

ME573 Homework Set # 9

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1 Problem 1

Show that for FTBS:

$$\frac{f_i^{n+1} - f_i^n}{\Delta t} + U \frac{f_i^n - f_{i-1}^n}{\Delta x} = 0 \quad (1)$$

$$\left| \frac{A^{n+1}}{A^n} \right|^2 = 1 - 2C_0(1 - C_0)(1 - \cos\theta) \quad (2)$$

From equation 1:

$$f_i^{n+1} = f_i^n - C_0(f_i^n - f_{i-1}^n) \quad (3)$$

And from Von-Neumann Stability Analysis:

$$f_i^n = A^n e^{Ii\theta} \quad (4)$$

Substitute 4 into 3:

$$A^{n+1} e^{Ii\theta} = A^n e^{Ii\theta} - C_0[A^n e^{Ii\theta} - A^n e^{I(i-1)\theta}] \quad (5)$$

$$\frac{A^{n+1}}{A^n} e^{Ii\theta} = e^{Ii\theta} - C_0[e^{Ii\theta} - e^{I(i-1)\theta}] \quad (6)$$

$$\frac{A^{n+1}}{A^n} = 1 - C_0[1 - e^{-I\theta}] \quad (7)$$

$$\left| \frac{A^{n+1}}{A^n} \right|^2 = -2C_0^2 e^{i\theta} + C_0^2 e^{2i\theta} + C_0^2 + 2C_0 e^{i\theta} - 2C_0 + 1 \quad (8)$$

2 Problem 2

Figure 2 displays results of the FTCS and FTBS schemes compared to the exact solution. This implementation of the scheme ($\Delta t = 0.01$ and $\Delta x = 0.05$) was stable ($C_0 = 0.628$)

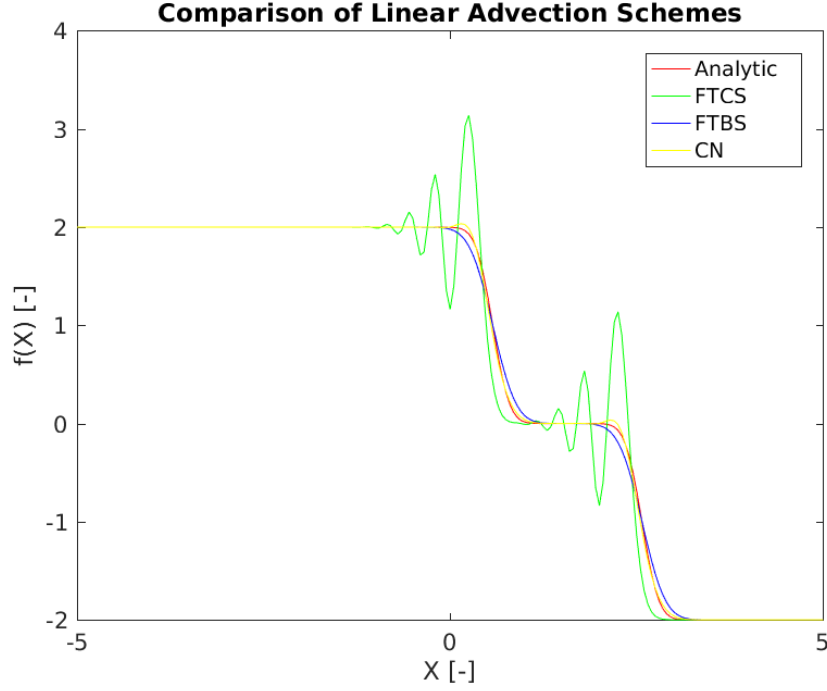


Figure 1:

The FTBS scheme was re-implemented in an unstable condition ($\Delta t = 0.02$ and $\Delta x = 0.025$), the Courant number in this state is 2.51. Figure 2 shows the results of this unstable solution.

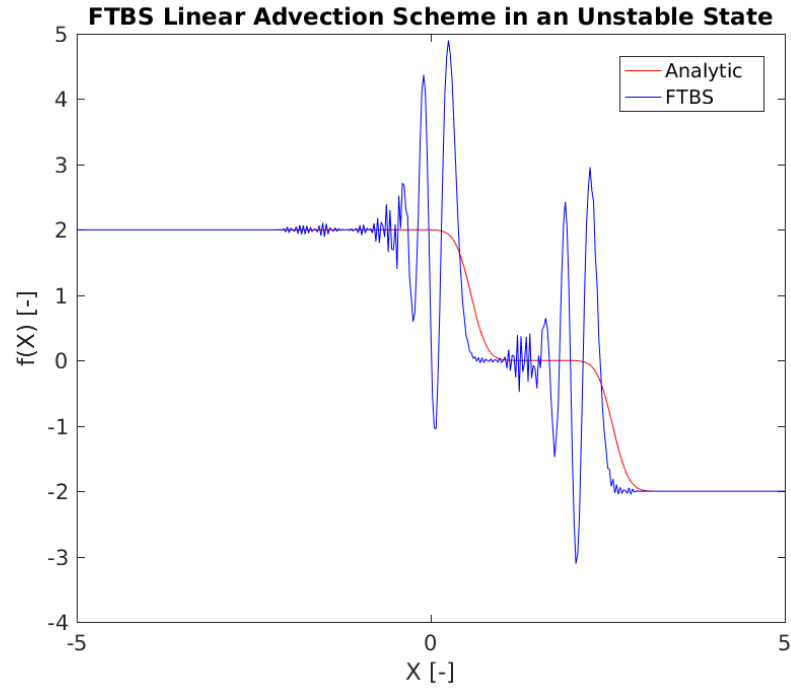


Figure 2:

3 Problem 3

Finally, the Crank-Nicolson Scheme was implemented to solve the same pure advection problem. Figure 3 shows the results of the CN approach compared to the FTBS scheme and Figure 3 shows the spatial error.

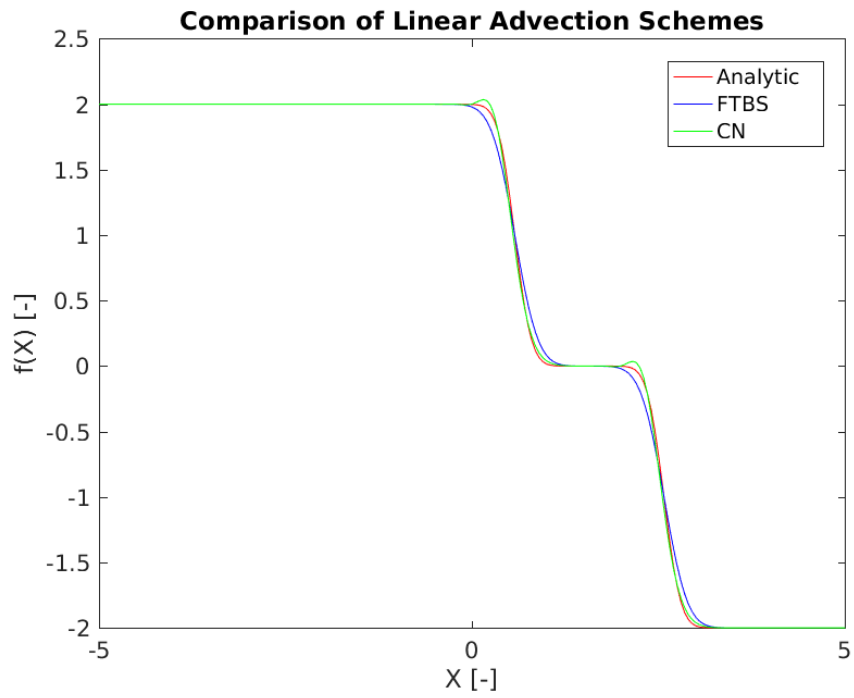


Figure 3:

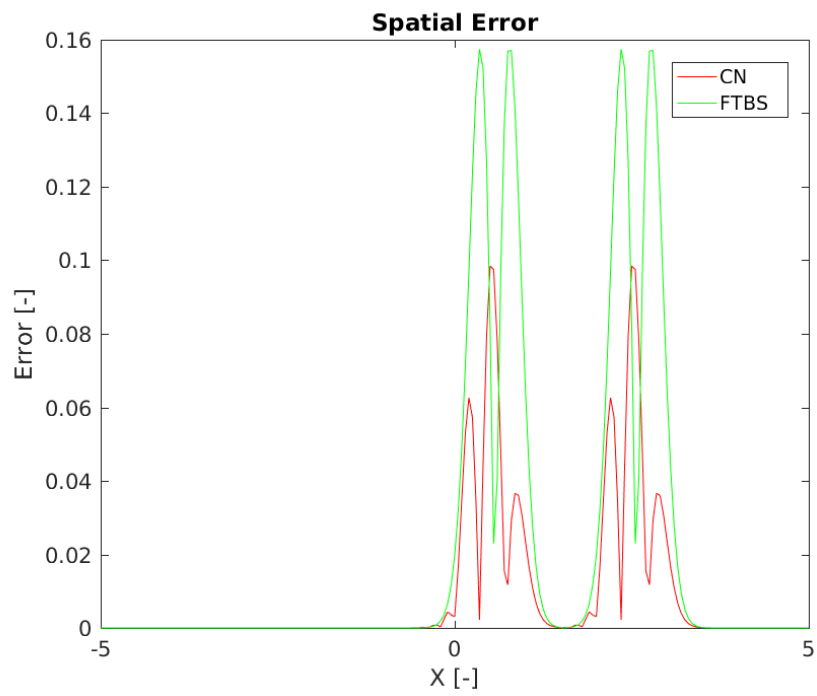


Figure 4: