Part 3

for nuetral steer $UG = 0$

$$C_{\alpha f} l_f = C_{\alpha R} l_r$$

$$l_r = w_f l$$

$$l_f = l - l_r$$

$$C_{\alpha f}(l - l_r) = C_{\alpha R}(w_f l)$$

$$C_{\alpha f}(l - w_f l) = C_{\alpha R}(w_f l)$$

$$l - w_f l = \frac{C_{\alpha R}(w_f l)}{C_{\alpha f}}$$

~~Divigs~~ding both sides by w_f

$$\frac{l}{w_f} - l = \frac{C_{\alpha R}\, l}{C_{\alpha f}}$$

$$\cancel{l}\left(\frac{1}{w_f} - 1\right) = \frac{C_{\alpha R}\, \cancel{l}}{C_{\alpha f}}$$

$$w_f = \frac{1}{\frac{C_{\alpha R}}{C_{\alpha f}} + 1}$$

Part 1, 2 ~~&~~ are in the Matlab code.

Part 4

let C_f & C_r be the % of ~~weight~~ cornering stiffness

$$C_f = \frac{C_{\alpha f}}{C_{\alpha f} + C_{\alpha R}}$$

$$C_R = \frac{C_{\alpha R}}{C_{\alpha f} + C_{\alpha R}}$$

for neutral steer

$$C_{\alpha f} l_f = C_{\alpha R} l_r$$

dividing both sides by $C_{\alpha f} + C_{\alpha R}$

$$\frac{C_{\alpha f} l_f}{C_{\alpha f} + C_{\alpha R}} = \frac{C_{\alpha R} l_r}{C_{\alpha f} + C_{\alpha R}}$$

$$C_f l_f = C_R l_r \quad \text{—— ①}$$

we know $C_f + C_R = 1$

$C_R = 1 - C_f$

substituting in ①

$$C_f l_f = (1 - C_f) l_r$$

$$C_f l_f + C_f l_r = l_r$$

$$C_f (l_f + l_r) = l_r$$

$$C_f = \frac{l_r}{l_f + l_r}$$

Part 5

$V_{\cancel{corr}char} = \sqrt{\dfrac{l}{UG}}$

$$UG = \frac{l}{V_{car}^2} \quad — \text{①}$$

we know $UG = \dfrac{m}{l} \cdot \dfrac{C_\alpha l_r - C_{\alpha f} l_f}{C_{\alpha R} C_{\alpha f}}$

& $l_r = w_f l$
$l_f = l - l_r$

Substituting these values in ①

$$\frac{m}{l} \cdot \frac{C_{\alpha R}(w_f l) - C_{\alpha f}(l - w_f l)}{C_{\alpha R} C_{\alpha f}} = \frac{l}{V_{char}^2}$$

$\cancel{w_f (m C_{\alpha R} l - m C_{\alpha f} l + m)}$

$$C_{\alpha R} w_f l + C_{\alpha f} w_f l = \frac{l^2 C_{\alpha R} C_{\alpha f}}{V_{char}^2 \cancel{C_{\alpha R} C_{\alpha f}} \cdot m} + m C_{\alpha f} l$$

$$w_f = \frac{l^2 C_{\alpha R} C_{\alpha f}}{\cancel{C_{\alpha R}^2 C_{\alpha f}^2}} + m C_{\alpha f} l$$

$$w_f = \frac{l^2 C_{\alpha R} C_{\alpha f}}{V_{char}^2 (\cancel{C_{\alpha R} C_{\alpha f} m})(C_{\alpha R} l + C_{\alpha f} l) m} + \frac{\cancel{m} C_{\alpha f} l}{C_{\alpha R} l + C_{\alpha f} l}$$

$$w_f = \frac{l\, C_{\alpha f} C_{\alpha R}}{V_{char}^2 (C_{\alpha R} + C_{\alpha R}) m} + \frac{C_{\alpha f}}{C_{\alpha R} + C_{\alpha R}}$$

Part 6

$$V_{crit} = \sqrt{-\frac{l}{4G}}$$

$$4G = -\frac{l}{V_{crit}^2} \quad \text{—— (1)}$$

$$4G = \frac{m}{l} \cdot \frac{C_{\alpha R} l_r - C_{\alpha f} l_f}{C_{\alpha R}\, C_{\alpha f}}$$

$l_r = w_f l$

$l_f = l - l_r$

substituting these values in (1)

$$\frac{m}{\cancel{4}l} \cdot \frac{C_{\alpha R} l_r - C_{\alpha f} l_f}{C_{\alpha R}\, C_{\alpha f}} = \frac{-\cancel{4}l}{V_{crit}^2}$$

$$m\, C_{\alpha R} l_r - C_{\alpha f} l_f = -\frac{C_{\alpha R} C_{\alpha f}\, l^2}{V_{crit}^2}$$

$$m\, C_{\alpha R}(w_f l) - m C_{\alpha f}(l - w_f l) = -\frac{C_{\alpha R} C_{\alpha f}\, l^2}{V_{crit}^2}$$

$$m\, l\, (C_{\alpha R}\, w_f - (C_{\alpha f}(1 - w_f))) = -\frac{C_{\alpha R} C_{\alpha f}\, l^2}{V_{crit}^2}$$

$$w_f C_{\alpha R} - C_{\alpha f} + w_f C_{\alpha f} = -\frac{C_{\alpha R} C_{\alpha f}\, l^2}{m\, l \cdot V_{crit}^2}$$

$$w_f (C_{\alpha R} + C_{\alpha f}) = -\frac{C_{\alpha R} C_{\alpha f}\, l^{\cancel{2}}}{\cancel{l} m\, V_{crit}^2} + C_{\alpha f}$$

$$w_f = -\frac{C_{\alpha R} C_{\alpha f}\, l}{m V_{crit}^2 (C_{\alpha R} + C_{\alpha f})} + \frac{C_{\alpha f}}{C_{\alpha R} + C_{\alpha f}}$$