ME573 Homework Set # 10

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1 Problem 1

See attached screenshots from Mathematica for problem 1. The screenshots support the exact solutions validity as a solution to the underlying PDE.

2 Problem 2

Displayed below is the matrix form of the first two steps in the algorithm.

2.1 (1-a)

$$\begin{bmatrix} 4(1+\alpha_x) & (\frac{u^n_{2,j}\Delta t}{\Delta x}-2\alpha) & & \\ -(\frac{u^n_{3,j}\Delta t}{\Delta x}+2\alpha) & 4(1+\alpha_x) & (\frac{u^n_{3,j}\Delta t}{\Delta x}-2\alpha) & \\ & \ddots & & \\ & & \ddots & (\frac{u^n_{N-2,j}\Delta t}{\Delta x}-2\alpha) \\ & & -(\frac{u^n_{3,j}\Delta t}{\Delta x}+2\alpha) & 4(1+\alpha_x) \end{bmatrix} \begin{bmatrix} f^*_{2,j} \\ f^*_{3,j} \\ \vdots \\ f^*_{Nx-2,j} \\ f^*_{Nx-1,j} \end{bmatrix} = \begin{bmatrix} b^n_{2,j} \\ b^n_{3,j} \\ \vdots \\ b^n_{N-2,j} \\ b^n_{N,j} \end{bmatrix}$$

Where:

$$b^n_{i,j} = (\frac{u^n_{i,j}\Delta t}{\Delta x}+2\alpha_x)u^n_{i-1,j} + 4(1-\alpha_x)u^n_{i,j} + (2\alpha_x - \frac{u^n_{i,j}\Delta t}{\Delta x})u^n_{i+1,j} \tag{1}$$

$$b_{2,j} = (\frac{u^n_{2,j}\Delta t}{\Delta x}+2\alpha_x)u^*_{1,j} \tag{2}$$

$$b_{N-1,j} = (2\alpha_x - \frac{u^n_{N-1,j}\Delta t}{\Delta x}+)u^*_{N,j} \tag{3}$$

2.2 (1-b)

$$\begin{bmatrix} 4(1+\alpha_y) & (\frac{v^n_{2,j}\Delta t}{\Delta x}-2\alpha) & & \\ -(\frac{v^n_{3,j}\Delta t}{\Delta x}+2\alpha) & 4(1+\alpha_y) & (\frac{v^n_{3,j}\Delta t}{\Delta x}-2\alpha) & \\ & \ddots & & \\ & & \ddots & (\frac{v^n_{N-2,j}\Delta t}{\Delta x}-2\alpha) \\ & & -(\frac{v^n_{3,j}\Delta t}{\Delta x}+2\alpha) & 4(1+\alpha_y) \end{bmatrix} \begin{bmatrix} f^*_{2,j} \\ f^*_{3,j} \\ \vdots \\ f^*_{Nx-2,j} \\ f^*_{Nx-1,j} \end{bmatrix} = \begin{bmatrix} b^n_{2,j} \\ b^n_{3,j} \\ \vdots \\ b^n_{N-2,j} \\ b^n_{N,j} \end{bmatrix}$$

Where:

$$b^n_{i,j} = (\frac{v^n_{i,j}\Delta t}{\Delta x}+2\alpha_y)v^n_{i-1,j} + 4(1-\alpha_y)v^n_{i,j} + (2\alpha_y - \frac{v^n_{i,j}\Delta t}{\Delta x})v^n_{i+1,j} \tag{4}$$

$$b_{2,j} = (\frac{v^n_{2,j}\Delta t}{\Delta x}+2\alpha_y)v^*_{1,j} \tag{5}$$

$$b_{N-1,j} = (2\alpha_y - \frac{v^n_{N-1,j}\Delta t}{\Delta x}+)v^*_{N,j} \tag{6}$$