Homework 2

1) $\lambda = 0.3$

$\lambda - 0.3 = 0$ — Multiply by $A\lambda^k$

$A\lambda^{k+1} - A\lambda^k 0.3 = 0$

$A\lambda^k = x_k \quad A\lambda^{k+1} = x_{k+1}$

$x_{k+1} = 0.3\, x_k$

2) $\lambda = 0.3 \pm 0.5i$

$\lambda^2 - 0.6\lambda + 0.34 = 0$ → Multiply by $A\lambda^{k-1}$

$A\lambda^{k+1} - A0.6\lambda^k + A0.34\lambda^{k-1} = 0$

$A\lambda^k = x_k \quad A\lambda^{k+1} = x_{k+1} \quad A\lambda^{k-1} = x_{k-1}$

$x_{k+1} = 0.6\, x_k \mp 0.34\, x_{k-1}$

3) $y_k = 1.273 y_{k-1} - 0.81 y_{k-2} + 4_k$ for $k = 0:40$.

$y_k = 1.273 y_{k-1} - 0.81 y_{k-2} + 1$

~~$y_k = A\lambda^k \quad y_{k-1} = A\lambda^{k-1} \quad y_{k+1} = A\lambda^{k+2}$~~

~~$A\lambda^k = 1.273 A\lambda^{k-1} - 0.81 A\lambda^{k-2} + 4$~~

$\lim_{k\to\infty} y_k \to$ 10.8

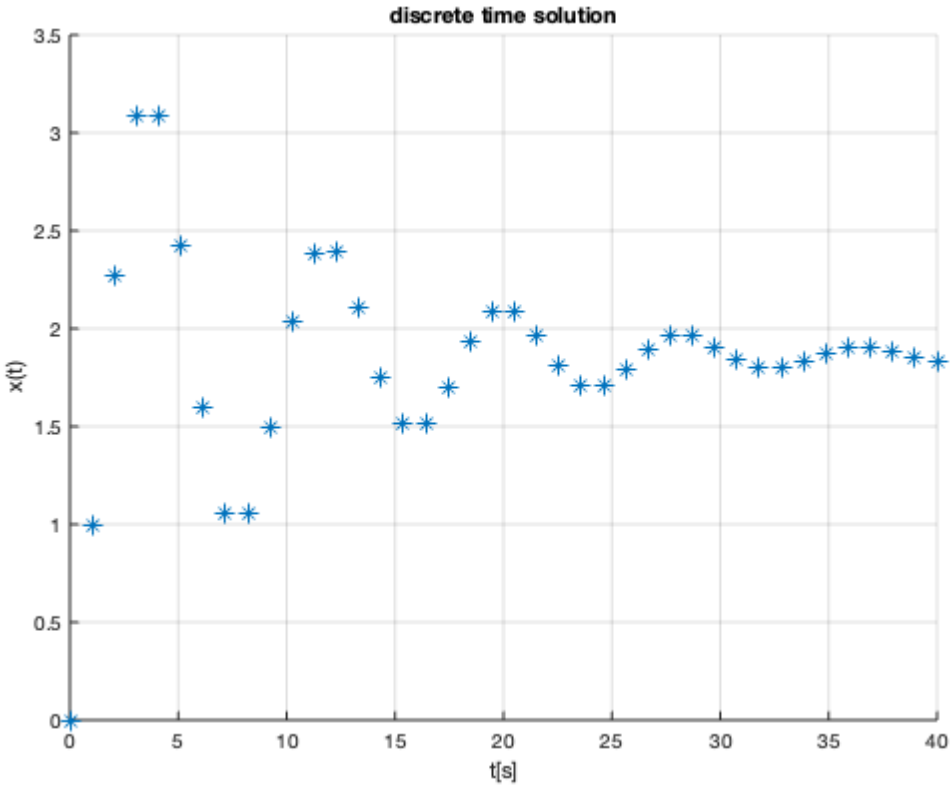
A should be 0.545 for $\lim_{k\to\infty} y_k \to 1$

```
k_1 = linspace(0,40,40);
y_k = zeros(40,1);
y_k(1) = 0;
y_k(2) = 1;
u_k_1 = ones(40,1);

for n = 3:40;

    y_k(n) = 1.273*y_k(n-1)-0.81*y_k(n-2)+u_k_1(n);

end
figure
scatter(k_1,y_k,'*')
ylabel('x(t)')
xlabel('t[s]')
title('discrete time solution')
grid on
```




```
u_k = ones(50,1);
u_k(1) = 1;
u_k(2) = 1;
x_k = zeros(50,1);
x_k(1) = 0;
x_k(2) = 0.6;
for a = 3:50
    x_k(a) = 1.542*x_k(a-1)-0.792*x_k(a-2)+0.6*u_k(a-1)+0.65*u_k(a-2);

end
k = linspace(0,50,50);
t = k*0.5;
```

Problem 4

4) $T = 0.1s$

$t_s = 10s$

$\omega_d = 2\pi$ rad/s

$$t_s = \frac{4}{\zeta \omega_n} \quad (1) \qquad \omega_d = \omega_n \sqrt{1-\zeta^2} \quad (2)$$

$$\omega_n = \frac{4}{\zeta t_s} \rightarrow \text{Put that in } (2)$$

$$\omega_d = \frac{4}{\zeta t_s} \sqrt{1-\zeta^2}$$

$$\frac{\omega_d \cdot t_s}{4} = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\frac{2\pi \cdot 10}{4} = \frac{1-\zeta^2}{\zeta^2} \rightarrow 15.707\,\zeta^2 = 1-\zeta^2$$

$$15.707\,\zeta^2 + \zeta^2 = 1$$

$$\zeta^2 = \frac{1}{16.707}$$

$$\boxed{\zeta = 0.244}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{2\pi}{\sqrt{1-0.24^2}} = 6.683 \frac{rad}{s}$$

$$\lambda_c = 1.630 \pm 2\pi i$$

$$\lambda d = e^{-\xi\omega nT}\cos(\omega d \cdot T) \pm e^{-\xi\omega nT}\sin(\omega d \cdot T)\, i$$

~~$\lambda d = 0.84885 \pm$~~

$\lambda d = 0.687 \pm 0.499\, i$

$(\lambda d - 0.687 + 0.499)(\lambda d - 0.687 - 0.499)$

$\lambda d^2 - 1.374\lambda + 0.5339 = 0$

Multiply by $A\lambda d^{k-2}$

$A\lambda d^{k} - 1.374 A\lambda d^{k-1} + 0.5339 A\lambda d^{k-2}$

$x_k = A\lambda d^{k} \quad x_{k-1} = A\lambda d^{k-1} \quad x_{k-2} = A\lambda d^{k-2}$

$x_k - 1.374\, x_{k-1} + 0.539\, x_{k-2}$

$x_k = 1.3474\, x_{k-1} - 0.539\, x_{k-2}$

Problem 5

5) $T = 0.2s$

$t_R = 1s$

$M_p = 0.2$

$$\zeta = \sqrt{\frac{[\ln(M_p)]^2}{[\ln(M_p)]^2 + \pi^2}} = 0.455$$

$$t_r = \frac{1}{\omega_d}\left(\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$$

$$\omega_d = \frac{1}{t_r}\left(\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$$

$$\omega_d = \frac{1}{1}\left(\pi - \tan^{-1}\left(\frac{\sqrt{1-0.455^2}}{0.455}\right)\right)$$

$$\omega_d = 2.092 \text{ rad/s}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 2.349 \text{ rad/s}$$

$$\lambda d = e^{-\zeta\omega_n T}\cos(\omega_d T) \pm e^{-\zeta\omega_n T}\sin(\omega_d T)\, i$$

$$\lambda d = 0.737 \pm 0.327\, i$$

$$(\lambda d - 0.737 + 0.327i)(\lambda d - 0.737 - 0.327i) = 0$$

$$\lambda d^2 - 1.474\lambda d + 0.650 = 0$$

Multiply $A\lambda^{k-2}$ ~~$R=2$~~

$$A\lambda d^k - 1.474\, A\lambda d^{k-1} + 0.650\, A\lambda d^{k-2}$$

$$A\lambda d^k = x_k \quad A\lambda d^{k-1} = x_{k-1} \quad A\lambda d^{k-2} = x_{k-2}$$

$$x_k = 1.474\, x_{k-1} - 0.650\, x_{k-2}$$

Problem 6

we can see that $M_p = \frac{7.5-5}{5}$

$= 0.5$

$$\zeta = \sqrt{\frac{[\ln(M_p)]^2}{[\ln(M_p)]^2 + \pi^2}} = 0.215$$

$T_d = 6s$

$$\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{6} = 1.047 \text{ rad/s}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 1.072 \text{ rad/s}$$

$T = 0.5s$

$$\lambda_d = e^{-\zeta\omega_n T}(\cos(\omega_d T)) \pm e^{-\zeta\omega_n T}\sin(\omega_d T)\, i$$

$$= 0.771 \pm 0.445i$$

$$(\lambda_d - 0.771 + 0.445i)(\lambda_d - 0.771 - 0.445i) = 0$$

$$\lambda_d^2 - 1.542\lambda_d + 0.792 = 0$$

Multiply $A\lambda_d^{k-2}$

$$A\lambda_d^k - 1.542 A_d^{k-1} + 0.792 A\lambda_d^{k-2} = 0$$

$A\lambda d^R = x_R$

$$x_k - 1.542\, x_{R-1} + 0.792\, x_{R-2} = 0$$

$$x_R = 1.542\, x_{R-1} - 0.792\, x_{R-2}$$

c) $x_{zR} = a\, x_{R-1} + b x_{R-2} + C_1 u_{R-1} + C_2 u_{R-2}$

~~@ k=0 $x_R = 0$ $x_{R-1} =$~~

@ steady state

~~$x \Rightarrow$~~ $x_R \simeq x_{R-1} \simeq x_{R-2} = 5$

$$5 = 1.542(5) - 0.792(5) + C_1(1) + C_2(1)$$

~~5 =~~

$$1.25 = C_1 + C_2$$

@ $k = 1$

~~$x_R = 0$~~ $x_{R-1} = 0$ $x_{R-2} = 0$

$u_{R1} = 1$ $u_{R-2} = 0$

x_R

$= 1.542(0) - 0.792(0) + C_1(1) + C_2(0)$

$C_1 = x_R = 0.6$

hence $C_2 = 0.65$

$$A\lambda_d^k - 1.542A_d^{k-1} + 0.792A\lambda_d^{k-2} = 0$$

$$A\lambda_d^k = x_k$$

$$x_k - 1.542x_{k-1} + 0.792x_{k-2} = 0$$

$$x_k = 1.542x_{k-1} - 0.792x_{k-2}$$

c) $$x_{zk} = ax_{k-1} + bx_{k-2} + C_1u_{k-1} + C_2u_{k-2}$$

~~@ k=0 $x_k = 0$ $x_{k-1} =$~~

@ steady state

~~$x_k \Rightarrow$~~ $x_k \simeq x_{k-1} \simeq x_{k-2} = 5$

$$5 = 1.542(5) - 0.792(5) + C_1(1) + C_2(1)$$

~~5 =~~

$$1.25 = C_1 + C_2$$

@ k = 1

~~$x_k = 0$~~ $x_{k-1} = 0$ $x_{k-2} = 0$

$u_{k-1} = 1$ $u_{k-2} = 0$

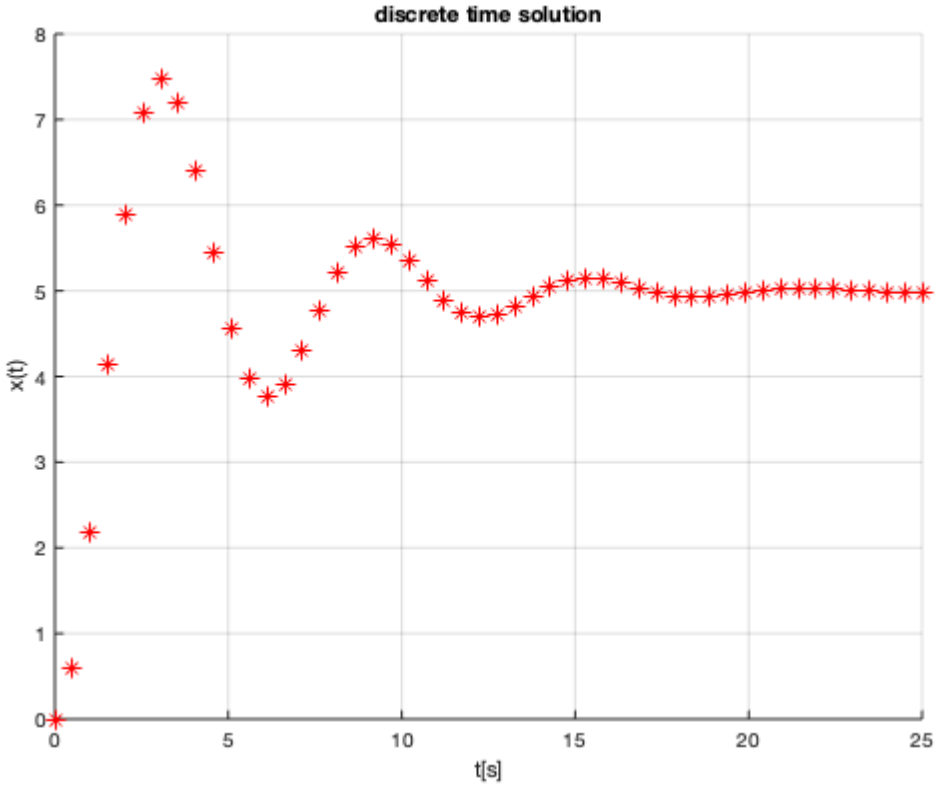
x_k

@ $= 1.542(0) - 0.792(0) + C_1(1) + C_2(0)$

$C_1 = x_k = 0.6$

hence $C_2 = 0.65$

```
figure
scatter(t,x_k,'*','r')
ylabel('x(t)')
xlabel('t[s]')
title('discrete time solution')
grid on
```



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