ME573 Homework Set # 8

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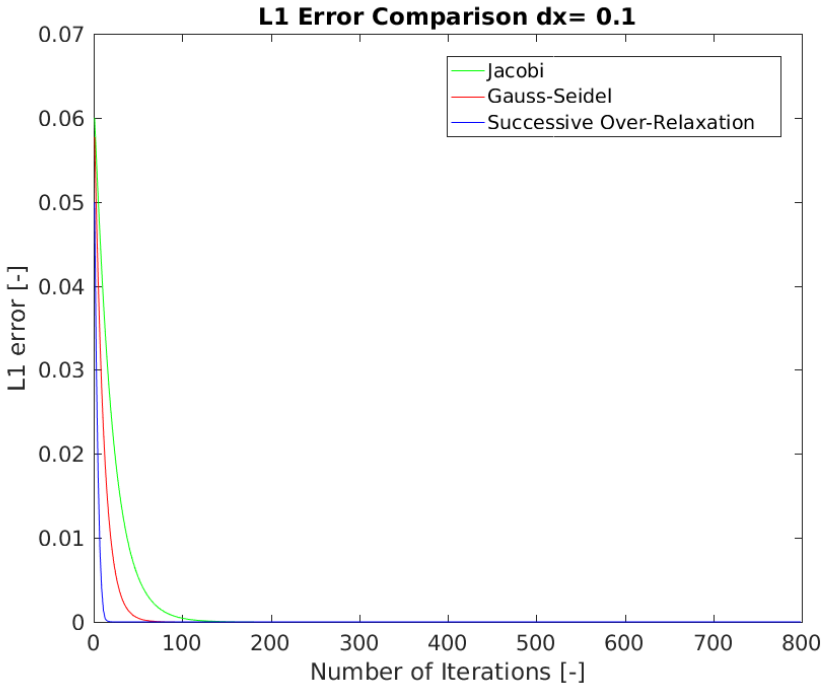
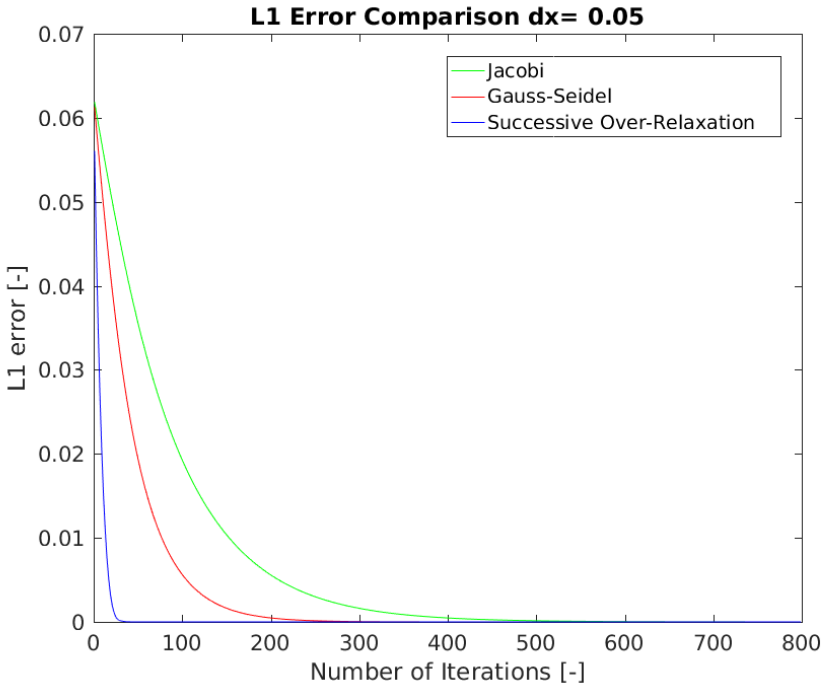
1 Introduction

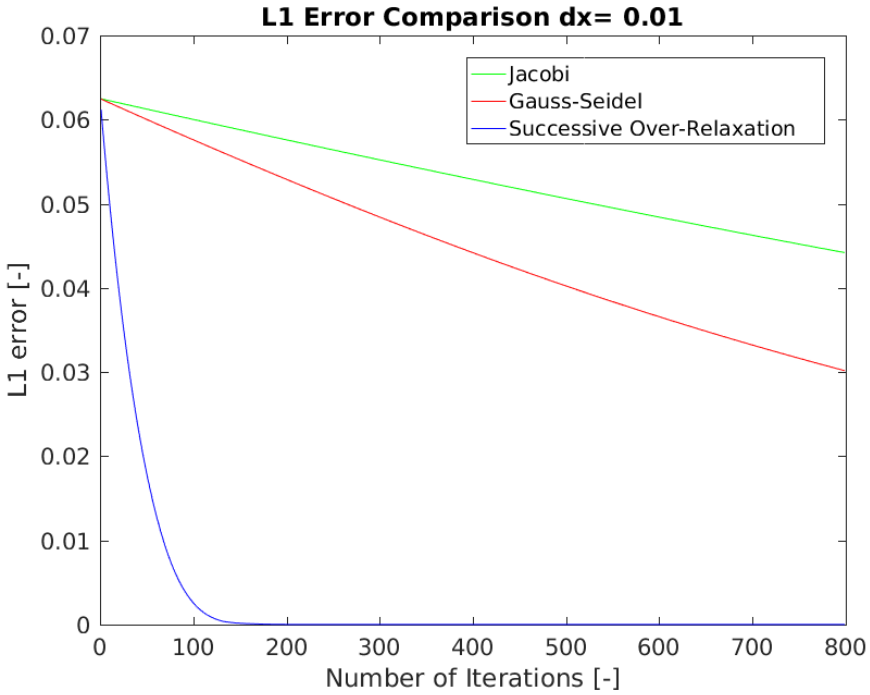
This coding assignment involved solving a Second-Order PDE with iterative methods. The three iterative methods tested were: Jacbobi iteration, Gauss-Seidel, and Successive Over-Relaxation.

2 Problem 1

Each method was tested on a unit square. The L1 error of each method was calculated with increasing number of iterations from 2-800. The impact of number of iterations on the L1 error is shown in the following three plots. Each plot shows the L1 error as a function of N iterations at different dx values. All three methods were tested using MATLAB (2016a).







3 Problem 2

The following iteration decomposition was considered and shown to hold for SOR.

$$\mathbf{M}\mathbf{u}^{k+1} = \mathbf{B}\mathbf{u}^{k} + \mathbf{b} \tag{1}$$

$$\mathbf{A}\mathbf{u} = \mathbf{b} \tag{2}$$

$$\mathbf{A}\mathbf{u} = \mathbf{b} \tag{3}$$

$$\mathbf{A} = \mathbf{D}\text{-}\mathbf{L}\text{-}\mathbf{U} \tag{4}$$

Show the following holds.

$$\mathbf{M} = \frac{1}{\omega}(\mathbf{D} - \omega\mathbf{L}) \tag{5}$$

$$\mathbf{N} = \frac{1}{\omega}[(1-\omega)\mathbf{D} + \omega\mathbf{U}] \tag{6}$$

First, start with some substitution (5 & 6 into 1)

$$\frac{1}{\omega}(\mathbf{D} - \omega\mathbf{L})\mathbf{u}^{k+1} = \frac{1}{\omega}[(1-\omega)\mathbf{D} + \omega\mathbf{U}]\mathbf{u}^{k} + \mathbf{b} \tag{7}$$

$$(\mathbf{D} - \omega\mathbf{L})\mathbf{u}^{k+1} = [(1-\omega)\mathbf{D} + \omega\mathbf{U}]\mathbf{u}^{k} + \omega\mathbf{b} \tag{8}$$

$$(\mathbf{D} - \omega\mathbf{L})\mathbf{u}^{k+1} = \mathbf{D}\mathbf{u}^{k} - \omega\mathbf{D}\mathbf{u}^{k} + \omega\mathbf{U}\mathbf{u}^{k} + \omega\mathbf{D}\mathbf{u}^{k} - \omega\mathbf{L}\mathbf{u}^{k} - \omega\mathbf{U}\mathbf{u}^{k} \tag{9}$$

$$(\mathbf{D} - \omega\mathbf{L})\mathbf{u}^{k+1} = \mathbf{D}\mathbf{u}^{k} - 2\omega\mathbf{U}\mathbf{u}^{k} - \omega\mathbf{L}\mathbf{u}^{k} \tag{10}$$

$$\mathbf{u}^{k+1} = \frac{\mathbf{D}\mathbf{u}^{k} - 2\omega\mathbf{U}\mathbf{u}^{k} - \omega\mathbf{L}\mathbf{u}^{k}}{(\mathbf{D} - \omega\mathbf{L})} \tag{11}$$