Introduction to Probability

Course: Computational Mathematics
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AGENDA

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- **Classic Defination Probability**
- **Additional Theorem of Probability**
- **Multiplication Theorem of Probability**
- **Conditional Probability**
- **Baye's Theorem**

INTRODUCTION

Probability

- Probability is used to **measure** the degree of **certainty or uncertainty** of the occurrence of **events**.
- Hence, if y is, the total number of outcomes and x is the favorable number of outcomes then probability of occurrence of an event A is denoted by P(A) and given by; **P(A) = x/y**
- When **toss a Coin**, the **probability** of **Head** or **Tails** is equal to **1/2**.
- When we **roll a dice**, the **probability** of each number is equal to **1/6** of the equal.
- Probability is always less than or equal to 1; **0≤P(A) ≤ 1**

TERMS USED IN PROBABILITY

Random experiment: It is an experiment which can be **performed**, **a number of times** in some conditions but **whose outcome cannot be predicted** exactly beforehand.

Event: Every possible outcome of a random experiment is called an event or a sample point.

Sample space: The **set formed** by sample points (event) is called sample space.

Example: If coin toss twice, sample space will be

{HH, HT, TH, TT}

TERMS USED IN PROBABILITY

Exhaustive :

- Exhaustive outcomes means that **all possible outcomes** have been **taken into consideration**.
- Thus, in the toss of a coin, there are only two outcomes, head and tail, in the case of throw of a dice there are six outcomes 1, 2, 3, 4, 5, 6. No other is possible.

Mutually exclusive:

- Mutually exclusive means that **not more than one outcome takes place**.
- In the toss of a coin, coming of head and tail simultaneously is not possible.
- A person is either alive or dead, not both. One excludes the other.

TERMS USED IN PROBABILITY

Equally likely:

- Equally likely means that we have no reason to **assume that one outcome is more likely to happen** than the other.
- If a coin is fair, and then likelihood of head or tail coming up are both equal. Coming of number in throw of cubical die, all numbers are equally possible.

CLASSICAL DEFINATION OF PROBABILITY

Probability:

If there is **y; exhaustive, mutually exclusive and equally likely outcomes** of a **random experiment** and **x** of them are **favorable** to the happening of an event A, then the **probability of A** is;

$$P(A)=x/y$$

ADDITION THEOREM OF PROBABILITY

Let A and B be two events, then P(A + B) or P(AUB) is the probability of happening of A or B or both A and B.

Hence, **P(AUB) = P(A) + P(B) - P(A n B).**
Where, AnB denotes the happening of both A and B simultaneously.
If A and B are mutually exclusive then P(AnB) = 0

Question: If the probability of solving a problem by two students George and James are 1/2 and 1/3 respectively then what is the probability of the problem to be solved.

MULTIPLICATION THEOREM

- P(AnB) = probability of A and B both simultaneously.
- If A and B are mutually exclusive, then P(AnB)=0. P(AnB) is sometimes written as P(AB).
- Question: Shareen has to select two students from a class of 23 girls and 25 boys. What is the probability that both students chosen are boys?

CONDITIONAL PROBABILITY

- Let A and B two events of a random experiment. The **probability of occurrence of A if B has already occurred** and P(B) ≠ 0 is known as conditional probability. This is **denoted by P(A/B)**.
- Also, conditional probability can be defined as the probability of occurrence of B if A has already occurred and P(A) ≠ 0. This is denoted by P(B/A).

$$P(A \vee B) = \frac{P(A \cap B)}{P(B)}$$

BAYE'S THEOREM

Bayes' Theorem states that the conditional probability of an event, based on the occurrence of another event, is equal to the likelihood of the second event given the first event multiplied by the probability of the first event.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) x P(B/A)}{P(B)}$$

BAYE'S THEOREMS EXAMPLE

Example: A man is known to speak the truth 2 out of 3 times. He throws a die and reports that the number obtained is a four. Find the probability that the number obtained is actually a four.

Solution:

Let **A** be the **event** that the man reports that **number four** is obtained.

Let **E1** be the event that four is obtained and E2 be its complementary event.

Then, **P(E1)** = Probability that four occurs = **1/6.**

P(E2) = Probability that four does not occur = **1- P(E1)** = **1 – (1/6) = 5/6**.

Also, **P(A|E1)**= Probability that man reports four and it is actually a four = **2/3**

P(A|E2) = Probability that man reports four and it is not a four = **1/3**.

BAYE'S THEOREMS EXAMPLE

By using Bayes' theorem, probability that number obtained is actually a four,

$$P(E_1/A) = \frac{P(E_1)\,x\,P(A/E_1)}{P(E_1)\,x\,P(A/E_1) + P(E_2)\,x\,P(A/E_2)} = \frac{\frac{1}{6}x\frac{2}{3}}{(\frac{1}{6}x\frac{2}{3}) + (\frac{5}{6}x\frac{1}{3})}$$

= 2/7

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Any Queries!

Thank You!