HW 1

problem 1

a) $z_k = -0.8 \cancel{k-1} z_{k-1}$

Homogeneous solution is of the form

$z_k = A\lambda^k \quad z_{k-1} = A \lambda^{k-1}$

$\cancel{A}\lambda^k = -0.8 \cancel{A}\lambda^{k-1}$

$\lambda^k + 0.8\lambda^{k-1} = 0$

multiply by λ^{1-k}

$\lambda + 0.8 = 0$ — charecteristic equation

b) $x_k = 1.1_{k-1} - 0.3_{k-2}$

$x_k = A\lambda^k \quad x_{k-1} = A\lambda^{k-1} \quad x_{k-2} = A\lambda^{k-2}$

$\cancel{A}\lambda^k = 1.1\cancel{A}\lambda^{k-1} - 0.3\cancel{A}\lambda^{k-2}$

$\lambda^k - 1.1\lambda^{k-1} + 0.3\lambda^{k-2} = 0$

multiply by λ^{2-k}

$\lambda^2 - 1.1\lambda + 0.3 = 0$ — Charecteristic equation

c) $y_k = 1.3816 y_{k-1} - 0.5625 y_{k-2}$

$y_k = A\lambda^k \quad y_{k-1} = A\lambda^{k-1} \quad y_{k-2} = A\lambda^{k-2}$

$\cancel{A}\lambda^k = 1.3816 A\lambda^{k-1} - 0.5625 A\lambda^{k-2}$

$\lambda^k - 1.3816\lambda^{k-1} - 0.5625\lambda^{k-2} = 0$

multiply by λ^{2-k}

$\lambda^2 - 1.3816\lambda - 0.5625 = 0 \rightarrow$ charecteristic equation.

d) $w_k = 2.8380\, w_{k-1} - 2.2323\, w_{k-2} + 0.576\, w_{k-3}$

$w_k = A\lambda^k \quad w_{k-1} = A\lambda^{k-1} \quad w_{k-2} = A\lambda^{k-2}, \quad w_{k-3} = A\lambda^{k-3}$

$A\lambda^k = 2.380\, A\lambda^{k-1} - 2.2323\, A\lambda^{k-2} + 0.576\, A\lambda^{k-3}$

$\lambda^k - 2.380\lambda^{k-1} + 2.2323\lambda^{k-2} - 0.576\lambda^{k-3} = 0$

multiply both sides by λ^{3-k}

$\lambda^3 - 2.380\lambda^2 + 2.2323\lambda - 0.576 = 0$

— charecteristic equation.

a) — roots — $-0.8 \rightarrow$ stable

b) roots — 0.6000 & $0.5000 \rightarrow$ stable

c) roots — 1.7105 & $-0.3289 \rightarrow$ root greater than 1 hence unstable

d) roots — 0.3999 & 0.9901 ± 0.6784

Part 4

a) $\lambda + 0.8 = 0$

$\lambda = -0.8$

$z_k = \cancel{0.8}\, A_1(\lambda)^k$

$z_k = A_1(-0.8)^k$

$z_0 = 10$

$10 = A_1(-0.8)^0$

$A_1 = 10$

$z_k = -10(0.8)^k$

b) ~~xkxk~~

$\lambda^2 - 1.1\lambda + 0.3 = 0$

$\lambda_1 = 0.6 \quad \lambda_2 = 0.5$

$x_k = A_1(0.6)^k + A_2(0.5)^k$

$x_0 = 3 \quad x_1 = 1.3$

$3 = A_1(0.6)^{\cancel{k}0} + A_2(0.5)^0$

$3 = A_1 + A_2$

$1.3 = A_1(0.6)^1 + A_2(0.5)^1$

$1.3 = A(0.6) + A_2(0.5)$

$$\begin{bmatrix} 1 & 1 \\ 0.6 & 0.5 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1.3 \end{bmatrix}$$

$A_1 = -2 \qquad A_2 = +5$

$x_k = -2(0.6)^k + (+5)(0.5)^k$

c) $\lambda^2 - 1.3816\lambda - 0.5625 = 0$

$\lambda_1 = 1.7105 \qquad \lambda_2 = -0.3289$

$y_k = A_1(\lambda_1)^k + A_2(\lambda_2)^k$

$y_k = A_1(1.7105)^k + A_2(-0.3289)^k$

$y_0 = 8 \qquad y_1 = 6.4026$

$\cancel{y_k =}$

$y_0 = A_1(1.7105)^{0} + A_2(-0.3289)^{0} = 8$

$A_1 + A_2 = 8$

$y_1 = 6.4026 = A_1(1.7105)^1 + A_2(-0.3289)^1$

$$\begin{bmatrix} 1 & 1 \\ 1.7105 & -0.3289 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6.4026 \end{bmatrix}$$

$A_1 = 4.4296 \qquad A_2 = 3.5704$

$y_k = 4.4296(1.7105)^k + 3.5704(-0.3289)^k$

d $\lambda^3 - 2.380\lambda^2 + 2.\cancel{3}^{2}323\lambda - 0.576$

$\lambda_1 = 0.3999$

$\lambda_{2,3} = 0.9901 \pm 0.6784i$

magnitude of $\lambda_{2,3} = 1.2002$

Phase $= \pm 0.6008$

$w_k = A_1(0.3999)^k + A_2(1.2002)e^{0.6008ki}$

$+ A_3(1.2002)e^{-0.6008k}$

$w_k = A_1(0.3999)^k + (1.2002)^k(\hat{A}_2\cos(0.6008k)$
$+ \hat{A}_3\sin(0.6008k))$

$w_0 = 8$

$w_1 = 6.152$

$w_2 = 3.089$

$w_0 = A_1(0.3999)^0 + (1.2002)^0(A_2\cos(0) + \hat{A}_3\sin(0)$

~~w_0~~

$8 = A_1 + A_2$

$w_1 = A_1(0.3999)^1 + (1.2002)^1((\hat{A}_2\cos(0.6008)(1)$
$+ \hat{A}_3\sin(0.6008(1)))$

$= 6.152$

$w_2 = A_1(0.3999^2) + (1.2002)^2(A_2\cos(1.2016) +$
$A_3\sin(1.2016))$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0.399 & \overset{0.990}{\cancel{0.678}} & 0.678 \\ 0.159 & 0.519 & 1.343 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6.152 \\ 3.089 \end{bmatrix}$$

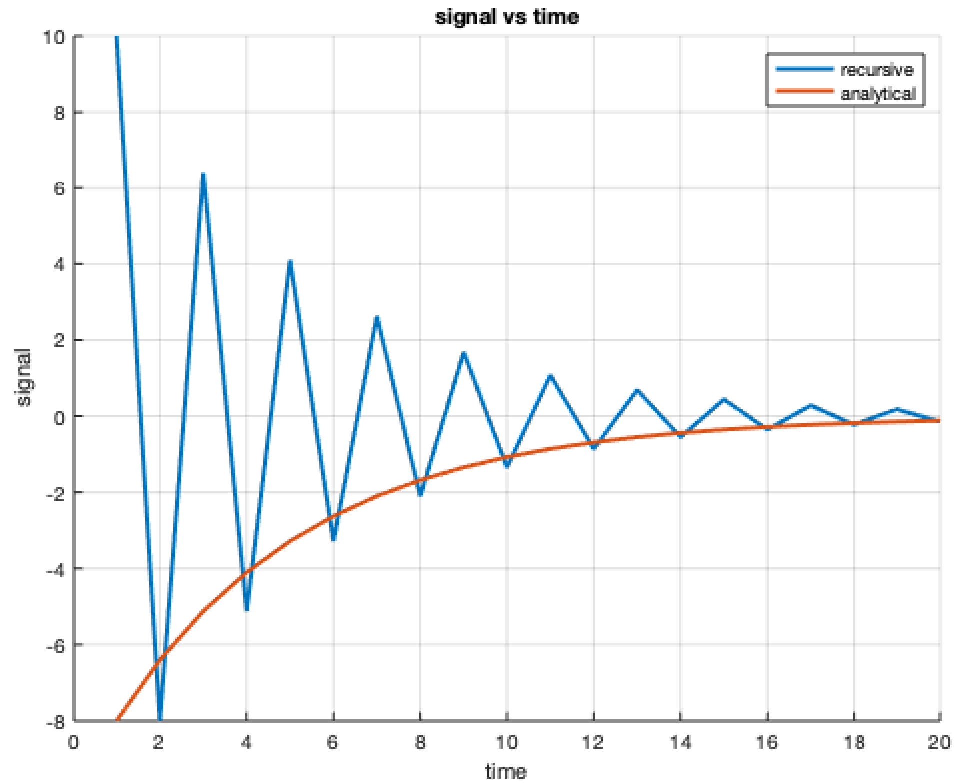
$A_1 = 3.0087$
$A_2 = 4.9913$
$A_3 = 0.0150$

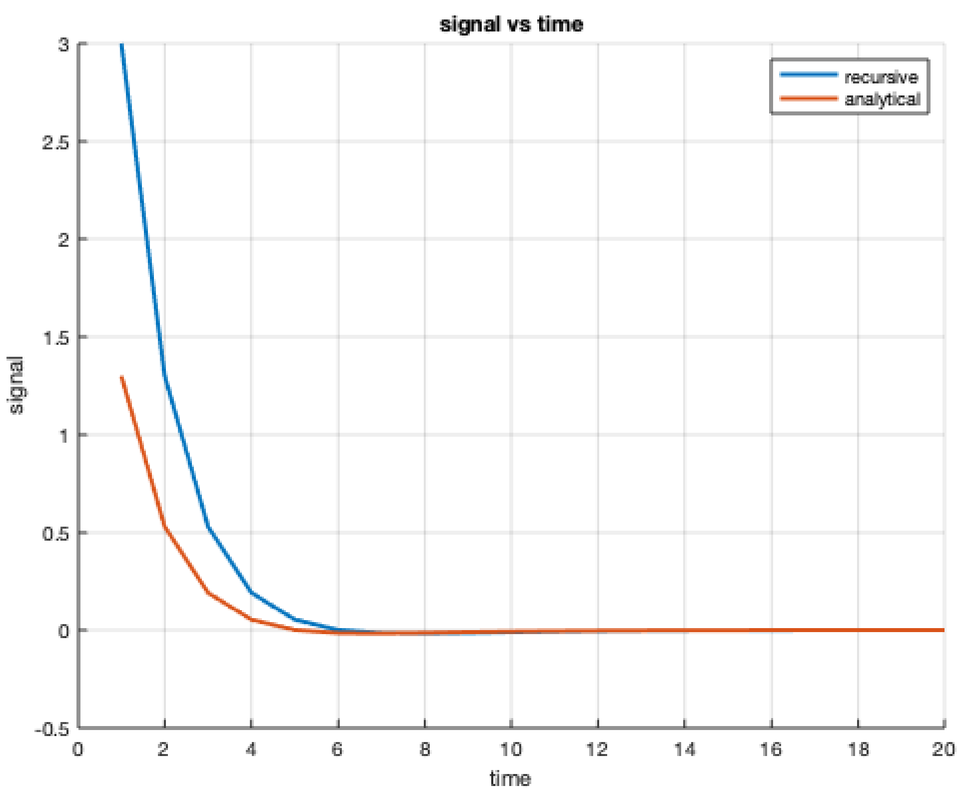
$$\omega_k = (3.0087)(0.399)^k + (1.2002)^k(4.9913\cos(0.6008k)$$
$$+ \cancel{A_3}\ 0.0150\sin(0.6008k)$$

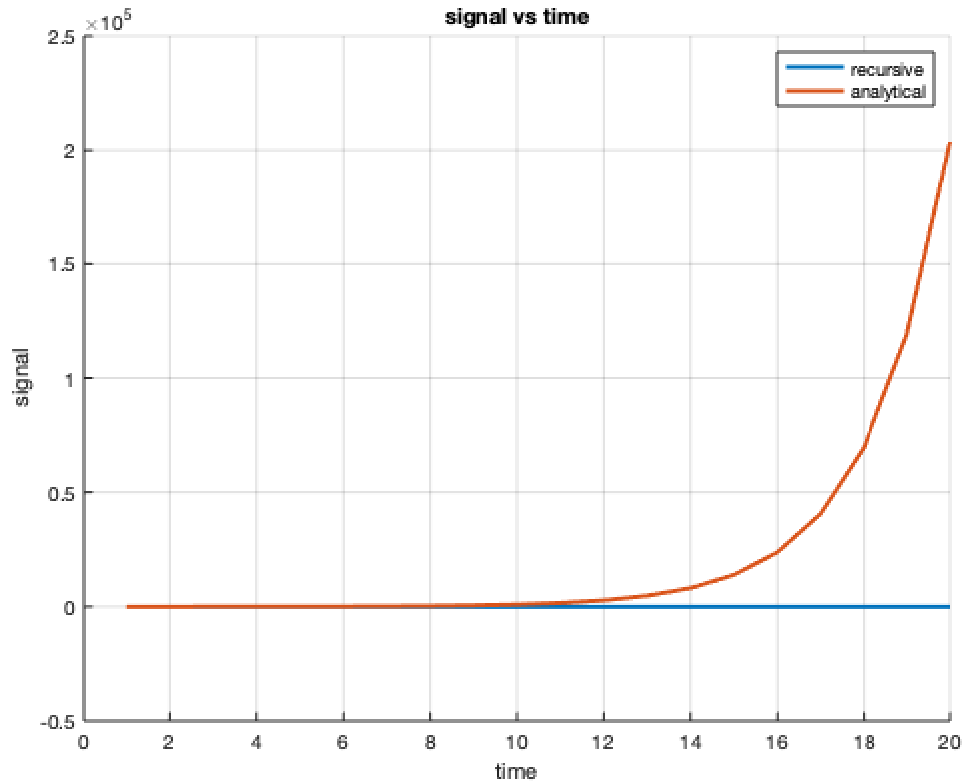
```
z_char = [1,0.8];
x_char = [1,-1.1,0.3];
y_char = [1,-1.3816,-0.5625];
w_char = [1,-2.380,2.2323,-0.576];

z_roots = roots(z_char)
x_roots = roots(x_char)
y_roots = roots(y_char)
w_roots = roots(w_char)

z_k(1)= 10;
x_k(1) = 3;
x_k(2) = 1.3;
y_k(1) = 8;
y_k(2) = 6.4026;
w_k(1) = 8;
w_k(2) = 6.152;
w_k(3) = 3.089;

for z = 2:20;
    z_k(z) = -0.8*z_k(z-1);
end

for x = 3:20;
    x_k(x) = 1.1*x_k(x-1)-0.3*x_k(x-2);
end
for y = 3:20;
    y_k(y) = 1.3816*y_k(y-1)-0.5625*y_k(y-2);
end
for w = 4:20;
    w_k(w) = 2.380*w_k(w-1)-2.2323*w_k(w-2)+0.576*w_k(w-3);
end
k = linspace(1,20,20);
z = -10*(0.8).^k;
x = -2*(0.6).^k + 5*(0.5).^k;
y = 4.4296*(1.7105).^k + 3.5704*(-0.3289).^k;
for n = 1:20;
w(n) = (3.0087)*(0.399)^k(n) + (1.2002)^k(n) * (4.9913*cos(0.6008*k(n))+0.0150*sin(0.6008*k(n)));
end
mag_w = abs(w_roots);
phase = angle(w_roots);

figure
hold on
plot(k,z_k,'linew',2)
plot(k,z,'linew',2)
xlabel('time')
ylabel('signal')
title('signal vs time')
legend('recursive','analytical')
grid on
hold off

figure
hold on
plot(k,x_k,'linew',2)
plot(k,x,'linew',2)
xlabel('time')
ylabel('signal')
```

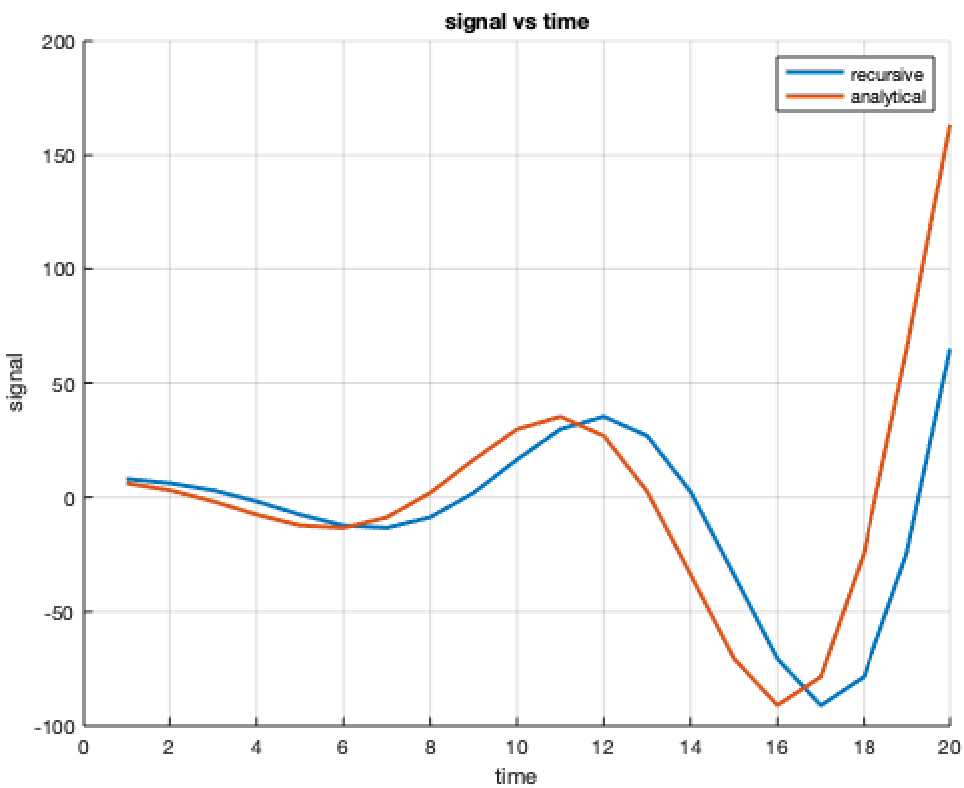
```
title('signal vs time')
legend('recursive','analytical')
grid on
hold off

figure
hold on
plot(k,y_k,'linew',2)
plot(k,y,'linew',2)
xlabel('time')
ylabel('signal')
title('signal vs time')
legend('recursive','analytical')
grid on
hold off

figure
hold on
plot(k,w_k,'Linew',2)
plot(k,w,'linew',2)
xlabel('time')
ylabel('signal')
title('signal vs time')
legend('recursive','analytical')
grid on
hold off
```

```
z_roots =

   -0.8000


x_roots =

    0.6000
    0.5000


y_roots =

    1.7105
   -0.3289


w_roots =

   0.9901 + 0.6784i
   0.9901 - 0.6784i
   0.3999 + 0.0000i
```


signal vs time
recursive
analytical
signal
time
10
8
6
4
2
0
-2
-4
-6
-8
0
2
4
6
8
10
12
14
16
18
20


signal vs time
recursive
analytical
signal
time
3
2.5
2
1.5
1
0.5
0
-0.5
0
2
4
6
8
10
12
14
16
18
20





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