Tutorial - 1

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①
```
int a=0, b=0;
for (i=0; i<N; i++)
{
  a = a+rand();
}
for (j=0; j<M; j++)
{
  b = b+rand();
}
```

Sol^n

O(N+M) - T.C
O(1) - S.C

The first loop is O(N)
The second loop is O(M)
Since, we don't know whether N>M or M>N and no extra space so, S.C is O(1) and T.C is O(N+M)

②
```
int sum=0, i;
for (i=0; i<n; i=i+2)
{
  sum = sum+i;
}
```

$\underline{i}$

0
2
4
6
⋮
n

$0 + 2 + 4 + 6 + \cdots n$

$a = 0$

$d = 2$

$S = 0 + \left(\left(\frac{n}{2}\right) - 1\right) 2$

$= n$

$\underline{O(n)}$.

③
```
int sum=0, i;
for (i=0; i<n; i=i*2)
sum+=i;
```

2^0
2^1
2^2
⋮
2^{K-1}

$2^{K-1} >= n$

Taking log.

$(K-1)\log_2 2 >= \log n$

$K-1 >= \log n$

$(K >= \log n + 1)$

$\underline{O(\log n)}$

④
```
int sum=0, i;
for (i=0; i*i<n; i++)
{
 sum+=i;
}
```

$K*K >= n$

$K^2 >= n$

$(K >= \sqrt{n})$ $= \underline{O(\sqrt{n})}$.

(5)

```
int j=1, i=0;
while (j<=n)
{
  i=i+j;
  j++;
}
```

At ith iteration, we are getting sum of first i nos.

$$\frac{K(K+1)}{2} >= n$$

$$K^2 >= n$$

$$K >= \sqrt{n}$$

$$\boxed{O(\sqrt{n})}$$

(6)

$$T(n) = T(n-1) + T(n-1) + 1$$
$$= 2T(n-1)$$
$$= 2 \cdot 2 \cdot T(n-2)$$
$$= 2 \cdot 2 \cdot 2T(n-3)$$
$$= \vdots$$
$$2^n$$

$$\Rightarrow \underline{O(2^n)}$$

(7) $O(\log n)$

$$T(n) = 2T(n/2) + 1$$

$a = b$

$a = b = 2$

$\log_2 2 = 1$

$f(n) = 1$

$g(n) = 1$

$$\underline{T.C = O(n \log(n))}$$

(8) (i) $T(n) = T(n-1) + 1$ $\qquad T(n) = \begin{cases} 1 & n = 0 \\ T(n-1)+1 & n > 0 \end{cases}$

$T(n-1) = T(n-2) + 1$

$T(n-2) = T(n-3) + 1$

$T(n) = [T(n-2)+1] + 1$

$T(n) = T(n-3) + 3$

$\vdots$

$T(n) = T(n-k) + k$

Let $n-k$ be 0 $\qquad n-k = 0 \Rightarrow n = k$

$T(n) = T(0) + n$

$T(n) = 1 + n \longrightarrow \underline{O(n)}$

if $T(1) = 1+1 = 2$

(ii) $T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + n - 1$

$T(n-2) = T(n-3) + n - 2$

$T(n) = T(n-3) + (n-2) + (n-1) + n$

$\vdots$

$T(n) = T(n-k) + (n-(k-1)) + (n-(k-2)) + \cdots + (n-1) + n$

Assume, $n-k = 0$

$n = k$

$T(n) = T(0) + (\cancel{n} - \cancel{n} + 1) + (\cancel{n} - \cancel{n} + 2) + \cdots + (n-1) + 2$

$T(n) = T(0) + 1 + 2 + 3 + \dots + n-1 + n$

$T(n) = 1 + \frac{n(n+1)}{2} \longrightarrow \underline{O(n^2)}$

$T(n) = 1 + (1)\frac{\cancel{2}}{\cancel{2}} = 2$

(iii) $T(n) = T(n/2) + 1$

$a = 1$

$b = 2$

$K = \log_2 1 = 0$

$n^0 = 1 \cong 1$

so,

$T(n) = O(1 \log n)$

$= O(\log n)$

(iv) $T(n) = 2T(n/2) + 1$

$T(n) = O(n \log n)$

(v) $O(2^n)$

(vi) $O(3^n)$

(vii) $T(n) = T(\sqrt{n}) + 1$

let $n = 2^m$

$T(2^m) = T(2^{m/2}) + 1$

$S(m) = T(2^m)$

$S(m) = S(m/2) + 1$

$K = \log_2 1$

$K=0,\ m^K=1$

$T(n)=O(\log(\log n))$

(viii) $T(n)=T(\sqrt{n})+n$

Let $n=2^m$

$T(2^m)=T(2^{m/2})+2^m$

$S(m)=T(2^m)$

$S(m)=S(m/2)+2^m$

$K=\log_2 1=0$

$m^0=1\neq 2^m$

$O(2^m)=O(n)$

(9)
```
int sum=0, i;
for(i=0; i<n; i++)
  sum+=i;
```

$0+1+2+\dots$ n times

$\Rightarrow \underline{O(n)}$

(10) $N+(N-1)+(N-2)+\dots+1+0$

$=N(N+1)/2=\underline{O(N^2)}$

(11) For n, j runs $O(\log n)$ times

i runs $n/2$ times

T.C $=O(n/2 * \log(n))$

$=O(n\log n)$

(12) x is asymtotically more efficient than y, means x is better choice for large inputs

(a) correct.

(13) $i = N$

$N/2$

$N/4$

$\vdots$

$N/2^{K-1}$

$\frac{N}{2^{K-1}} >= 1$

$N >= 2^{K-1}$

$\log N = \log_2 2(K-1)$

$K-1 = \log_2 N$

$K = (\log_2 N+1)$

$\boxed{T.C = O(\log n)}$

(14) $T(n) = 7T(n/2) + \underbrace{3n^2+2}_{f(n)}$

$a = 7, \ b = 2$

$K = \log_2 7 = 2.81$

$n^K = n^{2.81}$

a) $O(n^{2.81})$

b) $O(n^3)$

c) $\Theta(n^{2.81})$

(15) $f2 > f4 > f3 > f1$

$2^n > n^\wedge 10 * 2^\wedge(n/2) > (1.0001)^n > n^{\sqrt{n}}$

b) $2^{n/2} * 2^{n/2} > n^{10} * 2^\wedge n/2 > f3 > f1$

(16) $f(n) = 2^\wedge(2n)$

$f(n) >= c * 2n$

b) $\Omega(2^\wedge n)$

(17) $b = 2$, $a = 1$ $f(n) = n^2$

$\log_2 1 = 0 = k$

$n^0 = 1$

$n^2 > 1$

(a) $O(n^2)$

(18) $O(\log n)$

(19) $O(N^2)$

$O(N)$

$\Rightarrow O(N^2 + N) = O(N^2)$