Simplified version of MG2008 Appendix A

Note that $\delta = q_v - q_s$ where the saturation mixing ratio, $q_s = q_{sat,liq}(p,T) = \epsilon e_{sat,liq}(T)/(p - e_{sat,liq}(T))$.

Neglecting the effects of mixing and radiation (as in pyrcel), the evolution equations for a broad set of quantities (not including the cloud droplet number concentration) can be written:

$$\frac{d}{dt}\begin{pmatrix} z \\ p \\ T \\ q_v \\ q_c \\ q_i \\ \delta \end{pmatrix} = \begin{pmatrix} w \\ -\rho_{air} g w \\ -\frac{gw}{c_p} + \frac{L_v}{c_p} C \\ -C \\ C \\ 0 \\ A_\delta w - \frac{\delta}{\tau} \end{pmatrix} \tag{1}$$

where the condensation rate C is defined

$$C = \frac{\delta}{\tau\Gamma} \tag{2}$$

and the thermodynamic constants, A_δ and Γ, are

$$A_\delta = \left(\frac{g}{c_p}\right)\frac{dq_{sat}}{dT} - \frac{q_s \rho_{air} g}{p - e_{sat}} \tag{3}$$

$$\Gamma = 1 + \frac{L}{C_p}\frac{dq_s}{dT}. \tag{4}$$

Last, the relaxation timescale for the supersaturation is

$$\frac{1}{\tau_c} = 4\pi D_v N_c \langle r \rangle_c \tag{5}$$

Formally, $\langle r \rangle_c$ is the number-weighted mean radius:

$$\langle r \rangle_c = \frac{\int n_c(r) r\, dr}{\int n_c(r)\, dr} \tag{6}$$

where it should be noted that $N_c = \int n_c dr$. For simplicity, we will approximate $\langle r \rangle_c$ as the volume mean radius, r_v, which is defined by:

$$\frac{4\pi r_v^3}{3} N_c\, \rho_{liquid} = \rho_{air}\, q_c \tag{7}$$

The computation of N_c is diagnostic rather than predicted by an evolution equation like those in (1) above.

TODO:

- computation of N_c following MG2008, sec. 2.3, and including parameters as derived in MG200X and KC199X.
- Add proper citations.