

$$V_{\text{char}} = \sqrt{\frac{l}{u_g}}$$

$$u_g = \frac{l}{V_{\text{char}}^2} \quad \text{--- (1)}$$

We know $u_g = \frac{m}{l} \cdot \frac{C_{\alpha} l_r - C_{\alpha} l_f}{C_{\alpha} C_{\alpha f}}$

$$\begin{aligned} l_r &= w_f l \\ l_f &= l - l_r \end{aligned}$$

Substituting these values in (1)

$$\frac{m}{l} \cdot \frac{C_{\alpha} (w_f l) - C_{\alpha} (l - w_f l)}{C_{\alpha} C_{\alpha f}} = \frac{l}{V_{\text{char}}^2}$$

$$w_f (m C_{\alpha} - m C_{\alpha f}) + m C_{\alpha f} = m$$

$$C_{\alpha} w_f l + C_{\alpha f} w_f l = \frac{l^2 C_{\alpha} C_{\alpha f}}{V_{\text{char}}^2 C_{\alpha} C_{\alpha f} \cdot m} + m C_{\alpha f} l$$

$$w_f = \frac{l^2 C_{\alpha} C_{\alpha f}}{C_{\alpha}^2 C_{\alpha f}^2} + m C_{\alpha f} l$$

$$w_f = \frac{l^2 C_{\alpha} C_{\alpha f}}{V_{\text{char}}^2 (C_{\alpha} C_{\alpha f} m) (C_{\alpha} + C_{\alpha f}) m} + \frac{m C_{\alpha f}}{C_{\alpha} l + C_{\alpha f} l}$$

$$w_f = \frac{l C_{\alpha} C_{\alpha f}}{V_{\text{char}}^2 (C_{\alpha} + C_{\alpha f}) m} + \frac{m C_{\alpha f}}{C_{\alpha} + C_{\alpha f}}$$