Analysis of Electricity Price in Competitive Markets A Study in Singapore Market

G. B. Shrestha, *Senior Member* and Qiao Songbo

Abstract— **Market participants have to deal with constantly changing electricity price in competitive electricity markets in the course of their everyday system operation. Although these prices in a particular market do exhibit some characteristics they are stochastic in nature and the prices cannot be predicted or forecasted for an individual hour. Yet, reasonable predictions or representations of such prices are very crucial for the operations of many market participants, such as Gencos. Many studies have adopted suitable statistical representation of the price in order to carry out system operation problems. However, justification for such models on the basis of real market prices is rarely found.**

This paper presents the initial findings of the study conducted to characterize the statistical behavior of electricity price in competitive markets. The published electricity prices from Singapore market have been used in these studies. The preliminary analysis is carried out using the price data from the first three years after the introduction deregulation in the market. It is used to identify the general behavior of the mean values of electricity price which show that it is possible to establish reasonable relationship between the market price and many system related factors.

Recent historical data from the same market are used to investigate the statistical nature of the electricity price. Several statistical distribution functions are identified through the analyses of the historical price data which are likely to represent the stochastic nature of the electricity price. Graphical and statistical analyses are used for this purpose. Statistical analysis and testing have indicated that the behavior of the market price may resemble log-normal distribution. Further refinements in the representation of the market price are being studied.

Index Terms — **competitive electricity market, electricity price, statistical distribution – normal, log-normal, and Weibull distributions, statistical tests.**

I. Introduction

THE fluctuating market prices has become a crucial factor for the participants in a competitive power market. Pool prices will vary each hour and will be determined by the market clearing. While the level of demand can be reasonably estimated, the availability of generation capacity in the market is less predictable. As the highest marginal bid operating at any point of time determines the pool price, the market price is very uncertain. Uncertainties in the price arise due to various reasons such as fluctuations in plant availability due to outages (generation and transmission), fluctuation in plant capacity due to water inflow uncertainties (hydro) or demand fluctuations due to weather conditions, etc.

The uncertainty in price is especially serious in high load situations or with unexpected generation or transmission outages. These situations are critical to Gencos as a bad strategy can mean a huge loss in profit.
The use of appropriate parameters to represent the market price is very important in developing proper generation schedules. But competitive markets are undergoing constant changes both in terms of market structures and the trading rules. Hence, the parameters can vary significantly from one market to another or even within the same market with evolving structural and regulatory changes.

Proper representation of the spot price distribution is very important for Gencos because it has a direct impact on the performance of the optimal solution provided by the planning models. It should be noted that the statistical distribution of price may not be the same in all markets. Whatever may the distribution of price be, its impact on decision-making is significant. Some studies have investigated the relationship between the market prices and the system operating conditions [1]. The uncertainty in prices has been represented in many ways in different contexts [2-6]. Price taker bidding strategy under price uncertainty has been studied in [2] considering next day price as log-normally distributed [7]. A unique approach to build bidding curves using error scenarios is presented in [3]. Although these price representations/models appear to be conceptually reasonable, there have been few reported studies which verify these models on the basis of actual price data from real power markets.

This paper presents the initial studies conducted to investigate the nature of the stochastic spot price in electricity markets using the actual market prices. The price data from Singapore power market spanning over a numbers have been used for different stages of this study.

The organization of the paper is as follows: Section II details the preliminary analyses of electricity prices highlighting how the relationships could be observed when dealing with the mean values of the prices. . Sections III details the statistical analyses conducted to identify the proper statistical distribution of the electricity price. The summary of findings and conclusions are discussed in

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Section IV. Appendix is included in Section VIII.

II. Preliminary Analysis

The initial study investigated the trends of the electricity price in the market in the first three years since de-regulation, using the actual published electricity prices. The electricity price data were obtained from the website of Singapore Electricity Pool [8] which consisted of half-hourly electricity prices from the first day of de-regulation, 1 April 1998 till 31 December 2000. Salient trends are noted and correlated to relevant parameters such as power demand, time of day and fuel oil price. The scope of analysis was narrowed to study prices behavior during peak hours and off-peak hours of weekdays. A series of statistical tests were conducted to establish various relationships discussed below. Analyses were using MATLAB and Microsoft Excel for data manipulations and necessary statistical analyses.

The average hourly electricity prices in three years are plotted in Figure 1. It may be noted that the shape of the average price roughly follows the shape of the load curve of the system. Cursory look at the trend of the electricity price trend may indicate that the electricity price went down in 1999 but went up significantly higher in the subsequent year 2000. However, a little more analysis incorporating the increase in fuel oil price indicated that the average electricity price had a down ward trend throughout.

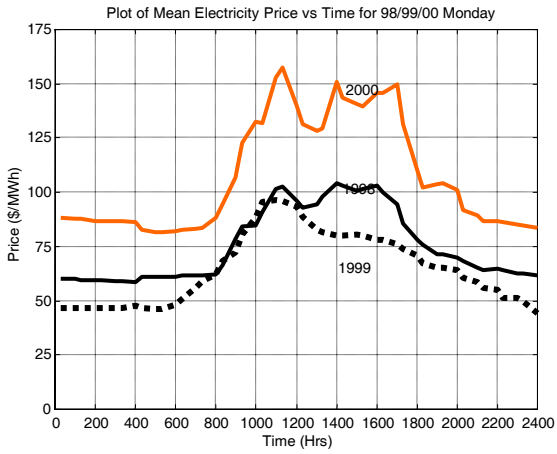

Fig. 1 Plot of mean electricity price vs time for 98/99/00 Monday

The trends of the electricity prices over three years have been studied by investigating the monthly average and the variance of the price as shown in Figure 2. Electricity prices were relatively stable in 1998 and 1999, and one year after de-regulation, 1999 saw a decline in electricity prices. Subsequently, however, electricity prices steadily increased in 2000. It was statistically verified that electricity prices in 1998 was more than those in 1999, whilst electricity prices in 1998 were less than those in 2000. Electricity prices in 2000 became unstable, with exceptionally drastic increases toward the end of the year - October, November and December. It should also be noted that the variance of the price also was quite high in the year 2000, compared to the previous two years.

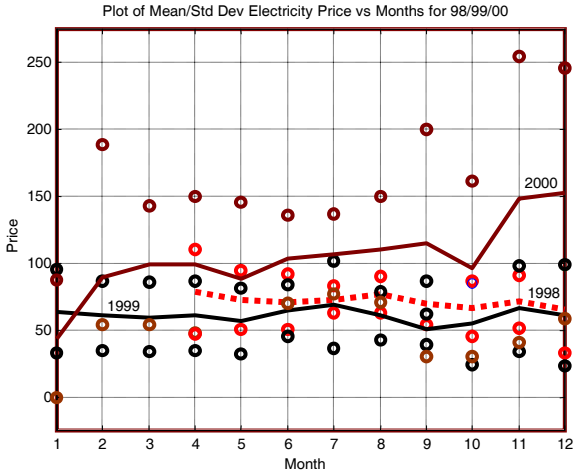

Fig 2. Plot of mean/st dev of electricity-price by months for 98/99/00

As the electricity price is determined by the equilibrium point between the demand and the supply, it is well accepted that the available generation capacity has a large impart on the electricity price. Figure 3 shows the variation of the spare generation capacity and the electricity price (USEP – Uniform Standard Electricity Price) for the entire year 2006. However, the focus of this study is not to study such individual relationships, but to develop suitable stochastic representations for electricity price in the competitive market, which will be dealt in the next section.

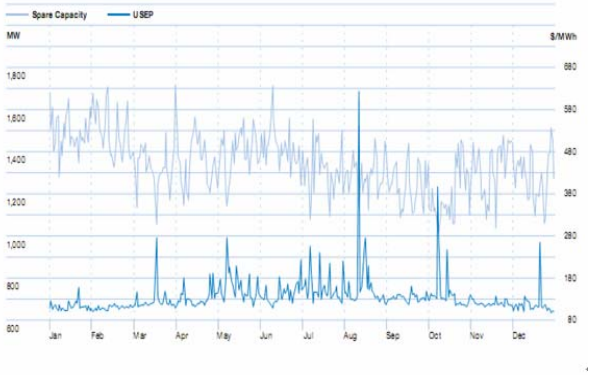

Fig. 3 Variation of spare capacity and USEP in 2006

III. Statistical Analysis of Price Behavior

It is easier to appreciate the relationships observed in Section I, between the electricity price and other relevant factors only when the mean values of these variables are considered. However, it is the individual spot price at a particular interval which is crucial for the market participants for optimizing their system operation, which are observed to be stochastic in nature. Therefore, the proper way to represent the price at any one interval may be to treat it as a stochastic variable and deal with it probabilistically.

Thus it is attempted study the market price as a stochastic behavior and to investigate whether the price follows a particular statistical distribution at any one given period on the basis of the historical data available from actual electricity markets. If the price could be shown to follow a specific distribution, it would be invaluable to represent the

electricity price probabilistically, just using a few parameters such as the mean and the standard deviation which could be readily estimated from sample price data.

This paper discusses the initial finding of an attempt to model the statistical distribution of the electricity price at any one given period on the basis of the historical data available collected from actual electricity markets.

A plot of the price variation at different periods of the day for a full month, along with the mean price, is shown in Figure 4. It shows the electricity prices at half-hourly intervals along with the mean price in each month at every period. Similarly, Figure 5 shows the mean prices for four consecutive weeks. It is found that the mean value of the price at any one given period of the day remains fairly constant. However the values of individual prices show wide variations. For example, the instantaneous price peaks to a high value on Aug 12 at 11 AM reaching above 3000$/MWh. The individual price is very volatile with a very wide range. Therefore, while forecasting the mean values of prices in the near future do not pose serious difficulties, it is the variations of individual price values from the mean, which has been difficult to predict.

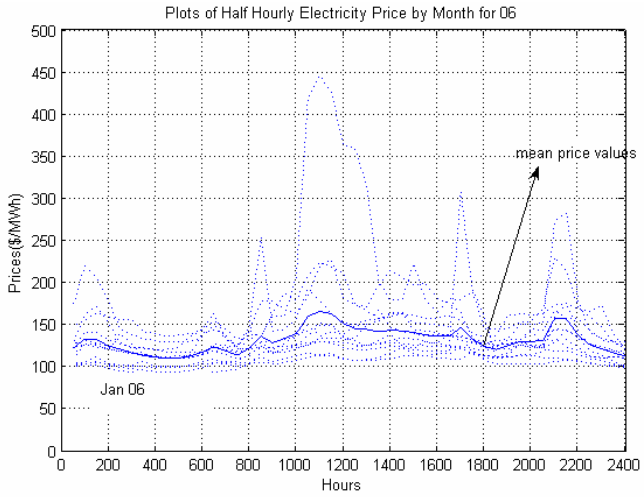

Fig 4. Variation of half hourly electricity price by month

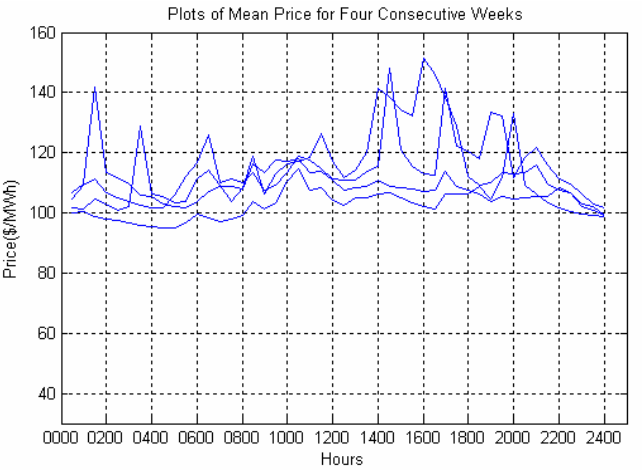

Fig 5. Variation of mean price for 4 consecutive weeks.

Thus it becomes more important to investigate the variation of individual prices from the mean. The histograms approximating the statistical distributions for two consecutive years 2005 and 2006 are shown in Figure 6.

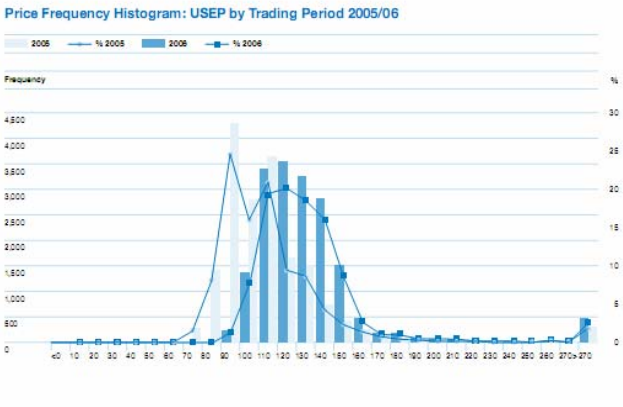

Fig 5. Price frequency histogram

The distribution of the USEP by trading period for 2006 centered on a mean of $32.42/MWh with a standard deviation of $95.68/MWh. The mean as well as the standard deviation for 2006 exhibited was higher indicating higher price dispersion. However, the histograms showed similar characteristics. In order to identify the likely statistical distribution, the histograms were prepared for the prices at different individual time intervals. Figure 7 shows the histogram for period 1.

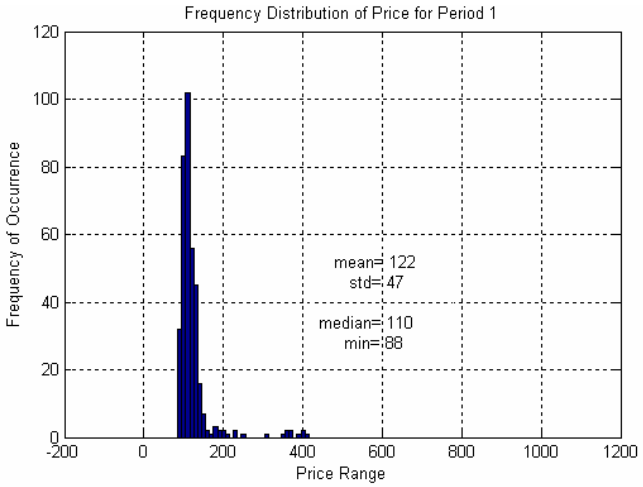

Fig 7. The histogram of price distribution for period 1

Studies conducted on numerous histograms drawn for different periods of the day for various days and months exhibit similar general shape. As presumed in several reported studies, these plots indicate that these histograms fit quite well with to Weibull distribution or lognormal distribution. The basic characteristics, the shape and the parameters of these two distributions along with the Normal distribution are listed in the Appendix.

The statistical parameters of the price data at any one interval could be easily computed from the available data. Then for a presumed statistical distribution, the cumulative probabilities can be easily computed for different price levels. Figure 8 shows the cumulative probabilities for period 1 computed from the actual sample data along with those obtained assuming normal, log-normal, and Weibull

distributions and the mean and standard deviation computed from the actual data. The best match between the actual data and the other distributions can be used to identify the more likely distributions. The series of such plots for different periods also indicate the distribution are more likely to follow log-normal or Weibull.

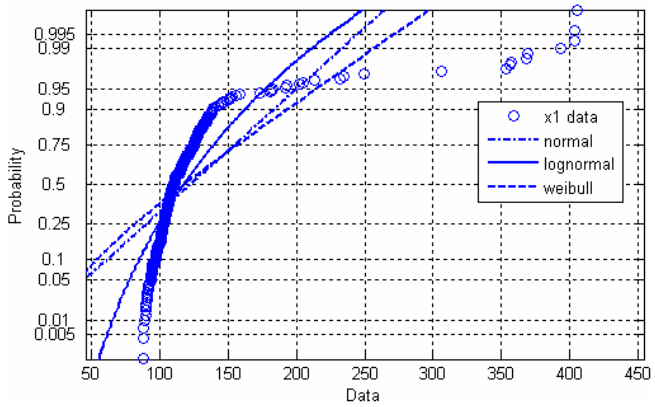

Fig 8. Probability plots of three distributions for period 1

The comparison of the histogram and the density functions using the parameters (ie, the mean and standard deviation) calculated from the actual data will also reveal the better match. The density functions adopting Weibull distribution, Normal distribution, and lognormal distribution along with the actual histogram for period 1 is shown in Figure 9. It is seen that lognormal distribution is the best fit followed by Weibull distribution.

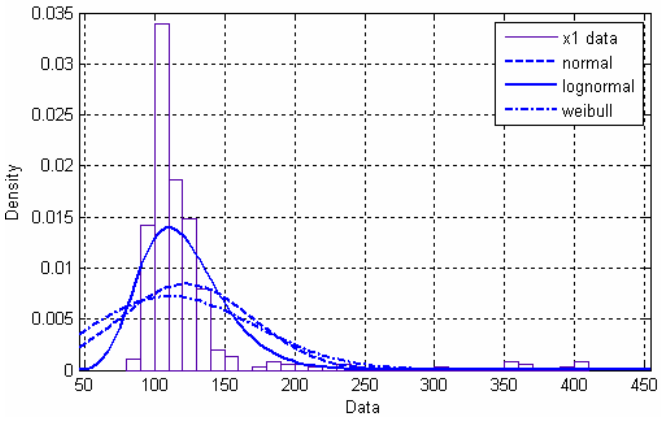

Fig 10. Comparison of three density functions for a period 1

IV. Conclusions

Investigations into the nature of electricity price in a competitive market have been presented based solely on the published historical market price. Section 2 presented the temporal variation of the mean values of electricity prices and their relationships with relevant factors in the system. The published electricity prices in the first three years after deregulation were used for these analyses. It was shown that the general nature of the price variation could be related to these factors. For example, although the electricity price appeared to increase after deregulation of the system, it could be explained in terms of the increase in the fuel prices. Such analyses, however, are not helpful in representing the behavior of electricity price at any one instant (ie, spot price) for planning and scheduling purposes.

The analyses in Section 3 presented systematic analyses of the published electricity price for two years to investigate the stochastic nature of the spot price. Sequential analyses of electricity prices were discussed to show the likely probability distribution of the electricity price. Three distributions, namely Normal, Log-normal and Weibull, were identified as the possible alternatives. Initial observations from the preliminary plots showed that the histogram of price distribution resembled the lognormal distribution. Strictly according to the probability plots of the three distributions, the collected data does not strictly comply with any one of the distributions. However, the lognormal distribution fitted the data lot better.

Although the findings of Section 2 and 3 are based on considerable amount of graphical and statistical analyses, more rigorous and thorough analysis are necessary to establish the statistical nature of the electricity price behavior in competitive market. Several, possible refinements in the modeling have been observed which are still being tested. The results observed so far have been encouraging and it is expected these approaches will lead to statistical representation of electricity price which can be established purely with the help of historical data but will prove very useful in planning exercises of market participants.

V. Acknowledgment

The authors would like to thank Pauline Won Pui Shan and Chong Chee Teng for conducting the preliminary test case studies.

VI. References

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VII. BIOGRAPHIES

G. B. Shrestha (S'88–M'90–SM'92) received B.E. (Honors) in electrical engineering from Jadavpur University (India) in 1975, MBA from University of Hawaii in 1985, M.S. in electrical power engineering from RPI in 1986, and Ph.D. in electrical engineering from Virginia Tech in 1990.
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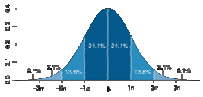
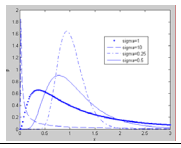
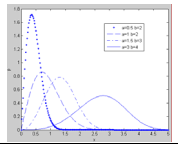
VIII. APPENDIX

Basic properties of some statistical distribution functions suitable for electricity price behavior.

Distribution	Shape of Density function	Parameters	Range support	cdf	Mean	Median	Variance
Normal		μ and σ	$x \in (-\infty;+\infty)$	$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x} e^{(\frac{(u-\mu)^2}{2\sigma^2})} du$	μ	μ	σ^2
Log Normal		$\sigma \geq 0$, $-\infty \leq \mu \leq \infty$	$x \in [0;+\infty)$	$\frac{1}{2}+\frac{1}{2}erf[\frac{\ln(x)-\mu}{\sigma\sqrt{2}}]$	$e^{[\mu+(\sigma^2/2)]}$	e^{μ}	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$
Weibull		$\lambda > 0$ scale (real) k>0 shape (real)	$x \in [0;+\infty)$	$1-e^{-(x/\lambda)^k}$	$\lambda\Gamma(1+\frac{1}{k})$	$\lambda \ln(2)^{1/k}$	$\lambda^2\Gamma(1+\frac{2}{k})-\mu^2$