ME573 Homework Set # 7

Alexander Swenson – aaswenson@wisc.edu

November 1, 2017

---

1 Introduction

This coding assignment involved modeling 2D heat diffusion with the Alternating Direct Implicit Method (ADI) algorithm. The algorithm was implemented in MATLAB (see attached code).

2 Part A

The L_{inf} norm was calculated between the initial conditions ans the exact solution (evaluated at t=0). The norm was: 1.2053e-05

```
A_x =

    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0
    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0
         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0         0
         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0
         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0
         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0
         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0
         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0
         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0
         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0
         0         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0
         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000
         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000

A_y =

    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0
    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0
         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0         0
         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0         0
         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0         0
         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0         0
         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0         0
         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0         0
         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0         0
         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0         0
         0         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000         0
         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000   -0.5000
         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0         0    0.5000    3.0000
```

3 Part B

Displayed below is the matrix form of the first two steps in the ADI algorithm.

3.1 (1-a)

$$\begin{bmatrix} 2(1+\alpha_x) & -\alpha_x & & & \\ -\alpha_x & 2(1+\alpha_x) & -\alpha_x & & \\ & \ddots & & & \\ & & \ddots & & -\alpha_x \\ & & & -\alpha_x & 2(1+\alpha_x) \end{bmatrix} \begin{bmatrix} f_{2,j}^{n+\frac{1}{2}} \\ f_{3,j}^{n+\frac{1}{2}} \\ \vdots \\ f_{Nx-2,j}^{n+\frac{1}{2}} \\ f_{Nx-1,j}^{n+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} (2+\alpha_y\Delta_y^2)f_{2,j}^n \\ (2+\alpha_y\Delta_y^2)f_{3,j}^n \\ \vdots \\ (2+\alpha_y\Delta_y^2)f_{Nx-2,j}^n \\ (2+\alpha_y\Delta_y^2)f_{Nx-1,j}^n \end{bmatrix}$$

3.2 (1-b)

$$\begin{bmatrix} 2(1+\alpha_y) & -\alpha_y & & & \\ -\alpha_y & 2(1+\alpha_y) & -\alpha_y & & \\ & \ddots & & & \\ & & \ddots & & -\alpha_y \\ & & & -\alpha_y & 2(1+\alpha_y) \end{bmatrix} \begin{bmatrix} f_{i,2}^{n+\frac{1}{2}} \\ f_{i,3}^{n+\frac{1}{2}} \\ \vdots \\ f_{i,Ny-2}^{n+\frac{1}{2}} \\ f_{i,Ny-1}^{n+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} (2+\alpha_y\Delta_x^2)f_{i,2}^n \\ (2+\alpha_y\Delta_x^2)f_{i,3}^n \\ \vdots \\ (2+\alpha_y\Delta_x^2)f_{i,Ny-2}^n \\ (2+\alpha_y\Delta_x^2)f_{i,Ny-1}^n \end{bmatrix}$$

4 Part C

The following figures contain the b vectors used to solve the problem. They are the vectors used in the first time step (1-a) and (1-b).

```
b =

         0
    0.0048
    0.0095
    0.0143
    0.0190
    0.0237
    0.0284
    0.0331
    0.0376
    0.0420
    0.0460
    0.0497
    0.0526
    0.0546
    0.0554
    0.0544
    0.0511
    0.0449
    0.0350
    0.0204
         0
```

```
b =

  Columns 1 through 11

         0    0.0048    0.0090    0.0128    0.0160    0.0188    0.0210    0.0228    0.0240    0.0248    0.0250

  Columns 12 through 21

    0.0248    0.0240    0.0228    0.0210    0.0188    0.0160    0.0128    0.0090    0.0048         0
```

5 Part D

The following Figure 5 shows the error between the exact solution and the ADI method at the end of the time steps.