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# NUMERICAL IMPLEMENTATION OF THE KINEMATICS FOR A 3-DOF PARALLEL ROBOT USING MATLAB

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***Abstract:*** *This paper presents a numerical implementation of inverse and direct kinematics of a 3-DOF parallel robot with R-P-S (Revolute-Prismatic-Spherical) joint structure using Matlab. The robot has two degrees of orientation freedom and one degree of translatory freedom. The inverse kinematics was expressed in term of Z-Y-Z Euler angles and direct kinematics was solved numerically using Newton-Kantorovich (N-K) method. A GUI was created using GUIDE tool from Matlab. Finally the results where verified using the GUI.*
***Keywords:*** *parallel robots, R-P-S joint structure, inverse kinematics, direct kinematics*

## 1. INTRODUCTION

Industrial robots have traditionally been used as general-purpose positioning devices and are anthropomorphic open-chain mechanisms which generally have the links actuated in series. The open kinematic chain manipulators usually have longer reach, larger workspace, and more dexterous maneuverability in reaching small space. Due to several increasingly important classes of robot applications, especially automatic assembly, data-driven manufacturing and reconfigurable jigs and fixtures assembly for high precision machining, significant effort has been directed towards finding techniques for improving the effective accuracy of the open-chain manipulator with calibration methods [4], compliance methods [5],[9],[13],[15] and endpoint sensing methods [3], [11]. Recently, some effort has been directed towards the investigation of alternative manipulator designs based on the concepts of closed kinematic chain due to the following advantages as compared to the traditional open kinematic chain manipulators: more rigidity and accuracy due to the lack of cantilever-like structure, high force/ torque capacity for the number of actuators as the actuators are arranged in parallel rather than in series, and relatively simpler inverse kinematics which is an advantage in real-time computer online control. The closed kinematic chain manipulators have potential applications where the demand on workspace and maneuverability is low but the dynamic loading is severe and high speed and precision motion are of primary concerns. The applications of parallel manipulators include airplane simulators, walking machines, adjustable articulated trusses, mining machines, pointing devices, etc. Gough and Whitehall [16] devised a six-linear jack system for use as Universal-tire testing machine. The six degree of freedom (DOF) Stewart platform was originally designed for use as an aircraft simulator [2], later as a robot wrist and as a tendon actuated in-parallel manipulator. Since then, parallel manipulators have been studied extensively by many researchers. But the spatial type parallel manipulators with less than 6-DOF gained prominence because of complicated analysis and structure of Stewart–Gough type and also several applications require fewer than 6-DOF.
A number of spatial parallel manipulators with different architectures are developed for the applications for which 6-DOF are not necessary. In particular, spatial 3-DOF manipulators have attracted much attention. Several types of spatial 3-DOF parallel manipulators, developed for different applications, have been reported in the literature. There are 3-DOF spatial manipulators, called rotational parallel manipulators (RPM) that allow the moving platform to rotate about a fixed point. Some manipulators, called translational parallel manipulators (TPM) provide the moving platform with pure translational motion and may be more useful in the fields of automated assembly and machine tools as alternatives to serial manipulator systems. Some other manipulators allow the platform to rotate and translate. A spatial 3-RPS parallel manipulator with three identical limbs, developed by Lee and Shah [6,7], is one such manipulator with 3-DOF.
Yang et al. [10] developed a low-cost driving simulator using the 3-RPS manipulator. Kinematic and dynamic characteristics of the 3-RPS manipulator have been studied by Lee and Shah [6,7] and Song and Zhang [14]. Joshi and Tsai [12] determined two singular positions of the 3-RPS manipulator from the Jacobian, determined by making use of the theory of reciprocal screws, while the other (Direct) singular positions of the manipulator were determined by Liu and Cheng [1].

## 2. THE STRUCTURE OF 3-RPS PARALLEL ROBOT

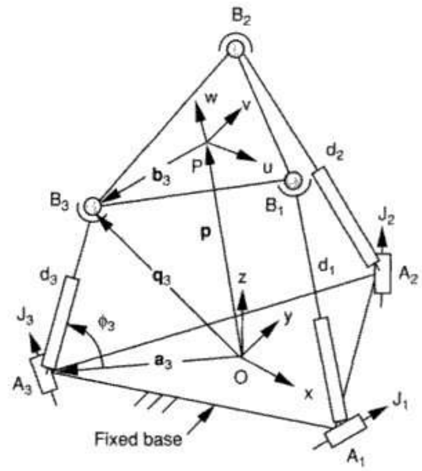


**Figure 1:** 3-RPS parallel robot

The robot consists of a moving platform $B_1B_2B_3$, a base $A_1A_2A_3$ and three extensible links, $A_1B_1$, $A_2B_2$, $A_3B_3$ of lengths $d_1$, $d_2$, $d_3$ respectively. The moving platform is connected to three extensible links (Prismatic joints) via three Spherical joints $B_1$, $B_2$ and $B_3$, which are equally spaced by $120^\circ$ at a radius $h$ from the mass center $P$ of the moving platform. The legs are connected to the base trough three revolute joints $A_1$, $A_2$ and $A_3$ at a distance $g$ from the center $O$ of the base platform. Two coordinate systems $A(X,Y,Z)$ and $B(x,y,z)$ are attached to the base platform and moving platform, respectively, in which the origin $P$ coincides with the mass center of the moving platform, the axis $x$ is along $\overline{PB_1}$, the points $A_1$, $A_2$ and $A_3$ lie the $X-Y$ plane, and the vector $\overline{OA_1}$ coincides with the axis $X$. The directions of the three revolute joints $J_i$ (where $i=1,2,3$) are perpendicular to $\overline{OA_1}$ respectively.

The revolute joints $A_1$, $A_2$ and $A_3$, with respect to the base coordinate system have the following position vectors:

$$a_1=[g,0,0]^T,\ a_2=[-\frac{1}{2}g,\frac{\sqrt{3}}{2}g,0]^T,\ a_3=[-\frac{1}{2}g,-\frac{\sqrt{3}}{2}g,0]^T. \quad \textbf{(1)}$$

Similarly, the position vectors of the spherical joints $B_1$, $B_2$ and $B_3$ with respect to the frame of the moving platform are:

$$^Bb_1=[h,0,0]^T,\ ^Bb_2=[-\frac{1}{2}h,\frac{\sqrt{3}}{2}h,0]^T,\ ^Bb_3=[-\frac{1}{2}h,-\frac{\sqrt{3}}{2}h,0]^T. \quad \textbf{(2)}$$

The position of the spherical joints with respect to the base frame is expressed as:

$$q_i=p+{}^AR_B\,{}^Bb_i,\ i=1,2,3, \quad \textbf{(3)}$$

where $p=[p_x,p_y,p_z]^T$ is the position vector of the end-effector and $^AR_B$ is a $3x3$ rotation matrix from the frame of the moving platform to the base one, that has the form:

$$^AR_B=\begin{pmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{pmatrix}, \quad \textbf{(4)}$$

where $(u_x,u_y,u_z)^T$, $(v_x,v_y,v_z)^T$ and $(w_x,w_y,w_z)^T$ are the directional cosines of the axes of frame $B(x,y,z)$ with respect to the frame $A(X,Y,Z)$. As the unit vectors $u$, $v$ and $w$ form an orthonormal set, there are six equations on the above nine elements:

$$u_x^2+u_y^2+u_z^2=1,\ v_x^2+v_y^2+v_z^2=1,\ w_x^2+w_y^2+w_z^2=1. \quad \textbf{(5)}$$

$$u_x v_x + u_y v_y + u_z v_z = 0\,,\ u_x w_x + u_y w_y + u_z w_z = 0\,,\ v_x w_x + v_y w_y + v_z w_z = 0\,. \qquad \textbf{(6)}$$

Substituting equations (2), (4) into equation (3) we obtain the position vectors of points $B_1$, $B_2$ and $B_3$:

$$q_1 = \begin{bmatrix} p_x + hu_x \\ p_y + hu_y \\ p_z + hu_z \end{bmatrix},\ q_2 = \begin{bmatrix} p_x - \frac{1}{2}hu_x + \frac{\sqrt{3}}{2}hv_x \\ p_y - \frac{1}{2}hu_y + \frac{\sqrt{3}}{2}hv_y \\ p_z - \frac{1}{2}hu_z + \frac{\sqrt{3}}{2}hv_z \end{bmatrix},\ q_3 = \begin{bmatrix} p_x - \frac{1}{2}hu_x - \frac{\sqrt{3}}{2}hv_x \\ p_y - \frac{1}{2}hu_y - \frac{\sqrt{3}}{2}hv_y \\ p_z - \frac{1}{2}hu_z - \frac{\sqrt{3}}{2}hv_z \end{bmatrix}. \qquad \textbf{(7)}$$

The constraint equations imposed by the revolute joints are ($J_i$ is perpendicular to $\overline{OA_i}$ for $i = 1, 2, 3$):

$$q_{1y} = 0, \qquad \textbf{(8)}$$

$$q_{2y} = -\sqrt{3}q_{2x}, \qquad \textbf{(9)}$$

$$q_{3y} = +\sqrt{3}q_{3x}. \qquad \textbf{(10)}$$

**2.1 INVERSE KINEMATICS**

The inverse kinematics equations that define the leg lengths $d_i$ for the prescribed position of the platform are obtained as the distance between points $A_i$, $B_i$:

$$d_i^2 = [q_i - a_i]^T [q_i - a_i]\,. \qquad \textbf{(11)}$$

Substituting equations (1) and (7) into equation (11) we obtain the inverse kinematic equations which define the actuating length of the links for a prescribed position and orientation of the moving platform:

$$d_1^2 = p_x^2 + p_y^2 + p_z^2 + 2h(p_x u_x + p_y u_y + p_z u_z) - 2gp_x - 2ghu_x + g^2 + h^2, \qquad \textbf{(12)}$$

$$d_1^2 = p_x^2 + p_y^2 + p_z^2 - h(p_x u_x + p_y u_y + p_z u_z) + \sqrt{3}h(p_x v_x + p_y v_y + p_z v_z) \qquad \textbf{(13)}$$

$$+ g(p_x - \sqrt{3}p_y) - \frac{1}{2}gh(u_x - \sqrt{3}u_y) + \frac{1}{2}gh(\sqrt{3}v_x - 3v_y) + g^2 + h^2,$$

$$d_1^2 = p_x^2 + p_y^2 + p_z^2 - h(p_x u_x + p_y u_y + p_z u_z) - \sqrt{3}h(p_x v_x + p_y v_y + p_z v_z) \qquad \textbf{(14)}$$

$$+ g(p_x + \sqrt{3}p_y) - \frac{1}{2}gh(u_x + \sqrt{3}u_y) - \frac{1}{2}gh(\sqrt{3}v_x + 3v_y) + g^2 + h^2\,.$$

A more compact form of solutions for the link lengths can be obtained by expressing the directional cosines in term of Z-Y-Z Euler angles ($\alpha, \beta, \gamma$) as:

$$d_1^2 = g^2 + h^2 + p_x^2 + p_y^2 + p_z^2 - 2gp_x + 2h(C_\alpha^2 C_\beta + S_\alpha^2)(p_x - g) \qquad \textbf{(15)}$$

$$+ h(C_\beta - 1)S_{2\alpha}p_y - 2hS_\beta C_\alpha p_z\,,$$

$$d_2^2 = g^2 + h^2 + p_x^2 + p_y^2 + p_z^2 + gp_x - \sqrt{3}gp_y - h(C_\alpha^2 C_\beta + S_\alpha^2 - \qquad \textbf{(16)}$$

$$\sqrt{3}C_\alpha S_\alpha(C_\beta - 1))(p_x + \frac{1}{2}g) - h((S_\alpha C_\alpha(C_\beta - 1) - \sqrt{3}(S_\alpha^2 C_\beta + C_\alpha^2))$$

$$(p_y - \frac{\sqrt{3}}{2}g) + hS_\beta(C_\alpha - \sqrt{3}S_\alpha)p_z\,,$$

$$d_3^2 = g^2 + h^2 + p_x^2 + p_y^2 + p_z^2 + gp_x + \sqrt{3}gp_y - h(C_\alpha^2 C_\beta + S_\alpha^2 + \qquad \textbf{(17)}$$

$$\sqrt{3}C_\alpha S_\alpha(C_\beta - 1))(p_x + \frac{1}{2}g) - h((S_\alpha C_\alpha(C_\beta - 1) + \sqrt{3}(S_\alpha^2 C_\beta + C_\alpha^2))$$

$$(p_y + \frac{\sqrt{3}}{2}g) + hS_\beta(C_\alpha + \sqrt{3}S_\alpha)p_z.$$

## 2.2 DIRECT KINEMATICS

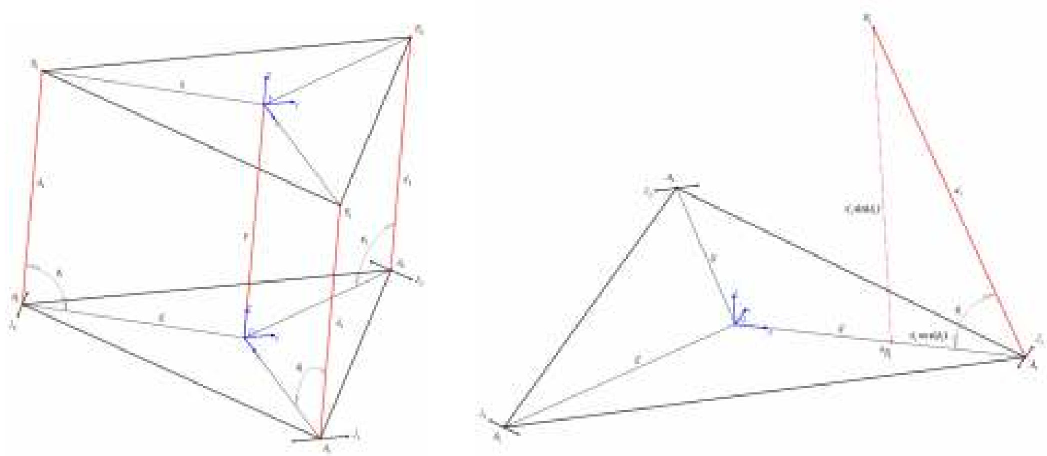

**Figure 2:** $\phi_1, \phi_2, \phi_3$ angles and point $B_1$ coordinates

The direct kinematics involves solving six simultaneous equations for the position/orientation in terms of the given lengths. For solving the direct kinematics problem of a 3-RPS parallel robot, the system of kinematic equations is reduced to a 16th degree polynomial using Sylvester dialytic elimination method [8]. But this method suggests 16 solutions of which only one solution indicates the actual configuration of the robot and the remaining solutions are to be filtered by examining the corresponding configurations. To overcome this difficulty, in this paper the forward kinematic equations are solved using numerical method Newton-Kantorovich (N-K) for non-linear system of equations.
The angles $\phi_1, \phi_2, \phi_3$ are defined to be the angles between the links $d_1$, $d_2$, $d_3$ and the base platform, respectively. As the distance between any two adjacent ball joints is $\sqrt{3}h$, $\theta_i$ can be related to $d_i$ implicitly using:

$$d_1^2 + d_2^2 + 3g^2 - 3h^2 + d_1 d_2 \cos(\phi_1)\cos(\phi_2) - 2d_1 d_2 \sin(\phi_1)\sin(\phi_2) \\ -3g d_1 \cos(\phi_1) - 3g d_2 \cos(\phi_2) = 0, \quad \textbf{(18)}$$

$$d_2^2 + d_3^2 + 3g^2 - 3h^2 + d_2 d_3 \cos(\phi_2)\cos(\phi_3) - 2d_2 d_3 \sin(\phi_2)\sin(\phi_3) \\ -3g d_2 \cos(\phi_2) - 3g d_3 \cos(\phi_3) = 0, \quad \textbf{(19)}$$

$$d_3^2 + d_1^2 + 3g^2 - 3h^2 + d_3 d_1 \cos(\phi_3)\cos(\phi_1) - 2d_3 d_1 \sin(\phi_3)\sin(\phi_1) \\ -3g d_3 \cos(\phi_3) - 3g d_1 \cos(\phi_1) = 0. \quad \textbf{(20)}$$

$$\phi_1 \in [0, \pi], \phi_2 \in [0, \pi], \phi_3 \in [0, \pi]. \quad \textbf{(21)}$$

Since ball joints are placed at the vertices of an equilateral triangle, the Cartesian position vector $p$ can be determined as:

$$p = \frac{1}{3}(q_1 + q_2 + q_3). \quad \textbf{(22)}$$

were the ball joint coordinates with respect to the base frame are:

$$q_1 = \begin{bmatrix} g - d_1 \cos(\phi_1) \\ 0 \\ d_1 \sin(\phi_1) \end{bmatrix}, \ q_2 = \begin{bmatrix} -\frac{1}{2}(g - d_2 \cos(\phi_2)) \\ \frac{\sqrt{3}}{2}(g - d_2 \cos(\phi_2)) \\ d_2 \sin(\phi_1) \end{bmatrix}, \ q_3 = \begin{bmatrix} -\frac{1}{2}(g - d_3 \cos(\phi_3)) \\ -\frac{\sqrt{3}}{2}(g - d_3 \cos(\phi_3)) \\ d_3 \sin(\phi_3) \end{bmatrix}. \quad \textbf{(23)}$$

With the position vectors of the ball joints defined in (23), the orientation of the moving platform can be expressed as:

$$u_x = (q_{1x} - p_x)/h,\ u_y = (q_{1y} - p_y)/h,\ u_z = (q_{1z} - p_z)/h. \quad \textbf{(24)}$$

$$v_x = u_y,\ v_y = \frac{\sqrt{3}(g - d_2\cos(\phi_2)) - 3p_y}{\sqrt{3}h},\ v_z = \frac{2d_2\sin(\phi_2) + d_1\sin(\phi_1) - 3p_z}{\sqrt{3}h}. \quad \textbf{(25)}$$

$$w_x = u_y v_z - v_y u_z,\ w_y = -u_x v_z + v_x u_z,\ w_z = u_x v_y - u_y v_x. \quad \textbf{(26)}$$

## 3. NUMERICAL IMPLEMENTATION OF THE KINEMATIC EQUATIONS

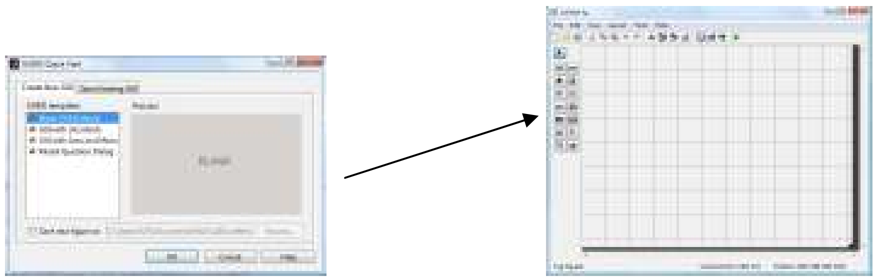

**Figure 3:** GUIDE tool from Matlab

GUIDE, the MATLAB graphical user interface development environment, provides a set of tools for creating graphical user interfaces (GUIs). These tools simplify the process of laying out and programming GUIs. Using the GUIDE Layout Editor, you can populate a GUI by clicking and dragging GUI components—such as axes, panels, buttons, text fields, sliders, and so on, into the layout area. You also can create menus and context menus for the GUI. From the Layout Editor, you can size the GUI, modify component look and feel, align components, set tab order, view a hierarchical list of the component objects, and set GUI options.

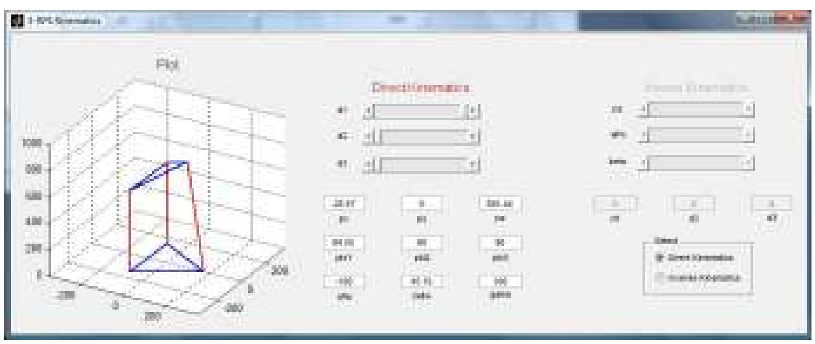


**Figure 4:** 3-RPS kinematics GUI

In figure 4 there is the GUI that calculates the kinematics for the studied robot. For exemplification we will use the following inputs, and in table 1 there are the results for three cases of the direct kinematics (1 means full open and 0 full retracted for the links lenghts):
Fixed platform radius g [mm] = 173.205,
Mobile platform radius h [mm] = 173.205,
Links length full retracted Lr [mm] = 620,
Links length full open Lo [mm] = 808.

**Table 1:** Direct Kinematics (N-K) Results

| Links length | $p_x$ [mm] | $p_y$ [mm] | $p_z$ [mm] | $\phi_1$ [degree] | $\phi_2$ [degree] | $\phi_3$ [degree] |
|---|---|---|---|---|---|---|
| d1=1,d2=0,d3=0 | -25.57 | 0 | 681.44 | 84.55 | 90 | 90 |
| d1=1,d2=1,d3=1 | 0 | 0 | 808 | 90 | 90 | 90 |
| d1=1,d2=1,d3=0 | 14.53 | 25.15 | 743.28 | 90 | 89.99 | 81.91 |

## 4. CONCLUSION

A numerical implementation of the kinematics for a 3-DOF parallel robot with R-P-S (Revolute-Prismatic-Spherical) joint structure using Matlab was presented in this paper. Firstly, the structure of the robot was presented, with her particularities. Then the inverse kinematics was expressed in terms of Z-Y-Z Euler angles and direct kinematics was solved with the numerical method Newton-Kantorovich (N-K) and it has been shown that this method is better than the Sylvester dialytic elimination method. A GUI was implemented with GUIDE tool from Matlab. Finally the results where verified using this GUI.

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## REFERENCES

[1] C.H. Liu, S. Cheng, Direct singular positions of 3RPS parallel manipulator, ASME J. Mech. Des. 126 (2004) 1006–1016.
[2] D. Stewart, A platform with six degrees of freedom, Proc. Inst. Mech. Eng., London 180 (15) (1965) 371–386.
[3] G. Beri, S. Hackwood, and W. S. Trimmer, “High-precision robot system for inspection and testing and electronic devices,” in Proc. IEEE Int. Conf. on Robotics (Atlanta, GA, Mar. 1984).
[4] H. McCallion and P. D. Truong, “The analysis of a six-degree-offreedom work station for mechanized assembly,” in froc. 5th World Congr. for the Theory of Machines and Mechanisms (an ASME publ.), pp. 611-616, 1979.
[5] K. Asakawa, F. Akiya, and F. Tabata, “A variable compliance device and its application for automatic assembly,’’ in froc. Autoface 5 Conf. (Detroit, MI, Nov. 14-17, 1983).
[6] K. Lee, D.K. Shah, Dynamic analysis of a three degrees of freedom in-parallel actuated manipulator, IEEE Trans. Rob. Autom. 4 (1988) 361–367.
[7] K. Lee, D.K. Shah, Kinematic analysis of a three degrees of freedom in-parallel actuated manipulator, in: Proceedings of the IEEE International Conference on Robotics and Automation, vol. 1, 1987, pp. 345–350.
[8] L.W. Tsai, Robot Analysis: The Mechanics of Serial and Parallel Manipulator, John Wiley & Sons, New York, 1999, pp. 142–151.
[9] M. T. Mason, “Compliance and force control for computer controlled manipulators,” IEEE Trans. Syst., Man., Cybern., vol. SMC-11 no. 6, pp. 418-432, 1981.
[10] P.H. Yang, K.J. Waldron, D.E. Orin, Kinematics of a three degrees-of-freedom motion platform for a low-cost driving simulator, in: J. Lenareie, V. Parenti-Castelli (Eds.), Recent Advances in Robot Kinematics, Kluwer Academic Publishers, London, 1996, pp. 89–98.
[11] R. H. Taylor, R. L. Hollis, and M. A. Lavin, “Precise manipulation with endpoint sensing,” IBM J. Res. Devel., vol. 29, no. 4, July 1985.
[12] S.A. Joshi, L.W. Tsai, Jacobian analysis of limited-DOF parallel manipulator, ASME J. Mech. Des. 124 (2002) 254–258.
[13] S. Drake, “Using compliance in lieu of sensory feedback for automatic assembly,” D.Sc. dissertation MIT, Cambridge, MA, 1977.
[14] S.M. Song, M.D. Zhang, A study of reactional force compensation based on three-degree-of-freedom parallel platforms, J. Rob. Syst. 12 (1995) 783–794.
[15] T. Lozaro-Perez, M. T. Mason, and R. H. Taylor, “Automatic synthesis of fine-motion strategies for manipulators,” Int. J. RoboticsRes., vol. 3, no. 1, 1984.
[16] V.E. Gough, S.G. Whitehall, Universal tyre test machine, in: Proceedings of the Ninth International Congress of F.I.S.T.T.A., vol. 117, 1962, pp. 117–135.