

Infectious diseases hazard map for India based on mobility networks

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- Acknowledgements :

Dr. G. J. Sreejith

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- Funded by

DST and SERB Special MATRICS
grant for COVID related research
(2020-21)

- *Current Science* **121**, 1208 (2021)

On COVID-19 pandemic

First case reported in December 2019 from Wuhan, China.
In India, first few cases started in February, 2020.

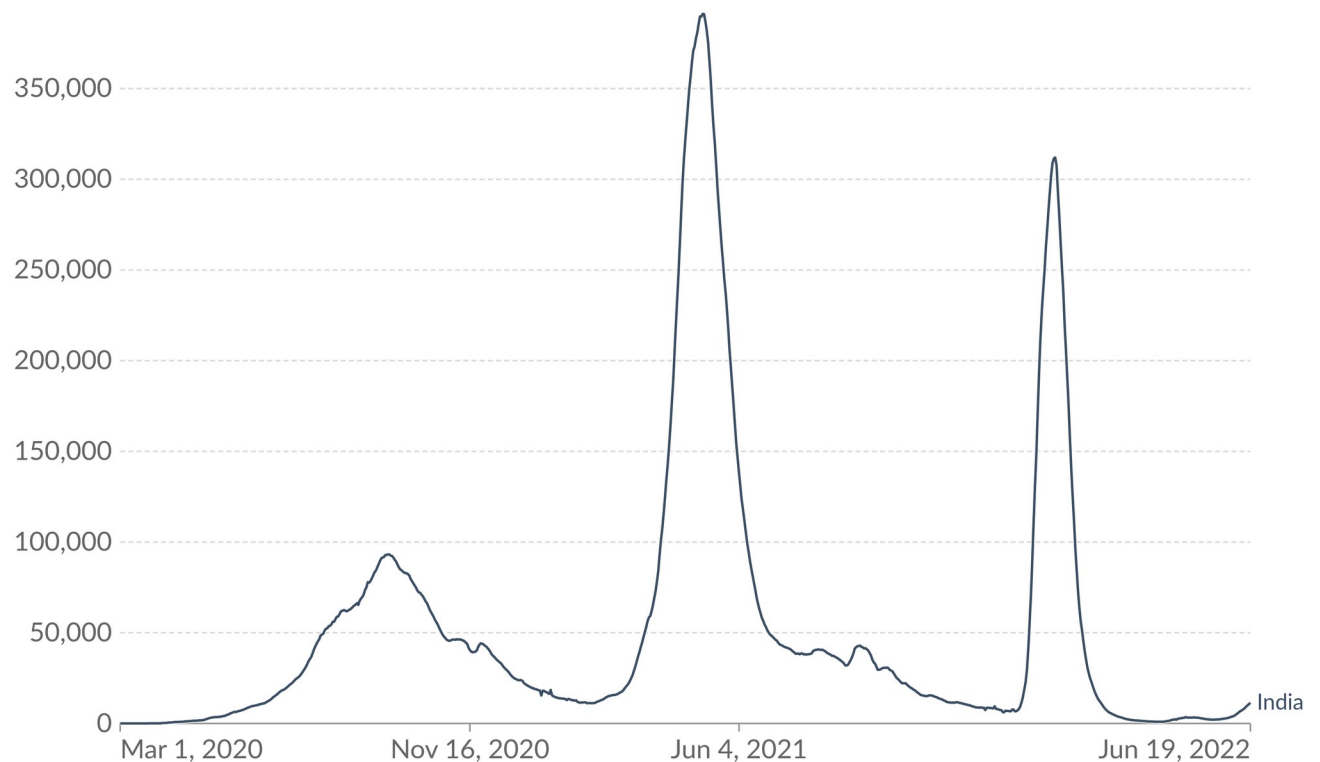
Total number cases : 4.33 crores (worldwide : 53.8 crores)

Deaths : 5.24 lakhs (worldwide : 63.2 lakhs)

Daily new confirmed COVID-19 cases

7-day rolling average. Due to limited testing, the number of confirmed cases is lower than the true number of infections.

Our World
in Data



Source: Johns Hopkins University CSSE COVID-19 Data

CC BY

The arrival of Spanish Flu (1918) in India

“

There is ample evidence during the first epidemic of the introduction of infection into a locality from another infected locality. The railway played a prominent part, as was inevitable. During the panic caused by the epidemic, the trains were filled with emigrants from infected centres, many of them being ill. The Post office also was an important agency in disseminating infection, also very largely through the Railway Postal Service. Lucknow, Lahore, Simla and other cities are said to have been infected in this manner

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there is ample evidence to prove that infection in India during the second epidemic was carried from province to province and place to place within each province by travellers by rail, riverboats, carts and on foot

“

ANNUAL REPORT OF THE SANITARY COMMISSIONER WITH THE GOVERNMENT OF INDIA

FOR

1918

WITH

APPENDICES AND RETURNS OF SICKNESS AND MORTALITY AMONG
EUROPEAN TROOPS, INDIAN TROOPS, AND PRISONERS
IN INDIA FOR THE YEAR.



CALCUTTA
SUPERINTENDENT GOVERNMENT PRINTING, INDIA
1920

Price, Two Rupees Eight Annas.

The arrival of Spanish Flu (1918) in India

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A Contribution to the Mathematical Theory of Epidemics.

By W. O. KERMACK and A. G. McKENDRICK.

(Communicated by Sir Gilbert Walker, F.R.S.—Received May 13, 1927.)

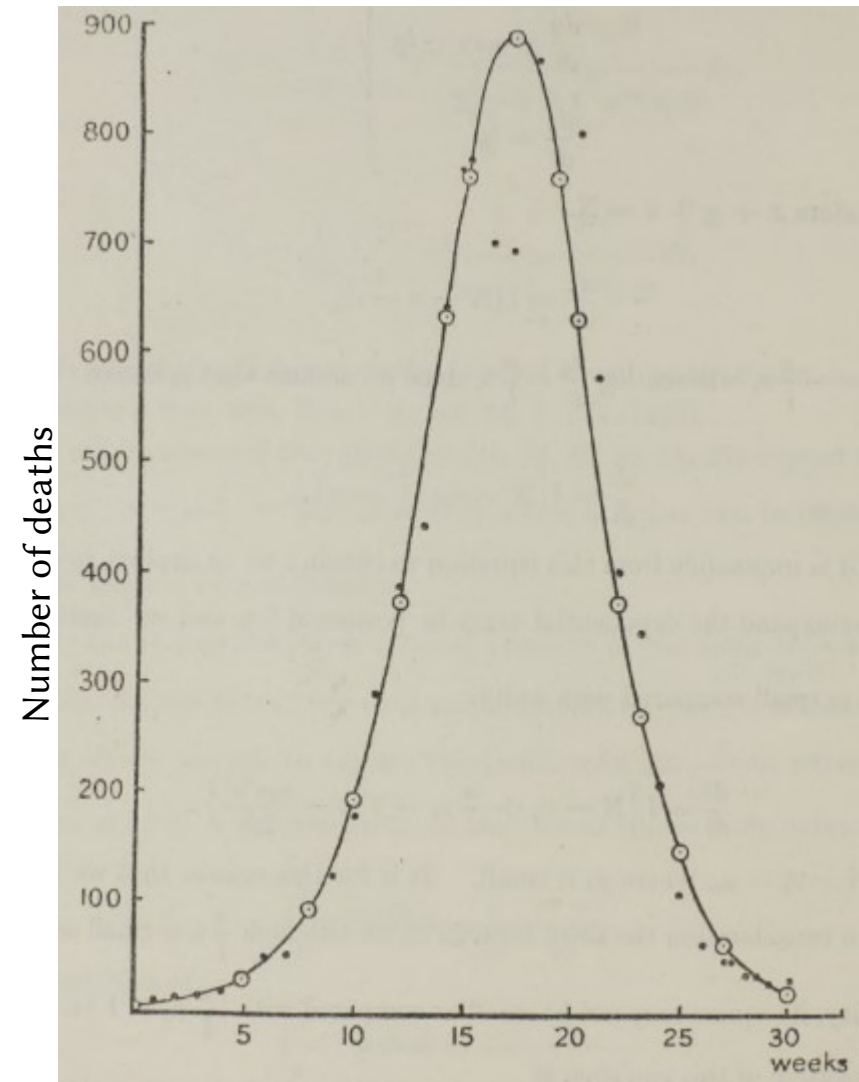
(From the Laboratory of the Royal College of Physicians, Edinburgh.)

Introduction.

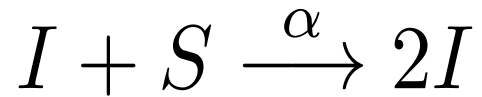
(1) One of the most striking features in the study of epidemics is the difficulty of finding a causal factor which appears to be adequate to account for the magnitude of the frequent epidemics of disease which visit almost every population. It was with a view to obtaining more insight regarding the effects of the various factors which govern the spread of contagious epidemics that the present investigation was undertaken. Reference may here be made to the work of Ross

W. O. Kermack and A. G. McKendrick
Proc. Royal. Soc. A **115**, 700 (1927)

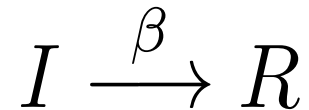
Plague in Bombay,
Dec 1905 to Jul 1907



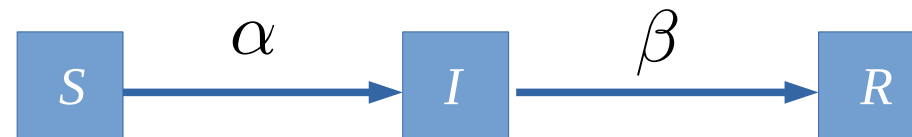
Susceptible-Infected-Recovered (SIR) model



Infection rate α



Recovery rate β



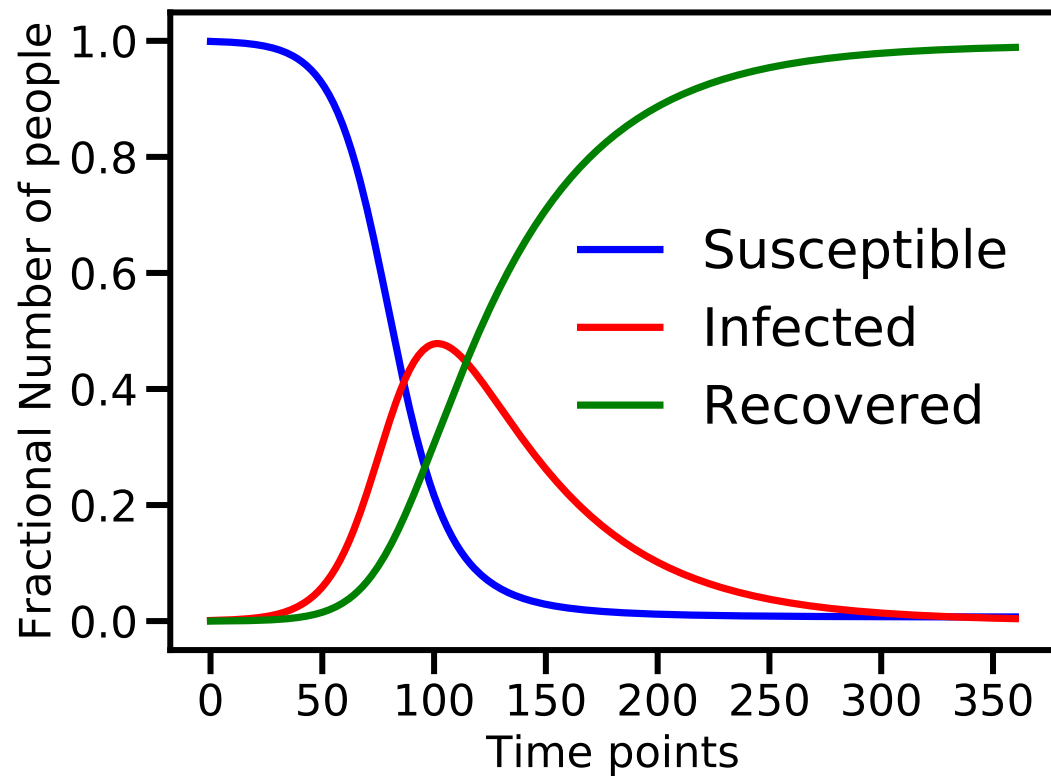
$$\begin{aligned}\frac{dS(t)}{dt} &= -\alpha \frac{S(t)I(t)}{N}, \\ \frac{dI(t)}{dt} &= +\alpha \frac{S(t)I(t)}{N} - \beta I(t), \\ \frac{dR(t)}{dt} &= +\beta I(t).\end{aligned}$$

$$S(t) + I(t) + R(t) = N$$

\downarrow
constant

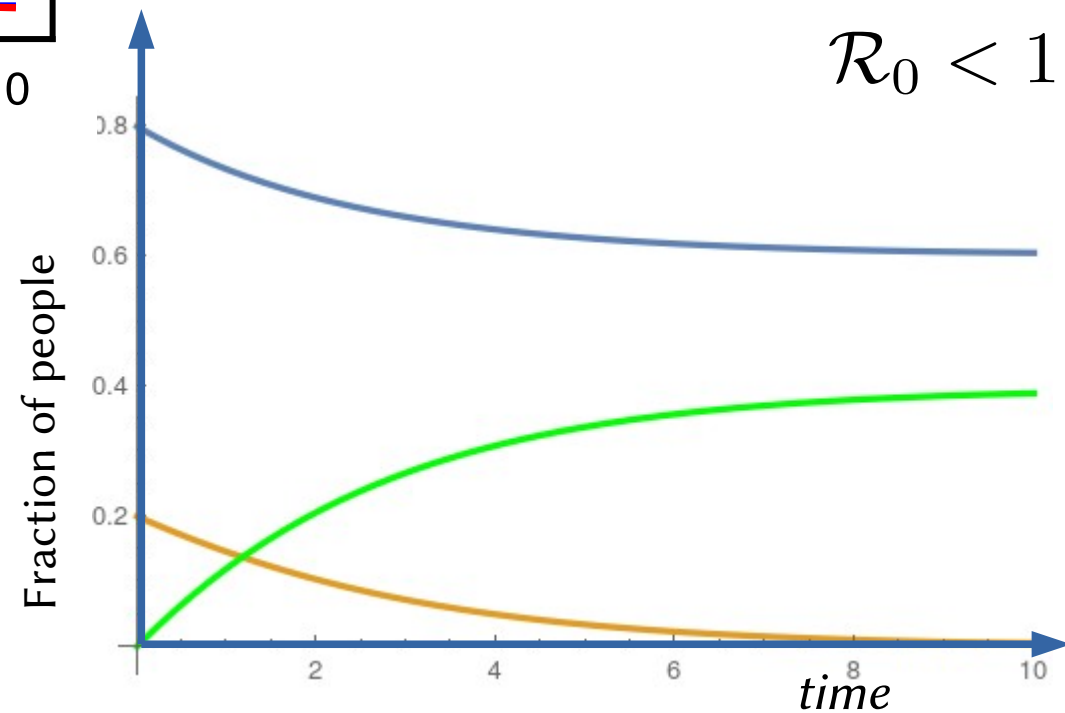
Compartmental model with 3 compartments

Two different scenarios from SIR model : numerical solutions

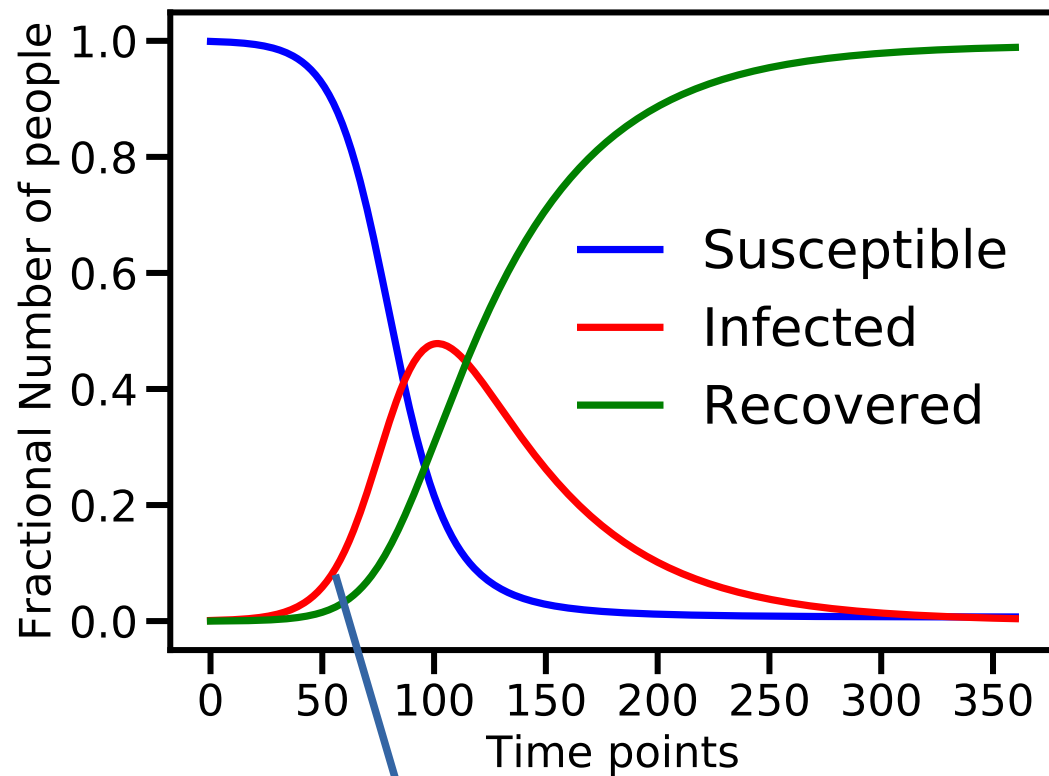


$$\alpha = 1.5, \quad \beta = 0.7$$
$$\mathcal{R}_0 > 1$$

$$\alpha = 0.5, \quad \beta = 1.0$$
$$\mathcal{R}_0 < 1$$

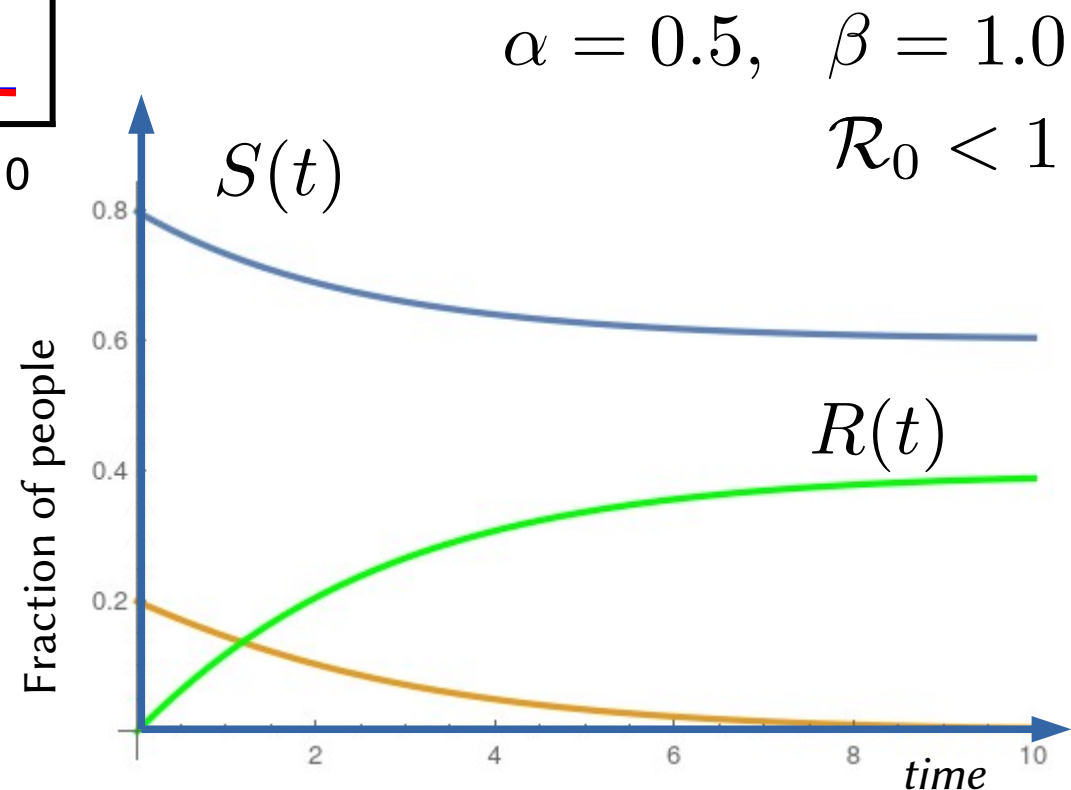


Two different scenarios from SIR model : numerical solutions



$$\alpha = 1.5, \quad \beta = 0.7$$
$$\mathcal{R}_0 > 1$$

Regime of exponential
increase



For very short times : $S(t) \approx N$

$$\frac{\partial I(t)}{\partial t} \approx (\alpha - \beta)I(t)$$

$$I(t) \sim I_0 e^{(\alpha - \beta)t}$$

$\alpha > \beta$ Exponential increase in infected cases

$\alpha < \beta$ Exponential increase in infected cases

- Reduce SIR model to

$$\frac{dr}{dt} = \beta \left(1 - r - s_0 r e^{-(\alpha/\beta)r} \right) \quad r = \frac{R}{N}$$

- $t \rightarrow \infty, \quad \frac{dr}{dt} \rightarrow 0, \quad r_\infty = \text{constant}$

$$r_\infty = 1 - e^{-\mathcal{R}_0 r_\infty}, \quad \mathcal{R}_0 = \frac{\alpha}{\beta}$$

Basic reproduction number

When does a solution exist ?

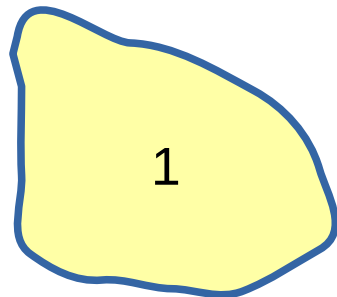
$$r_{\infty} = 1 - e^{-R_0 r_{\infty}}, \quad R_0 = \frac{\alpha}{\beta}$$

- No epidemic (decay of infections)

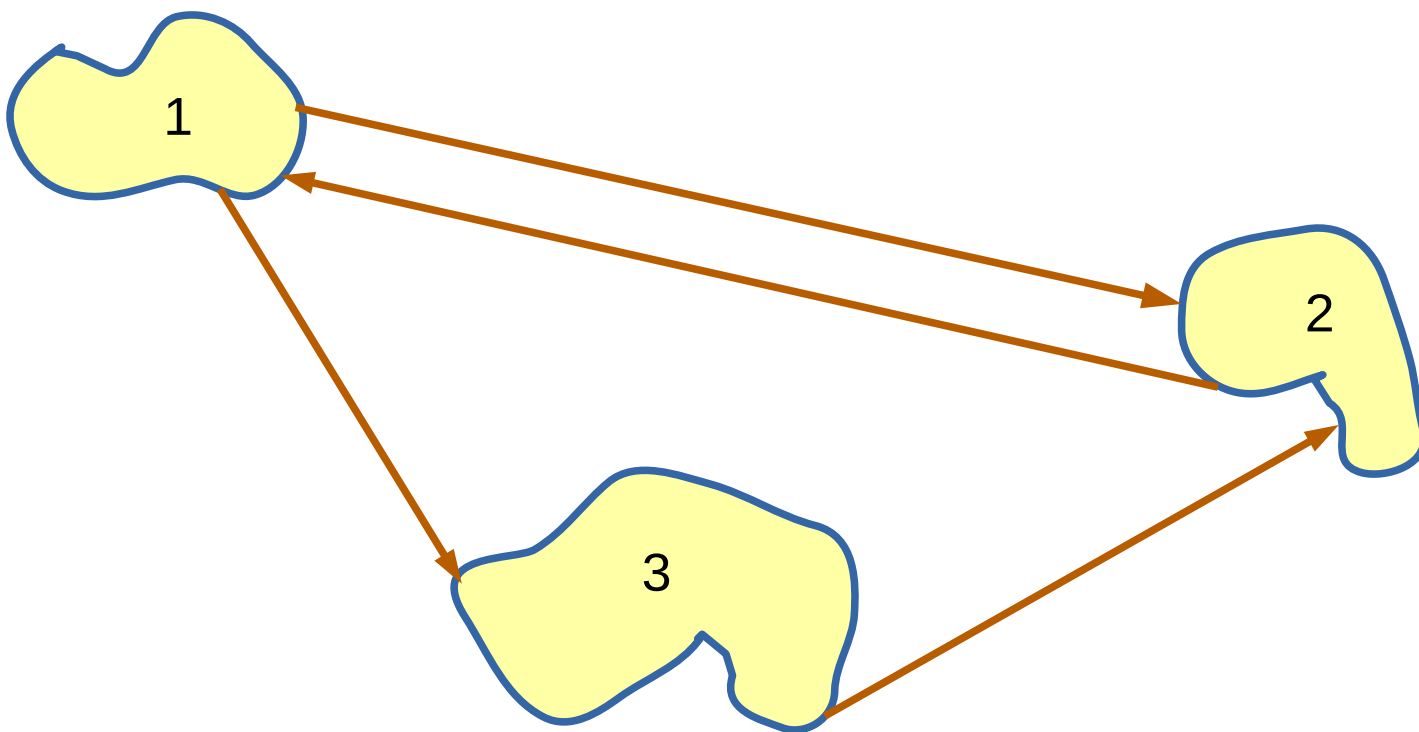
$$R_0 < 1, \quad \alpha < \beta, \quad r_{\infty} \rightarrow 0$$

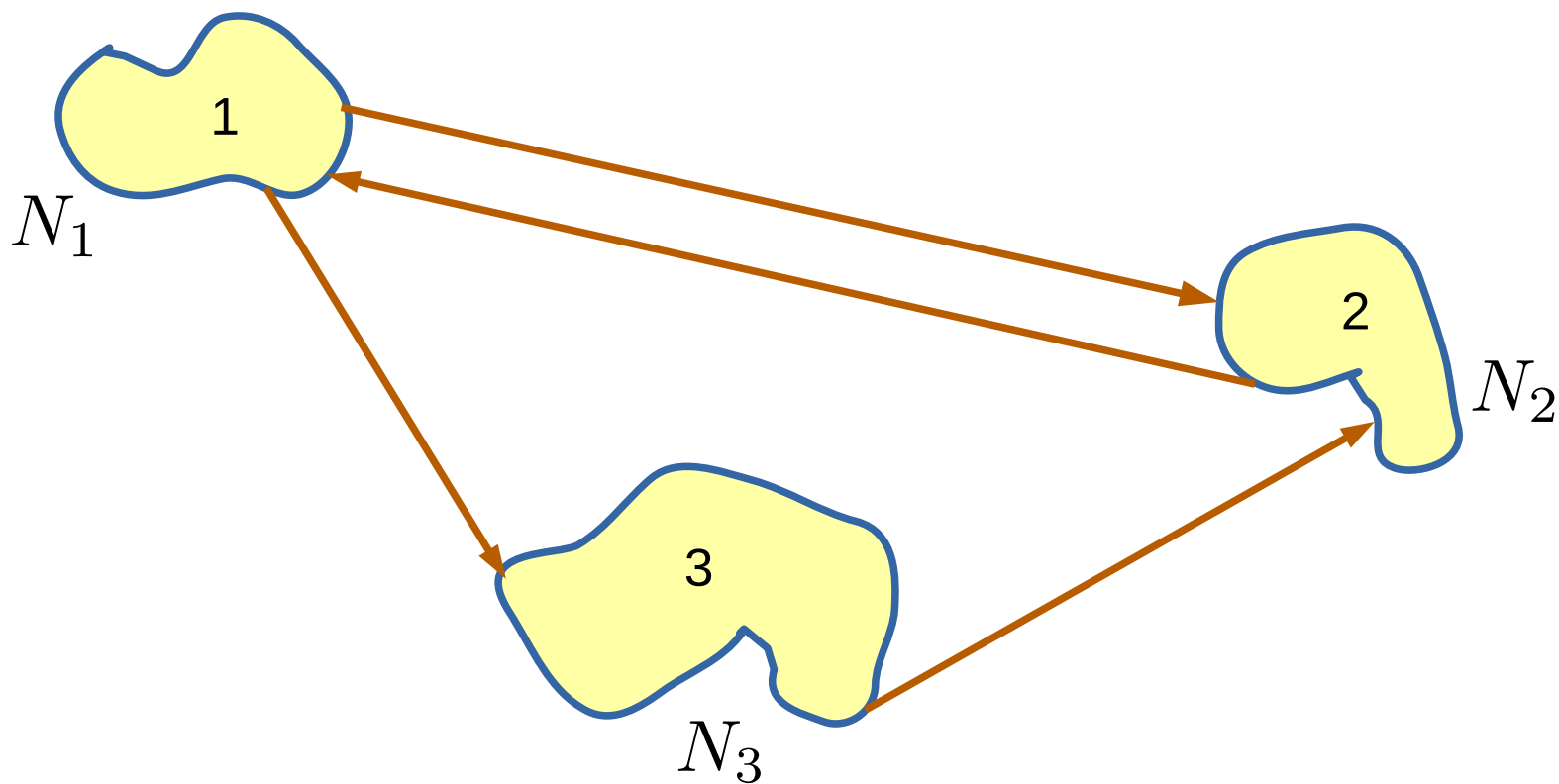
- Epidemic (infections increasing)

$$R_0 > 1, \quad \alpha > \beta, \quad r_{\infty} \rightarrow \text{constant} > 0$$



Well-mixed population within one city or town





nodes	1,	2,	3,	M
population	N_1	N_2	N_3	N_M
total population	$N = N_1 + N_2 + \dots N_M$			

- **Characterization of the second wave of COVID-19 in India**

Rajesh Ranjan^{1,*}, Aryan Sharma² and Mahendra K. Verma²

¹Department of Aerospace Engineering, Indian Institute of Technology, Kanpur 208 016, India

²Department of Physics, Indian Institute of Technology, Kanpur 208 016, India

Current Science (2021)

- **Dynamical modelling and analysis of COVID-19 in India**

R. Gopal¹, V. K. Chandrasekar^{1,*} and M. Lakshmanan²



¹Centre for Nonlinear Science and Engineering, School of Electrical and Electronics Engineering, SASTRA Deemed University, Thanjavur 613 401, India

²Department of Nonlinear Dynamics, School of Physics, Bharathidasan University, Tiruchirappalli 620 014, India

- Modelling and forecasting of COVID-19 pandemic in India
Chaos, Solitons and Fractals 139, 110049 (2020).

PAPER

- **Digital herd immunity and COVID-19**

Vir B Bulchandani^{1,2} , Saumya Shivam³ , Sanjay Moudgalya^{3,4,5} and S L Sondhi^{6,3}

Published 23 June 2021 • © 2021 IOP Publishing Ltd

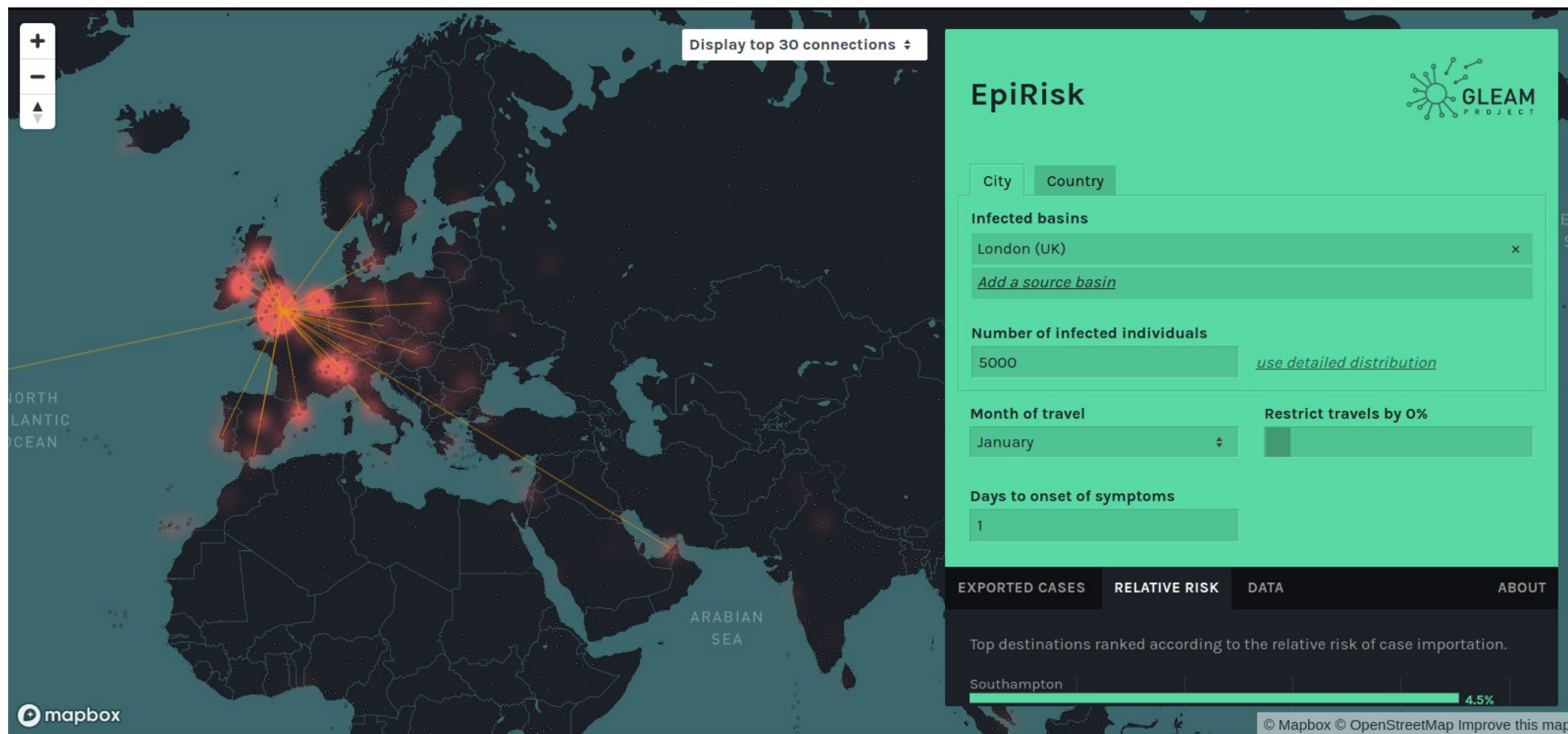
[Physical Biology](#), [Volume 18](#), [Number 4](#)

The basic premise is that irrespective of the severity of the infectious disease, its spread is caused by the mobility of the people via the transportation networks.

- Central question :

In a network of N cities/towns given by $X_1, X_2, X_3, \dots, X_N$, and if the infection outbreak location is declared to be X_1 , can a hazard value be assigned to all other cities/towns reflecting their risk of catching the infection ?

This is not about predicting the infection case loads and its evolution.

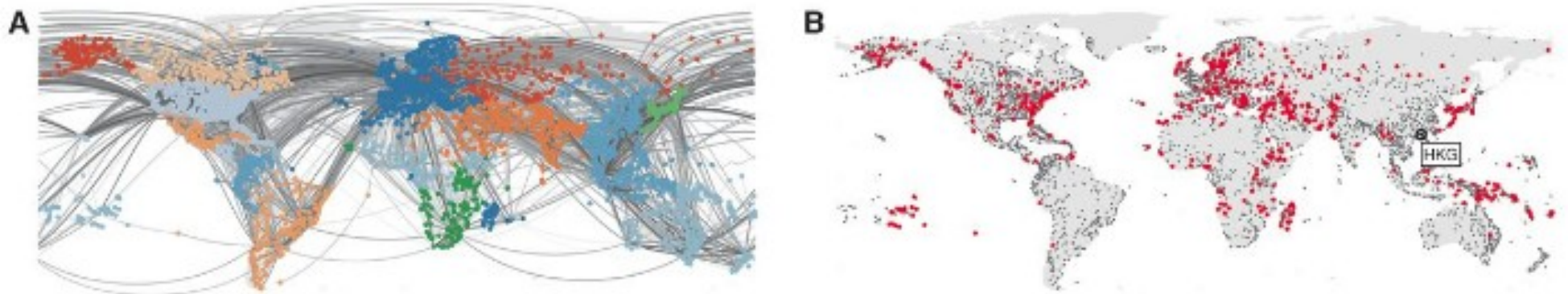


epirisk.net

The Hidden Geometry of Complex, Network-Driven Contagion Phenomena

DIRK BROCKMANN AND DIRK HELBING

SCIENCE • 13 Dec 2013 • Vol 342, Issue 6164 • pp. 1337-1342 • DOI: 10.1126/science.1245200




D. Brockmann and D. Helbing,
Science **342**, 1337 (2013)

F. Iannelli, *Phys. Rev. E* **95**, 012313 (2017)

Movement kinetics

$$\frac{\partial X_n}{\partial t} = \sum_{m \neq n} \mu_{nm} X_m - \mu_{mn} X_n$$

$$X = S, I, R$$


Mobility rate for travelling
from city m to n

- $F_m^n \rightarrow$ Number of people going from city m to n in unit time
- $F_m = \sum_n F_m^n \rightarrow$ Total outflux from city m
- $P_m^n = \frac{F_m^n}{F_m} \rightarrow$ Probability for an individual to go from city m to city n

SIR model augmented with mobility

$$\frac{\partial S_n(t)}{\partial t} = -\alpha \frac{S_n(t)I_n(t)}{N_n} + \sum_m \left[\frac{F_m^n}{N_m} S_m(t) - \frac{F_n^m}{N_n} S_n(t) \right]$$

$$\frac{\partial I_n(t)}{\partial t} = +\alpha \frac{S_n(t)I_n(t)}{N_n} - \beta I_n(t) + \sum_m \left[\frac{F_m^n}{N_m} I_m(t) - \frac{F_n^m}{N_n} I_n(t) \right]$$

$$\frac{\partial R_n(t)}{\partial t} = +\beta I_n(t) + \sum_m \left[\frac{F_m^n}{N_m} R_m(t) - \frac{F_n^m}{N_n} R_n(t) \right]$$

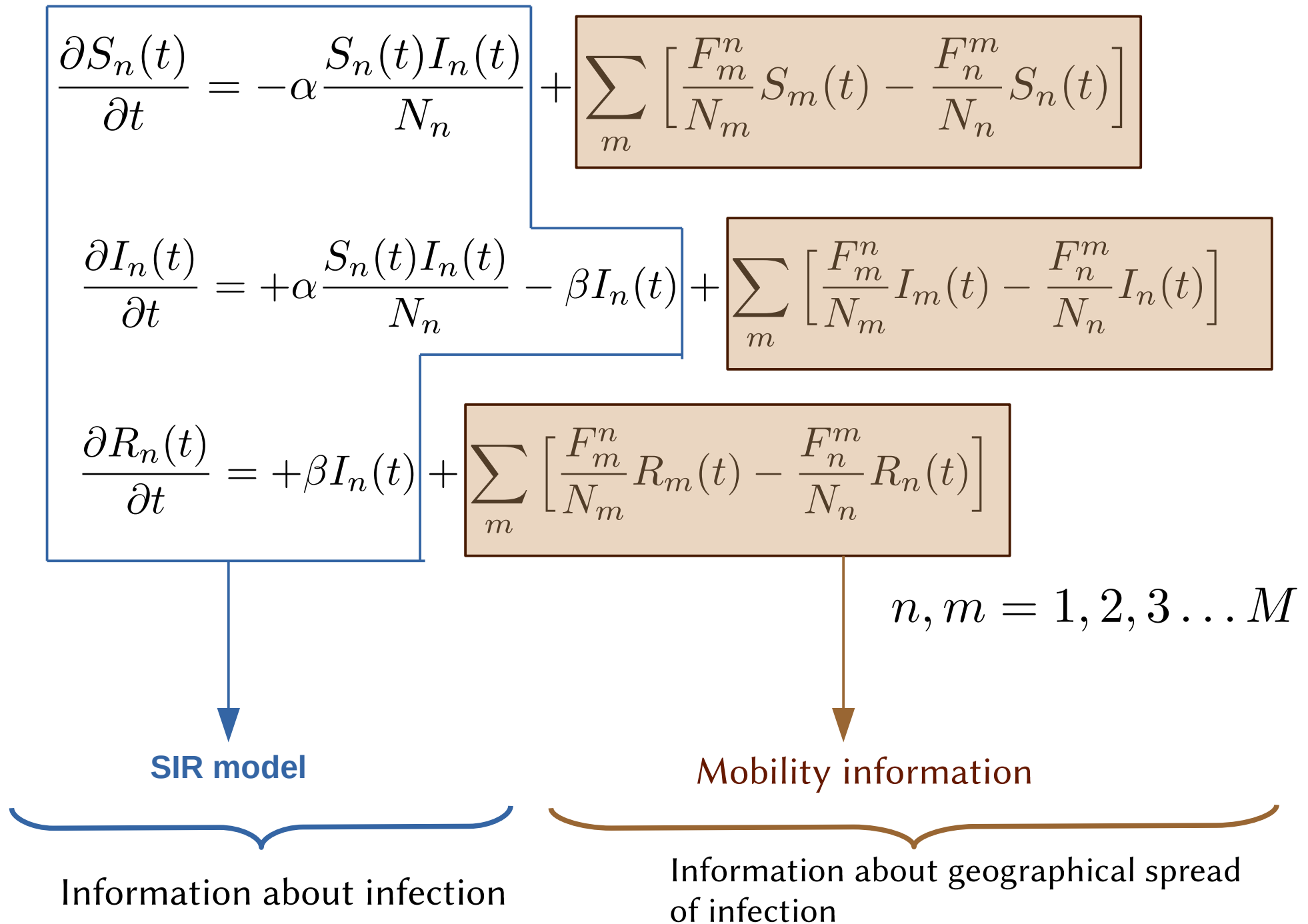
$\alpha \rightarrow$ Infection rate

$\beta \rightarrow$ Recovery rate

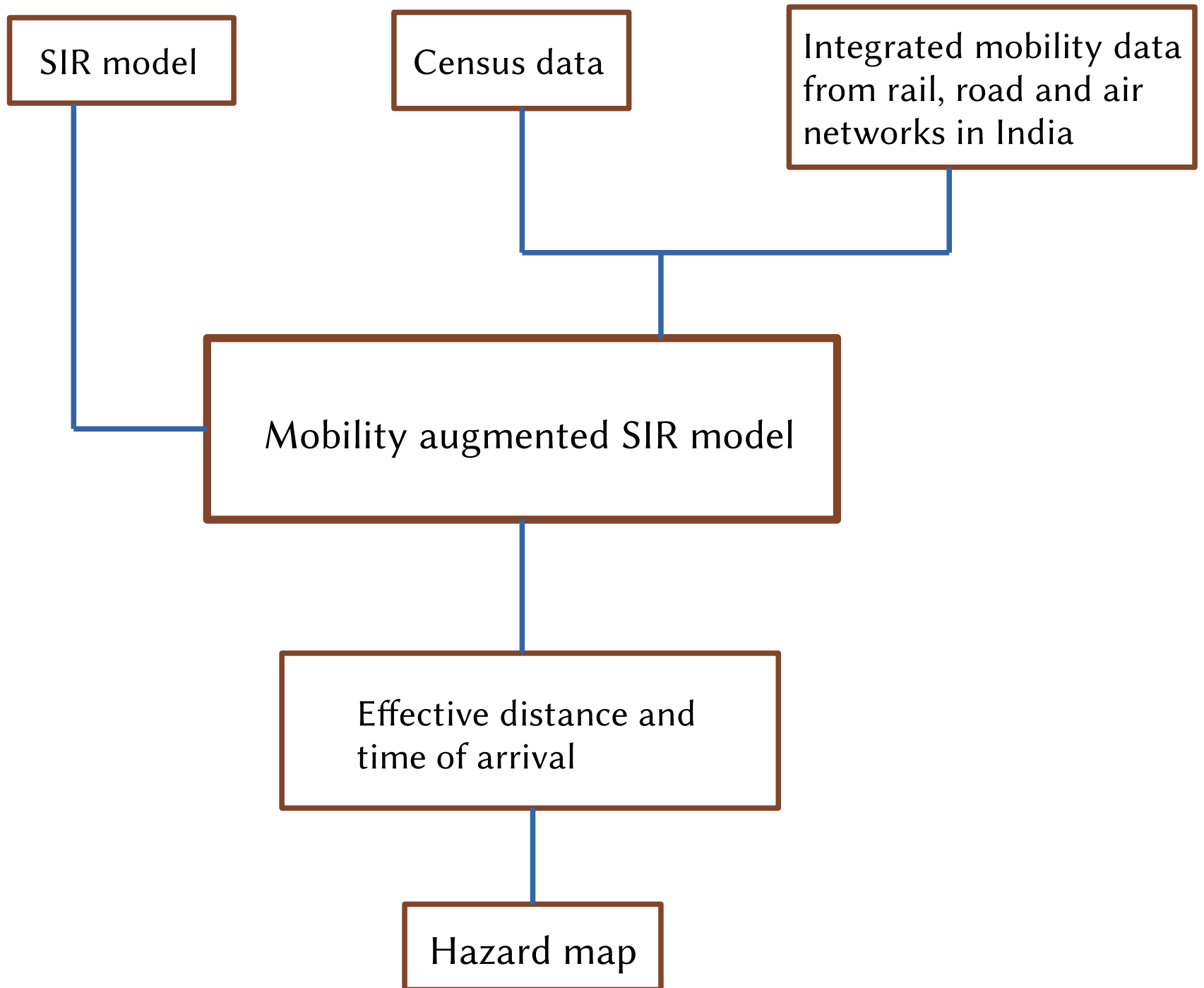
$$n, m = 1, 2, 3 \dots M$$

$$N = S(t) + I(t) + R(t)$$

SIR model augmented with mobility



algorithm behind the hazard map : top level view




effective distance

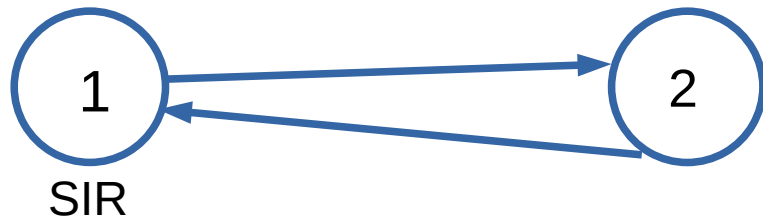
- **Effective distance**
between i -th and j -th
cities

$$D_{\text{eff}}^{i \rightarrow j} = 1 - \log P_i^j = 1 - \log(F_i^j / F_i)$$

One-step conditional probability
that a person leaving city i travels
to city j


mobility
data

Two-city model



α Infection rate

$Q \rightarrow$ transition rate from 1 to 2

What is the distribution of first arrival time of infection ?

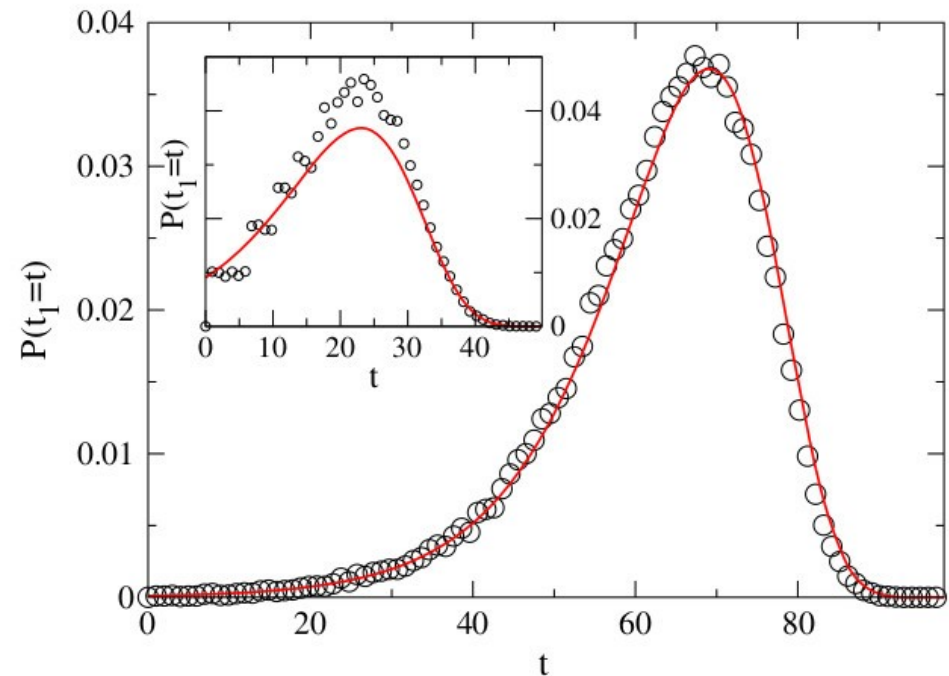
Gumbel distribution.

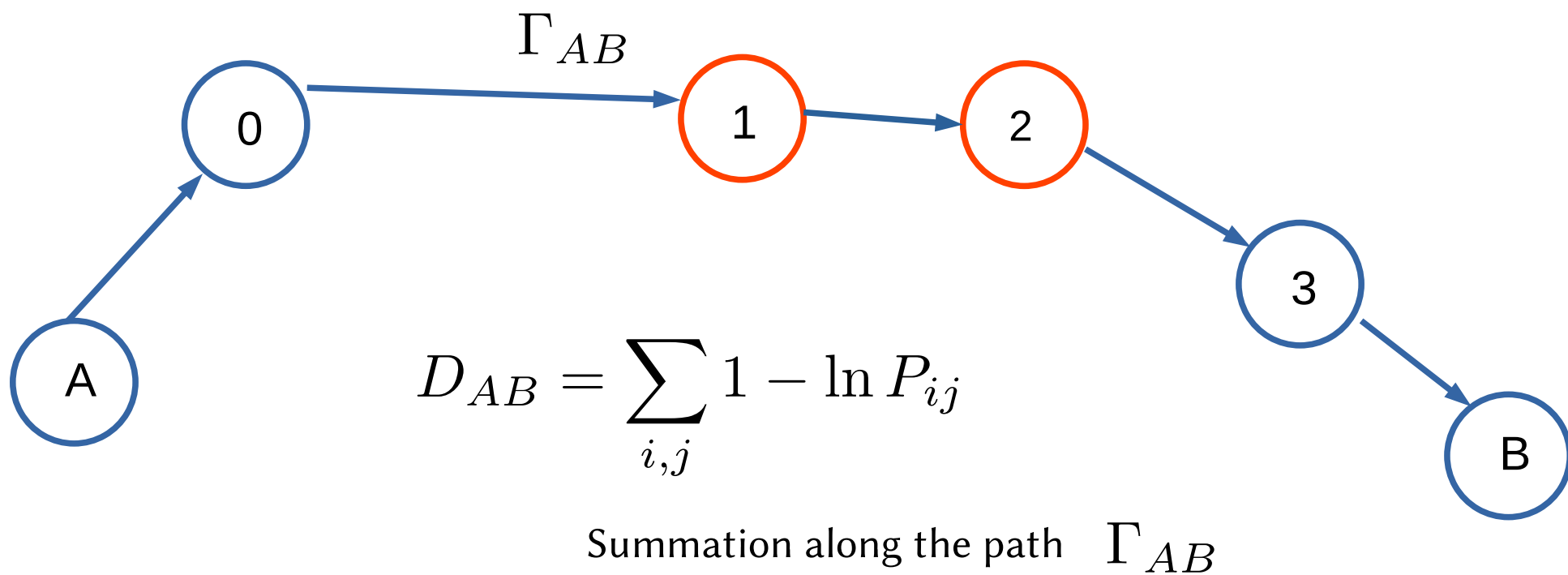
$$P(t) = Q \exp \left(\alpha t - \frac{Q}{\alpha} e^{\alpha t} \right)$$

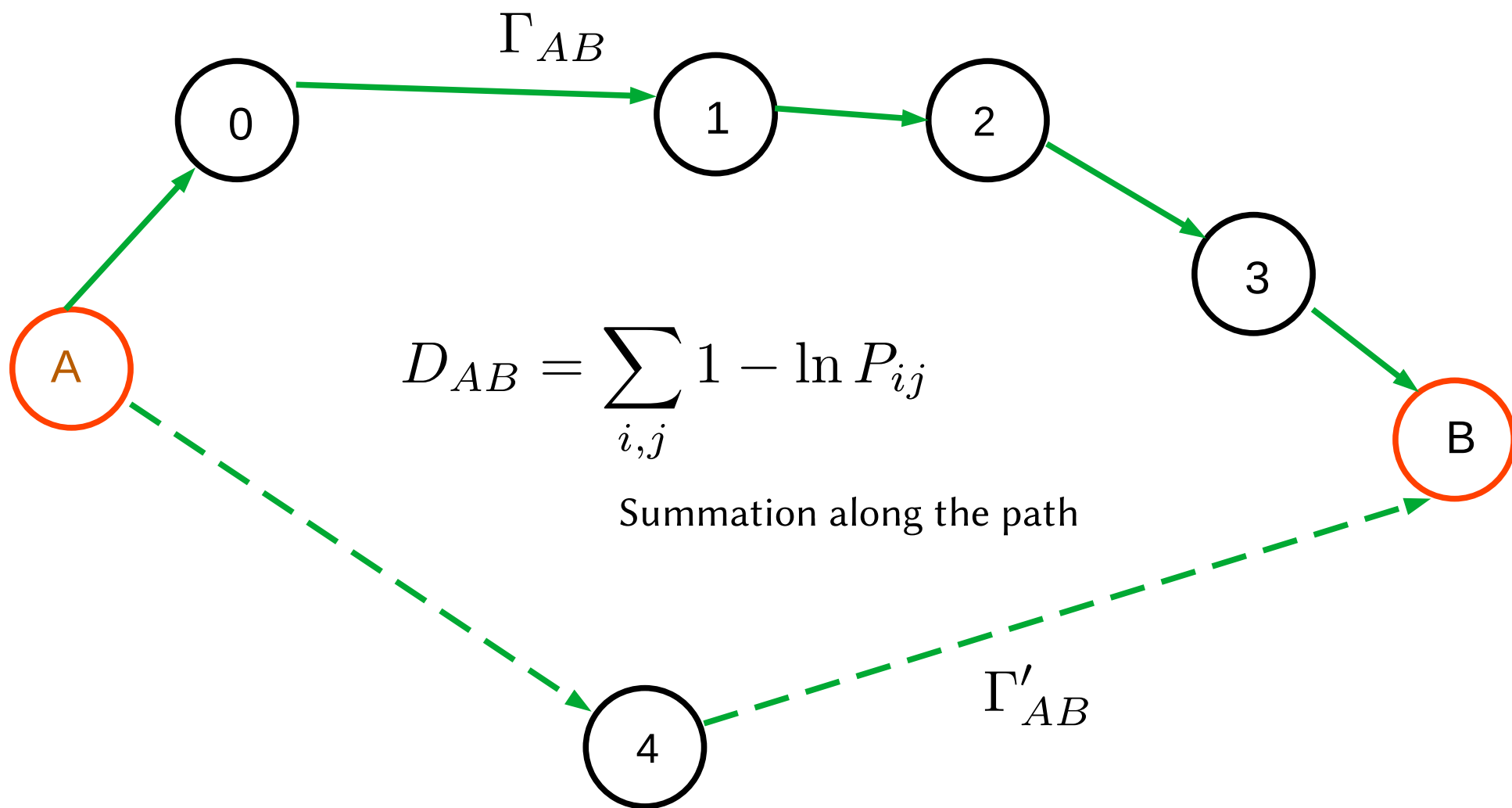
Mean hitting time

$$\langle t \rangle = \frac{1}{\alpha} \left(\ln \frac{\alpha}{Q} - \gamma \right)$$

$$D = 1 - \ln P_{ij}$$







● Effective distance

$$D_{\text{eff}}^{A \rightarrow B} = \min_{\Gamma} \sum_{i,j} 1 - \ln P_{ij}$$

effective distance and ToA

- Effective distance between i -th and j -th cities

$$D_{\text{eff}}^{i \rightarrow j} = 1 - \log P_i^j = 1 - \log(F_i^j / F_i)$$

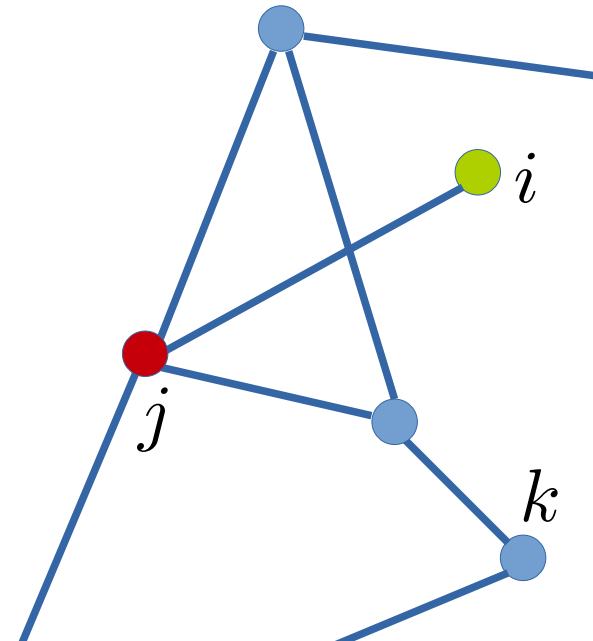
One-step conditional probability that a person leaving city i travels to city j

mobility data

- Time of arrival of an infection is when the infection cases $I(t)$ exceed a threshold value for the first time

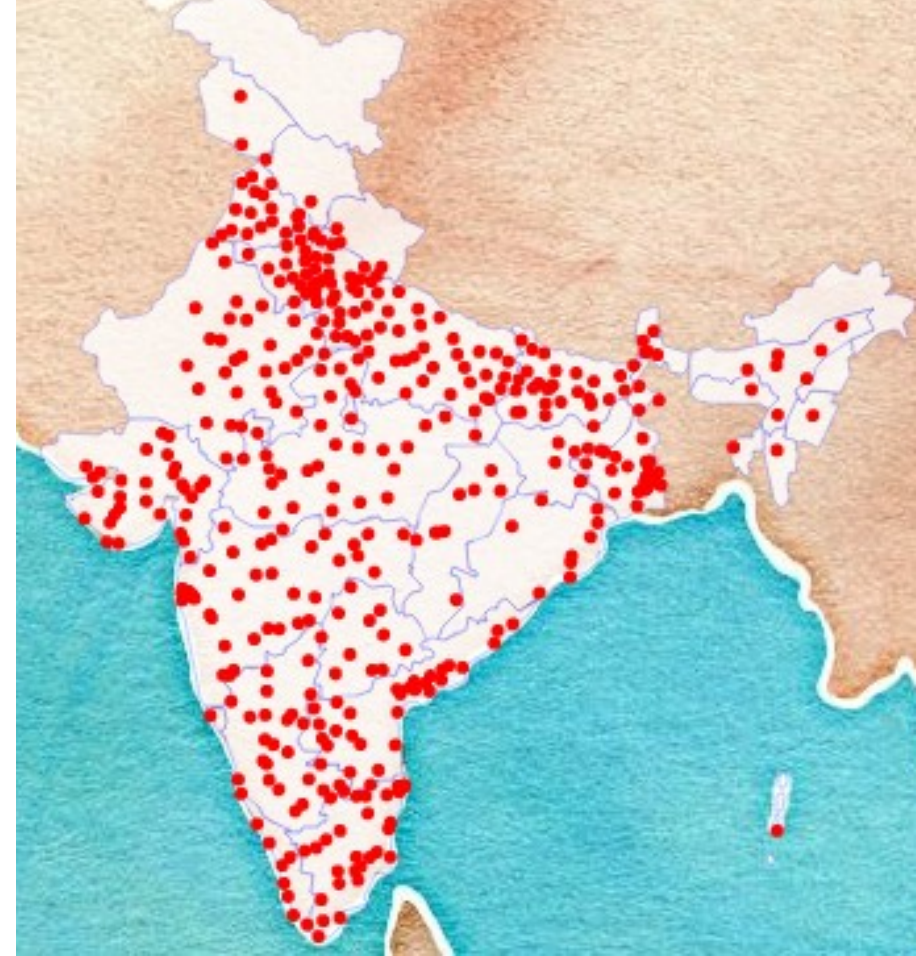
$$\text{At } t = T_A, \quad I(t) > I_c$$

Central result : $T_A \propto D_{\text{eff}}^{i \rightarrow j}$



network and mobility data

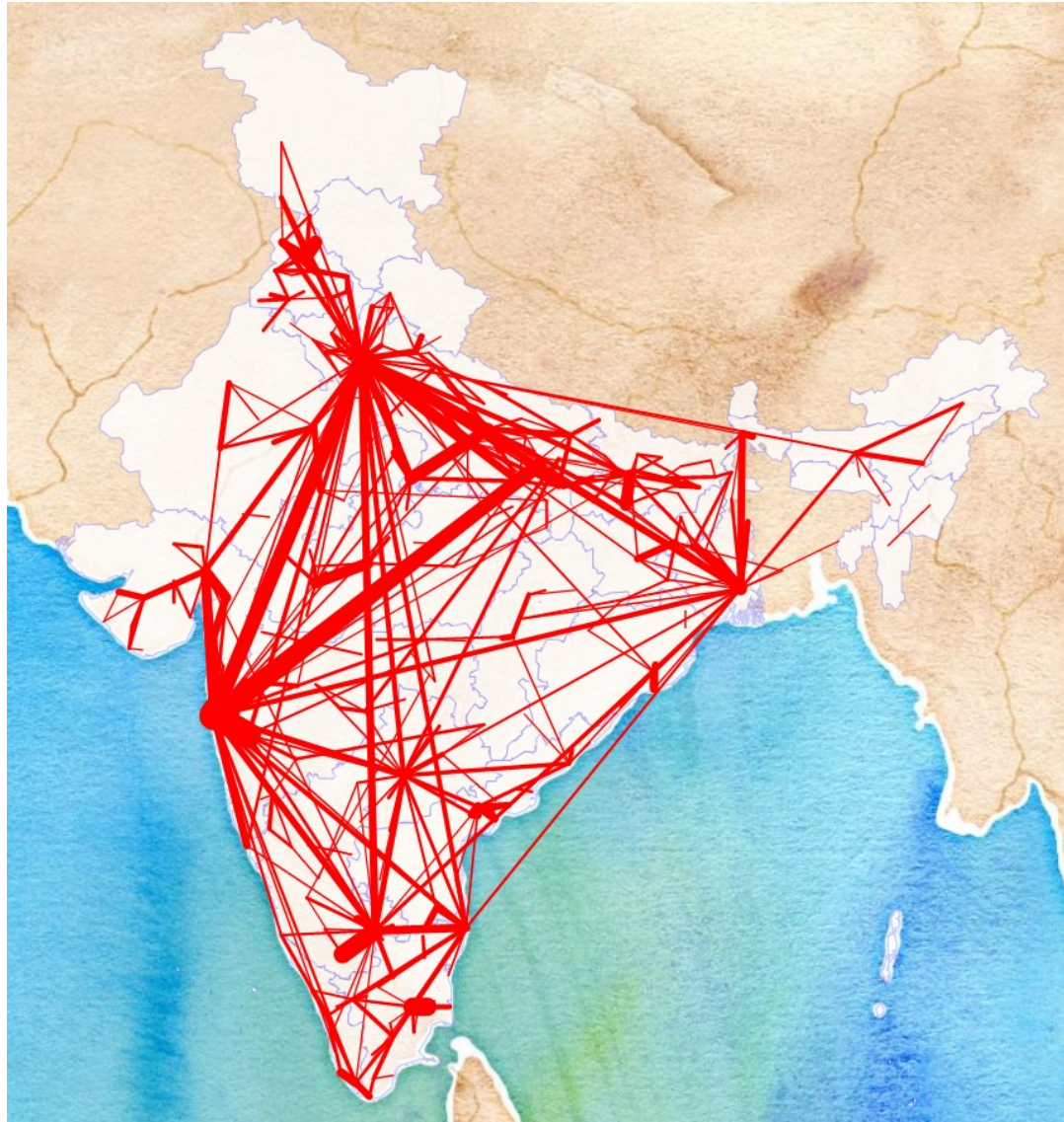
- 446 cities/towns in India with a population of 1 lakh or more (based on 2011 census).
- Data of train, air and road traffic considered.
- Train data for 435 cities/towns collected. Data from Indian Railway time table, Live train status websites used.
- Flight schedules and passenger data from airline websites used.
- NHAI website data was used for road traffic. Gaps in road traffic data. Estimated used.
- Geographical location of cities obtained using python library *geopy*.
- All local mobility discarded. For instance, Mumbai local train passengers ignored.



How India travels
(A statistical summary)

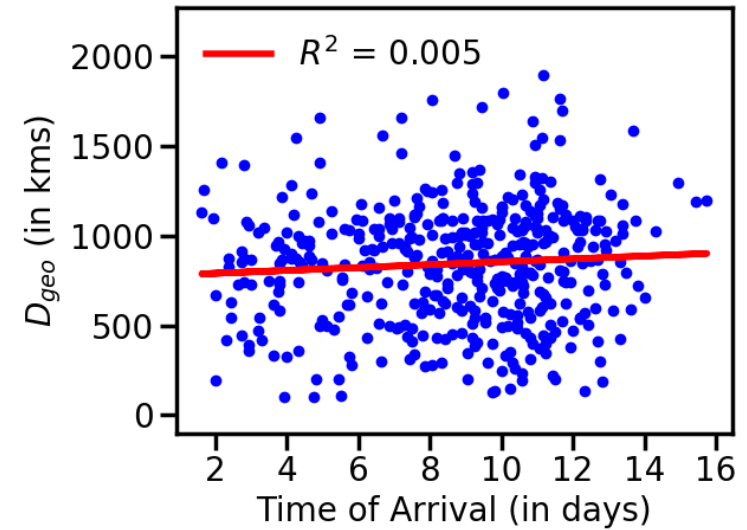
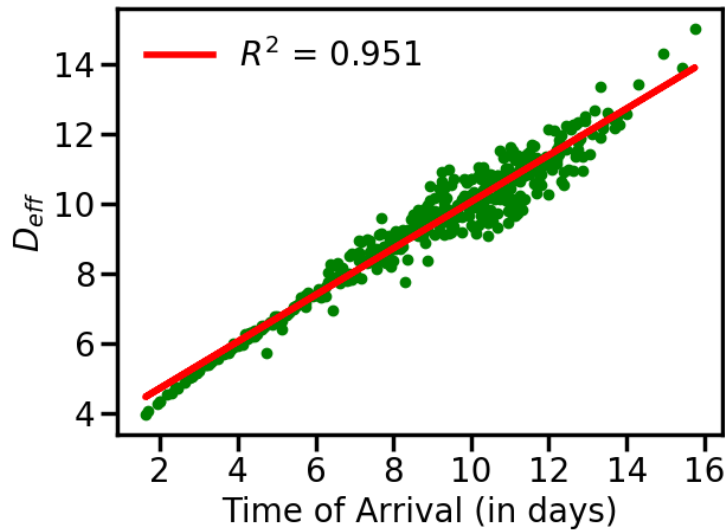
Property	Airway	Railway	Roadway	Combined
Number of Nodes	85	435	446	446
Number of Edges	1182	41594	9128	46448
Average Degree	13	95	20	104
Route symmetry index	1	0.9875	1	0.9878
Locality of Mobility	Same	Different	Same	Different
Passengers/day	7.5×10^5	8.8×10^6	2.5×10^6	1.2×10^7
Fraction of total	0.06	0.73	0.21	1.0

How India travels

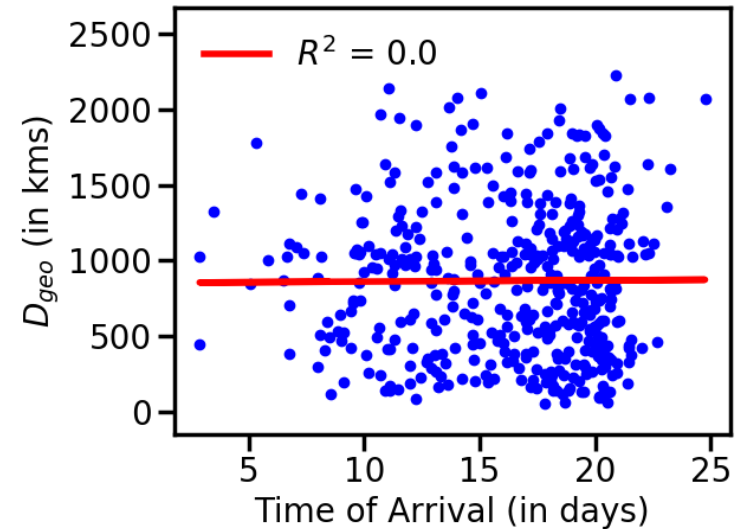
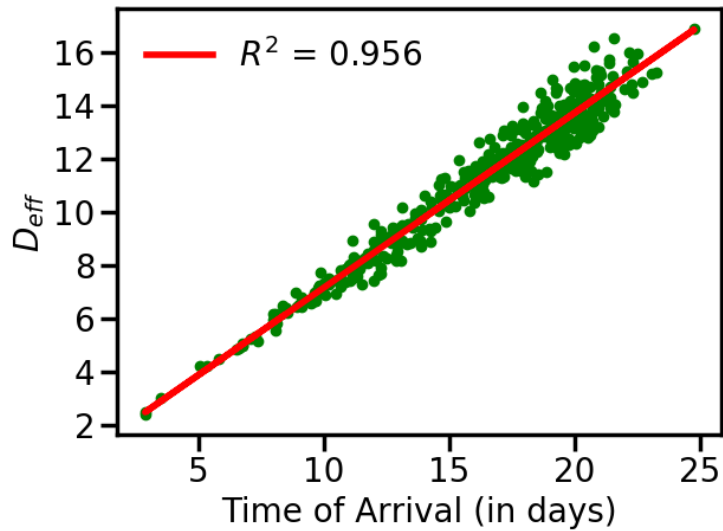


- Data from rail, road and air. Top 500 of 23224 connections shown here. Thicker lines indicate more mobility of people in that route.

Outbreak location : *Delhi*



Outbreak location : *Tirupati*



$$T_A \propto D_{\text{eff}}^{i \rightarrow j}$$

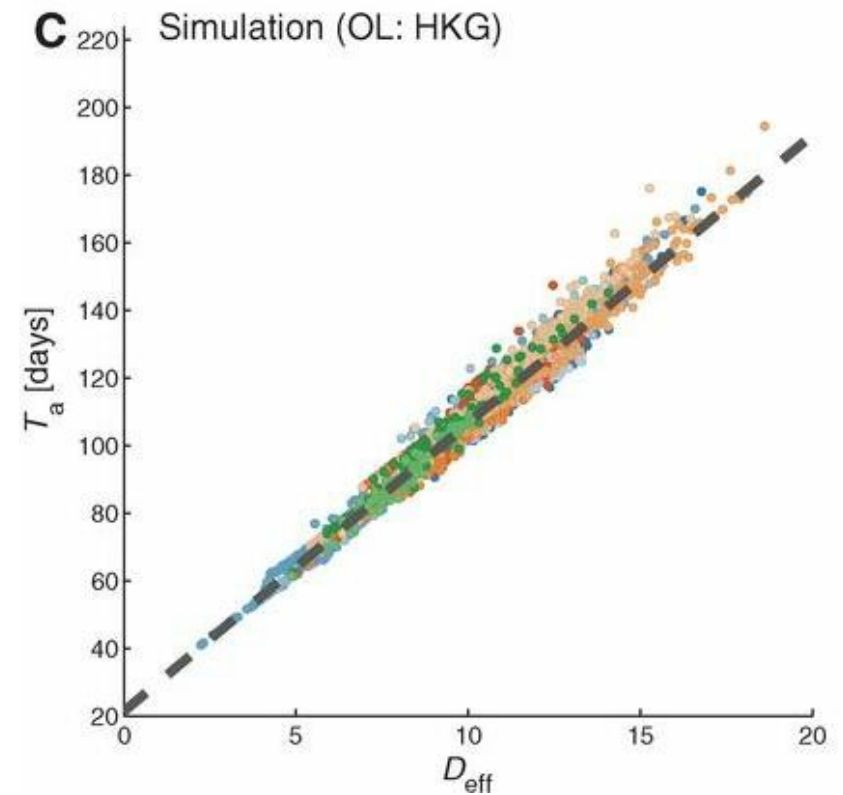
The Hidden Geometry of Complex, Network-Driven Contagion Phenomena

DIRK BROCKMANN AND DIRK HELBING

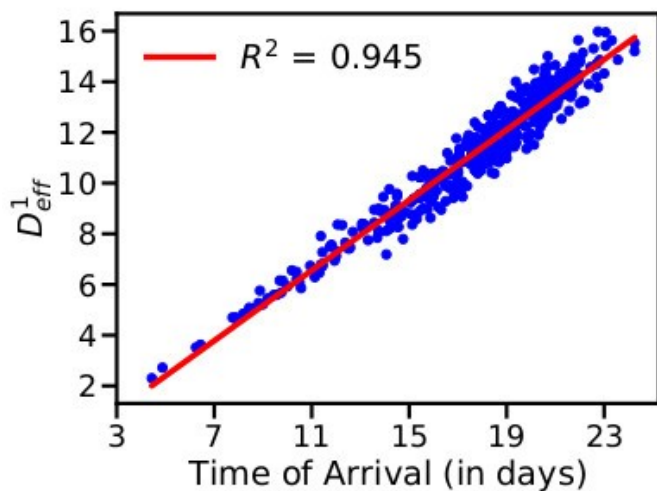
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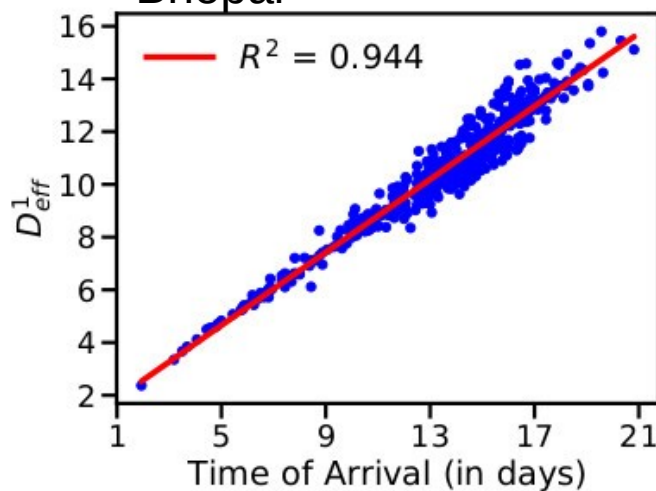
Based on airline traffic data



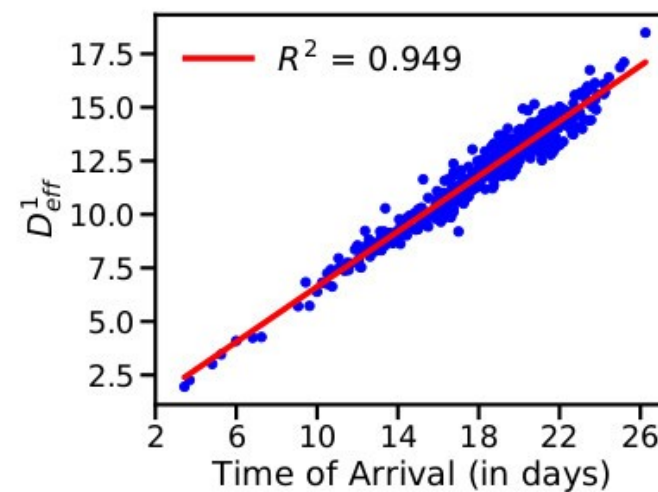
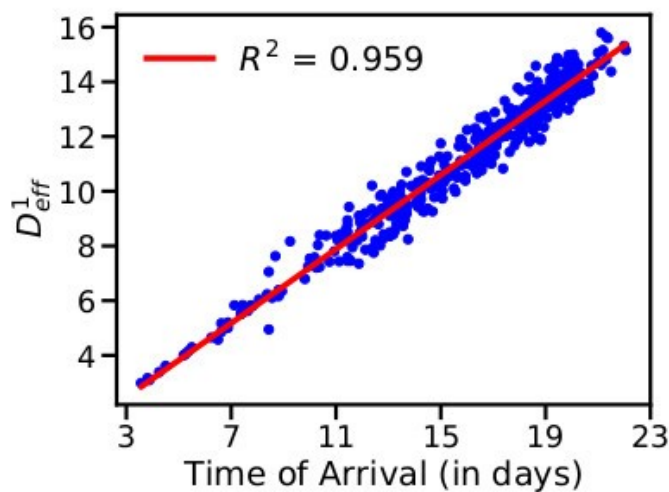
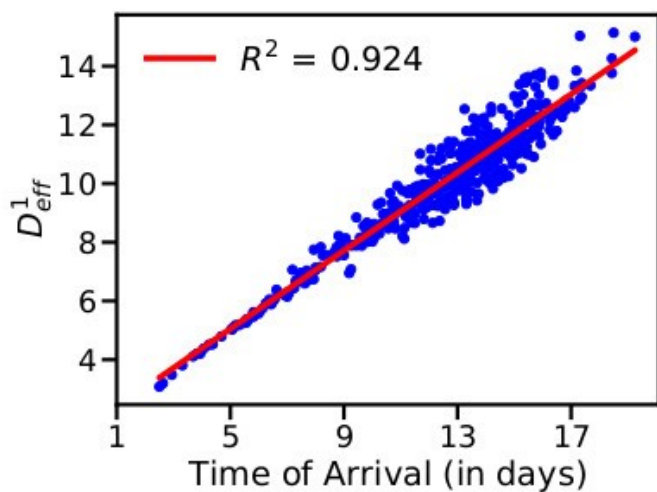
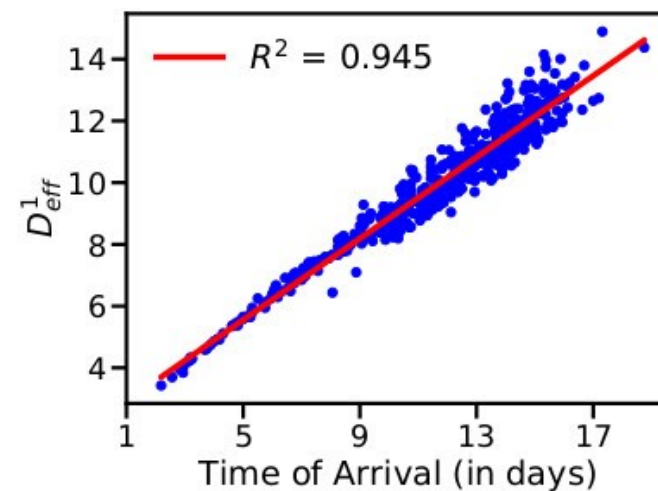
Ahmednagar



Bhopal



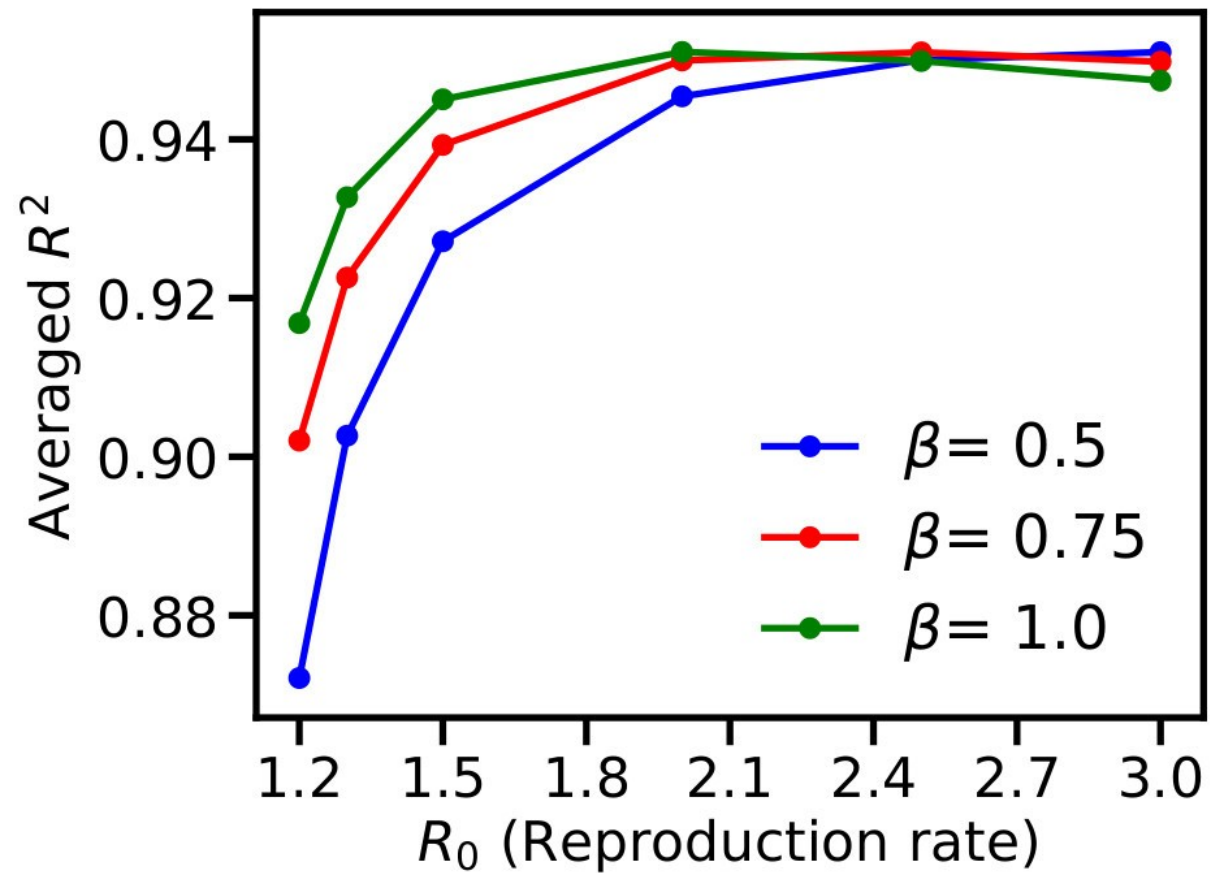
Chennai



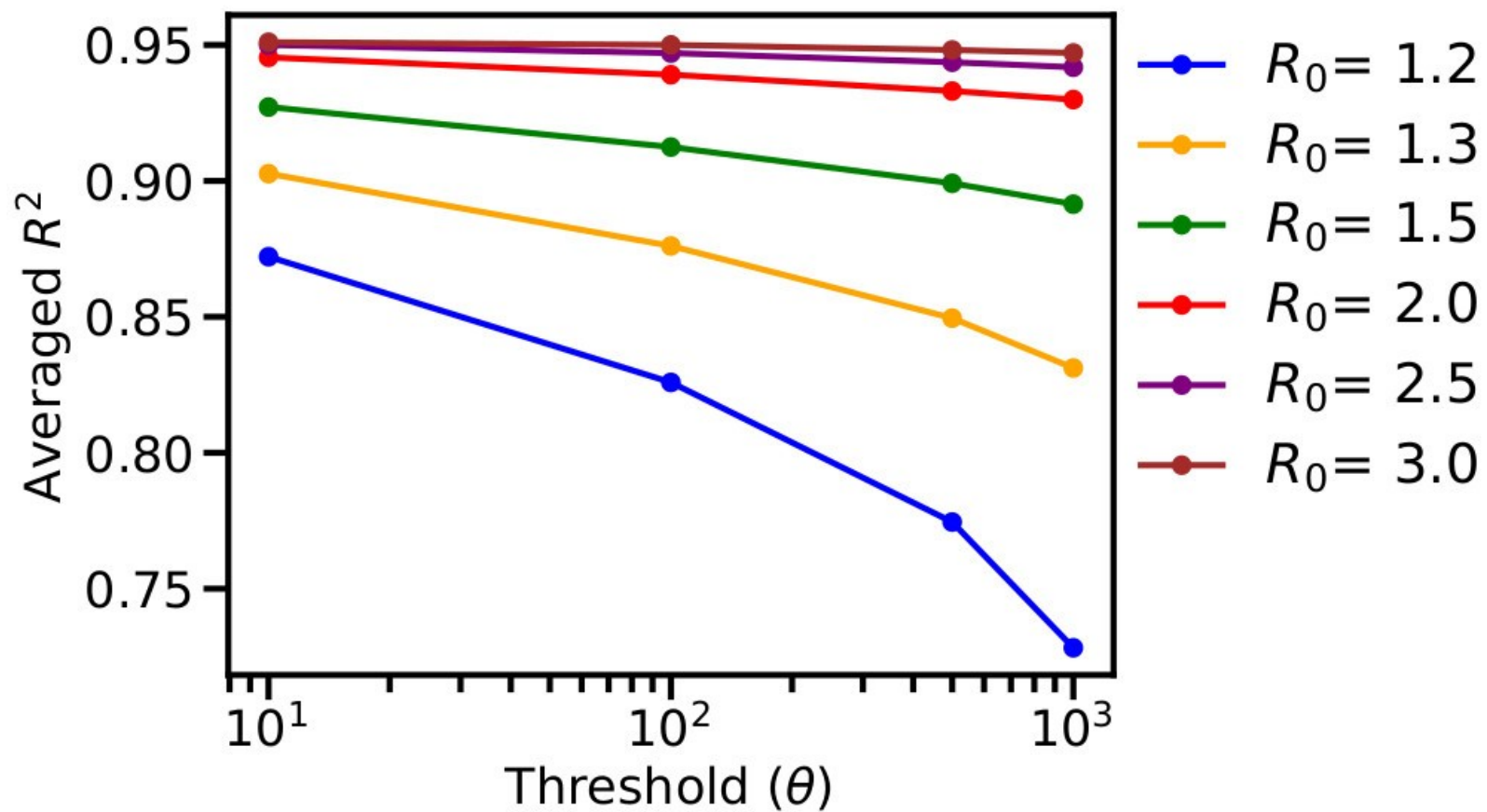
Jaipur

Mangalore

Pathankot



Averaged over all outbreak locations as a function of α and β .



Averaged over all outbreak locations as a function of α and β .

hazard value

$$h = \exp(-D_{\text{eff}})$$

Given an outbreak location, if effective distance is small, infection arrives faster. Hence higher hazard.

Patna	
City	TOA
Gaya	2.06
Dinapur Nizamat	2.50
Arrah	2.75
Delhi	2.81
Bhagalpur	3.25
Kolkata	3.25
Darbhanga	3.88
Biharsharif	3.94
Jehanabad	3.94
Begusarai	4.00
Muzaffarpur	4.06
Varanasi	4.06

Tirupati	
City	TOA
Chittoor	2.88
Chennai	2.88
Hyderabad	3.50
Bangalore	5.06
Vellore	5.31
Tiruvannamalai	5.81
Kadapa	6.50
Vijayawada	6.62
Visakhapatnam	6.75
Anantapur	6.75
Madanapalle	6.75
Nellore	7.06

Mumbai	
City	TOA
Thane	1.00
Pune	1.19
Delhi	1.62
Ahmedabad	2.00
Surat	2.00
Pimpri Chinchwad	2.19
Nashik	2.25
Bangalore	2.38
Vasai	2.38
Hyderabad	2.56
Chennai	2.94
Vasco Da Gama	3.00

TOA in days

hazard value

$$h = \exp(-D_{\text{eff}})$$

Given an outbreak location, if effective distance is small, infection arrives faster. Hence higher hazard.

Hazard rank

Patna		
	City	TOA
1	Gaya	2.06
2	Dinapur Nizamat	2.50
3	Arrah	2.75
4	Delhi	2.81
5	Bhagalpur	3.25
	Kolkata	3.25
	Darbhanga	3.88
	Biharsharif	3.94
	Jehanabad	3.94
	Begusarai	4.00
	Muzaffarpur	4.06
	Varanasi	4.06

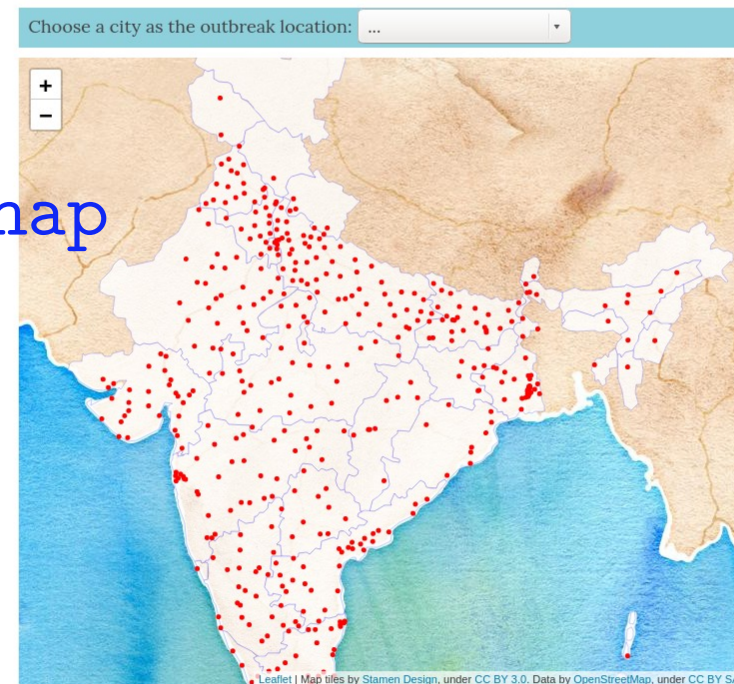
Tirupati		
	City	TOA
1	Chittoor	2.88
2	Chennai	2.88
3	Hyderabad	3.50
4	Bangalore	5.06
5	Vellore	5.31
	Tiruvannamalai	5.81
	Kadapa	6.50
	Vijayawada	6.62
	Visakhapatnam	6.75
	Anantapur	6.75
	Madanapalle	6.75
	Nellore	7.06

Mumbai		
	City	TOA
1	Thane	1.00
2	Pune	1.19
3	Delhi	1.62
4	Ahmedabad	2.00
5	Surat	2.00
	Pimpri Chinchwad	2.19
	Nashik	2.25
	Bangalore	2.38
	Vasai	2.38
	Hyderabad	2.56
	Chennai	2.94
	Vasco Da Gama	3.00

TOA in days



Welcome to the infectious diseases hazard map project. The map below shows 446 cities/towns in India with a population of more than 1 lakh. Hover the mouse over any red dot to get its average hazard rank. Click on it, and you can make that city/town as the outbreak location for an infectious disease. This will take you to the hazard map with the chosen outbreak location. You can also choose the outbreak location using the following dropdown menu.



sites.iiserpune.ac.in/~hazardmap

Choose any city/town as outbreak location
and see the hazard map.

You can get more details [here](#). Average hazard rank indicates the relative risk faced by a city/town due to any infectious disease. Smaller the rank, more the risk. Thus, rank 3 is more at risk compared to rank 4, and so on. For more details, see [arXiv/2105.15123](#). Note that the location of some cities on the map may not be accurate.

Contact: hazardmap@acads.iiserpune.ac.in

Supported by a special MATRICS grant from SERB, Govt. of India, and INSPIRE, DST, Govt. of India and IISER Pune.



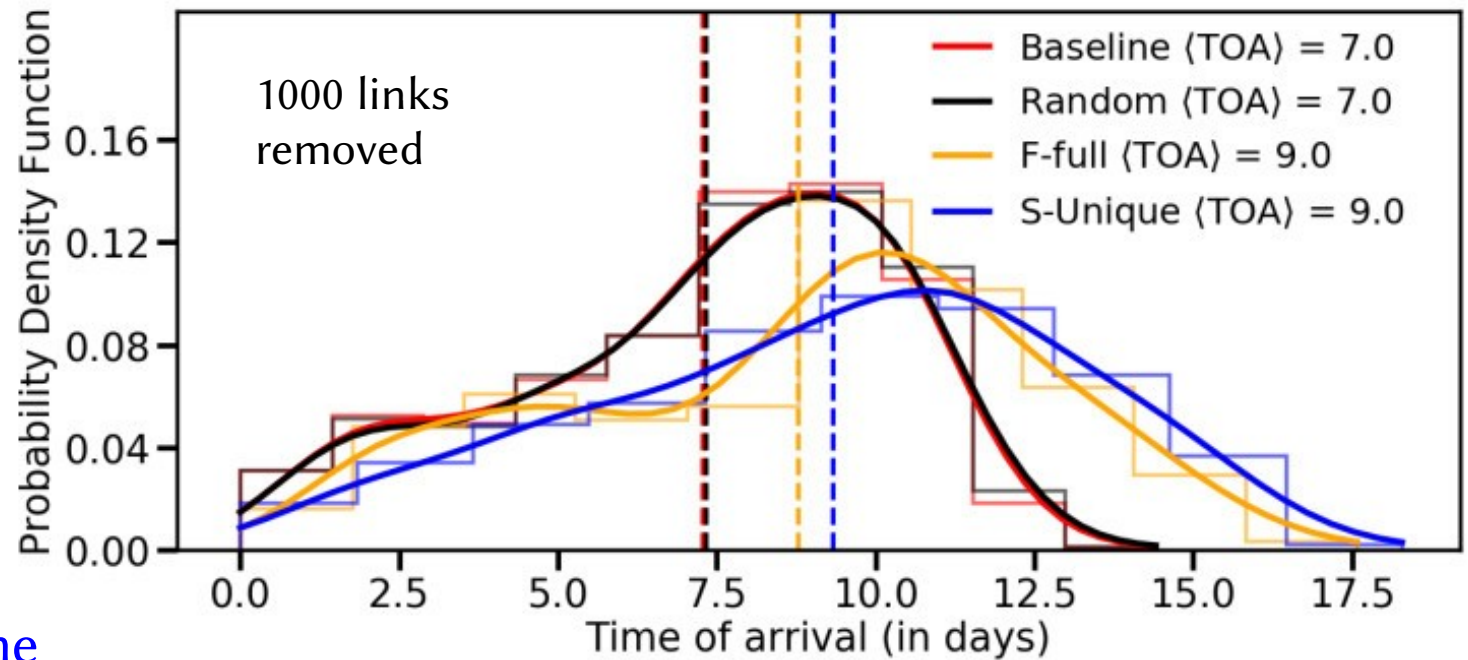
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comparison with real data

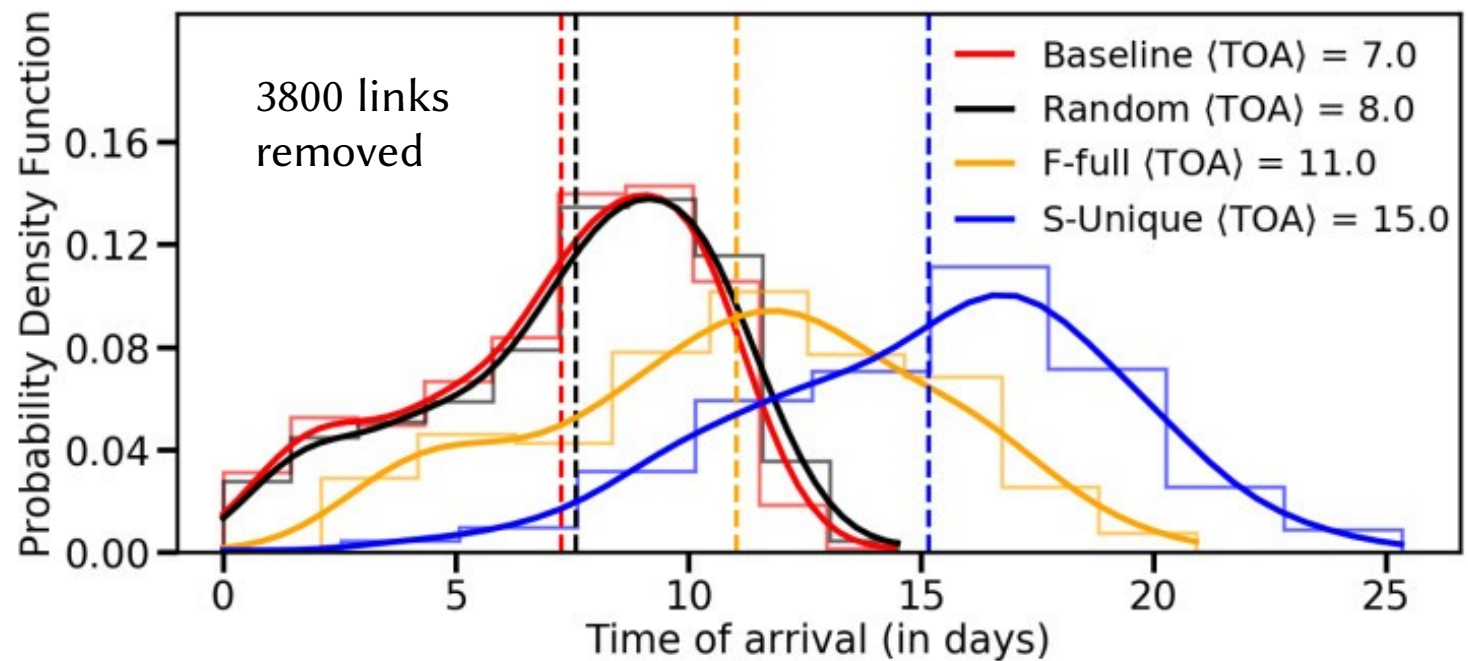
Based on observed data
From March – June 2020

City	Simulation TOA	City	Real TOA
➡ Mumbai	1	Mumbai	1
➡ Pune	22	Delhi	11
➡ Thane	23	Ahmedabad	13
➡ Delhi	25	Chennai	16
➡ Ahmedabad	26	Thane	25
Surat	26	Pune	46
➡ Bangalore	27	Hyderabad	57
Pimpri Chinchwad	28	Bangalore	65
➡ Hyderabad	29	Guwahati	70
➡ Nashik	29	Kolkata	79
Vasai	30	Nashik	85
➡ Chennai	31	Guntur	88

9 out of 12 cities are common to both the lists.



Can we slow down the spread of Infection ?



Bangalore as outbreak location

If β is small, then infection would have spread to all the places by the time $R \rightarrow 0$.

$$S_n + I_n \approx N_n$$

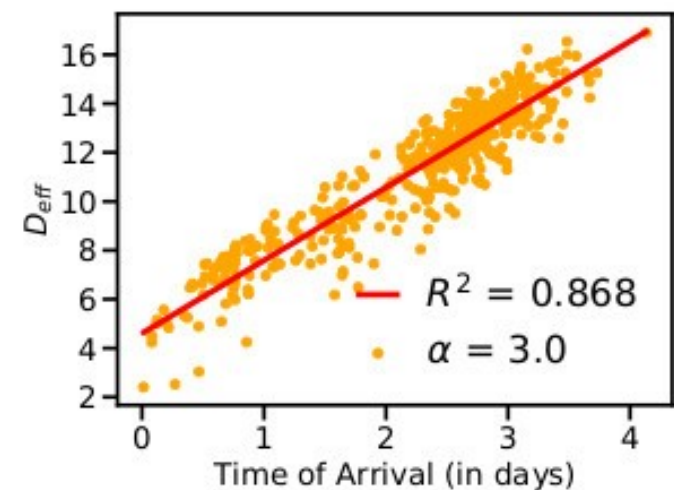
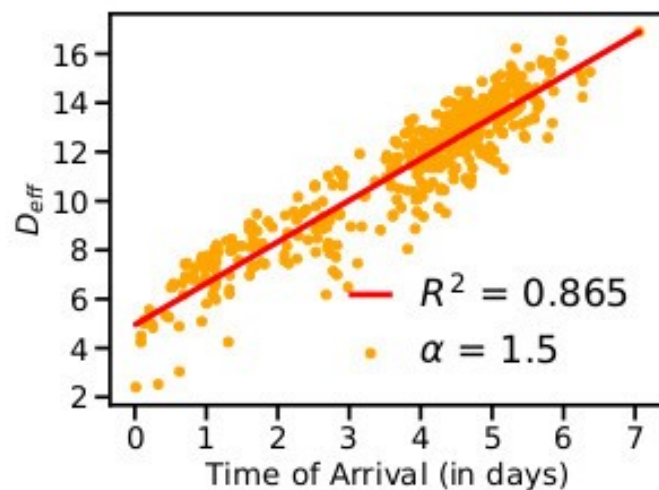
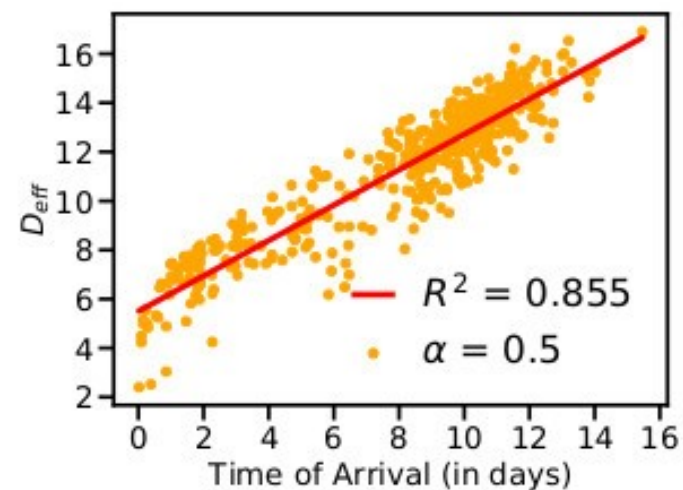
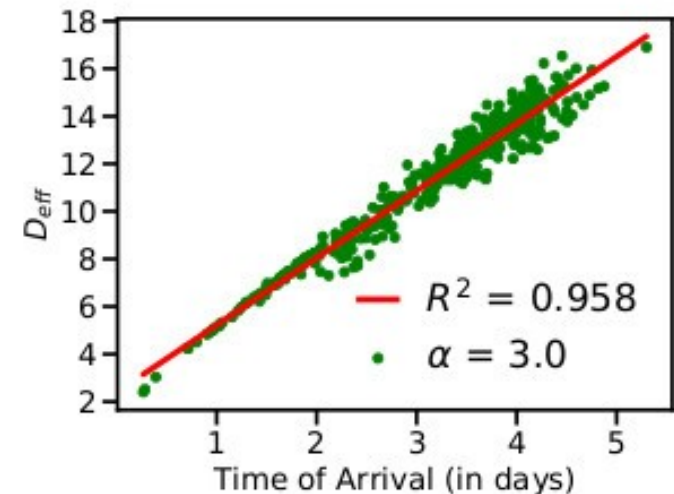
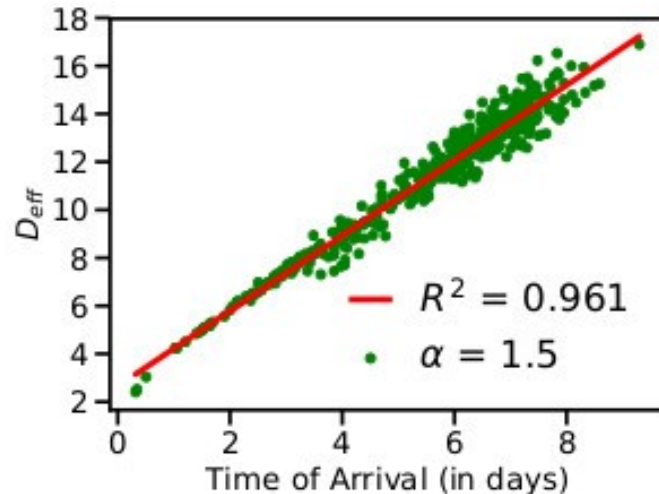
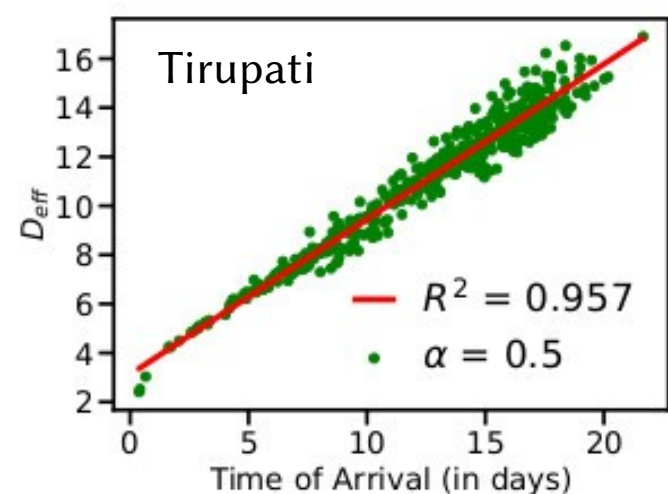
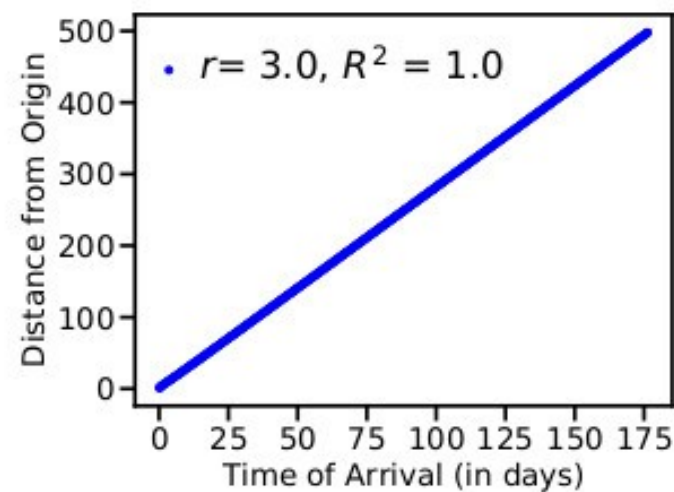
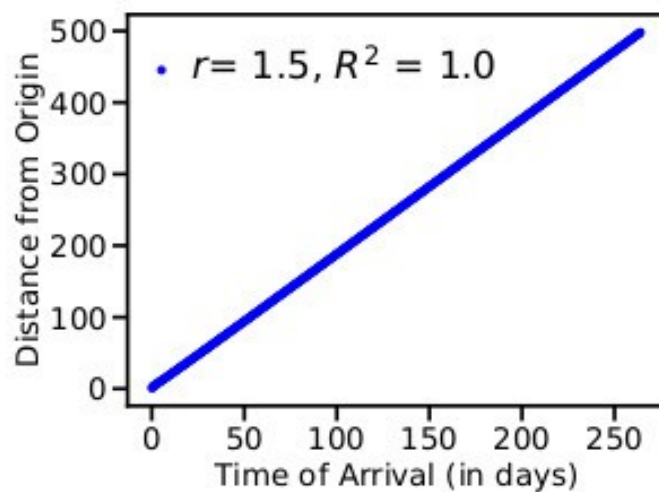
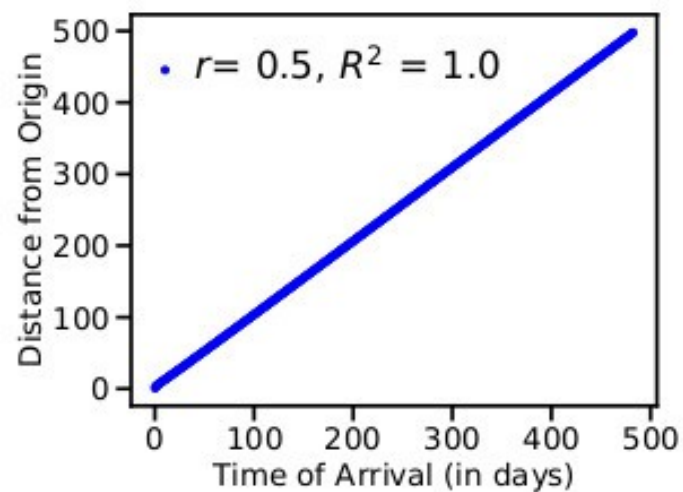
Since city population is a constant, only one variable is involved in the equations.

$$\frac{\partial I_n}{\partial t} = \alpha I_n \left(1 - \frac{I_n}{N_n}\right) - \gamma_n I_n + \sum_m \frac{F_m^n}{N_m} I_m, \quad n, m = 1, 2, \dots, M. \quad (\text{SI model})$$

$$\frac{\partial i_n}{\partial t} = \alpha i_n (1 - i_n) + \frac{1}{N_n} \sum_m \left[\gamma_m P_m^n N_m i_m - \gamma_n P_n^m N_n i_n \right], \quad n, m = 1, 2, \dots, M.$$

$$\frac{\partial u}{\partial t} = ru(1 - u) + D\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u_n}{\partial t} = ru_n(1 - u_n) + D\sum_m \left[\mathcal{P}_m^n u_m - \mathcal{P}_n^m u_n \right], \quad n, m = 1, 2, \dots, M,$$



hazard map

- Effective distance is a useful metric in the context of spreading processes on complex networks.
- A framework created for a hazard map based on mobility data in India.

. . . . and future extensions

- Improve mobility data used in the model. More data from Indian Railways, NHAI, DGCA is needed. Reduce estimates as much as possible.
- This system works for multiple outbreak locations.
- This framework is not specific to COVID-19. It is a useful resource for other infectious diseases as well.

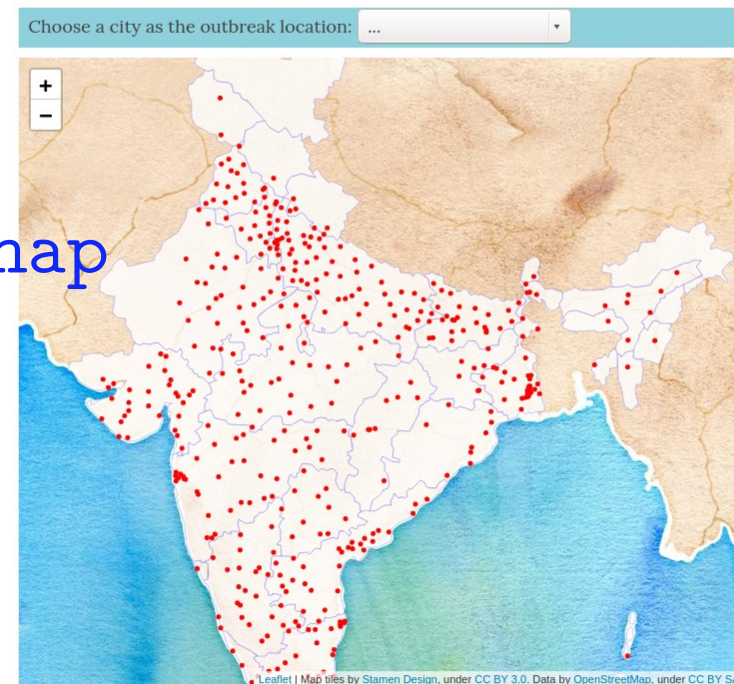


THANK YOU . . .

sites.iiserpune.ac.in/~hazardmap

- Choose any city/town as outbreak Location and see the estimated hazard for your city/town.
- *Current Science* **121**, 1208 (2021)

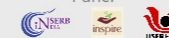
Welcome to the infectious diseases hazard map project. The map below shows 446 cities/towns in India with a population of more than 1 lakh. Hover the mouse over any red dot to get its average hazard rank. Click on it, and you can make that city/town as the outbreak location for an infectious disease. This will take you to the hazard map with the chosen outbreak location. You can also choose the outbreak location using the following dropdown menu.



You can get more details [here](#). Average hazard rank indicates the relative risk faced by a city/town due to any infectious disease. Smaller the rank, more the risk. Thus, rank 3 is more at risk compared to rank 4, and so on. For more details, see [arXiv/2105.15123](#). Note that the location of some cities on the map may not be accurate.

Contact: hazardmap@acads.iiserpune.ac.in

Supported by a special MATRICS grant from SERB, Govt. of India, and INSPIRE, DST, Govt. of India and IISER Pune.



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mathematical framework

Movement kinetics :

$$\frac{\partial N_n(t)}{\partial t} = \sum_{m=1}^M \left[W_m^n N_m(t) - W_n^m N_n(t) \right], \quad n, m = 1, 2, \dots, M,$$

Rate of people going from i -th to j -th city

Population in n -th city at time t

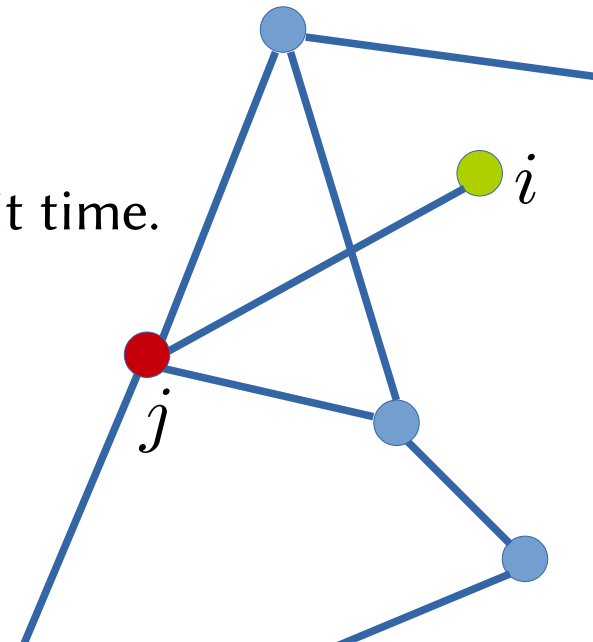
Total number of cities/towns.

- Mobility is accounted through traffic matrix \mathbf{F}

F_i^j : number of people going from i -th to j -th city/town.

$F_i = \sum_j F_i^j$: total number of people leaving i -th city per unit time.

$$\frac{\partial N_n(t)}{\partial t} = \sum_m [F_m^n - F_n^m]$$



Random matrix theory related work :

- Symmetry deduction from spectral fluctuations in complex quantum systems
S. Harshini Tekur and M. S. Santhanam
Physical Review Research (Rapid Communications) **2**, 032063 (2020)
- Scaling in the eigenvalue fluctuations of the empirical correlation matrices
Udaysinh T. Bhosale, S. Harshini Tekur and M. S. Santhanam
Phys. Rev. E **98**, 052133 (2018).
- Higher order spacing ratios in random matrix theory and complex quantum systems
S. Harshini Tekur, Udaysinh T. Bhosale and M. S. Santhanam
Phys. Rev. B. **98**, 104305 (2018).