ROAD VEHICLE DYNAMICS
NOTES: 17, F 11/6

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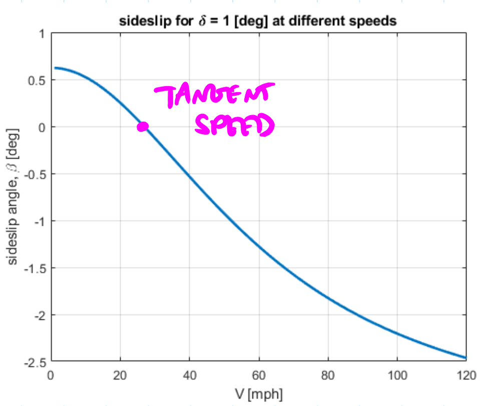
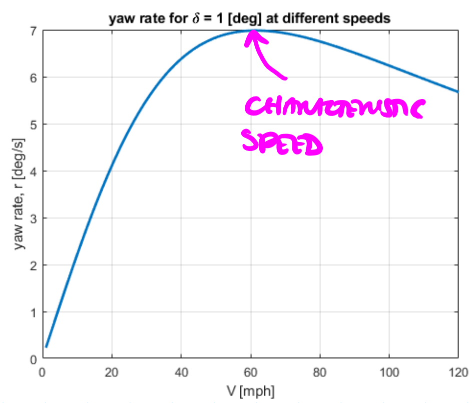
THE UNDERSTEER GRADIENT

- USING OUR ALGEBRAIC EXPRESSIONS FOR STEADY-STATE BEHAVIOR, WE CAN QUICKLY PLOT HOW ALL OF THESE QUANTITIES CHANGE W/ VELOCITY, WITHOUT INTEGRATING THE EQUATIONS OF MOTION A BUNCH OF TIMES!

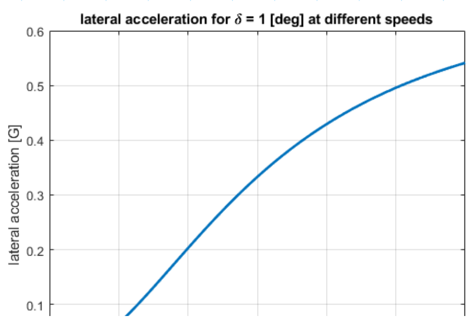
STEADY STATE SIDESLIP VS. SPEED



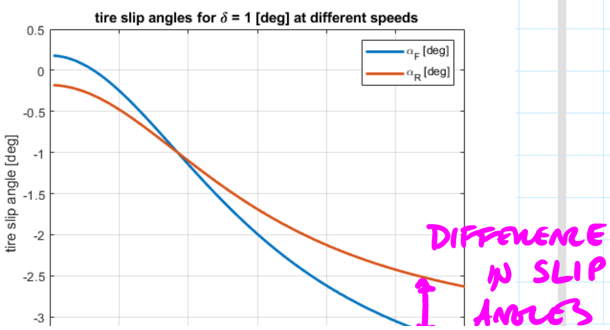
STEADY-STATE YAW RATE VS. SPEED

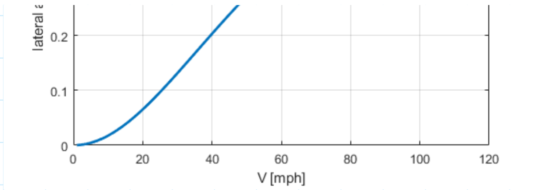


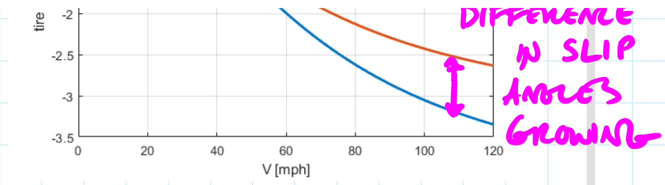
STEADY-STATE LATERAL ACCELERATION VS. SPEED



STEADY-STATE FRONT AND REAR TIRE SLIP ANGLES VS. SPEED





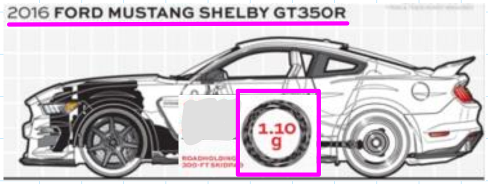


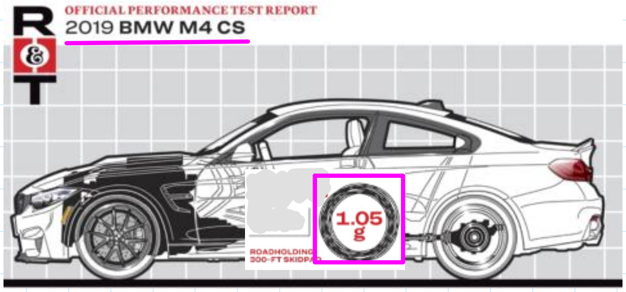
THE UNDERSTEER GRADIENT

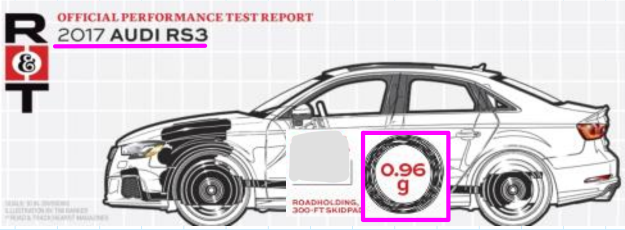
- DURING A "SKID PAD" TEST, A VEHICLE WILL CORNER A CIRCULAR RADIUS AT EVER-INCREASING SPEED.

- ONE GOAL OF THIS TEST IS TO FIND THE MAXIMUM LATERAL ACCELERATION A VEHICLE CAN PRODUCE. BECAUSE, THE LATERAL FORCE MUST INCREASE TO HOLD THE VEHICLE IN THE CORNER AT HIGHER SPEEDS, AND EVENTUALLY THE FRONT OR REAR TIRES WILL LOSE GRIP AND START TO SLIDE!

- ROAD AND TRACK MAGAZINE RUNS 150-ft RADIUS SKID PAD TESTS FOR MANY VEHICLES. BELOW ARE A COUPLE EXAMPLES, W/ MAXIMUM LATERAL ACCELERATIONS, IN [G's].







2008

Car	Skidpad Grip (g)
1.) Katech Chevrolet Corvette Z06 ClubSport	1.12
2.) Dodge Viper SRT10 ACR	1.08
3.) Chevrolet Corvette ZR-1	1.07
4.) Bugatti Veyron	1.00
5.) Nissan GT-R	0.99
6.) *(tie)* Porsche 911 GT2	0.98
Ferrari 599GTB Fiorano	0.98
8.) *(tie)* Lotus Elise SC	0.97
Aston Martin DBS	0.97
Mitsubishi Lancer Evolution GSR	0.97

- ANOTHER PURPOSE OF A SKID-PAD TEST IS TO CHARACTERIZE THE UNDERSTEER GRADIENT OF A VEHICLE.

- THE UNDERSTEER GRADIENT, UG, CAN BE DEFINED AS:
  - "THE CHANGE IN STEADY-STATE STEER ANGLE PER INCREASE IN LATERAL ACCELERATION WHILE CORNERING A CONSTANT RADIUS"

- OR,

$$UG = \frac{\partial \delta^*}{\partial a_y^*}$$

STEADY-STATE STEER ANGLE

STEADY-STATE LATERAL ACCELERATION

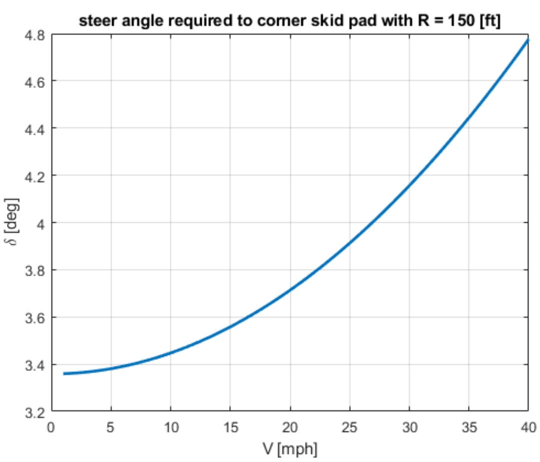
- THERE ARE THREE CLASSIFICATIONS:

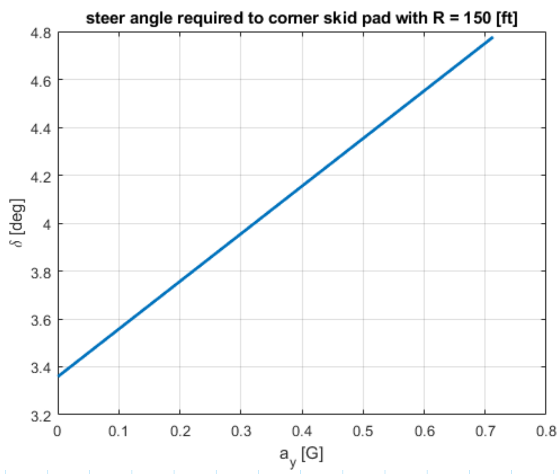
$$UG > 0 \quad \text{"UNDERSTEER"}$$

$$UG = 0 \quad \text{"NEUTRAL STEER"}$$

$$UG < 0 \quad \text{"OVERSTEER"}$$

- LET'S CONSIDER THE PASSENGER VEHICLE WE WORKED WITH LAST CLASS. HERE, WE LOOK AT THE STEADY-STATE STEERING ANGLE AS WE DRIVE IT AT INCREASING SPEED ON A 150-ft RADIUS SKID-PAD:





- DERIVATION OF THE UNDERSTEER GRADIENT.
    - LET US START WITH THE RELATIONSHIP BETWEEN THE STEADY-STATE LATERAL ACCELERATION AND THE STEADY-STATE STEER ANGLE:

$$\lim_{t\to\infty} a_y = a_y^* = V \cdot \frac{N_\beta Y_\delta - N_\delta Y_\beta}{N_r Y_\beta - N_\beta Y_r + N_\beta m V} \delta^*$$

- OR, EQUIVALENTLY,

$$\delta^* = \frac{1}{V} \frac{N_r Y_\beta - N_\beta Y_r + N_\beta m V}{N_\beta Y_\delta - N_\delta Y_\beta} a_y^*$$

- IF WE WRITE THE LATERAL FORCE DERIVATIVES (Y_β, Y_r, Y_δ) AND YAW MOMENT DERIVATIVES (N_β, N_r, N_δ) IN TERMS OF VEHICLE PARAMETERS,

THIS CAN BE SIMPLIFIED TO:

$$\delta^* = \frac{l}{R} + \frac{m}{l} \cdot \frac{C_{\alpha_R} l_R - C_{\alpha_F} l_F}{C_{\alpha_R} C_{\alpha_F}} a_y^*$$

- THEN, THE <u>UNDERSTEER GRADIENT</u> IS GIVEN BY

$$\boxed{KG = \frac{\partial \delta^*}{\partial a_y^*} = \frac{m}{l} \cdot \frac{C_{\alpha_R} l_R - C_{\alpha_F} l_F}{C_{\alpha_R} C_{\alpha_F}}}$$

- THE KEY COMPONENT IS THE TERM

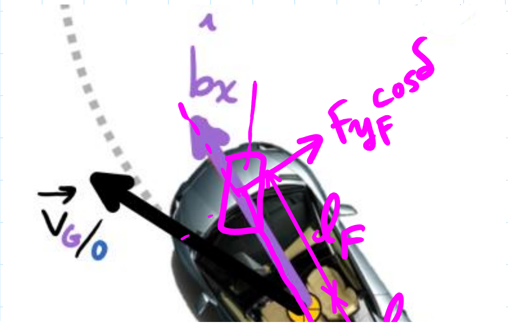
$$C_{\alpha_R} l_R - C_{\alpha_F} l_F$$

WHICH IS ALSO DEFINED AS N_β, THE DIRECTIONAL STABILITY DERIVATIVE.

  - **UNDERSTEER:** $K_{UG} > 0 \rightarrow C_{\alpha_R} l_R > C_{\alpha_F} l_F$
  - **NEUTRAL STEER:** $K_{UG} = 0 \rightarrow C_{\alpha_R} l_R = C_{\alpha_F} l_F$
  - **OVERSTEER:** $K_{UG} < 0 \rightarrow C_{\alpha_R} l_R < C_{\alpha_F} l_F$

- HERE IS A WAY TO THINK ABOUT N_β AND ITS RELATIONSHIP TO UNDERSTEER / OVERSTEER:

  - AT STEADY-STATE, THE YAW MOMENT CREATED BY THE FRONT TIRES MUST BALANCE THE YAW MOMENT CREATED BY THE REAR TIRES (OTHERWISE THE VEHICLE WOULD BE SPINNING OUT ONE DIRECTION OR ANOTHER):



MOMENT BALANCE:

$$\underbrace{l_F F_{y_F} \overbrace{\cos\delta}^{\approx 1}}_{\text{FRONT YAW MOMENT}} = \underbrace{l_R F_{y_R}}_{\text{REAR YAW MOMENT}}$$

FRONT YAW MOMENT REAR YAW MOMENT

$$\rightarrow -l_F C_{\alpha F}\alpha_F = -l_R C_{\alpha R}\alpha_R$$ ← CONDITION FOR MOMENTS TO BALANCE

- FOR UNDERSTEER, $C_{\alpha R} l_R > C_{\alpha F} l_F$

→ THIS EFFECTIVELY MEANS THE REAR TIRES PRODUCE A STRONGER YAW MOMENT, PER SLIP ANGLE, THAN THE FRONT TIRES.

→ FOR MOMENTS TO BALANCE, THE FRONT TIRES MUST HAVE A LARGER SLIP ANGLE THAN THE REAR TIRES

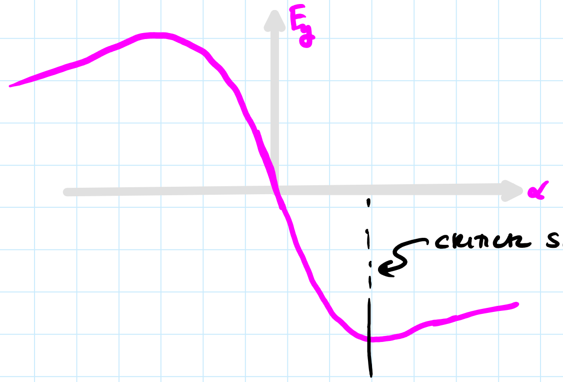
$$\rightarrow |\alpha_F| > |\alpha_R|$$

→ AS SPEED IS INCREASED, THE DIFFERENCE BETWEEN THE SLIP ANGLES MUST GROW!

→ THE LATERAL ACCELERATION LIMIT IS REACHED WHEN THE FRONT TIRES REACH THE CRITICAL SLIP ANGLE, BEYOND WHICH LATERAL TIRE FORCE STARTS TO DECREASE.

→ THE FRONT TIRES GIVE OUT BEFORE THE REAR TIRES FOR

AN UNDERSTEER VEHICLE.



- FOR AN OVERSTEER VEHICLE $C_{\alpha_R} l_R < C_{\alpha_F} l_F$
    - USING SIMILAR REASONING:

→ FRONT TIRES PRODUCE A STRONGER YAW MOMENT PER SLIP ANGLE

→ $|\alpha_R| > |\alpha_F|$

→ THE REAR TIRES WILL GENERALLY REACH THE LIMIT BEFORE THE FRONT TIRES.