ME 573 Fall 2017 Homework #5 Due Oct. 13th 11PM

Problem 1:

Consider the FTCS discretization of the heat equation

$$\frac{f_i^{n+1}-f_i^n}{\Delta t}=\kappa\frac{f_{i+1}^n-2f_i^n+f_{i-1}^n}{\Delta x^2}$$

where the domain size is $x\in[-3;3]$ and $\Delta x=[0.5,0.25,0.05]$, $\Delta t=0.01$, and $\kappa=10^{-3}$.

Initial condition is given by:

$$f(x,0)=\begin{cases} U_o=const=2 & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases} \quad \text{and } \kappa=10^{-3},$$

and analytical solution is

$$f(x,t)=\frac{U_o}{2}\left[erf\left(\frac{(1-x)}{2\sqrt{\kappa t}}\right)-erf\left(-\frac{(x+1)}{2\sqrt{\kappa t}}\right)\right]$$

(a) Code the explicit FTCS algorithm and compare with the analytical solution presented in lecture at time $t=10^2$. In one plot put the three curves corresponding to the three levels of grid refinement against the analytical solution.

Problem 2:

Solve the heat (diffusion) equation, with the same initial conditions, physical domain, and $\kappa\left(=10^{-3}\right)$ as problem 1.

(a) Use both the BTCS and FTCS discretization, and plot 3 curves: i) BTCS results, ii) FTCS results, and iii) exact solution

The BTCS discretization is given by

$$\frac{f_i^{n+1}-f_i^n}{\Delta t}=\kappa\frac{f_{i+1}^{n+1}-2f_i^{n+1}+f_{i-1}^{n+1}}{\Delta x^2}$$

$\rightarrow$ Use $\Delta t=1.0$ and $\Delta x=0.05$ and compute the solution until the final time $t_f=10^2$.

(b) Repeat (a) but with $(\Delta t=2.0 \text{ and } \Delta x=0.05)$

Problem 3:

(a) Derive the amplitude ratio G for the Crank-Nicolson scheme, as shown in Eq. 12.2.31 (5_pozrikidis...pdf).

(b) Is the method stable? unstable? conditional?