I. EXPRESSIONS FOR $\bar{H}$ (UP TO TRIPLE COMMUTATORS)

A. E_{gr}

The ground state energy comprises the Hartree-Fock energy and the correlation energy. The latter is the fully contracted part of $\bar{H}$. We collect all contirbutions up to triple commutators for E_{gr}. $[O, \sigma]$ can yield a fully contracted contribution only when O is non-diagonal. The expression for E_{gr} is given by

$$E_{gr} = E_{\rm HF} + \left(\sum_{ijab} \frac{1}{8} \langle ij||ab\rangle \sigma_{ij}^{ab} + \sum_{ijab} \frac{1}{12} \langle ij||ab\rangle \sigma_i^a \sigma_j^b + h.c. \right) \tag{1}$$

$$+ \Bigg(- \sum_{ijklabcd} \frac{1}{12} (\sigma_{kl}^{cd})^* \langle ij||ab\rangle \sigma_{ik}^{ac} \sigma_{jl}^{bd} - \sum_{ijklabcd} \frac{1}{24} (\sigma_{kl}^{cd})^* \langle ij||ab\rangle \sigma_{ij}^{ac} \sigma_{kl}^{bd}$$

$$- \sum_{ijklabcd} \frac{1}{24} (\sigma_{kl}^{cd})^* \langle ij||ab\rangle \sigma_{ik}^{ab} \sigma_{jl}^{cd} - \sum_{ijklabcd} \frac{1}{96} (\sigma_{kl}^{cd})^* \langle ij||ab\rangle \sigma_{ij}^{cd} \sigma_{kl}^{ab} + h.c. \Bigg) \tag{2}$$

$$+ \Bigg(\sum_{ijklabc} \frac{1}{4} (\sigma_{jl}^{bc})^* \langle ik||aj\rangle \sigma_i^a \sigma_{kl}^{bc} - \sum_{ijkabcd} \frac{1}{4} (\sigma_{jk}^{cd})^* \langle ic||ab\rangle \sigma_i^a \sigma_{jk}^{bd}$$

$$+ \sum_{ijklabc} \frac{1}{2} (\sigma_{il}^{bc})^* \langle kj||ai\rangle \sigma_j^b \sigma_{lk}^{ca} - \sum_{ijkabcd} \frac{1}{2} (\sigma_{jk}^{cd})^* \langle ic||ab\rangle \sigma_j^b \sigma_{ik}^{ad}$$

$$+ \sum_{ijklabc} \frac{1}{12} (\sigma_{il}^{bc})^* \langle jk||ia\rangle \sigma_l^c \sigma_{kj}^{ab} - \sum_{ijkabcd} \frac{1}{12} (\sigma_{jk}^{cd})^* \langle ic||ab\rangle \sigma_k^d \sigma_{ij}^{ab}$$

$$+ \sum_{ijklabc} \frac{1}{8} (\sigma_{il}^{cb})^* \langle kj||ia\rangle \sigma_l^a \sigma_{kj}^{cb} - \sum_{ijkabcd} \frac{1}{8} (\sigma_{jk}^{dc})^* \langle ci||ab\rangle \sigma_i^d \sigma_{kj}^{ab} + h.c. \Bigg) \tag{3}$$

$$+ \Bigg(- \sum_{ijkabc} \frac{1}{12} (\sigma_k^c)^* \langle ij||ab\rangle \sigma_i^a \sigma_{jk}^{bc} - \sum_{ijkabc} \frac{1}{12} (\sigma_k^c)^* \langle ij||ab\rangle \sigma_i^c \sigma_{jk}^{ba}$$

$$- \sum_{ijkabc} \frac{1}{12} (\sigma_k^c)^* \langle ij||ab\rangle \sigma_k^a \sigma_{ij}^{cb} - \sum_{ijkabc} \frac{1}{3} (\sigma_{ik}^{bc})^* \langle jb||ai\rangle \sigma_j^c \sigma_k^a$$

$$- \sum_{ijklab} \frac{1}{12} (\sigma_{ij}^{ab})^* \langle kl||ij\rangle \sigma_k^a \sigma_l^b - \sum_{ijabcd} \frac{1}{12} (\sigma_{ij}^{cd})^* \langle cd||ab\rangle \sigma_j^b \sigma_i^a + h.c. \Bigg) \tag{4}$$

$$+ \Bigg(+ \sum_{ijkab} \frac{1}{3} (\sigma_k^b)^* \langle ij||ak\rangle \sigma_i^a \sigma_j^b - \sum_{ijabc} \frac{1}{3} (\sigma_j^a)^* \langle ai||bc\rangle \sigma_j^b \sigma_i^c + h.c. \Bigg). \tag{5}$$

B. $\bar{H}_{ai}$

$$\bar{H}^{[1]}_{ai} = \sum_{jb}\langle aj||ib\rangle\sigma^b_j + \sum_{jb}\frac{1}{2}(\sigma^b_j)^*\langle ab||ij\rangle + \sum_{jbc}\frac{1}{2}\langle aj||cb\rangle\sigma^{cb}_{ij} - \sum_{jkb}\frac{1}{2}\langle kj||ib\rangle\sigma^{ba}_{jk}. \tag{6}$$

$$\begin{aligned}\bar{H}^{[2]}_{ai} = &\Bigg(\sum_{jbc}\langle aj||cb\rangle\sigma^b_j\sigma^c_i - \sum_{jkb}\langle kj||ib\rangle\sigma^b_j\sigma^a_k + \sum_{jbc}\frac{1}{2}(\sigma^b_j)^*\langle ab||cj\rangle\sigma^c_i - \sum_{jkb}\frac{1}{2}(\sigma^b_j)^*\langle kb||ij\rangle\sigma^a_k \\ &+ \sum_{jbc}\frac{1}{2}(\sigma^c_j)^*\langle ac||ib\rangle\sigma^b_j - \sum_{jkb}\frac{1}{2}(\sigma^b_j)^*\langle ak||ij\rangle\sigma^b_k\Bigg) \qquad (7)\\ &+\Bigg(\sum_{jkbc}\frac{5}{12}\langle jk||bc\rangle\sigma^b_j\sigma^{ac}_{ik} - \sum_{jkbc}\frac{1}{3}\langle jk||bc\rangle\sigma^a_k\sigma^{cb}_{ij} - \sum_{jkbc}\frac{1}{3}\langle jk||bc\rangle\sigma^c_i\sigma^{ba}_{jk} \\ &- \sum_{jkbc}\frac{1}{2}(\sigma^c_k)^*\langle cj||ib\rangle\sigma^{ba}_{jk} - \sum_{jkbc}\frac{1}{2}(\sigma^c_k)^*\langle aj||kb\rangle\sigma^{cb}_{ij} - \sum_{jkbc}\frac{1}{3}(\sigma^{cb}_{jk})^*\langle ab||ij\rangle\sigma^c_k \\ &- \sum_{jkbc}\frac{1}{6}(\sigma^{bc}_{jk})^*\langle bc||ji\rangle\sigma^a_k - \sum_{jkbc}\frac{1}{6}(\sigma^{bc}_{jk})^*\langle ab||kj\rangle\sigma^c_i + \sum_{jbcd}\frac{1}{4}(\sigma^c_j)^*\langle ac||bd\rangle(\sigma^{bd}_{ij})^* \\ &+ \sum_{jklb}\frac{1}{4}(\sigma^b_k)^*\langle jl||ik\rangle(\sigma^{ab}_{jl})^*\Bigg) \qquad (8)\\ &+\Bigg(-\sum_{jklbc}\frac{1}{2}(\sigma^{bc}_{jk})^*\langle al||ik\rangle\sigma^{bc}_{jl} + \sum_{jkbcd}\frac{1}{2}(\sigma^{bd}_{jk})^*\langle ad||ic\rangle\sigma^{bc}_{jk} - \sum_{jklbc}(\sigma^{bc}_{jk})^*\langle bl||ji\rangle\sigma^{ca}_{kl} \\ &+ \sum_{jkbcd}(\sigma^{bc}_{jk})^*\langle ab||dj\rangle\sigma^{cd}_{ki} - \sum_{jklbc}\frac{1}{4}(\sigma^{bc}_{jk})^*\langle bl||jk\rangle\sigma^{ac}_{il} + \sum_{jkbcd}\frac{1}{4}(\sigma^{bd}_{jk})^*\langle bd||jc\rangle\sigma^{ac}_{ik} \\ &+ \sum_{jklbc}\frac{1}{4}(\sigma^{bd}_{jk})^*\langle bd||ic\rangle\sigma^{ca}_{jk} - \sum_{jkbcd}\frac{1}{4}(\sigma^{bc}_{jk})^*\langle al||jk\rangle\sigma^{cb}_{il}\Bigg). \qquad (9)\end{aligned}$$

$$\begin{aligned}\bar{H}^{[3]}_{ai} = &\Bigg(-\sum_{jkbc}\frac{7}{12}\langle jk||bc\rangle\sigma^a_j\sigma^b_i\sigma^c_k + \sum_{jklb}\frac{1}{2}(\sigma^b_l)^*\langle jk||il\rangle\sigma^b_k\sigma^a_j + \sum_{jbcd}\frac{1}{2}(\sigma^b_j)^*\langle ab||cd\rangle\sigma^d_j\sigma^c_i \\ &- \sum_{jkbc}\frac{1}{3}(\sigma^c_k)^*\langle jk||bi\rangle\sigma^b_j\sigma^a_k - \sum_{jkbc}\frac{1}{3}(\sigma^c_k)^*\langle ja||bk\rangle\sigma^b_j\sigma^c_i - \sum_{jkbc}\frac{1}{6}(\sigma^c_k)^*\langle jc||bk\rangle\sigma^b_i\sigma^a_j \\ &+ \sum_{jkbc}\frac{1}{6}(\sigma^c_k)^*\langle ja||bi\rangle\sigma^c_j\sigma^b_k + \sum_{jkbc}\frac{1}{2}(\sigma^c_k)^*\langle jc||bi\rangle\sigma^a_j\sigma^b_k + \sum_{jkbc}\frac{1}{2}(\sigma^c_k)^*\langle ja||bk\rangle\sigma^c_j\sigma^b_i\end{aligned}$$

$$+\sum_{jkbc}\frac{1}{8}(\sigma_k^c)^*(\sigma_j^b)^*\langle bc||ji\rangle\sigma_k^a-\sum_{jkbc}\frac{1}{8}(\sigma_k^c)^*(\sigma_j^b)^*\langle ab||kj\rangle\sigma_i^c+\sum_{jkbc}\frac{1}{6}(\sigma_k^b)^*(\sigma_j^c)^*\langle ab||ij\rangle\sigma_k^c\Bigg) \tag{10}$$

$$\begin{aligned}+\Bigg(&\sum_{jklbc}\frac{1}{12}(\sigma_l^c)^*\langle jk||bl\rangle\sigma_{ij}^{ab}\sigma_k^c-\sum_{jkbcd}\frac{1}{12}(\sigma_k^c)^*\langle jc||bd\rangle\sigma_{ij}^{ab}\sigma_k^d+\sum_{jklbc}\frac{1}{6}(\sigma_l^c)^*\langle jk||bl\rangle\sigma_{ij}^{cb}\sigma_k^a\\
&-\sum_{jkbcd}\frac{1}{6}(\sigma_k^c)^*\langle jc||bd\rangle\sigma_{jk}^{ba}\sigma_i^d+\sum_{jklbc}\frac{1}{6}(\sigma_l^c)^*\langle jk||bi\rangle\sigma_{jl}^{ba}\sigma_k^c+\sum_{jkbcd}\frac{1}{6}(\sigma_k^d)^*\langle ja||bc\rangle\sigma_{ij}^{db}\sigma_k^c\\
&+\sum_{jklbc}\frac{1}{12}(\sigma_l^c)^*\langle jk||bl\rangle\sigma_{ki}^{ca}\sigma_j^b-\sum_{jkbcd}\frac{1}{12}(\sigma_k^c)^*\langle jc||bd\rangle\sigma_{ki}^{da}\sigma_j^b+\sum_{jklbc}\frac{1}{12}(\sigma_l^c)^*\langle jk||bl\rangle\sigma_{jk}^{ac}\sigma_i^b\\
&-\sum_{jkbcd}\frac{1}{12}(\sigma_k^c)^*\langle jc||bd\rangle\sigma_{ik}^{bd}\sigma_j^a-\sum_{jklbc}\frac{1}{12}(\sigma_l^c)^*\langle jk||bi\rangle\sigma_{kj}^{ac}\sigma_l^b+\sum_{jkbcd}\frac{1}{12}(\sigma_k^d)^*\langle ja||bc\rangle\sigma_{ik}^{cb}\sigma_j^d\\
&+\sum_{jklbc}\frac{1}{6}(\sigma_{kl}^{cb})^*\langle jb||kl\rangle\sigma_j^a\sigma_i^c-\sum_{jkbcd}\frac{1}{6}(\sigma_{kj}^{bc})^*\langle bc||dj\rangle\sigma_k^a\sigma_i^d-\sum_{jklbc}\frac{1}{3}(\sigma_{kl}^{bc})^*\langle jb||ik\rangle\sigma_j^a\sigma_l^c\\
&+\sum_{jkbcd}\frac{1}{3}(\sigma_{jk}^{bd})^*\langle ab||cj\rangle\sigma_i^c\sigma_k^d+\sum_{jklbc}\frac{1}{3}(\sigma_{kl}^{bc})^*\langle jb||ik\rangle\sigma_j^c\sigma_l^a-\sum_{jkbcd}\frac{1}{3}(\sigma_{jk}^{bd})^*\langle ab||cj\rangle\sigma_k^c\sigma_i^d\\
&+\sum_{jkbc}\frac{1}{6}(\sigma_{kl}^{cb})^*\langle ja||lk\rangle\sigma_j^b\sigma_i^c+\sum_{jkbcd}\frac{1}{6}(\sigma_{jk}^{bc})^*\langle bc||di\rangle\sigma_j^d\sigma_k^a\\
&+\sum_{jklbc}\frac{1}{2}(\sigma_k^c)^*(\sigma_l^b)^*\langle jb||kl\rangle\sigma_{ij}^{ac}-\sum_{jkbcd}\frac{1}{2}(\sigma_k^b)^*(\sigma_j^c)^*\langle bc||dj\rangle\sigma_{ik}^{ad}+\sum_{jklbc}\frac{1}{4}(\sigma_k^b)^*(\sigma_l^c)^*\langle jb||ik\rangle\sigma_{jl}^{ac}\\
&-\sum_{jkbcd}\frac{1}{4}(\sigma_k^d)^*(\sigma_j^b)^*\langle ab||cj\rangle\sigma_{ik}^{cd}+\sum_{jklbc}\frac{1}{4}(\sigma_k^b)^*(\sigma_l^c)^*\langle ja||ki\rangle\sigma_{jl}^{bc}-\sum_{jkbcd}\frac{1}{4}(\sigma_k^d)^*(\sigma_j^b)^*\langle ab||ic\rangle\sigma_{jk}^{cd}\Bigg)\end{aligned} \tag{11}$$

$$\begin{aligned}+\Bigg(&-\sum_{jklbcd}\frac{1}{8}(\sigma_l^d)^*\langle jk||bc\rangle\sigma_{ij}^{ab}\sigma_{kl}^{cd}-\sum_{jklbcd}\frac{1}{24}(\sigma_l^d)^*\langle jk||bc\rangle\sigma_{kj}^{ca}\sigma_{li}^{db}+\sum_{jklbcd}\frac{1}{48}(\sigma_l^d)^*\langle jk||bc\rangle\sigma_{ik}^{bc}\sigma_{lj}^{da}\\
&-\sum_{jklbcd}\frac{1}{6}(\sigma_j^b)^*\langle kl||cd\rangle\sigma_{ik}^{bc}\sigma_{lj}^{da}+\sum_{jklbcd}\frac{1}{24}(\sigma_l^d)^*\langle jk||bc\rangle\sigma_{il}^{bc}\sigma_{kj}^{da}\\
&-\sum_{jklmbc}\frac{1}{12}(\sigma_{lm}^{cb})^*\langle jk||il\rangle\sigma_{jk}^{bc}\sigma_m^a-\sum_{jkbcde}\frac{1}{12}(\sigma_{jk}^{eb})^*\langle ab||cd\rangle\sigma_{kj}^{dc}\sigma_i^e+\sum_{jklmbc}\frac{1}{6}(\sigma_{lm}^{bc})^*\langle jk||li\rangle\sigma_{jk}^{ba}\sigma_m^c\\
&+\sum_{jkbcde}\frac{1}{6}(\sigma_{kj}^{eb})^*\langle ab||cd\rangle\sigma_{ij}^{cd}\sigma_k^e+\sum_{jklmbc}\frac{5}{24}(\sigma_{ml}^{cb})^*\langle jk||lm\rangle\sigma_{ki}^{ca}\sigma_j^b+\sum_{jkbcde}\frac{5}{24}(\sigma_{jk}^{bc})^*\langle bc||de\rangle\sigma_{ik}^{ae}\sigma_j^d\\
&+\sum_{jklmbc}\frac{1}{2}(\sigma_{lm}^{bc})^*\langle jk||il\rangle\sigma_{mk}^{cb}\sigma_j^a+\sum_{jkbcde}\frac{1}{2}(\sigma_{jk}^{be})^*\langle ab||cd\rangle\sigma_{jk}^{de}\sigma_i^c-\sum_{jklmbc}\frac{1}{6}(\sigma_{ml}^{cb})^*\langle jk||lm\rangle\sigma_{ik}^{bc}\sigma_j^a\\
&-\sum_{jkbcde}\frac{1}{6}(\sigma_{jk}^{bc})^*\langle bc||ed\rangle\sigma_{jk}^{ad}\sigma_i^e+\sum_{jklmbc}(\sigma_{lm}^{bc})^*\langle jk||il\rangle\sigma_{jm}^{ac}\sigma_k^b+\sum_{jkbcde}(\sigma_{jk}^{be})^*\langle ab||cd\rangle\sigma_{ik}^{ce}\sigma_j^d\end{aligned}$$

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\begin{aligned}
&- \sum_{jklbcd} \frac{1}{6}(\sigma_{kl}^{cd})^*\langle jc||bi\rangle\sigma_{kl}^{ad}\sigma_j^b - \sum_{jklbcd} \frac{1}{6}(\sigma_{kl}^{cd})^*\langle ja||bk\rangle\sigma_{il}^{cd}\sigma_j^b - \sum_{jklbcd} \frac{1}{3}(\sigma_{kl}^{cd})^*\langle jc||bk\rangle\sigma_{jl}^{ba}\sigma_i^d \\
&- \sum_{jklbcd} \frac{1}{3}(\sigma_{kl}^{cd})^*\langle jc||bk\rangle\sigma_{ji}^{bd}\sigma_l^a - \sum_{jklbcd} \frac{1}{3}(\sigma_{kl}^{cd})^*\langle jc||bi\rangle\sigma_{jk}^{ba}\sigma_l^d - \sum_{jklbcd} \frac{1}{3}(\sigma_{kl}^{cd})^*\langle ja||bk\rangle\sigma_{ij}^{cb}\sigma_l^d \\
&- \sum_{jklbcd} \frac{1}{3}(\sigma_{kl}^{cd})^*\langle ja||bk\rangle\sigma_{jl}^{bd}\sigma_i^c - \sum_{jklbcd} \frac{1}{3}(\sigma_{kl}^{cd})^*\langle jc||bi\rangle\sigma_{jl}^{bd}\sigma_k^a + \sum_{jklbcd} \frac{1}{2}(\sigma_{kl}^{cd})^*\langle ja||bi\rangle\sigma_{jl}^{cd}\sigma_k^b \\
&+ \sum_{jklbcd} \frac{1}{3}(\sigma_{kl}^{cd})^*\langle ja||bi\rangle\sigma_{kl}^{bd}\sigma_j^c + \sum_{jklbcd} \frac{1}{3}(\sigma_{il}^{cd})^*\langle jc||bi\rangle\sigma_{kl}^{bd}\sigma_j^a + \sum_{jklbcd} \frac{1}{2}(\sigma_{kl}^{cd})^*\langle ja||bk\rangle\sigma_{jl}^{cd}\sigma_i^b \\
&- \sum_{jklbcd} \frac{2}{3}(\sigma_{kl}^{cd})^*\langle jc||bk\rangle\sigma_{jl}^{ad}\sigma_i^b + \sum_{jklbcd} \frac{2}{3}(\sigma_{kl}^{cd})^*\langle jc||bk\rangle\sigma_{il}^{bd}\sigma_j^a + \sum_{jklbcd} (\sigma_{kl}^{cd})^*\langle jc||bi\rangle\sigma_{jl}^{ad}\sigma_k^b \\
&+ \sum_{jklbcd} (\sigma_{kl}^{cd})^*\langle ja||bk\rangle\sigma_{li}^{db}\sigma_j^c - \sum_{jklbcd} \frac{1}{2}(\sigma_{kl}^{cd})^*\langle jc||bi\rangle\sigma_{kl}^{ab}\sigma_j^d - \sum_{jklbcd} \frac{1}{2}(\sigma_{kl}^{cd})^*\langle ja||bk\rangle\sigma_{ij}^{cd}\sigma_l^b \\
&+ \sum_{jklbcd} \frac{5}{12}(\sigma_{kl}^{cd})^*\langle jc||bk\rangle\sigma_{ij}^{ad}\sigma_l^b + \sum_{jklbcd} \frac{5}{12}(\sigma_{kl}^{cd})^*\langle jc||bk\rangle\sigma_{il}^{ab}\sigma_j^d \\
&+ \sum_{jklbcd} \frac{1}{16}(\sigma_j^b)^*(\sigma_{kl}^{cd})^*\langle bc||ji\rangle\sigma_{kl}^{ad}\Bigg) \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
+\Bigg(&- \sum_{jklmbcd} \frac{1}{24}(\sigma_{ml}^{dc})^*\langle jk||bl\rangle\sigma_{jk}^{bc}\sigma_{mi}^{da} + \sum_{jklbcde} \frac{1}{24}(\sigma_{kl}^{ce})^*\langle jc||bd\rangle\sigma_{jk}^{bd}\sigma_{li}^{ea} + \sum_{jklmbcd} \frac{1}{12}(\sigma_{lm}^{cd})^*\langle jk||bl\rangle\sigma_{jk}^{ba}\sigma_{im}^{cd} \\
&- \sum_{jklbcde} \frac{1}{12}(\sigma_{kl}^{ce})^*\langle jc||bd\rangle\sigma_{ij}^{db}\sigma_{kl}^{ae} - \sum_{jklmbcd} \frac{1}{4}(\sigma_{lm}^{cd})^*\langle jk||bl\rangle\sigma_{ji}^{ba}\sigma_{km}^{cd} + \sum_{jklbcde} \frac{1}{4}(\sigma_{kl}^{ce})^*\langle jc||bd\rangle\sigma_{ij}^{ab}\sigma_{kl}^{de} \\
&+ \sum_{jklmbcd} \frac{1}{3}(\sigma_{ml}^{dc})^*\langle jk||bi\rangle\sigma_{lj}^{ab}\sigma_{km}^{cd} - \sum_{jklbcde} \frac{1}{3}(\sigma_{lk}^{ed})^*\langle ja||bc\rangle\sigma_{ji}^{bd}\sigma_{kl}^{ce} - \sum_{jklmbcd} \frac{2}{3}(\sigma_{lm}^{cd})^*\langle jk||bi\rangle\sigma_{jl}^{bc}\sigma_{km}^{ad} \\
&+ \sum_{jklbcde} \frac{2}{3}(\sigma_{kl}^{de})^*\langle ja||bc\rangle\sigma_{jk}^{bd}\sigma_{il}^{ce} - \sum_{jklmbcd} (\sigma_{lm}^{cd})^*\langle jk||bl\rangle\sigma_{ij}^{cb}\sigma_{km}^{ad} - \sum_{jklbcde} (\sigma_{kl}^{ce})^*\langle jc||bd\rangle\sigma_{jk}^{ba}\sigma_{il}^{de} \\
&- \sum_{jklmbcd} \frac{1}{2}(\sigma_{ml}^{dc})^*\langle jk||bl\rangle\sigma_{jm}^{bd}\sigma_{ik}^{ac} + \sum_{jklbcde} \frac{1}{2}(\sigma_{kl}^{ce})^*\langle jc||bd\rangle\sigma_{jl}^{be}\sigma_{ik}^{ad} \\
&+ \sum_{jklmbcd} \frac{1}{2}(\sigma_{lm}^{cd})^*\langle jk||bl\rangle\sigma_{jm}^{ba}\sigma_{ik}^{dc} - \sum_{jklbcde} \frac{1}{2}(\sigma_{kl}^{ce})^*\langle jc||bd\rangle\sigma_{ji}^{be}\sigma_{kl}^{da} + \sum_{jklmbcd} \frac{1}{2}(\sigma_{lm}^{cd})^*\langle jk||bl\rangle\sigma_{im}^{bd}\sigma_{jk}^{ac} \\
&- \sum_{jklmbcd} \frac{1}{2}(\sigma_{kl}^{ce})^*\langle jc||bd\rangle\sigma_{jl}^{ae}\sigma_{ik}^{bd} - \sum_{jklmbcd} \frac{1}{8}(\sigma_{lm}^{cd})^*\langle jk||bl\rangle\sigma_{im}^{ab}\sigma_{jk}^{dc} + \sum_{jklmbcd} \frac{1}{8}(\sigma_{ml}^{ec})^*\langle jc||bd\rangle\sigma_{ij}^{ae}\sigma_{ml}^{bd} \\
&- \sum_{jklmbcd} \frac{1}{6}(\sigma_{ml}^{dc})^*\langle jk||bi\rangle\sigma_{jk}^{ca}\sigma_{ml}^{db} + \sum_{jklbcde} \frac{1}{6}(\sigma_{lk}^{ed})^*\langle ja||bc\rangle\sigma_{jl}^{de}\sigma_{ik}^{cb} + \sum_{jklmbcd} \frac{1}{12}(\sigma_{ml}^{dc})^*\langle jk||bi\rangle\sigma_{jk}^{dc}\sigma_{lm}^{ab}
\end{aligned}
$$

$$+\sum_{jklmbcd}\frac{1}{12}(\sigma_{kl}^{de})^*\langle ja||bc\rangle\sigma_{ij}^{de}\sigma_{lk}^{bc}\Bigg). \tag{13}$$

C. $\bar{H}_{ab,ij}$

$$\bar{H}_{ab,ij}^{[1]} = \sum_c f_{ac}\sigma_{ij}^{cb} - \sum_k f_{ki}\sigma_{kj}^{ab} - P(ab)\sum_k\langle ka||ji\rangle\sigma_k^b + P(ij)\sum_c\langle ab||ic\rangle\sigma_j^c + \sum_{kl}\frac{1}{2}\langle kl||ij\rangle\sigma_{kl}^{ab}$$
$$+\sum_{cd}\frac{1}{2}\langle ab||cd\rangle\sigma_{ij}^{cd} + P(ij)P(ab)\sum_{kc}\langle ak||ic\rangle\sigma_{jk}^{bc}. \tag{14}$$

$$\bar{H}_{ab,ij}^{[2]} = \Bigg(P(ab)\sum_{kl}\frac{1}{2}\langle kl||ij\rangle\sigma_k^a\sigma_l^b - P(ij)P(ab)\sum_{kc}\langle ak||cj\rangle\sigma_i^c\sigma_k^b + P(ij)\sum_{cd}\frac{1}{2}\langle ab||cd\rangle\sigma_i^c\sigma_j^d$$
$$- P(ab)\sum_{kc}\frac{1}{3}(\sigma_k^c)^*\langle ac||ij\rangle\sigma_k^b - P(ij)\sum_{kc}\frac{1}{3}(\sigma_k^c)^*\langle ab||ik\rangle\sigma_j^c\Bigg) \tag{15}$$
$$+\Bigg(-P(ij)\sum_{klc}(\sigma_l^c)^*\langle ck||lj\rangle\sigma_{ik}^{ab} + P(ab)\sum_{lcd}(\sigma_l^c)^*\langle bc||dl\rangle\sigma_{ij}^{ad} + P(ij)\sum_{lcd}\frac{1}{2}(\sigma_l^c)^*\langle ab||id\rangle\sigma_{jl}^{dc}$$
$$- P(ab)\sum_{klc}\frac{1}{2}(\sigma_l^c)^*\langle ak||ij\rangle\sigma_{kl}^{bc} + \sum_{klc}(\sigma_l^c)^*\langle ck||ji\rangle\sigma_{kl}^{ab} + P(ij)P(ab)\sum_{klc}(\sigma_l^c)^*\langle bk||li\rangle\sigma_{jk}^{ca}$$
$$- P(ij)P(ab)\sum_{lcd}(\sigma_l^c)^*\langle ac||dj\rangle\sigma_{il}^{db} - \sum_{lcd}(\sigma_l^c)^*\langle ab||dl\rangle\sigma_{ij}^{dc} - P(ij)\sum_{klc}\langle kl||cj\rangle\sigma_k^c\sigma_{il}^{ab}$$
$$+ P(ab)\sum_{kcd}\langle kb||cd\rangle\sigma_k^c\sigma_{ij}^{ad} - P(ij)P(ab)\sum_{klc}\langle kl||cj\rangle\sigma_l^b\sigma_{ik}^{ac} + P(ij)P(ab)\sum_{kcd}\langle kb||cd\rangle\sigma_j^d\sigma_{ik}^{ac}$$
$$+ P(ij)\frac{1}{2}\sum_{klc}\langle kl||ci\rangle\sigma_j^c\sigma_{kl}^{ba} - P(ab)\frac{1}{2}\sum_{kcd}\langle ka||cd\rangle\sigma_k^b\sigma_{ij}^{dc}\Bigg) \tag{16}$$
$$+\Bigg(P(ij)P(ab)\sum_{klcd}\frac{1}{3}\langle kl||cd\rangle\sigma_{ik}^{ac}\sigma_{jl}^{bd} + \sum_{klcd}\frac{1}{6}\langle kl||cd\rangle\sigma_{ij}^{cd}\sigma_{kl}^{ab} - P(ab)\sum_{klcd}\frac{1}{3}\langle kl||cd\rangle\sigma_{ij}^{ad}\sigma_{kl}^{cb}$$
$$- P(ij)\sum_{klcd}\frac{1}{3}\langle kl||cd\rangle\sigma_{il}^{ab}\sigma_{jk}^{dc} + P(ij)P(ab)\sum_{klcd}\frac{1}{3}(\sigma_{kl}^{cd})^*\langle ad||il\rangle\sigma_{jk}^{bc} + \sum_{klcd}\frac{1}{12}(\sigma_{kl}^{cd})^*\langle cd||ij\rangle\sigma_{kl}^{ab}$$
$$+ \sum_{klcd}\frac{1}{12}(\sigma_{kl}^{cd})^*\langle ab||kl\rangle\sigma_{ij}^{cd} - P(ab)\sum_{klcd}\frac{1}{6}(\sigma_{kl}^{cd})^*\langle ad||ij\rangle\sigma_{kl}^{cb} - P(ij)\sum_{klcd}\frac{1}{6}(\sigma_{kl}^{cd})^*\langle ab||il\rangle\sigma_{jk}^{dc}$$
$$- P(ij)\sum_{klcd}\frac{1}{6}(\sigma_{kl}^{cd})^*\langle cd||kj\rangle\sigma_{il}^{ab} - P(ab)\sum_{klcd}\frac{1}{6}(\sigma_{kl}^{cd})^*\langle bc||lk\rangle\sigma_{ij}^{ad}\Bigg). \tag{17}$$

$$
\begin{aligned}
\bar{H}^{[3]}_{ab,ij} = \Bigg(& P(ij)P(ab)\sum_{klc}\langle kl||ic\rangle\sigma^a_k\sigma^c_j\sigma^b_l - P(ij)P(ab)\sum_{kcd}\langle ak||dc\rangle\sigma^b_k\sigma^c_j\sigma^b_i \\
& + P(ij)P(ab)\sum_{klc}\frac{1}{3}(\sigma^c_l)^*\langle ka||jl\rangle\sigma^b_k\sigma^c_i - P(ij)\sum_{kcd}\frac{1}{3}(\sigma^d_k)^*\langle ab||kc\rangle\sigma^c_j\sigma^d_i \\
& + P(ab)\sum_{klc}\frac{1}{3}(\sigma^c_l)^*\langle kc||ji\rangle\sigma^b_k\sigma^a_l - P(ij)P(ab)\sum_{kcd}\frac{1}{3}(\sigma^d_k)^*\langle bd||ci\rangle\sigma^a_k\sigma^c_j \Bigg) \qquad (18) \\
+ \Bigg(& - P(ab)\sum_{klcd}\frac{7}{12}\langle kl||cd\rangle\sigma^{ac}_{ij}\sigma^b_k\sigma^d_l - P(ij)\sum_{klcd}\frac{7}{12}\langle kl||cd\rangle\sigma^{ab}_{ik}\sigma^c_j\sigma^d_l \\
& - P(ij)P(ab)\sum_{klcd}\frac{3}{4}\langle kl||cd\rangle\sigma^{ad}_{il}\sigma^c_j\sigma^b_k + P(ab)\sum_{klcd}\frac{3}{8}\langle kl||cd\rangle\sigma^{cd}_{ij}\sigma^a_k\sigma^b_l \\
& + P(ij)\sum_{klcd}\frac{3}{8}\langle kl||cd\rangle\sigma^{ab}_{kl}\sigma^c_i\sigma^d_j \\
& - P(ij)\sum_{klmc}\frac{1}{6}(\sigma^c_m)^*\langle kl||mj\rangle\sigma^{ab}_{kl}\sigma^c_i - P(ab)\sum_{kcde}\frac{1}{6}(\sigma^c_k)^*\langle bc||de\rangle\sigma^{de}_{ji}\sigma^a_k \\
& + P(ij)\sum_{klmc}(\sigma^c_m)^*\langle kl||mi\rangle\sigma^{ab}_{lj}\sigma^c_k + P(ab)\sum_{kcde}(\sigma^c_k)^*\langle ac||de\rangle\sigma^{db}_{ij}\sigma^e_k \\
& + P(ij)P(ab)\sum_{klmc}(\sigma^c_m)^*\langle kl||im\rangle\sigma^{cb}_{lj}\sigma^a_k + P(ij)P(ab)\sum_{kcde}(\sigma^c_k)^*\langle ac||de\rangle\sigma^{eb}_{kj}\sigma^d_i \\
& + P(ab)\sum_{klmc}\frac{1}{2}(\sigma^c_m)^*\langle kl||ij\rangle\sigma^{bc}_{lm}\sigma^a_k + P(ij)\sum_{kcde}\frac{1}{2}(\sigma^e_k)^*\langle ab||cd\rangle\sigma^{ed}_{kj}\sigma^c_i \\
& - \sum_{klmc}\frac{2}{3}(\sigma^c_m)^*\langle kl||ij\rangle\sigma^{ba}_{lm}\sigma^c_k - \sum_{kcde}\frac{2}{3}(\sigma^e_k)^*\langle ab||cd\rangle\sigma^{ed}_{ij}\sigma^c_k \\
& - P(ij)P(ab)\sum_{klcd}\frac{1}{3}(\sigma^d_l)^*\langle ka||cl\rangle\sigma^{bc}_{jk}\sigma^d_i - P(ij)P(ab)\sum_{klcd}\frac{1}{3}(\sigma^d_l)^*\langle kd||ci\rangle\sigma^{bc}_{jk}\sigma^a_l \\
& - P(ab)\sum_{klcd}\frac{1}{3}(\sigma^d_l)^*\langle kb||cl\rangle\sigma^{ad}_{ij}\sigma^c_k - P(ij)\sum_{klcd}\frac{1}{3}(\sigma^d_l)^*\langle kd||cj\rangle\sigma^{ab}_{il}\sigma^c_k \\
& - P(ij)\sum_{klcd}\frac{2}{3}(\sigma^d_l)^*\langle kd||cl\rangle\sigma^{ab}_{kj}\sigma^c_i - P(ab)\sum_{klcd}\frac{2}{3}(\sigma^d_l)^*\langle kd||cl\rangle\sigma^{cb}_{ij}\sigma^a_k \\
& + P(ij)P(ab)\sum_{klcd}\frac{2}{3}(\sigma^d_l)^*\langle ka||ci\rangle\sigma^{bd}_{jk}\sigma^c_l + P(ij)P(ab)\sum_{klcd}\frac{2}{3}(\sigma^d_l)^*\langle ka||ci\rangle\sigma^{bc}_{jl}\sigma^d_k \\
& - P(ij)P(ab)\sum_{klcd}\frac{1}{2}(\sigma^d_l)^*\langle ka||ci\rangle\sigma^{bd}_{kl}\sigma^c_j - P(ij)P(ab)\sum_{klcd}\frac{1}{2}(\sigma^d_l)^*\langle ka||ci\rangle\sigma^{cd}_{jl}\sigma^b_k
\end{aligned}
$$

$$
\begin{aligned}
&+ P(ij)P(ab)\sum_{klcd}(\sigma_l^d)^*\langle kd||ci\rangle\sigma_{lj}^{cb}\sigma_k^a + P(ij)P(ab)\sum_{klcd}(\sigma_l^d)^*\langle ka||cl\rangle\sigma_{jk}^{bd}\sigma_i^c \\
&+ P(ij)\sum_{klcd}(\sigma_l^d)^*\langle kd||ci\rangle\sigma_{kj}^{ab}\sigma_l^c + P(ab)\sum_{klcd}(\sigma_l^d)^*\langle ka||cl\rangle\sigma_{ij}^{cb}\sigma_k^d \\
&+ P(ij)\sum_{klcd}(\sigma_l^d)^*\langle kd||ci\rangle\sigma_{kl}^{ba}\sigma_j^c + P(ab)\sum_{klcd}(\sigma_l^d)^*\langle ka||cl\rangle\sigma_{ij}^{dc}\sigma_k^b \\
&- P(ij)P(ab)\sum_{klcd}\frac{1}{4}(\sigma_{kl}^{cd})^*\langle bc||jk\rangle\sigma_i^d\sigma_l^a + P(ij)\sum_{klcd}\frac{1}{8}(\sigma_{kl}^{cd})^*\langle ab||kl\rangle\sigma_i^c\sigma_j^d \\
&+ P(ab)\sum_{klcd}\frac{1}{8}(\sigma_{kl}^{cd})^*\langle cd||ij\rangle\sigma_l^b\sigma_k^a - P(ij)\sum_{klcd}\frac{1}{4}(\sigma_{kl}^{cd})^*\langle ab||lj\rangle\sigma_k^c\sigma_i^d \\
&- P(ab)\sum_{klcd}\frac{1}{4}(\sigma_{kl}^{cd})^*\langle bd||ji\rangle\sigma_l^a\sigma_k^c + P(ij)P(ab)\sum_{klcd}\frac{1}{2}(\sigma_k^d)^*(\sigma_l^c)^*\langle ad||il\rangle\sigma_{kj}^{cb} \\
&- P(ij)\sum_{klcd}\frac{1}{8}(\sigma_k^c)^*(\sigma_l^d)^*\langle ab||lj\rangle\sigma_{ki}^{cd} - P(ab)\sum_{klcd}\frac{1}{8}(\sigma_k^c)^*(\sigma_l^d)^*\langle db||ij\rangle\sigma_{lk}^{ac} \\
&+ \sum_{klcd}\frac{1}{4}(\sigma_k^c)^*(\sigma_j^d)^*\langle cd||ij\rangle\sigma_{kl}^{ab} + \sum_{klcd}\frac{1}{4}(\sigma_k^c)^*(\sigma_l^d)^*\langle ab||kl\rangle\sigma_{ij}^{cd} \\
&- P(ij)\sum_{klcd}\frac{7}{24}(\sigma_k^c)^*(\sigma_l^d)^*\langle cd||kj\rangle\sigma_{li}^{ba} - P(ab)\sum_{klcd}\frac{7}{24}(\sigma_k^c)^*(\sigma_l^d)^*\langle bc||lk\rangle\sigma_{ij}^{ad}\Bigg) \qquad (19) \\
+\Bigg(&- P(ij)P(ab)\sum_{klmcd}\frac{2}{3}(\sigma_m^d)^*\langle kl||cm\rangle\sigma_{ik}^{ac}\sigma_{jl}^{bd} + P(ij)P(ab)\sum_{klcde}\frac{2}{3}(\sigma_l^d)^*\langle kd||ce\rangle\sigma_{ik}^{ac}\sigma_{jl}^{be} \\
&- P(ij)P(ab)\sum_{klmcd}\frac{1}{2}(\sigma_m^d)^*\langle kl||cj\rangle\sigma_{ik}^{ac}\sigma_{lm}^{bd} + P(ij)P(ab)\sum_{klcde}\frac{1}{2}(\sigma_l^e)^*\langle kb||cd\rangle\sigma_{ik}^{ac}\sigma_{jl}^{de} \\
&- P(ij)\sum_{klmcd}\frac{1}{2}(\sigma_m^d)^*\langle kl||cj\rangle\sigma_{il}^{ab}\sigma_{km}^{cd} + P(ab)\sum_{klcde}\frac{1}{2}(\sigma_l^e)^*\langle bk||cd\rangle\sigma_{ij}^{ac}\sigma_{kl}^{de} \\
&+ P(ij)\sum_{klmcd}\frac{2}{3}(\sigma_m^d)^*\langle kl||ic\rangle\sigma_{km}^{ab}\sigma_{jl}^{dc} + P(ij)P(ab)\sum_{klmcd}\frac{2}{3}(\sigma_m^d)^*\langle kl||jc\rangle\sigma_{ik}^{ad}\sigma_{ml}^{bc} \\
&- P(ij)P(ab)\sum_{klcde}\frac{2}{3}(\sigma_l^e)^*\langle kb||cd\rangle\sigma_{il}^{ad}\sigma_{kj}^{ce} - P(ab)\sum_{klcde}\frac{2}{3}(\sigma_l^e)^*\langle ka||cd\rangle\sigma_{ij}^{de}\sigma_{kl}^{cb} \\
&+ P(ij)\sum_{klmcd}\frac{2}{3}(\sigma_m^d)^*\langle kl||cm\rangle\sigma_{il}^{ab}\sigma_{jk}^{dc} - P(ab)\sum_{klcde}\frac{2}{3}(\sigma_l^c)^*\langle kc||de\rangle\sigma_{ij}^{ae}\sigma_{kl}^{db} \\
&+ P(ij)\sum_{klmcd}\frac{1}{4}(\sigma_m^d)^*\langle kl||ci\rangle\sigma_{kl}^{ba}\sigma_{jm}^{cd} - P(ab)\sum_{klcde}\frac{1}{4}(\sigma_l^e)^*\langle ka||dc\rangle\sigma_{ij}^{cd}\sigma_{kl}^{be}
\end{aligned}
$$

$$
\begin{aligned}
&+ P(ab) \sum_{klmcd} \frac{1}{3} (\sigma_m^d)^* \langle kl||cm\rangle \sigma_{ij}^{ac} \sigma_{kl}^{bd} - P(ij) \sum_{klcde} \frac{1}{3} (\sigma_l^c)^* \langle ck||de\rangle \sigma_{ik}^{ab} \sigma_{jl}^{ed} \\
&- P(ij)P(ab) \sum_{klmcd} \frac{1}{3} (\sigma_m^d)^* \langle kl||cj\rangle \sigma_{kl}^{db} \sigma_{im}^{ac} + P(ij)P(ab) \sum_{klcde} \frac{1}{3} (\sigma_l^e)^* \langle kb||cd\rangle \sigma_{jl}^{dc} \sigma_{ik}^{ae} \\
&- \sum_{klmcd} \frac{1}{3} (\sigma_m^d)^* \langle kl||cm\rangle \sigma_{ij}^{dc} \sigma_{kl}^{ba} + \sum_{klcde} \frac{1}{3} (\sigma_l^c)^* \langle kc||de\rangle \sigma_{ij}^{ed} \sigma_{kl}^{ba} \\
&+ P(ab) \sum_{klmcd} \frac{1}{6} (\sigma_{ml}^{dc})^* \langle kc||ji\rangle \sigma_{ml}^{da} \sigma_k^b - P(ij) \sum_{klcde} \frac{1}{6} (\sigma_{lk}^{ed})^* \langle ab||kc\rangle \sigma_{il}^{de} \sigma_j^c \\
&- P(ij)P(ab) \sum_{klcde} \frac{1}{6} (\sigma_{kl}^{ce})^* \langle bc||jd\rangle \sigma_{kl}^{ae} \sigma_i^d + P(ij)P(ab) \sum_{klmcd} \frac{1}{6} (\sigma_{ml}^{dc})^* \langle kb||lj\rangle \sigma_{im}^{cd} \sigma_k^a \\
&- P(ij)P(ab) \sum_{klmcd} \frac{1}{3} (\sigma_{lm}^{cd})^* \langle kc||jl\rangle \sigma_{im}^{ad} \sigma_k^b + P(ij)P(ab) \sum_{klcde} \frac{1}{3} (\sigma_{kl}^{de})^* \langle bd||ck\rangle \sigma_{il}^{ae} \sigma_j^c \\
&- P(ij)P(ab) \sum_{klmcd} \frac{1}{3} (\sigma_{lm}^{cd})^* \langle ka||li\rangle \sigma_{mj}^{db} \sigma_k^c + P(ij)P(ab) \sum_{klcde} \frac{1}{3} (\sigma_{kl}^{ce})^* \langle ac||id\rangle \sigma_{lj}^{eb} \sigma_k^d \\
&- P(ab) \sum_{klmcd} \frac{1}{12} (\sigma_{lm}^{cd})^* \langle ka||ml\rangle \sigma_{ij}^{cd} \sigma_k^b + P(ij) \sum_{klcde} \frac{1}{12} (\sigma_{kl}^{dc})^* \langle cd||ei\rangle \sigma_{lk}^{ba} \sigma_j^e \\
&+ \sum_{klcde} \frac{1}{6} (\sigma_{kl}^{de})^* \langle ab||kc\rangle \sigma_{ij}^{de} \sigma_l^c - \sum_{klmcd} \frac{1}{6} (\sigma_{lm}^{cd})^* \langle kc||ji\rangle \sigma_{ml}^{ba} \sigma_k^d \\
&+ P(ij) \sum_{klmcd} \frac{1}{3} (\sigma_{lm}^{cd})^* \langle kc||il\rangle \sigma_{mj}^{ab} \sigma_k^d - P(ab) \sum_{klcde} \frac{1}{3} (\sigma_{kl}^{ce})^* \langle ac||dk\rangle \sigma_{ij}^{eb} \sigma_l^d \\
&+ P(ab) \sum_{klmcd} \frac{1}{6} (\sigma_{lm}^{dc})^* \langle ka||ml\rangle \sigma_{ij}^{db} \sigma_k^c - P(ij) \sum_{klcde} \frac{1}{6} (\sigma_{kl}^{cd})^* \langle cd||ei\rangle \sigma_{jl}^{ba} \sigma_k^e \\
&- P(ij) \sum_{klmcd} \frac{1}{2} (\sigma_{ml}^{dc})^* \langle kc||li\rangle \sigma_{kj}^{ab} \sigma_m^d + P(ab) \sum_{klcde} \frac{1}{2} (\sigma_{kl}^{ce})^* \langle ac||dk\rangle \sigma_{ij}^{db} \sigma_l^e \\
&- P(ij)P(ab) \sum_{klmcd} \frac{1}{2} (\sigma_{ml}^{dc})^* \langle ka||li\rangle \sigma_{kj}^{cb} \sigma_m^d + P(ij)P(ab) \sum_{klcde} \frac{1}{2} (\sigma_{kl}^{ce})^* \langle ac||id\rangle \sigma_{jk}^{bd} \sigma_l^e \\
&- P(ab) \sum_{klmcd} \frac{1}{6} (\sigma_{ml}^{dc})^* \langle ka||ji\rangle \sigma_{kl}^{bc} \sigma_m^d + P(ij) \sum_{klcde} \frac{1}{6} (\sigma_{kl}^{de})^* \langle ab||ic\rangle \sigma_{jk}^{cd} \sigma_l^e \\
&- \sum_{klmcd} \frac{1}{2} (\sigma_{ml}^{dc})^* \langle kc||ji\rangle \sigma_{kl}^{ba} \sigma_m^d - \sum_{klcde} \frac{1}{2} (\sigma_{li}^{ed})^* \langle ab||kc\rangle \sigma_{ji}^{cd} \sigma_l^e \\
&+ P(ij) \sum_{klmcd} \frac{1}{12} (\sigma_{ml}^{dc})^* \langle kc||ml\rangle \sigma_{kj}^{ab} \sigma_i^d - P(ab) \sum_{klcde} \frac{1}{12} (\sigma_{lk}^{cd})^* \langle cd||ek\rangle \sigma_{ji}^{be} \sigma_l^a
\end{aligned}
$$

$$
\begin{aligned}
&+ P(ij) \sum_{klmcd} \frac{1}{2} (\sigma^{dc}_{ml})^* \langle kc||jl\rangle \sigma^{ab}_{mk} \sigma^{d}_{i} - P(ab) \sum_{klcde} \frac{1}{2} (\sigma^{ec}_{lk})^* \langle bc||dk\rangle \sigma^{ed}_{ij} \sigma^{a}_{l} \\
&+ P(ij)P(ab) \sum_{klmcd} \frac{1}{2} (\sigma^{dc}_{ml})^* \langle kc||il\rangle \sigma^{bd}_{jk} \sigma^{a}_{m} - P(ij)P(ab) \sum_{klcde} \frac{1}{2} (\sigma^{ec}_{lk})^* \langle ac||dk\rangle \sigma^{db}_{lj} \sigma^{e}_{i} \\
&+ P(ij)P(ab) \sum_{klmcd} \frac{1}{4} (\sigma^{cd}_{ml})^* \langle ka||ml\rangle \sigma^{bc}_{jk} \sigma^{d}_{i} - P(ij)P(ab) \sum_{klcde} \frac{1}{4} (\sigma^{cd}_{kl})^* \langle cd||ei\rangle \sigma^{eb}_{kj} \sigma^{a}_{l} \\
&+ P(ij)P(ab) \sum_{klmcd} \frac{1}{2} (\sigma^{cd}_{lm})^* \langle kb||lj\rangle \sigma^{da}_{mk} \sigma^{c}_{i} - P(ij)P(ab) \sum_{klcde} \frac{1}{2} (\sigma^{ce}_{kl})^* \langle cb||dj\rangle \sigma^{de}_{il} \sigma^{a}_{k} \\
&+ P(ab) \sum_{klmcd} \frac{1}{2} (\sigma^{cd}_{lm})^* \langle kc||ji\rangle \sigma^{db}_{mk} \sigma^{a}_{l} - P(ij) \sum_{klcde} \frac{1}{2} (\sigma^{de}_{km})^* \langle ab||kc\rangle \sigma^{ce}_{jm} \sigma^{d}_{i} \\
&+ P(ab) \sum_{klmcd} \frac{1}{12} (\sigma^{cd}_{lm})^* \langle ka||ji\rangle \sigma^{dc}_{mk} \sigma^{b}_{l} - P(ij) \sum_{klcde} \frac{1}{12} (\sigma^{de}_{kl})^* \langle ab||ic\rangle \sigma^{ce}_{kl} \sigma^{d}_{j} \\
&+ P(ij)P(ab) \sum_{klmcd} \frac{1}{4} (\sigma^{cd}_{lm})^* \langle kb||lj\rangle \sigma^{dc}_{ik} \sigma^{a}_{m} - P(ij)P(ab) \sum_{klcde} \frac{1}{4} (\sigma^{ce}_{kl})^* \langle bc||jd\rangle \sigma^{da}_{kl} \sigma^{e}_{i} \Bigg) \quad (20) \\
+ \Bigg(&- P(ij) \sum_{klmncd} \frac{1}{12} (\sigma^{cd}_{mn})^* \langle kl||im\rangle \sigma^{dc}_{nj} \sigma^{ab}_{kl} - P(ab) \sum_{klcdef} \frac{1}{12} (\sigma^{cf}_{kl})^* \langle ac||de\rangle \sigma^{de}_{ij} \sigma^{bf}_{kl} \\
&+ P(ij)P(ab) \sum_{klmncd} \frac{1}{6} (\sigma^{cd}_{mn})^* \langle kl||im\rangle \sigma^{ac}_{kl} \sigma^{bd}_{jn} + P(ij)P(ab) \sum_{klcdef} \frac{1}{12} (\sigma^{cf}_{kl})^* \langle ac||de\rangle \sigma^{de}_{ik} \sigma^{fb}_{lj} \\
&- P(ij) \sum_{klmncd} \frac{1}{12} (\sigma^{cd}_{nm})^* \langle kl||jm\rangle \sigma^{ab}_{in} \sigma^{cd}_{kl} - P(ab) \sum_{klcdef} \frac{1}{12} (\sigma^{fc}_{kl})^* \langle bc||de\rangle \sigma^{ed}_{lk} \sigma^{af}_{ij} \\
&+ P(ij) \sum_{klmncd} \frac{1}{2} (\sigma^{cd}_{nm})^* \langle kl||jm\rangle \sigma^{ab}_{ik} \sigma^{cd}_{ln} + P(ab) \sum_{klcdef} \frac{1}{2} (\sigma^{cf}_{kl})^* \langle bc||de\rangle \sigma^{ef}_{kl} \sigma^{ad}_{ij} \\
&- \sum_{klmncd} \frac{1}{3} (\sigma^{dc}_{nm})^* \langle kl||ji\rangle \sigma^{ba}_{km} \sigma^{cd}_{ln} - \sum_{klcdef} \frac{1}{3} (\sigma^{ef}_{kl})^* \langle ab||cd\rangle \sigma^{cf}_{kl} \sigma^{ed}_{ij} \\
&+ P(ab) \sum_{klmncd} \frac{1}{2} (\sigma^{cd}_{mn})^* \langle kl||ij\rangle \sigma^{ac}_{km} \sigma^{bd}_{ln} + \sum_{klcdef} \frac{1}{2} (\sigma^{ef}_{kl})^* \langle ab||cd\rangle \sigma^{df}_{jl} \sigma^{ce}_{ik} \\
&+ P(ij)P(ab) \sum_{klmncd} \frac{2}{3} (\sigma^{cd}_{mn})^* \langle kl||mn\rangle \sigma^{ac}_{ik} \sigma^{bd}_{ln} + P(ij)P(ab) \sum_{klcdef} \frac{2}{3} (\sigma^{cf}_{kl})^* \langle cd||ef\rangle \sigma^{df}_{jl} \sigma^{ae}_{ik} \\
&- P(ij) \sum_{klmncd} \frac{1}{6} (\sigma^{cd}_{mn})^* \langle kl||mn\rangle \sigma^{ab}_{il} \sigma^{dc}_{jk} - P(ab) \sum_{klcdef} \frac{1}{6} (\sigma^{cd}_{kl})^* \langle cd||ef\rangle \sigma^{af}_{ij} \sigma^{be}_{lk} \\
&- P(ij) \sum_{klmncd} \frac{1}{2} (\sigma^{cd}_{mn})^* \langle kl||mi\rangle \sigma^{cd}_{kj} \sigma^{ab}_{ln} - P(ab) \sum_{klcdef} \frac{1}{2} (\sigma^{cf}_{kl})^* \langle ac||ed\rangle \sigma^{df}_{ij} \sigma^{eb}_{kl}
\end{aligned}
$$

$$
+ P(ij)P(ab) \sum_{klmncd} \frac{1}{3}(\sigma_{mn}^{cd})^* \langle kl||mn\rangle \sigma_{ik}^{ac} \sigma_{jl}^{bd} + P(ij)P(ab) \sum_{klcdef} \frac{1}{3}(\sigma_{kl}^{cd})^* \langle cd||ef\rangle \sigma_{ik}^{ae} \sigma_{jl}^{bf}
$$

$$
- P(ij)P(ab) \sum_{klmcde} \frac{1}{6}(\sigma_{ml}^{ed})^* \langle kd||cj\rangle \sigma_{ik}^{ac} \sigma_{ml}^{eb} - P(ij)P(ab) \sum_{klmcde} \frac{1}{6}(\sigma_{ml}^{ed})^* \langle kb||cl\rangle \sigma_{ik}^{ac} \sigma_{jm}^{de}
$$

$$
- P(ij)P(ab) \sum_{klmcde} \frac{1}{3}(\sigma_{ml}^{ed})^* \langle kd||ci\rangle \sigma_{mj}^{eb} \sigma_{lk}^{ac} - P(ij)P(ab) \sum_{klmcde} \frac{1}{3}(\sigma_{ml}^{ed})^* \langle ka||cl\rangle \sigma_{ik}^{dc} \sigma_{mj}^{eb}
$$

$$
+ P(ij) \sum_{klmcde} \frac{1}{6}(\sigma_{ml}^{ed})^* \langle kd||ci\rangle \sigma_{kj}^{ce} \sigma_{lm}^{ab} + P(ab) \sum_{klmcde} \frac{1}{6}(\sigma_{ml}^{ed})^* \langle ka||cl\rangle \sigma_{km}^{cb} \sigma_{ij}^{de}
$$

$$
- P(ij) \sum_{klmcde} \frac{1}{3}(\sigma_{ml}^{ed})^* \langle kd||cj\rangle \sigma_{mk}^{ec} \sigma_{il}^{ab} - P(ab) \sum_{klmcde} \frac{1}{3}(\sigma_{ml}^{ed})^* \langle kb||cl\rangle \sigma_{ij}^{ad} \sigma_{km}^{ce}
$$

$$
+ P(ij)P(ab) \sum_{klmcde} \frac{1}{3}(\sigma_{ml}^{ed})^* \langle ka||ci\rangle \sigma_{lj}^{cb} \sigma_{mk}^{ed} + P(ij)P(ab) \sum_{klmcde} \frac{1}{3}(\sigma_{ml}^{ed})^* \langle ka||ci\rangle \sigma_{kj}^{db} \sigma_{lm}^{ce}
$$

$$
+ P(ij) \sum_{klmcde} \frac{1}{2}(\sigma_{ml}^{ed})^* \langle kd||cj\rangle \sigma_{ik}^{ab} \sigma_{lm}^{ce} + P(ab) \sum_{klmcde} \frac{1}{2}(\sigma_{ml}^{ed})^* \langle kb||cl\rangle \sigma_{ij}^{ac} \sigma_{km}^{de}
$$

$$
- P(ij) \sum_{klmcde} \frac{2}{3}(\sigma_{ml}^{ed})^* \langle kd||cl\rangle \sigma_{ik}^{ab} \sigma_{jm}^{ce} - P(ab) \sum_{klmcde} \frac{1}{3}(\sigma_{ml}^{ed})^* \langle kd||cl\rangle \sigma_{ij}^{ac} \sigma_{km}^{be}
$$

$$
- P(ij)P(ab) \sum_{klmcde} \frac{2}{3}(\sigma_{ml}^{ed})^* \langle ka||ci\rangle \sigma_{jl}^{cd} \sigma_{km}^{be} + P(ij)P(ab) \sum_{klmcde} (\sigma_{ml}^{ed})^* \langle kd||cj\rangle \sigma_{il}^{ac} \sigma_{km}^{be}
$$

$$
+ P(ij)P(ab) \sum_{klmcde} (\sigma_{ml}^{ed})^* \langle kb||cl\rangle \sigma_{ik}^{ad} \sigma_{jm}^{ce} + P(ab) \sum_{klmcde} (\sigma_{ml}^{ed})^* \langle ka||cl\rangle \sigma_{ji}^{cd} \sigma_{km}^{be}
$$

$$
+ P(ij) \sum_{klmcde} (\sigma_{ml}^{ed})^* \langle kd||ci\rangle \sigma_{lk}^{ab} \sigma_{jm}^{ce} - P(ij)P(ab) \sum_{klmcde} \frac{1}{6}(\sigma_{ml}^{ed})^* \langle ka||ci\rangle \sigma_{ml}^{bc} \sigma_{jk}^{ed}
$$

$$
- P(ij)P(ab) \sum_{klmcde} \frac{1}{2}(\sigma_{ml}^{ed})^* \langle kd||cj\rangle \sigma_{ml}^{cb} \sigma_{ik}^{ae} - P(ij)P(ab) \sum_{klmcde} \frac{1}{2}(\sigma_{ml}^{ed})^* \langle kb||cl\rangle \sigma_{jk}^{de} \sigma_{im}^{ac}
$$

$$
+ P(ij)P(ab) \sum_{klmcde} \frac{2}{3}(\sigma_{ml}^{ed})^* \langle kd||cl\rangle \sigma_{jk}^{be} \sigma_{im}^{ac} + \sum_{klmcde} \frac{2}{3}(\sigma_{ml}^{ed})^* \langle kd||cl\rangle \sigma_{ij}^{ec} \sigma_{mk}^{ab} \Bigg). \qquad (21)
$$