineteenth century, arriving at the concept of modern (abstract) mathematical constructions. For example, the empirical observation that set algebraic expressions can be manipulated by transforming them into Boolean algebraic expressions is described in modern terms as set algebra being Boolean algebra. In fact, M. H. Stone proved in 1936 that all Boolean algebras are isomorphic to sets.

In the 1930s, while studying circuits, Claude Shannon observed that the rules of Boolean algebra were applicable to this environment, and he used algebraic means to analyze circuits and identify logic gates. Introduced switching algebra to design. Since Shannon already had an abstract mathematical apparatus at his disposal, he formulated the switching algebra as a two-element Boolean algebra. In modern circuit technology, “switching algebra” and “Boolean algebra” are often used synonymously because there is little need to consider other Boolean algebras.

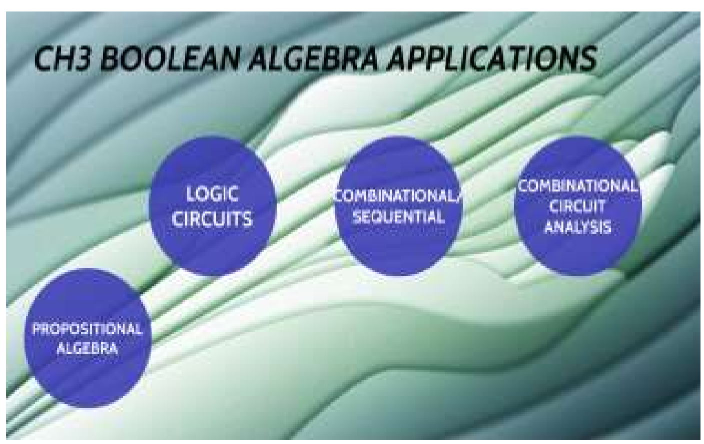


Efficient implementation of Boolean functions is a fundamental problem in the design of combinational logic circuits. Modern electronic design automation tools for VLSI circuits often rely on efficient representations of Boolean functions known as (reduced-order) binary decision diagrams (BDDs) for logic synthesis and formal verification. Logical propositions that can be expressed in classical propositional calculus have equivalent expressions in Boolean algebra. Boolean logic is therefore sometimes used to represent propositional computations performed in this way.

Although the development of mathematical logic did not follow Boole's program, his relationship between algebra and logic was later laid on a firm foundation within the framework of algebraic logic. Given the problem of determining whether the variables of a Boolean (propositional) expression can be assigned in such a way that the expression evaluates to true is called the Boolean satisfiability problem (SAT) and is related to theoretical computer science. It turned out to be NP-complete. A closely related model of computation known as a Boolean circuit relates time complexity (of an algorithm) to circuit complexity.

APPLICATIONS

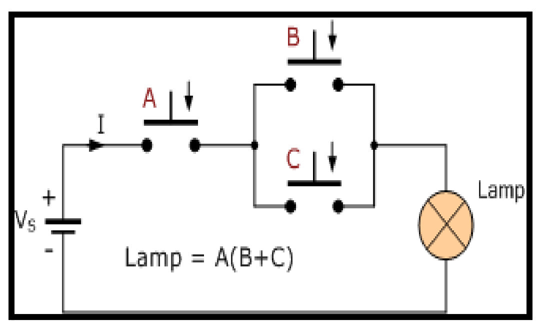
Boolean algebra is frequently used in computer manufacturing, where it is implemented by a large number of logic gates in container-shaped integrated circuits.An external connection of pins that are connected to the individual gate's input and output lines facilitates communication between the gates.The printed circuit board, which has metallic strips, is the most common type.After the integrated circuits have been programmed, the internal connections can be made more workable by using gate networks.Because of this, it is now ready to be integrated into the systems.The computer will be able to perform logical and arithmetic operations once it is incorporated into the computer systems.The machine can use algebra to either save a value into the storage unit or determine the value of an output signal (one or zero). An analysis of flight accidents has been developed using boolean algebra.Probabilistic and logical tools are used to incorporate the occurrences that result in failures in complex systems.Human factors, environmental factors, and



equipment failures can all be identified thanks to the tools.The fault tree method is then used to group the accident's contributors into categories.Tests for maintenance, risk assessment, and probability maintenance all make use of the faulty tree for a specific purpose.The faulty tree employs a deductive method to demonstrate the relationship between the events that contributed to the accident's cause (i.e., undeveloped event, basic event, and intermediate event in that order). Flight management uses diagrams and the logical event relationship to find system failures. Search engines are one example of another system that makes use of boolean algebra.Boolean algebra is used by search engines like Google and DuckDuckGo to give users access to data whenever they request it or search for something.The logical AND, OR, and NOT have been used to implement the boolean search concept.The boolean search treats each Internet webpage as a part of a set.When data is requested, it is simple to retrieve it in this manner.

<u>Boolean algebra in the lighting system :</u>

Even in this century, the lighting system is very important to our day-to-day lives.Many homes around the world are illuminated by means of a lighting system that can be traced back to boolean algebra.We are now free to do things we could not before thanks to the lighting system.This is because longer working hours make it possible to complete certain tasks with light.Due to the availability of lighting systems, numerous businesses around the world have made significant advancements.Additionally, major cities are illuminated at night.The business has grown as a result of this.The number of hours worked increases with the size of the business.

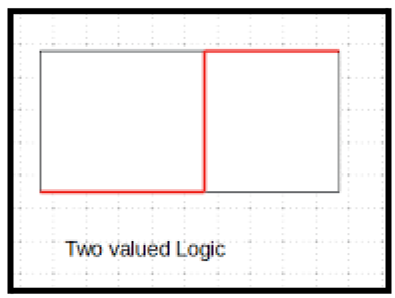


<u>Boolean algebra helps the development of modern technology :</u>

In most industries, the use of computers has reduced the need for human labor. It has made it possible to make robots that can do the work of humans. The use of computers makes it possible to automate tasks. Robots have been utilized in emergency clinics to complete clinical exploration and precise medical service arrangements. The researchers now have less work to do. Additionally, computers that are used to keep an eye on a variety of businesses to prevent losses have been developed using algebra. Because it now takes less time to complete a given task, the use of computers has had a significant impact on society. Specifically, employing robots rather than employees. Patients have received medical care that is more effective as a result of the accuracy of the machines. The amount of work required to analyze a system in order to identify a potential issue has decreased as a result of the use of computers in-flight management. The use of computers to manage flights has reduced the number of deaths caused by crashes on airplanes. Society has gained a lot from this. The most significant application of boolean algebra in our day-to-day lives is search engines. It spreads knowledge across a variety of fields. Due to the ease with which data can be retrieved, it has altered workplace and educational practices. This has made it easier to look for a particular idea in books. The issue of wasted time has been resolved by this because one uses a search engine to find the information they require.

Two-valued logic:

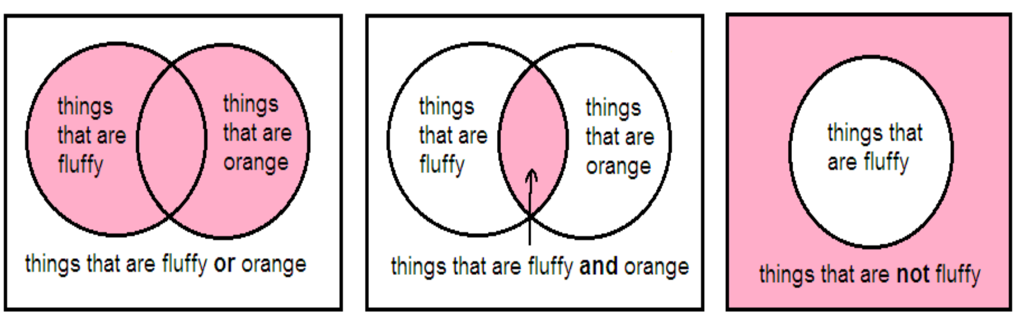
Math and the law are two additional fields in which choosing two values is advantageous. Nuanced or intricate responses like "maybe" or "only on the weekend" are acceptable in casual conversation. However, it is deemed advantageous to frame questions so as to admit a simple yes-or-no answer—is the defendant guilty or not guilty, is the proposition true or false—and to disallow any other answer in more focused situations like a court of law or theorem-based mathematics. The simple yes-no question principle has become a central feature of both judicial and mathematical logic, making two-valued logic deserving of organization and study in its own right, regardless of how much of a straitjacket this might prove to be in practice for the respondent. Membership is a key concept in set theory. Now, an organization can accept novice, associate, and full levels of membership. An element, on the other hand, is either in or out in sets. The candidates for membership in a group function similarly to digital computer wires: Just like each wire is either high or low, each candidate is either a member or not. Because algebra is a fundamental tool in any area that can be treated mathematically, these factors make the algebra of two values crucial to computer hardware, mathematical logic, and set theory. Multi-valued logic can be extended from two-valued logic by substituting the unit interval [0,1] for the Boolean domain "0, 1." In this case, instead of only



accepting values 0 and 1, any value between and including 0 and 1 can be assumed. Negation (NOT) is now represented algebraically by 1 x, conjunction (AND) by multiplication (XY), and disjunction (OR) by De Morgan's law. A multi-valued logic is produced when these values are interpreted as logical truth values. This multi-valued logic serves as the foundation for probabilistic and fuzzy logic. A value is interpreted as the "degree" of truth, or the probability that a proposition is true, according to these interpretations.

Boolean operations in Natural language :

There are words for several Boolean operations in natural languages like English, including conjunction (and), disjunction (or), negation (not), and implication (implies). However, not and not are synonymous terms. The meanings of these logical connectives frequently have the same meaning as their logical counterparts when they are used to combine situational assertions like "the block is on the table" and "cats drink milk," which are naively either true or false. However, with descriptions of behavior like "Jim walked through the door," one begins to notice differences like a failure of commutativity. For instance, the conjunction of "Jim opened the door" and "Jim walked through the door" in that order is not equivalent to their conjunction in the opposite order because typically means and then in such situations. Similar questions may be asked: the question in the order "Is the sky blue, and why is it blue?" makes more sense than putting things in reverse. Similar to behavioral assertions like getting dressed and going to



school, conjunctive commands about behavior.The implication that one option is less desirable causes disjunctive commands like "love me," "leave me," "fish," and "cut bait" to typically be asymmetric. Conjoined nouns like "tea" and "milk" typically mean "set union," whereas "tea" and "milk" mean "choice."However, these perceptions can be reversed by context, as when you say that your choices are coffee or tea (alternatives), which typically means the same thing. Double negation, such as "I don't like milk," rarely means "I like milk," but rather conveys some kind of hedging, as if to suggest that there is a third possibility." Not not P" can be interpreted loosely as "surely P," and although "not not P" necessarily implies "not not P," the opposite is questionable in English, similar to intuitionistic logic. Boolean algebra cannot be used as a reliable framework for interpreting conjunctions in natural languages because of their highly idiosyncratic usage.

Digital logic :

Boolean operations are used to combine the bits on individual wires and interpret them over "0,1" in digital logic. At the point when a vector of n indistinguishable double entryways is utilized to join worthless vectors every one of n bits, the singular digit tasks can be seen all in all as a solitary procedure on values from a Boolean variable-based math with 2n components.

Video cards :

A computer display based on raster graphics uses 256 elements of free Boolean algebra with three generators. These displays use bit blits to manipulate entire regions made up of pixels and rely on Boolean operations to specify how source regions and destinations are combined. A third area, called the mask, is usually used. For this purpose, all modern video cards offer his ternary operation of 223 = 256. The operation selection is a 1-byte (8-bit) parameter. Boolean operations like (SRCDST)&MSK (meaning XOR of source and destination, AND of result and mask) are computed at compile time using constants SRC = 0xaa or 10101010, DST = 0xcc or 11001100 can be written directly as a constant indicating the number of bytes and MSK = 0xf0 or 11110000. Regardless of the complexity of the representation, video cards interpret the bytes as raster operations specified by the original representation at runtime in a uniform manner that requires virtually no hardware and takes no time at all.

Computer-Aided Design (CAD):

Solid modeling systems provide a variety of methods for building objects from other objects, one of which is a combination of Boolean operations. Shapes are defined as subsets of S, and the space in which objects exist is understood to be a set S of voxels, which are the three-dimensional equivalent of pixels in two-dimensional graphics. This makes it possible to combine objects as sets through union, intersection, and other methods. Building a complex shape from simple shapes simply by combining them is one obvious application. Sculpting, which is defined as the removal of material, is another application: With the Boolean operation x y or x y, which in set theory is set to difference, remove the elements of y from those of x, any grinding, milling, routing, or drilling operation that can be performed with physical machinery on physical materials can be simulated on the computer. Given two shapes, one to be machined and the other to be removed, the result of machining the former to remove the latter is simply referred to as their set difference.

REFERENCES

- ***https://www.nios.ac.in/media/documents/330srsec/online_course_material_330/Theory/Lesson_02.pdf***
- ***https://www.sciencedirect.com/topics/engineering/binary-logic***
- ***https://www2.southeastern.edu/Academics/Faculty/kyang/2018/Spring/CMPS375/ClassNotes/CMPS375ClassNotesChap03.pdf***
- ***http://www0.cs.ucl.ac.uk/teaching/B261/binary_logic.html***
- ***https://www.sciencedirect.com/science/article/pii/B9780128150733000090***
- ***http://www.amsheela.org.in/dwn/ece/II%20Year%20ECE/ec1203_digital_electronics-Part-A_2.pdf***