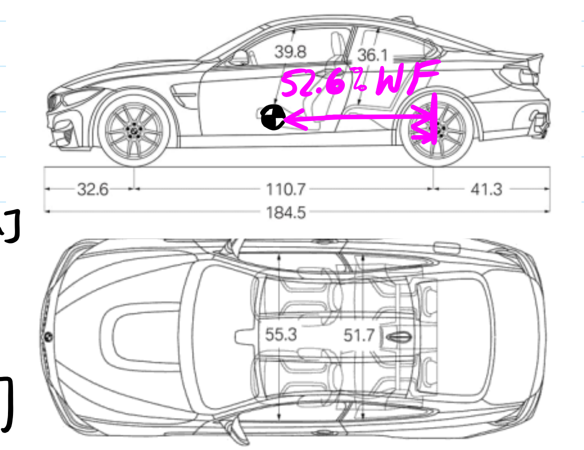
TANGENT SPEED, CHARACTERISTIC SPEED, AND CRITICAL SPEED

- BMW M4 VEHICLE PARAMETERS:



- $m = 1630$ [kg]
- $l = 2.81$ [m]
- $WF = 52.6\% \rightarrow l_F = 1.33$ [m], $l_R = 1.48$ [m]

ESTIMATED FROM SKID PAD:

- $C_{\alpha_F} = 84{,}316$ [N/rad], $C_{\alpha_R} = 91{,}177$ [N/rad]
- $UG = 0.0017 \frac{[\text{rad}]}{[\text{m/s}^2]} \rightarrow UG > 0 \rightarrow$ UNDERSTEER

TANGENT SPEED, V_T

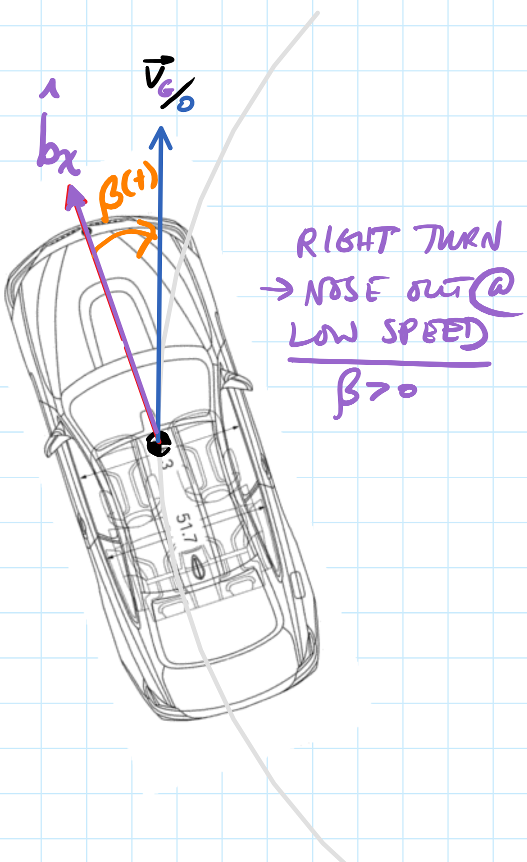
- THE TANGENT SPEED, V_T, IS THE SPEED AT WHICH THE NOSE OF THE VEHICLE IS ALIGNED WITH THE VELOCITY VECTOR ($\hat{b}_x$ IS ALIGNED WITH $\vec{V}_{G/O}$) WHILE CORNERING A CONSTANT RADIUS CORNER, AT CONSTANT SPEED

CORNER, AT CONSTANT SPEED

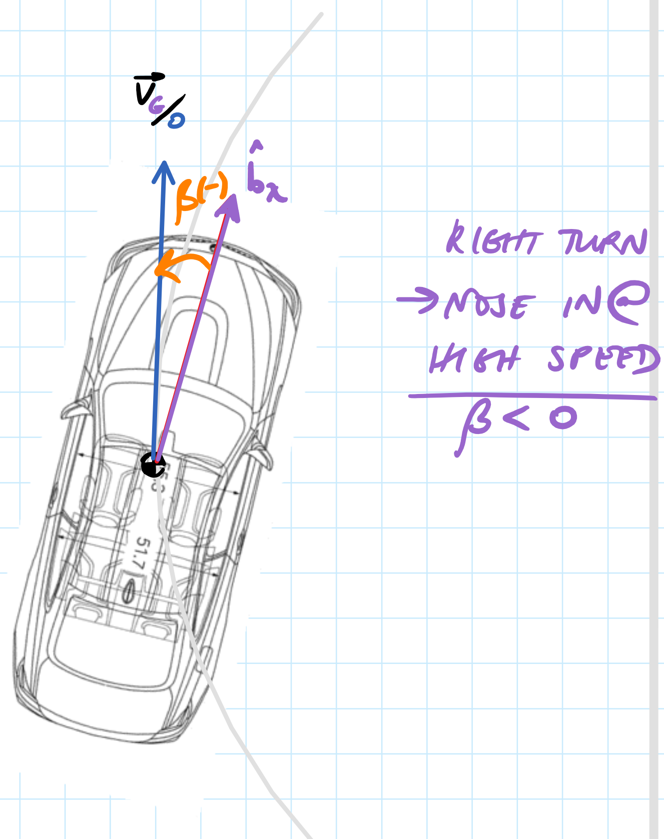
- IN OTHER WORDS, THIS IS THE SPEED AT WHICH A VEHICLE CORNERS WITH <u>NO</u> STEADY-STATE SIDESLIP.

- BELOW THE TANGENT SPEED, A VEHICLE WILL HAVE A "NOSE-OUT" ORIENTATION. ABOVE THE TANGENT SPEED, A VEHICLE WILL HAVE A NOSE-IN ORIENTATION.

BELOW TANGENT SPEED:



ABOVE TANGENT SPEED:



- INTERESTINGLY, THE TANGENT SPEED IS INDEPENDENT OF THE STEER ANGLE (FOR SMALL STEER ANGLES). THIS MEANS, REGARDLESS OF THE RADIUS YOU ARE CORNERING, THE TANGENT SPEED WILL BE THE SAME.

- WE CAN DERIVE THE TANGENT SPEED BY EXAMINING THE CONDITIONS FOR WHICH THE STEADY-STATE SIDESLIP ANGLE IS ZERO!

$$\lim_{t \to \infty} \beta(t) = \lim_{s \to 0} s\beta(s) = \beta^* = G_\beta^* \delta^*$$

- β^* IS ZERO IF $G_\beta^* = 0$. THEREFORE, AT THE TANGENT SPEED

$$G_\beta^* = \frac{N_\delta (Y_r - mV_T) - Y_\delta N_r}{N_r Y_\beta - N_\beta Y_r + N_\beta m V_T} = 0$$

- THEREFORE, THE NUMERATOR MUST BE ZERO

$$N_\delta (Y_r - mV_T) - Y_\delta N_r = 0$$

- ALSO, REMEMBER Y_r AND N_r ARE FUNCTIONS OF THE VEHICLE VELOCITY, SO LET US EXPAND EVERYTHING OUT:

$$C_{\alpha F} l_F \left(\frac{C_{\alpha R} l_R - C_{\alpha F} l_F}{V_T} - m V_T \right) + \frac{C_{\alpha F} \left(C_{\alpha F} l_F^2 + C_{\alpha R} l_R^2 \right)}{V_T} = 0$$

- MULTIPLYING BY V_T, AND DIVIDING BY $C_{\alpha F}$

$$\cancel{C_{\alpha F}} C_{\alpha R} l_F l_R - \cancel{C_{\alpha F}^2 l_F^2} - m V_T^2 \cancel{C_{\alpha F}} l_F + \cancel{C_{\alpha F}^2 l_F^2} + \cancel{C_{\alpha F}} C_{\alpha R} l_R^2 = 0$$

$$\rightarrow C_{\alpha R} l_R \underbrace{(l_F + l_R)}_{l} - m V_T^2 l_F = 0$$

$$\rightarrow \boxed{V_T = \sqrt{\frac{l}{m} \cdot \frac{l_R}{l_F} \cdot C_{\alpha R}}}$$

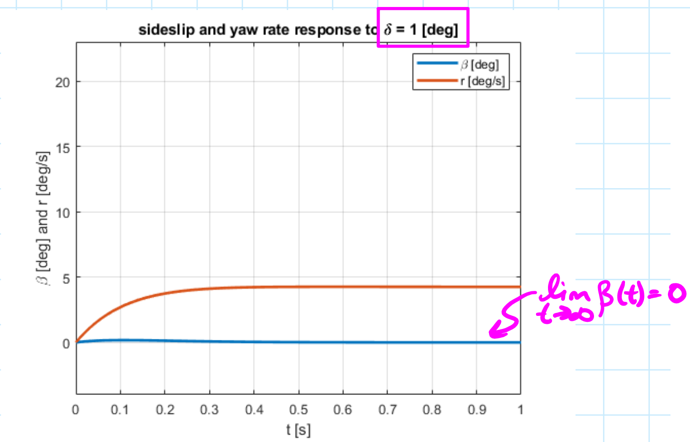
EXAMPLE 19-1: TANGENT SPEED BMW M4

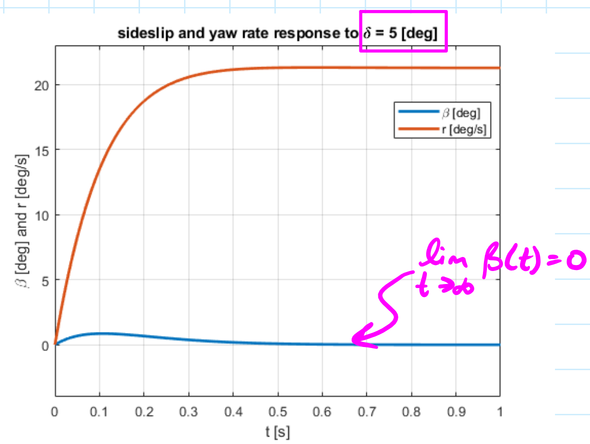
$$V_T = \sqrt{\frac{2.81\,[m]}{1630\,[kg]} \cdot \frac{1.48\,[m]}{1.33\,[m]} \cdot 91{,}177 \frac{[rad]}{[m/s^2]}}$$

$$V_T = \sqrt{\frac{\quad}{1630\,[kg]} \cdot \frac{\quad}{1.33\,[m]} \cdot 41{,}144\,[m/s^2]}$$

$$= \boxed{13.21\,[m/s] = 29.5\,[mph]}$$

- LOOKING AT STEP RESPONSE AT TANGENT SPEED, COMPARING $\delta = 1\,[deg]$ AND $\delta = 5\,[deg]$





- IN BOTH CASES, THE SIDESLIP IS ZERO, EVEN THOUGH THE YAW RATE AND LATERAL ACCELERATION ARE VERY DIFFERENT!

CHARACTERISTIC SPEED, V_{CHAR}

- THE CHARACTERISTIC SPEED IS THE SPEED AT WHICH AN UNDERSTEER CAR PRODUCES

WHICH AN UNDERSTEER CAR PRODUCES THE MAXIMUM YAW RATE PER STEER ANGLE

- THE CHARACTERISTIC SPEED CAN ALSO BE A MEASURE OF THE AMOUNT OF UNDERSTEER A CAR HAS: THE LOWER THE CHARACTERISTIC SPEED, THE MORE UNDERSTEER A CAR IS!
  - ~ THE CHARACTERISTIC SPEED OF MOST PASSENGER CARS IS AROUND 40-50 [mph]

- LET US DERIVE A GENERAL EXPRESSION FOR THE CHARACTERISTIC SPEED OF AN UNDERSTEER VEHICLE.

$$\lim_{t \to \infty} r(t) = \lim_{s \to 0} s\, r(s) = r^* = G_r^* \delta^*$$

- V_{CHAR} IS THE SPEED AT WHICH $G_r^* = \frac{r^*}{\delta^*}$ IS MAXIMIZED

$$G_r^* = \frac{N_\beta Y_\delta - N_\delta Y_\beta}{N_r Y_\beta - N_\beta Y_r + N_\beta m V}$$

$$= \frac{C_{\alpha R} C_{\alpha F}\, l}{C_{\alpha F} C_{\alpha R}\, l^2 \quad (C \quad l \quad C \quad l) \quad V} \cdot \frac{\frac{V}{C_{\alpha F} C_{\alpha R}\, l^2}}{\frac{V}{\quad}}$$

$$= \frac{[\text{illegible}]}{\frac{C_{\alpha F} C_{\alpha R} \ell^2}{V} + \left(C_{\alpha R}\ell_R - C_{\alpha F}\ell_F\right) m V} \cdot \frac{[\text{illegible}]}{\frac{V}{C_{\alpha F} C_{\alpha R} \ell^2}}$$

$$= \frac{\frac{V}{\ell}}{1 + \frac{\left(C_{\alpha R}\ell_R - C_{\alpha F}\ell_F\right) m V}{C_{\alpha F} C_{\alpha R} \ell^2}}$$

$$= \frac{\frac{V}{\ell}}{1 + \boxed{\frac{u_G}{\ell}} \cdot V^2}$$

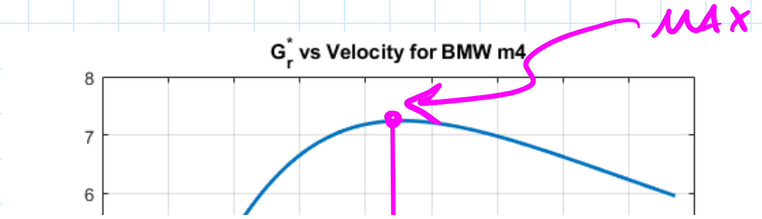
DEFINED AS THE "STABILITY FACTOR", K

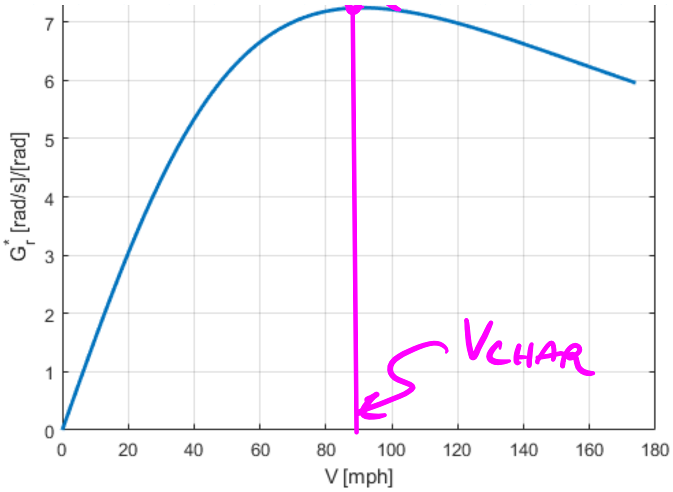
$$K \triangleq \frac{u_G}{\ell}$$

$$\rightarrow G_r^* = \frac{\frac{V}{\ell}}{1 + K V^2}$$

- LET US PLOT G_r^* FOR THE m4:

MAX G_r^*





- BY VISUAL INSPECTION, WE CAN IDENTIFY THE CHARACTERISTIC SPEED SOMEWHERE AROUND 90 [mph], BUT IT IS MORE INSTRUCTIVE TO DERIVE AN ANALYTICAL EXPRESSION FOR V_{CHAR}.

- G_r^* IS MAXIMIZED WHERE $\frac{\partial G_r^*}{\partial V} = 0$

$$\frac{\partial G_r^*}{\partial V} = \frac{\partial}{\partial V}\left(\frac{\frac{V}{l}}{1+kV^2}\right)$$

$$= \frac{1}{l}\left(1+kV^2\right)^{-1} - \frac{V}{l}\left(1+kV^2\right)^{-2} 2kV$$

$$= \frac{1-kV^2}{l\left(1+kV^2\right)^2}$$

$$\left.\frac{\partial G_r^*}{\partial V}\right|_{V=V_{CHAR}} = \frac{1-kV_{CHAR}^2}{\ell\left(1+kV_{CHAR}^2\right)^2} = 0$$

$$\rightarrow 1 - kV_{CHAR}^2 = 0$$

$$\rightarrow \boxed{V_{CHAR} = \sqrt{\frac{1}{k}} = \sqrt{\frac{\ell}{UG}}}$$

- **NOTICE:** V_{CHAR} INCREASES AS UG GETS SMALLER
  - A NEUTRAL STEER CAR HAS $V_{CHAR} \rightarrow \infty$
  - V_{CHAR} DOES NOT EXIST FOR OVERSTEER CARS! (BECAUSE $UG < 0$)

EXAMPLE 19-2: V_{CHAR} FOR BMW m4

- THE UNDERSTEER GRADIENT IS

$$UG = \frac{m}{\ell}\,\frac{C_{\alpha R}\ell_R - C_{\alpha F}\ell_F}{C_{\alpha R}C_{\alpha F}} = 0.0017\ \frac{[\text{rad}]}{[\text{m/s}^2]}$$

- THE STABILITY FACTOR IS:

- THE STABILITY FACTOR IS:

$$k = \frac{\mathcal{U}G}{\ell} = \frac{0.0017\,[\text{rad}]/[m/s^2]}{2.81\,[m]} = 6.0262\times10^{-4}\,\frac{[\text{rad}]}{[m^2/s^2]}$$

- THE CHARACTERISTIC VELOCITY IS:

$$\boxed{V_{CHAR} = \sqrt{\frac{1}{K}} = 40.74\,[m/s] = 91.1\,[mph]}$$

CRITICAL SPEED, V_{CRIT}

- THE CRITICAL SPEED, V_{CRIT}, ONLY APPLIES TO OVERSTEER CARS ($\mathcal{U}G < 0$)
- THE CRITICAL SPEED IS THE SPEED AT WHICH AN OVERSTEER CAR HAS A THEORETICALLY INFINITE YAW RATE IN RESPONSE TO ANY STEERING INPUT (THE SLIGHTEST STEERING DISTURBANCE WILL SPIN OUT THE CAR!)

- MATHEMATICALLY, THIS MEANS

$$G_r^*\Big|_{V_{CRIT}} = \frac{\frac{V_{CRIT}}{\ell}}{1 + K \cdot V_{CRIT}^2} = \infty$$

- THIS GIVES US THE CONDITION

$$1 + K \cdot V_{CRIT}^2 = 0$$

$$\rightarrow \boxed{V_{CRIT} = \sqrt{-\frac{1}{K}} = \sqrt{-\frac{\ell}{u_G}}}$$

THIS GIVES A REAL RESULT ONLY IF $u_G < 0$

EXAMPLE 19-3: MAKING THE m4 OVERSTEER

- WE COULD TRY TO MAKE THE BMW m4 OVERSTEER BY SIGNIFICANTLY INCREASING THE REAR TIRE PRESSURE TO DECREASE THE REAR CORNERING STIFFNESS.

- IN ORDER TO MAKE THE CAR OVERSTEER,

WE NEED

$$N_\beta = C_{\alpha_R} l_R - C_{\alpha_F} l_F < 0$$

$$\rightarrow C_{\alpha_R} l_R < C_{\alpha_F} l_F$$

$$\rightarrow C_{\alpha_R} < \frac{l_F}{l_R} C_{\alpha_F} = \frac{1.3328\,[m]}{1.479\,[m]}\, 84316 \left[\frac{N}{rad}\right]$$

$$\rightarrow C_{\alpha_R} < 75{,}981 \left[\frac{N}{rad}\right]$$

- LET US SAY WE REDUCE C_{α_R} TO $70{,}000 \left[\frac{N}{rad}\right]$.

- THE NEW UNDERSTEER GRADIENT IS

$$UG = \frac{m}{l} \frac{C_{\alpha_R} l_R - C_{\alpha_F} l_F}{C_{\alpha_R} C_{\alpha_F}}$$

$$= -8.686 \times 10^{-4} \quad \frac{[rad]}{[m/s^2]}$$

- THIS GIVES US

$$\boxed{V_{\cdot} = \sqrt{-\frac{l}{\;}} = 56.90\,[m/s] = 127.27\,[mph]}$$

$$\boxed{V_{CRIT} = \sqrt{-\frac{\ell}{uG}} = 56.90\,[m/s] = 127.27\,[mph]}$$

- IF WE PLOT $G_r^* = \frac{r^*}{\delta^*}$ VS. V FOR THIS OVERSTEER CONFIGURATION, WE EASILY SEE THE INSTABILITY