HW #4

$$y_k - 0.8y_{k-1} + 0.32y_{k-2} = u_{k-1} - 0.5u_{k-2}$$

a) Taking the z transform.

$$Y(z) - 0.8z^{-1}Y(z) + 0.32z^{-2}Y(z) = U(z)z^{-1} - 0.5\frac{U(z)}{z^{-2}}$$

$$Y(z)(1 - 0.8z^{-1} + 0.32z^{-2}) = U(z)(z^{-1} - 0.5z^{-2})$$

$$\frac{Y(z)}{U(z)} = \frac{z^{-1} - 0.5z^{-2}}{1 - 0.8z^{-1} + 0.32z^{-2}} \times \frac{z^2}{z^2}$$

$$\frac{Y(z)}{U(z)} = \frac{z - 0.5}{z^2 - 0.8z + 0.32}$$

b) controller transfer function.

~~$D(z) =$~~ $U(z) = K E(z)$

$$D(z) = \frac{U(z)}{E(z)} = K$$

c) $G_{OL} = G(z)\,D(z)$

$$G(z) = \frac{(z-0.5)}{z^2 - 0.8z + 0.32}$$

$D(z) = K$

$$G_{OL} = \frac{(z-0.5)K}{z^2 - 0.8z + 0.32}$$

d) $$G_{CL} = \frac{Y(z)}{R(z)}$$

$$Y(z) = G(z)\,U(z)$$
$$= G(z)\,D(z)\,E(z)$$
$$= G(z)\,D(z)\,(R(z) - Y(z))$$

$$\frac{Y(z)}{R(z)} = \frac{G(z)D(z)}{1 + G(z)D(z)}$$

$$= \frac{\dfrac{z-0.5}{z^2-0.8z+0.32}K}{1 + \dfrac{z-0.5}{z^2-0.8z+0.32}K}$$

$$= \frac{kz^3 - 1.3kz^2 + 0.72kz - 0.16kz}{kz^3 + z^4 - 1.3kz^2 - 1.6z^3 + 0.72kz + 1.28z^2 - 0.16k - 0.512z + 0.1024}$$

e) $$\frac{E(z)}{R(z)} = \frac{1}{1 + G(z)D(z)}$$

$$= \frac{1}{1 + \frac{z - 0.5K}{z^2 - 0.8z + 0.32}}$$

$$= \frac{z^2 - 0.8z + 0.32}{Kz + z^2 - 0.5K - 0.8z + 0.32}$$

f(z) $$K_p = \lim_{z \to 1} G(z) * D(z)$$

$$= \lim_{z \to 1} \frac{z - 0.5(K)}{z^2 - 0.8z + 0.32} = \frac{-0.5K}{0.52}$$

$$K_p = 0.96 \Rightarrow K \simeq K$$

$$\lim_{k\to\infty} e_k \quad \cancel{E(z) = F} = \lim_{z\to 1} \frac{\cancel{z-1}}{\cancel{z}} \left(\frac{1}{1+G(z)D(z)}\right)\left(\frac{z}{z-1}\right)$$

$$\lim_{k\to\infty} e_k = \lim_{z\to 1} \frac{z^2 - 0.8z + 0.32}{Kz + z^2 - 0.5K - 0.8z + 0.32}$$

$$\lim_{k\to\infty} e_k = \frac{1 - 0.8 + 0.32}{k + 1 - 0.5k - 0.8 + 0.32} = \frac{0.52}{0.5k + 0.52}$$

$$\lim_{k\to\infty} e_k = \frac{0.52}{0.5kK + 0.52}$$

g) $$\lim_{k\to\infty} e_k = \lim_{z\to 1} \left(\frac{\cancel{z-1}}{\cancel{z}}\right)\left(\frac{1}{1+G(z)D(z)}\right)\left(\frac{z}{(z-1)^{\cancel{z}}}\right)$$

$$= \lim_{z\to 1} \frac{1}{1+G(z)D(z)} \cdot \frac{1}{(z-1)} \cdot$$

$$= \lim_{z\to 1} \frac{z^2 - 0.8z + 0.3z}{Kz + z^2 - 0.5K - 0.8z + 0.32} \cdot \frac{T}{(z-1)} = \frac{1 - 0.8 + 0.32}{K - 1 - 0.5K - 0.8 + 0.32} \cdot$$

$$\cancel{= \lim_{z\to 1} \quad z^2 - 0.8z + 0.32}$$

$$= \lim_{z\to 1} \frac{(z^2 - 0.8z + 0.32)T}{Kz^2 + z^4 - 0.5kz - 0.8z^2 + 0.32z - Kz - z^2 + 0.5K + 0.8z - 0.32}$$

$$= \frac{1 - 0.8z + 0.32}{Kz + K - 0.5K - 0.8 + 0.32 - K - 1 + 0.5K + 0.8z - 0.32}$$

$$= \infty$$

$$\lim_{k \to \infty} e_k = \infty$$

$$K_p = \lim_{z \to 1}$$

$$K_v = \frac{1}{T} \lim_{z \to 1} (z-1)\, G(z) D(z)$$

$$K_v = \frac{1}{T} \left(\frac{(1-1)\ (1-0.5)K}{(1)^2 - 0.8z + 0.32} \right)$$

$$K_v = \frac{1}{T} \left(\frac{0}{1 - 0.8 + 0.32} \right) = 0$$

$$K_v = 0$$

Problem 2

$$G(z) = \frac{1}{z-0.2} \qquad D(z) = \frac{0.5}{z-0.3}$$

$$G_{OL} = G(z)D(z) = \frac{1}{(z-0.2)} \cdot \frac{0.5}{(z-0.3)}$$

$$= \frac{0.5}{z^2-0.3z-0.2z+0.06} = \frac{0.5}{z^2-0.5z+0.06}$$

There are two poles so this is a ~~type 2~~ 2nd order system.

b) $$K_p = \lim_{z \to 1} G(z)D(z) = \lim_{z \to 1} \left(\frac{0.5}{1^2-0.5(1)+0.06} \right)$$

$$K_p = 0.893$$

c) To increase K_p you

d) $\lim_{k \to \infty} e_k = \frac{1}{1+K_p} = \frac{1}{1+0.893} = 0.528$

Problem 3

$$D(z) = 1.2 \cdot \frac{z-0.5}{(z-0.3)(z-1)}$$

$$= 1.2 \cdot \frac{z-0.5}{z^2 - z - 0.3z + 0.3}$$

$$= 1.2 \cdot \frac{z-0.5}{z^2 - 1.3z + 0.3}$$

$$\frac{U(z)}{E(z)} = 1.2 \cdot \frac{z-0.5}{z^2 - 1.3z + 0.3}$$

~~$(z^2 - 1.3z + 0.3) E(z) =$~~

$(U(z)(z^2 - 1.3z + 0.3) U(z) = 1.2(z-0.5) E(z)$

~~U(z) U(z)~~ * Multiply both sides z^{-2}

$U(z)(1 - 1.3z^{-1} + 0.3z^{-2}) = 1.2(z^{-1} - 0.5z^{-2}) E(z)$

$$u_k - 1.3u_{k-1} + 0.3u_{k-2} = 1.2(e_{k-1} - 0.5e_{k-2})$$

$$\boxed{u_k = 1.3u_{k-1} - 0.3u_{k-2} + 1.2e_{k-1} - 0.6e_{k-2}}$$

Problem 4

$$G(z) = \frac{1}{(z-0.5)(z-1)} \qquad D(z) = K\,\frac{z+\alpha}{z+\beta}$$

closed loop poles @ $z = 0.5 \pm 0.2i$

$$G_{CL} = \frac{G(z)\,D(z)}{1 + G(z)\,D(z)}$$

$$G_{CL} = \frac{\frac{1}{(z-0.5)(z-1)}\left(\frac{z+\alpha}{z+\beta}\right)}{1 + \left(\frac{1}{(z-0.5)(z-1)}\right)\cdot\frac{z+\alpha}{z+\beta}}$$

$$G_{CL} = \frac{\frac{z+\alpha}{bz^2 + z^3 - 1.5bz - 1.5z^2 + 0.5b + 0.5z}}{1 + \frac{z+\alpha}{bz^2 + z^3 - 1.5bz - 1.5z^2 + 0.5b + 0.5z}}$$

$$G_{CL} =$$

Problem 4

$z^* = 0.5 \pm 0.2i$

z zero of $D(z)$ @ 0.68

$$D(z) = K \frac{z+\alpha}{z+\beta}$$

$$G(z) = \frac{1}{(z-0.5)(z-1)}$$

for zero of $D(z)$ @ 0.68

~~$z+\alpha = 0.68$~~

$z^* = 0.68$

$\alpha \not\equiv -0.68$

$$\therefore D(z) = K \frac{z-0.68}{z+\beta}$$

~~magn The each poles of $D(z)$ @ z^* of~~

$$G(z)D(z) = \frac{1}{(z-0.5)(z-1)} \cdot \frac{z-0.68}{(z+\beta)}$$

$$= \frac{z-0.68}{(z-0.5)(z-1)(z+\beta)}$$

~~$|G(z^*)D(z^*)| =$~~ $\frac{z-0.68}{}$

$$|G(z^*)\ D^*(z^*)| = \left| \frac{(0.5+0.2i)-0.68}{(0.5+0.2i-0.5)(0.5+0.2i-1)(0.5+0.2i+\beta)} \right|$$

$$k\,| = \left| \frac{-0.18+0.2i}{(0.2i)(-0.5+0.2i)(0.5+0.2i+\beta)} \right|$$

$$1 = \frac{-0.18+0.2i}{-0.046-0.008i-0.16i-0.05i}$$

$$1 = \frac{-0.18+0.2i}{-0.046-(0.08+0.16+0.05)i}$$

$$1 = \frac{-0.18+0.2i}{-0.046-(0.13+0.16)i}$$

$$1 = 0.1\ \frac{|-0.18+0.2i|}{|-0.046-(0.13+0.16)i|}$$

$$1 = \frac{\sqrt{(-0.18)^2+(0.2)^2}}{\sqrt{(-0.046)^2+(0.13+0.16)^2}}$$

$$\sqrt{(-0.046)^2+(0.13+0.16)^2} = \sqrt{(0.18)^2+(0.2)^2}$$

square both sides

$$(-0.046)^2 + (0.13 + 0.1b)^2 = 0.269$$

$$0.0016b^2 + 0.0169 + 0.032b + 0.01b^2 = 0.269$$

$$0.0116b^2 + 0.032b = 0.2521$$

$$b = 0.6395 \quad \text{or} \quad b = -3.3981$$

$$D(z) = K\frac{(z - 0.68)}{(z + 0.63)}$$

$$\angle GP(z^*) = \frac{\angle(0.5 \pm 0.2i - 0.68}{\angle 0.5 \pm 0.2i + b} = \pm 111.81$$

$\angle z$

$$\frac{\tan^{-1}\left(\frac{-0.18}{0.2\,1i}\right)}{\tan^{-1}\left(\frac{0.5 + b}{0.2}\right)} = \pm 111.81$$

$$\frac{-41.98}{\tan^{-1}\left(\frac{0.5+b}{0.2}\right)} = 111.81$$

$$\tan^{-1}\left(\frac{0.5+b}{0.2}\right) = -0.3755$$

$$b = 0.501$$