# Lecture 17: Attention

Admin: A4

Object Detection: FCOS, Faster R-CNN

Due Tuesday, 3/29/2022, 11:59pm ET

Updated A4 starter code out today:

- Incorporates clarifications / documentation improvements from Piazza
- No functional code changes: you can copy-paste all your code from previous to current version and everything should still work
- Optional: if you are not confused, can keep going with original release

Justin Johnson Lecture 17 - 2 March 21, 2022

# Admin: Project

Project details are available here:

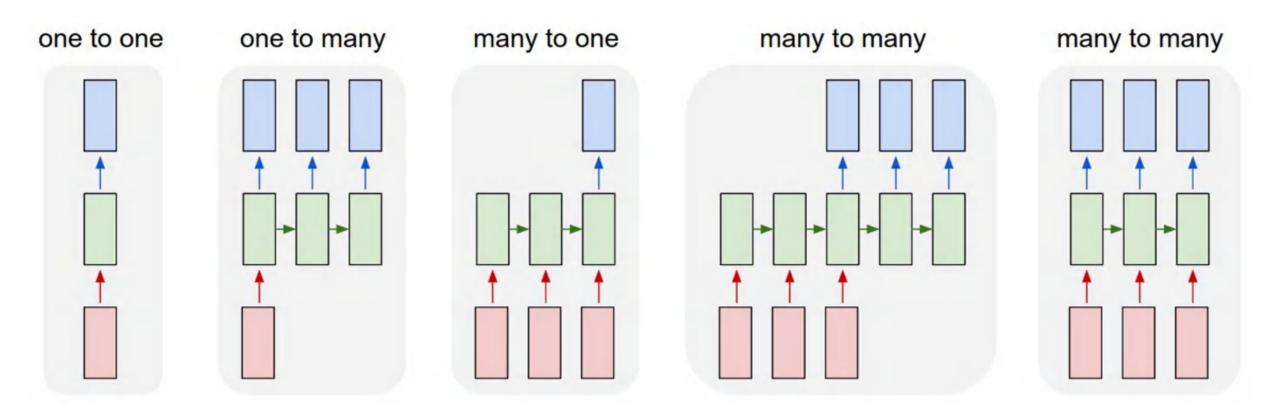
https://web.eecs.umich.edu/~justincj/teaching/eecs498/WI2022/project.html

#### Project options:

- Image Classification
- Single-Image Super-Resolution
- Novel View Synthesis with NeRF
- Choose Your Own

For Choose Your Own project: need to submit a **project proposal** by Friday April 1, 11:59 ET. Make a private post on Piazza under tag "project-proposal". This is not graded, but we need to ok the project.

#### Last Time: Recurrent Neural Networks

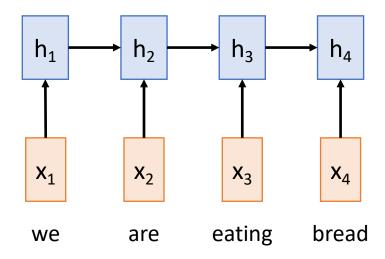


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**Input**: Sequence  $x_1, ... x_T$ 

**Output**: Sequence  $y_1, ..., y_{T'}$ 

**Encoder:**  $h_t = f_W(x_t, h_{t-1})$ 

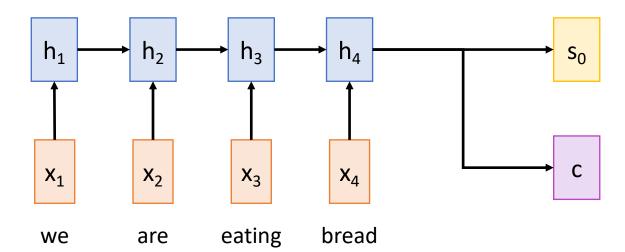


**Input**: Sequence  $x_1, ... x_T$ 

**Output**: Sequence  $y_1, ..., y_{T'}$ 

**Encoder:**  $h_t = f_W(x_t, h_{t-1})$ 

From final hidden state predict: Initial decoder state  $s_0$ Context vector c (often  $c=h_T$ )



**Input**: Sequence  $x_1, ... x_T$ 

**Decoder:**  $s_t = g_{ij}(y_{t-1}, s_{t-1}, c)$ 

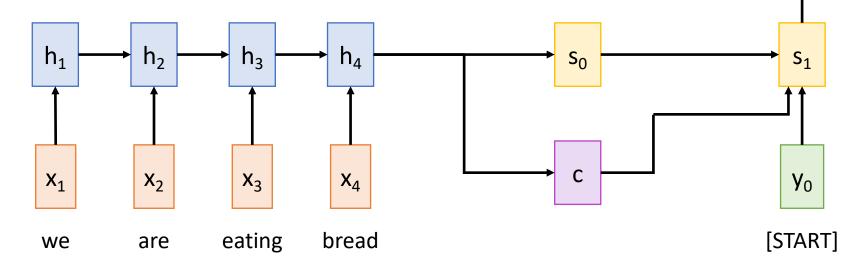
**Output**: Sequence  $y_1, ..., y_{T'}$ 

estamos

**y**<sub>1</sub>

**Encoder:**  $h_t = f_W(x_t, h_{t-1})$ 

From final hidden state predict: **Initial decoder state** s<sub>0</sub> **Context vector** c (often  $c=h_T$ )



**Input**: Sequence  $x_1, ... x_T$ 

Deco

**Decoder:**  $s_t = g_U(y_{t-1}, s_{t-1}, c)$ 

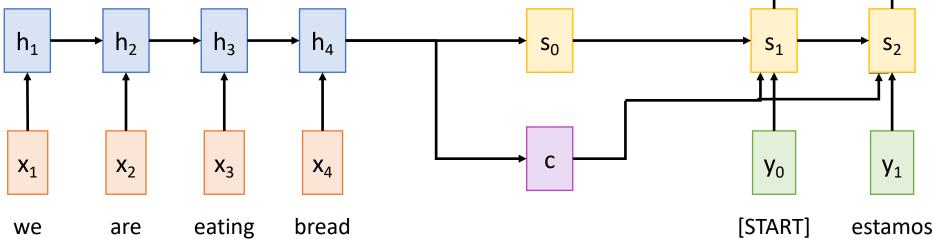
**Output**: Sequence  $y_1, ..., y_{T'}$ 

estamos comiendo

**Encoder:**  $h_t = f_W(x_t, h_{t-1})$ 

From final hidden state predict: Initial decoder state  $s_0$ Context vector c (often  $c=h_T$ )

 $y_1$   $y_2$   $h_T$ )



**Input**: Sequence  $x_1, ... x_T$ 

**Output**: Sequence  $y_1, ..., y_{T'}$ 

**Decoder:**  $s_t = g_U(y_{t-1}, s_{t-1}, c)$ 

comiendo [STOP] estamos pan From final hidden state predict: **y**<sub>1</sub> **y**<sub>2</sub> **y**<sub>3</sub> **y**<sub>4</sub> **Initial decoder state** s<sub>0</sub> **Encoder:**  $h_t = f_W(x_t, h_{t-1})$ **Context vector** c (often  $c=h_T$ )  $h_1$  $h_2$  $h_4$ h<sub>3</sub>  $S_4$  $S_0$  $S_2$  $S_3$  $X_2$  $X_3$  $X_4$ **y**<sub>1</sub>  $X_1$ y<sub>0</sub> **y**<sub>2</sub> **y**<sub>3</sub> eating bread [START] comiendo estamos we are pan

**Input**: Sequence  $x_1, ... x_T$ 

Output: Sequence  $y_1, ..., y_{T'}$ 

**Decoder:**  $s_t = g_U(y_{t-1}, s_{t-1}, c)$ 

comiendo [STOP] estamos pan From final hidden state predict: **y**<sub>1</sub> **y**<sub>2</sub> **y**<sub>3</sub> **y**<sub>4</sub> **Initial decoder state** s<sub>0</sub> **Encoder:**  $h_t = f_W(x_t, h_{t-1})$ **Context vector** c (often  $c=h_T$ )  $h_1$  $h_2$  $h_4$ h<sub>3</sub>  $S_4$  $S_0$  $S_2$  $S_3$  $X_3$  $X_4$  $X_1$  $X_2$ y<sub>0</sub> **y**<sub>1</sub> **y**<sub>2</sub> **y**<sub>3</sub> **Problem: Input sequence** [START] eating bread comiendo estamos we are pan bottlenecked through fixed-

Sutskever et al, "Sequence to sequence learning with neural networks", NeurIPS 2014

sized vector. What if T=1000?

**Input**: Sequence  $x_1, ... x_T$ 

**Output**: Sequence  $y_1, ..., y_{T'}$ 

h<sub>3</sub>

 $X_3$ 

eating

**Decoder:**  $s_t = g_U(y_{t-1}, s_{t-1}, c)$ 

pan

**y**<sub>2</sub>

comiendo

comiendo

estamos

y<sub>0</sub>

[START]

[STOP]

**y**<sub>3</sub>

pan

Encoder:  $h_t = f_W(x_t, h_{t-1})$ From final hidden state predict:

Initial decoder state  $s_0$ Context vector c (often  $c=h_T$ )

 $h_4$ 

 $X_4$ 

bread

 $y_1$   $y_2$   $y_3$   $y_4$   $y_4$ 

Problem: Input sequence bottlenecked through fixed-sized vector. What if T=1000?

 $S_0$ 

Idea: use new context vector at each step of decoder!

estamos

**y**<sub>1</sub>

Sutskever et al, "Sequence to sequence learning with neural networks", NeurIPS 2014

h<sub>1</sub>

 $X_1$ 

we

 $h_2$ 

 $X_2$ 

are

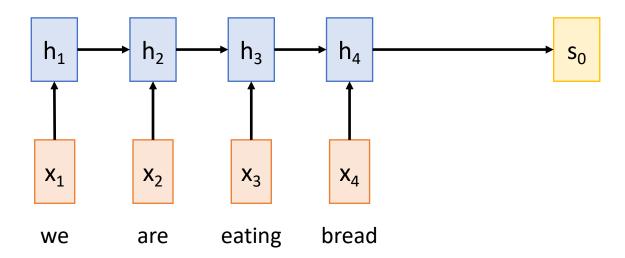
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**Input**: Sequence  $x_1, ... x_T$ 

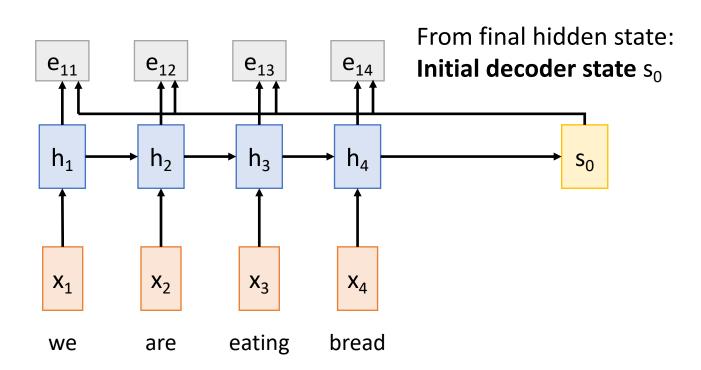
**Output**: Sequence  $y_1, ..., y_{T'}$ 

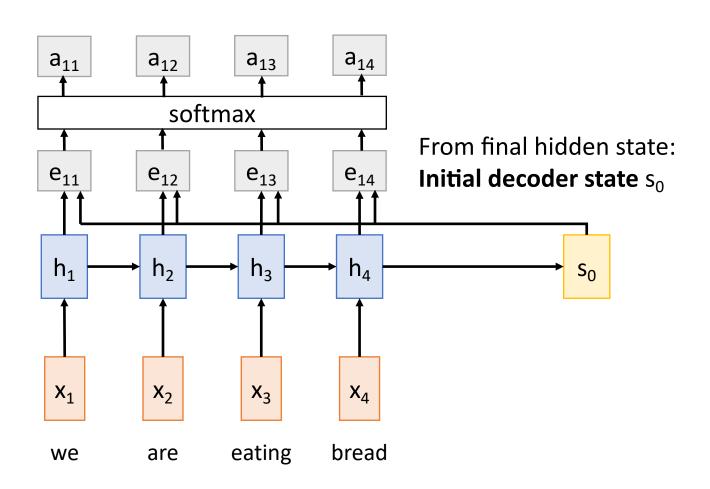
**Encoder:**  $h_t = f_W(x_t, h_{t-1})$ 

From final hidden state: **Initial decoder state** s<sub>0</sub>



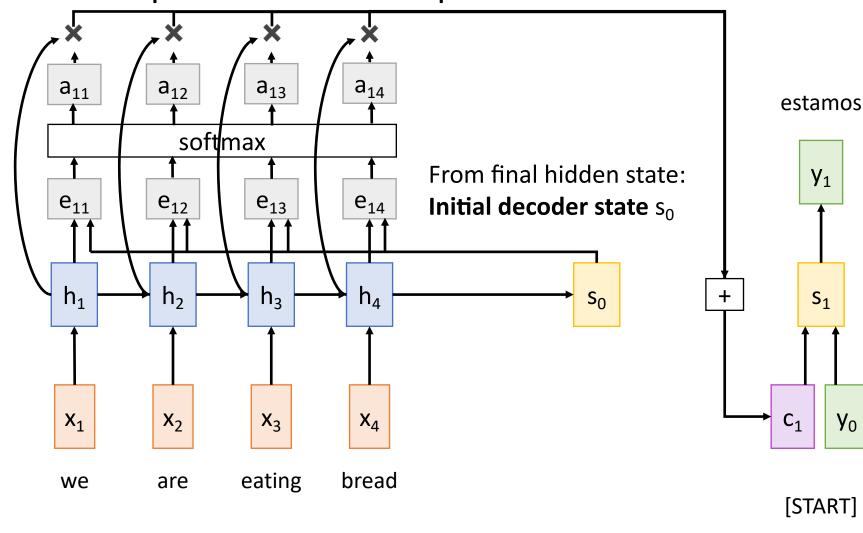
Compute (scalar) **alignment scores**  $e_{t,i} = f_{att}(s_{t-1}, h_i)$  ( $f_{att}$  is an MLP)





Compute (scalar) **alignment scores**  $e_{t,i} = f_{att}(s_{t-1}, h_i)$  ( $f_{att}$  is an MLP)

Normalize alignment scores to get **attention weights**  $0 < a_{t,i} < 1$   $\sum_{i} a_{t,i} = 1$ 



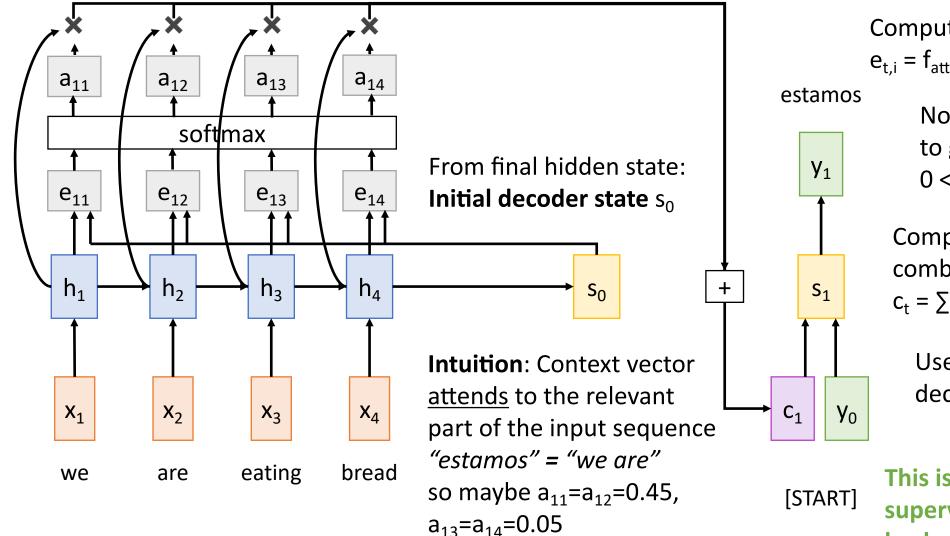
Compute (scalar) **alignment scores**  $e_{t,i} = f_{att}(s_{t-1}, h_i)$  ( $f_{att}$  is an MLP)

Normalize alignment scores to get **attention weights**  $0 < a_{t,i} < 1$   $\sum_{i} a_{t,i} = 1$ 

Compute context vector as linear combination of hidden states  $c_t = \sum_i a_{t,i} h_i$ 

Use context vector in decoder:  $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$ 

This is all differentiable! Do not supervise attention weights – backprop through everything



Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

Compute (scalar) **alignment scores**  $e_{t,i} = f_{att}(s_{t-1}, h_i)$  ( $f_{att}$  is an MLP)

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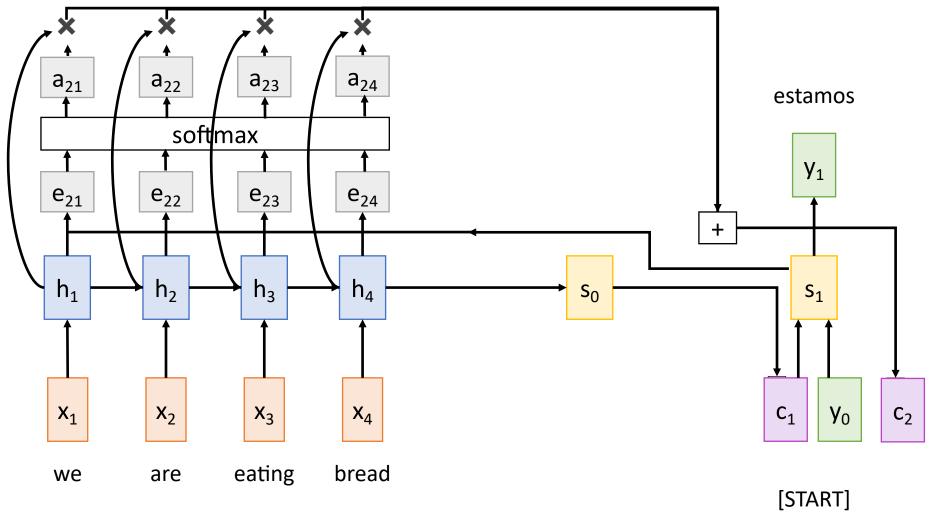
Compute context vector as linear combination of hidden states  $c_t = \sum_i a_{t,i} h_i$ 

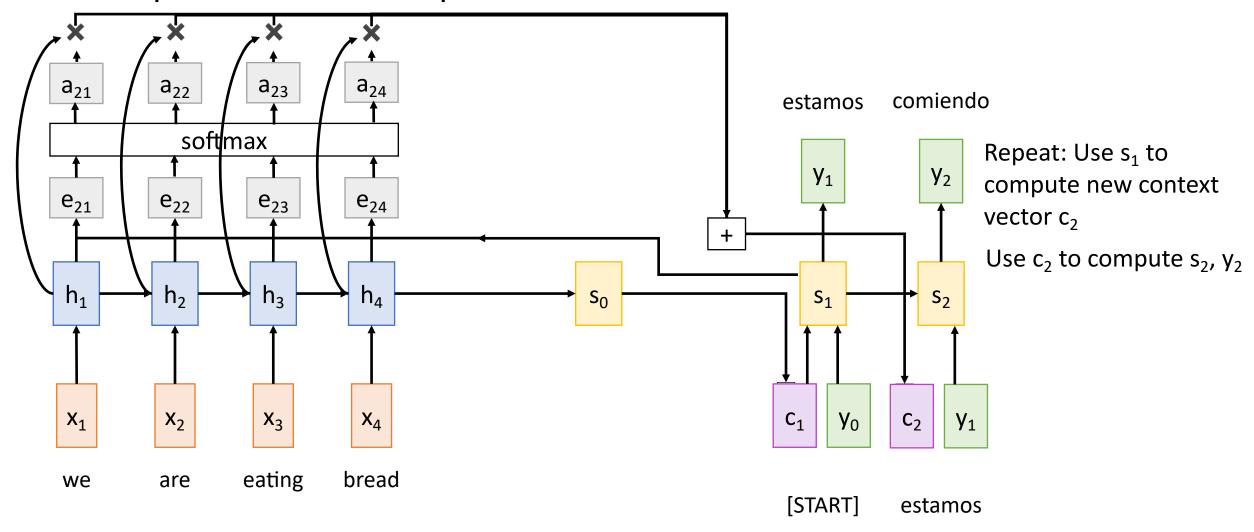
Use context vector in decoder:  $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$ 

This is all differentiable! Do not supervise attention weights – backprop through everything

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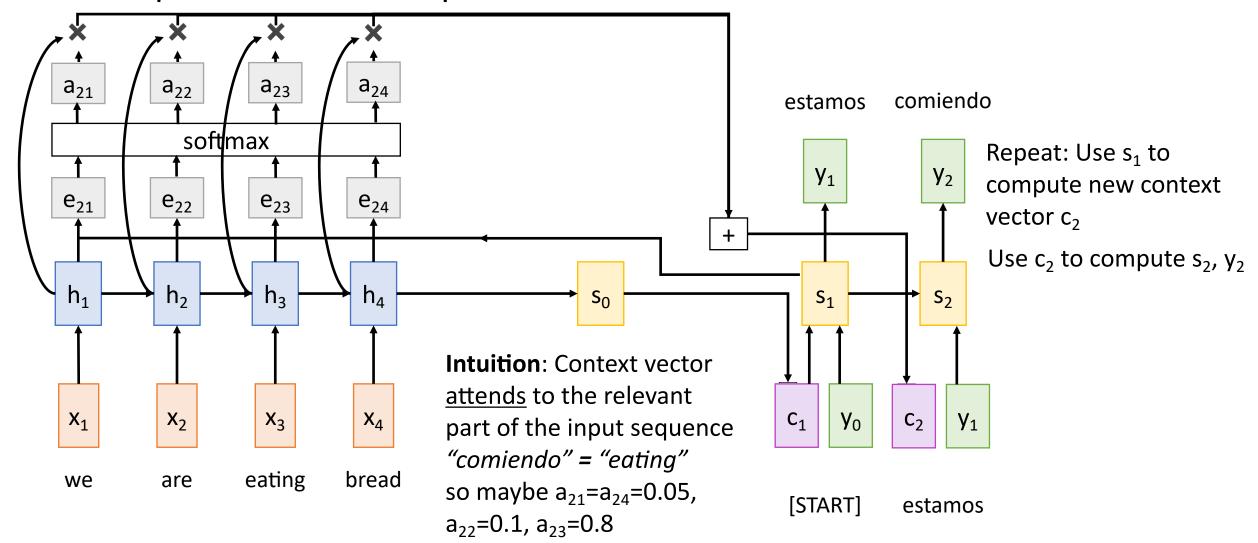
Repeat: Use  $s_1$  to compute new context vector  $c_2$ 





Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

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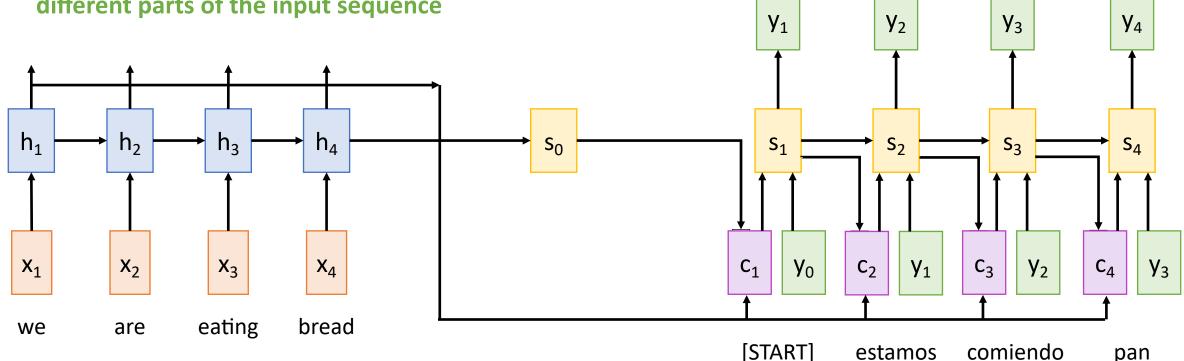
Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

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Use a different context vector in each timestep of decoder

Input sequence not bottlenecked through single vector

At each timestep of decoder, context vector "looks at" different parts of the input sequence



comiendo

pan

estamos

[STOP]

Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

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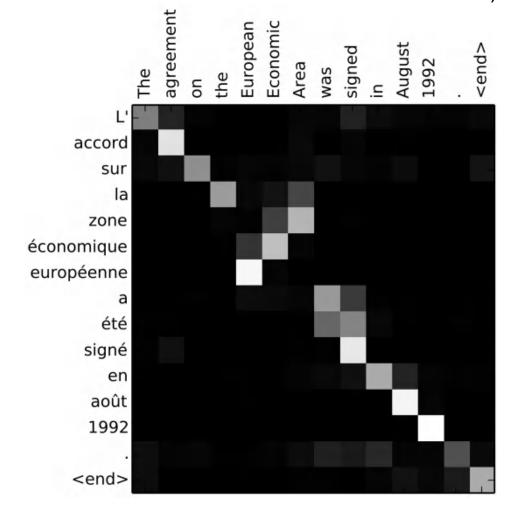
**Example**: English to French

translation

**Input**: "The agreement on the European Economic Area was signed in August 1992."

**Output**: "L'accord sur la zone économique européenne a été signé en août 1992."

Visualize attention weights a<sub>t,i</sub>



**Example**: English to French

translation

Input: "The agreement on the European Economic Area was signed in August 1992."

Output: "L'accord sur la zone économique européenne a été signé en août 1992."

Diagonal attention means words correspond in order

Diagonal attention means words correspond in order

Visualize attention weights att. accord sur la zone économique européenne été signé en août 1992 <end>

Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

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**Example**: English to French

translation

Input: "The agreement on the European Economic Area was signed in August 1992."

Output: "L'accord sur la zone économique européenne a été signé en août 1992."

**Diagonal attention means** accord words correspond in order sur la zone **Attention figures out** économique different word orders européenne été signé en août **Diagonal attention means** 1992 words correspond in order <end>

Visualize attention weights at i

**Example**: English to French

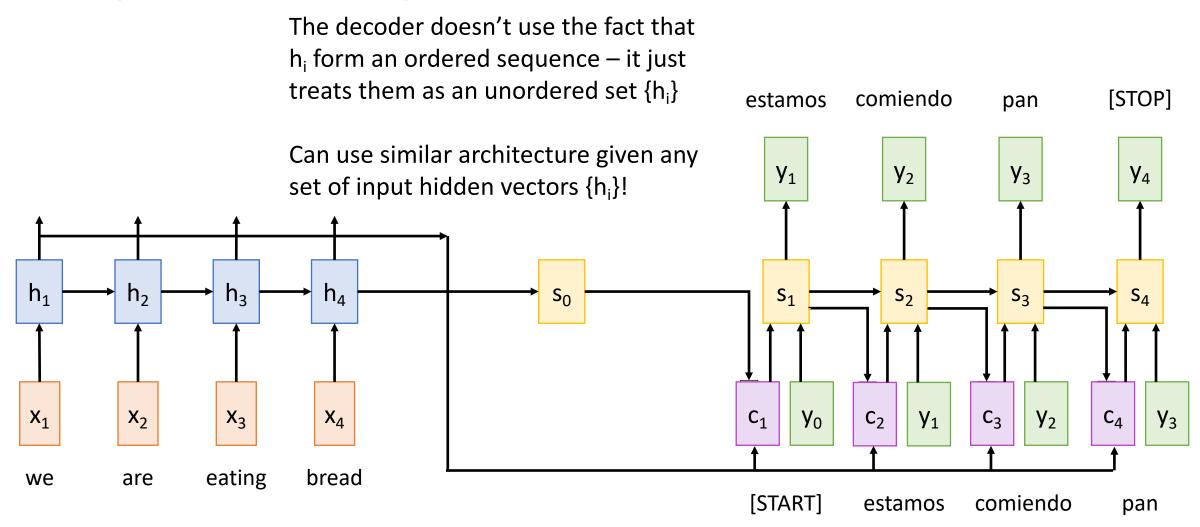
translation

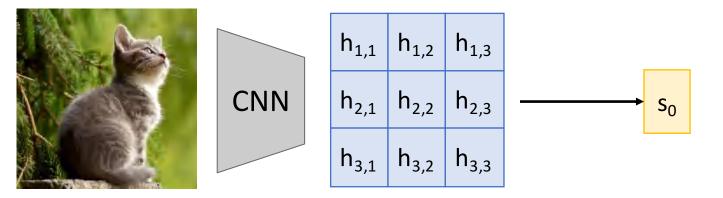
Input: "The agreement on the European Economic Area was signed in August 1992."

Output: "L'accord sur la zone économique européenne a été signé en août 1992."

**Diagonal attention means** accord words correspond in order sur la zone **Attention figures out** économique different word orders européenne été Verb conjugation signé en août **Diagonal attention means** 1992 words correspond in order <end>

Visualize attention weights at i



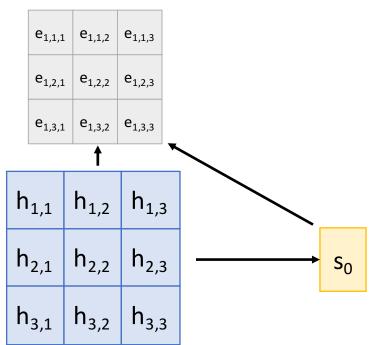


Use a CNN to compute a grid of features for an image

Cat image is free to use under the Pixabay License

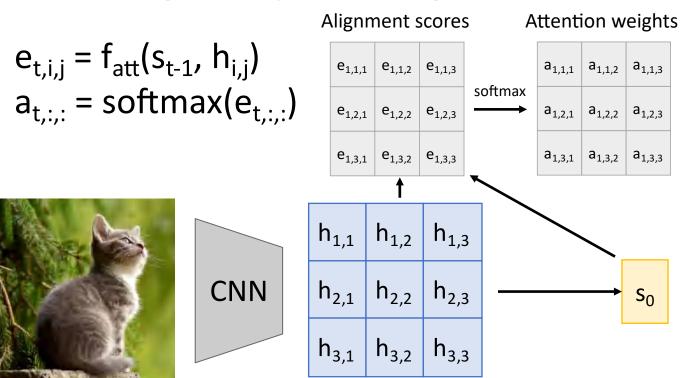


#### Alignment scores

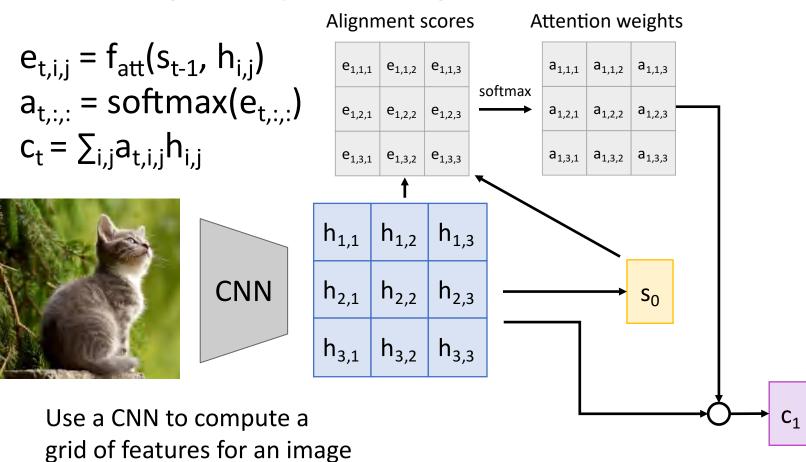


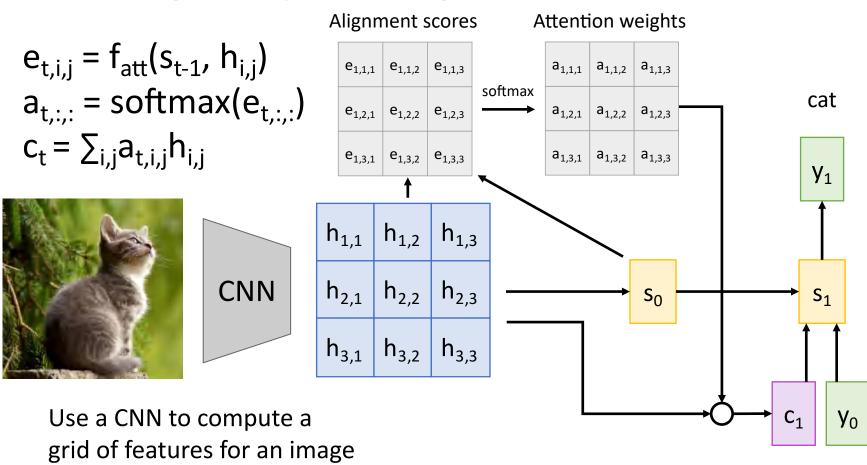


Use a CNN to compute a grid of features for an image

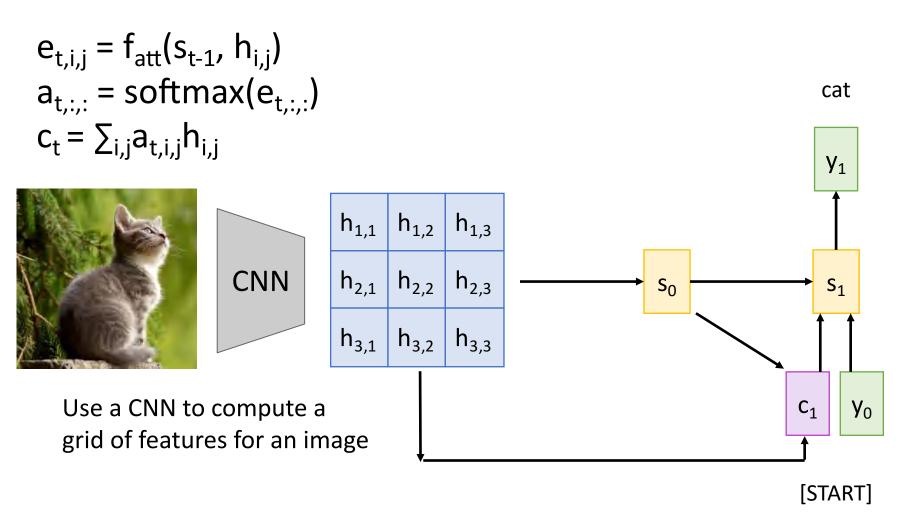


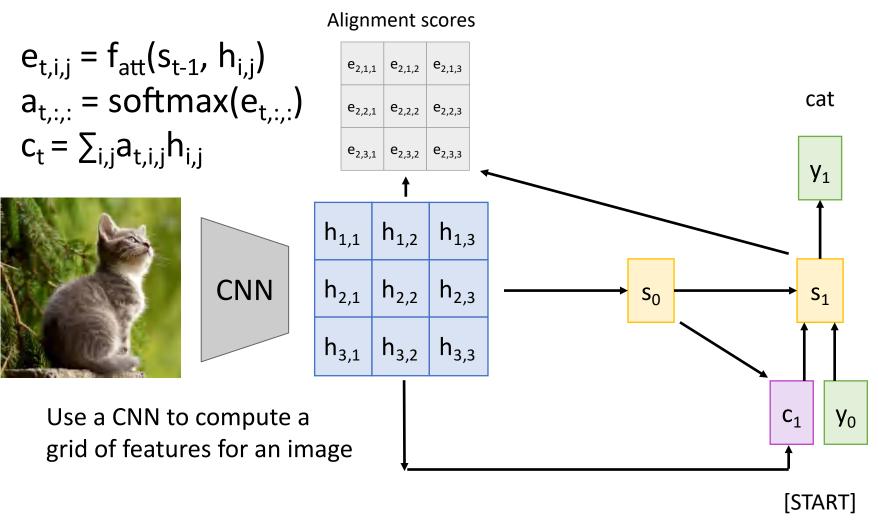
Use a CNN to compute a grid of features for an image

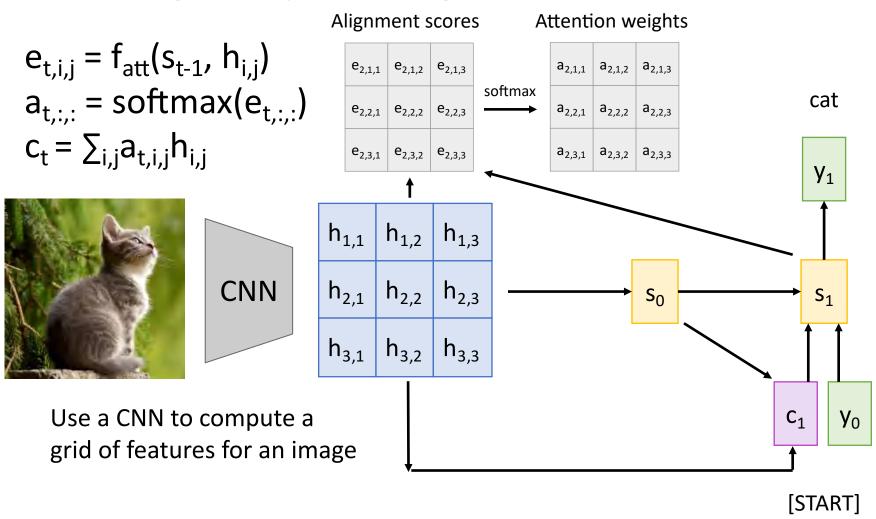


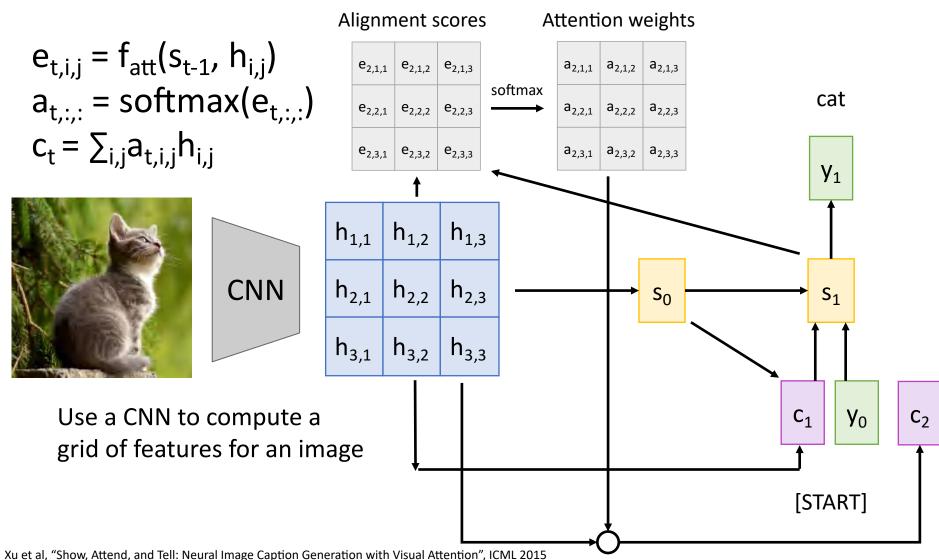


[START]

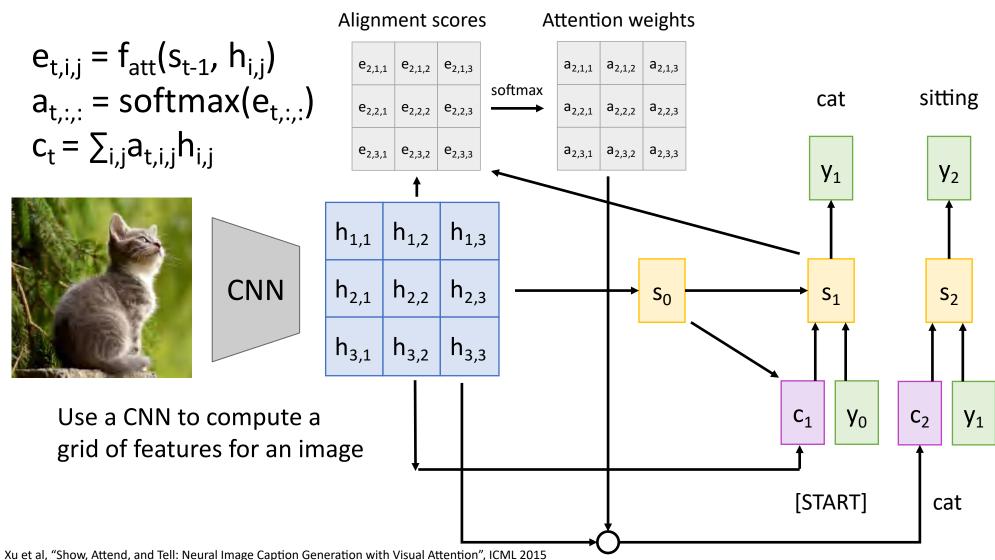




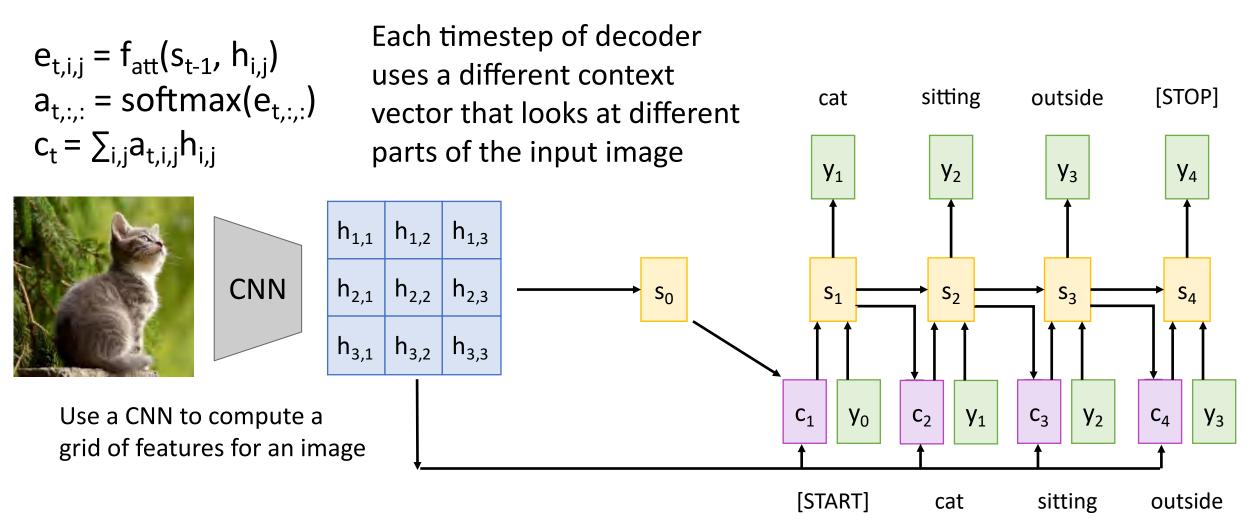




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# Image Captioning with RNNs and Attention



Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

# Image Captioning with RNNs and Attention



A dog is standing on a hardwood floor.



A <u>stop</u> sign is on a road with a mountain in the background.



A group of <u>people</u> sitting on a boat in the water.

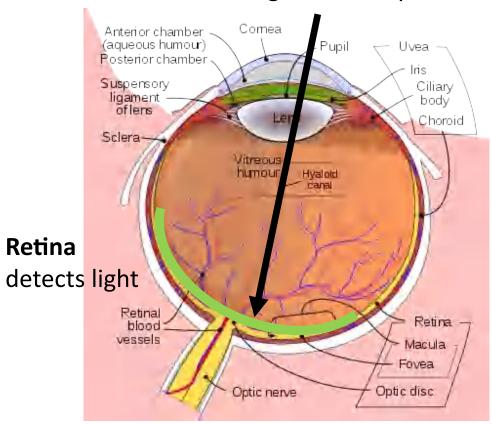


A giraffe standing in a forest with trees in the background.

Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

# Human Vision: Fovea

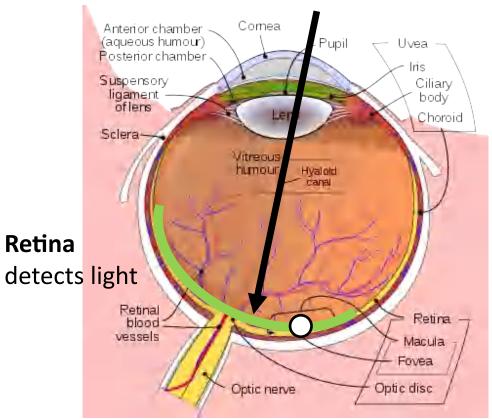
## Light enters eye



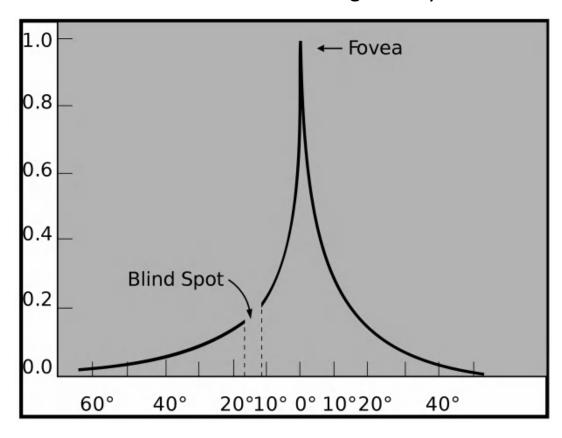
Acuity graph is licensed under CC A-SA 3.0 Unported

# Human Vision: Fovea





The **fovea** is a tiny region of the retina that can see with high acuity

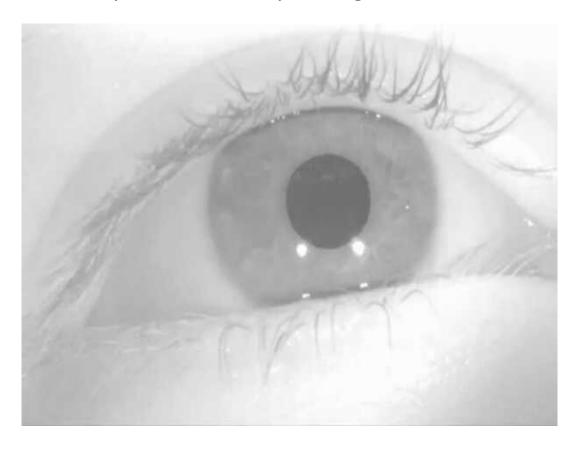


Eye image is licensed under CC A-SA 3.0 Unported (added black arrow, green arc, and white circle)

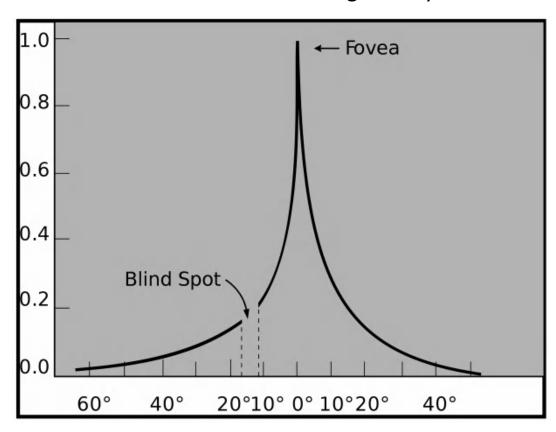
Acuity graph is licensed under CC A-SA 3.0 Unported (No changes made)

# Human Vision: Saccades

Human eyes are constantly moving so we don't notice



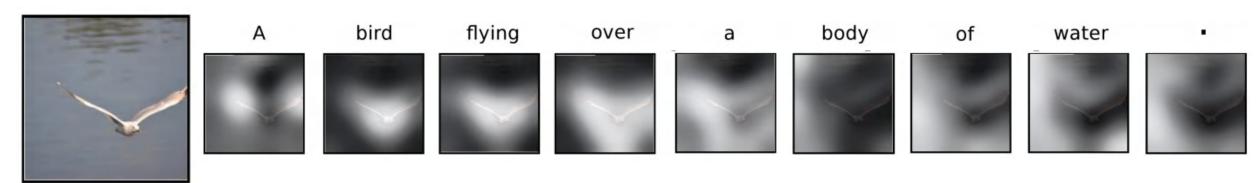
The **fovea** is a tiny region of the retina that can see with high acuity



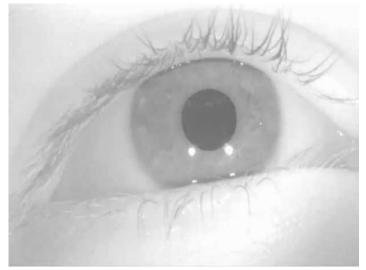
<u>Saccade video</u> is licensed under <u>CC A-SA 4.0 International</u> (no changes made)

Acuity graph is licensed under CC A-SA 3.0 Unported (No changes made)

# Image Captioning with RNNs and Attention



Attention weights at each timestep kind of like saccades of human eye



Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Saccade video is licensed under CC A-SA 4.0 International (no changes made)

# X, Attend, and Y

"Show, attend, and tell" (Xu et al, ICML 2015)
Look at image, attend to image regions, produce question

"Ask, attend, and answer" (Xu and Saenko, ECCV 2016)

"Show, ask, attend, and answer" (Kazemi and Elqursh, 2017)

Read text of question, attend to image regions, produce answer

"Listen, attend, and spell" (Chan et al, ICASSP 2016)
Process raw audio, attend to audio regions while producing text

"Listen, attend, and walk" (Mei et al, AAAI 2016)
Process text, attend to text regions, output navigation commands

"Show, attend, and interact" (Qureshi et al, ICRA 2017)
Process image, attend to image regions, output robot control commands

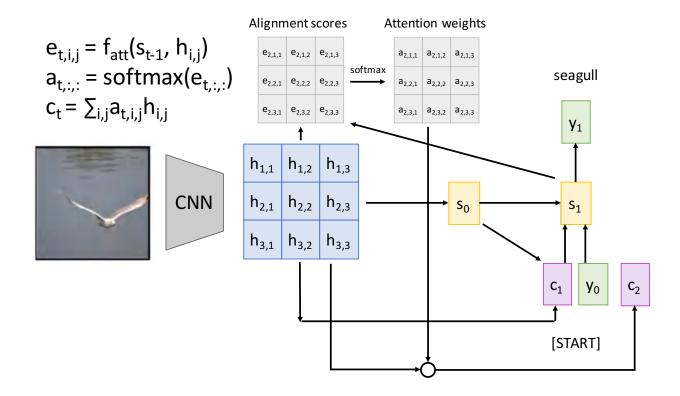
"Show, attend, and read" (Li et al, AAAI 2019)
Process image, attend to image regions, output text

Inputs:

**Query vector**: **q** (Shape: D<sub>Q</sub>)

**Input vectors**: X (Shape:  $N_X \times D_X$ )

**Similarity function**: f<sub>att</sub>



# **Computation**:

**Similarities**: e (Shape:  $N_X$ )  $e_i = f_{att}(q, X_i)$ 

**Attention weights**: a = softmax(e) (Shape:  $N_x$ )

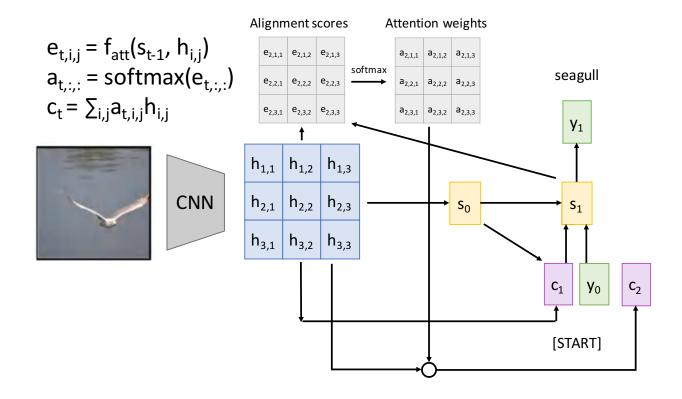
**Output vector**:  $y = \sum_i a_i X_i$  (Shape:  $D_X$ )

Inputs:

**Query vector**:  $\mathbf{q}$  (Shape:  $D_Q$ )

**Input vectors**: X (Shape:  $N_X \times D_Q$ )

Similarity function: dot product



## **Computation**:

**Similarities**: e (Shape:  $N_X$ )  $e_i = \mathbf{q} \cdot \mathbf{X_i}$ 

**Attention weights**: a = softmax(e) (Shape:  $N_x$ )

**Output vector**:  $y = \sum_i a_i X_i$  (Shape:  $D_X$ )

# Changes:

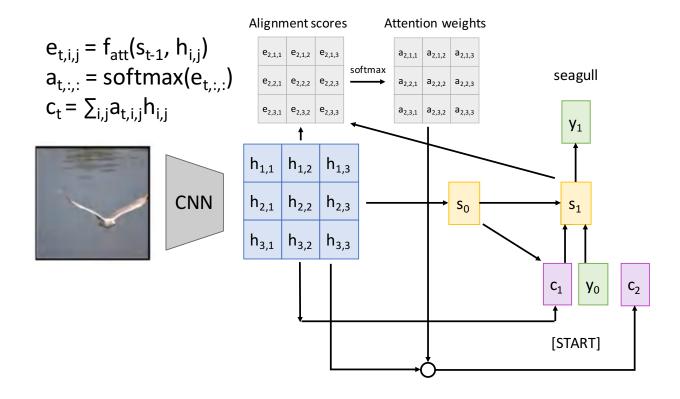
Use dot product for similarity

Inputs:

**Query vector**: **q** (Shape: D<sub>Q</sub>)

Input vectors: X (Shape:  $N_X \times D_Q$ )

Similarity function: scaled dot product



## **Computation**:

**Similarities**: e (Shape:  $N_X$ )  $e_i = \mathbf{q} \cdot \mathbf{X}_i / \sqrt{D_Q}$ 

**Attention weights**: a = softmax(e) (Shape:  $N_x$ )

Output vector:  $y = \sum_i a_i X_i$  (Shape:  $D_X$ )

## Changes:

Use scaled dot product for similarity

## Inputs:

**Query vector**: **q** (Shape: D<sub>Q</sub>)

Input vectors: X (Shape:  $N_X \times D_Q$ )

Similarity function: scaled dot product

Large similarities will cause softmax to saturate and give vanishing gradients

Recall  $a \cdot b = |a||b| \cos(angle)$ 

Suppose that a and b are constant vectors of

dimension D

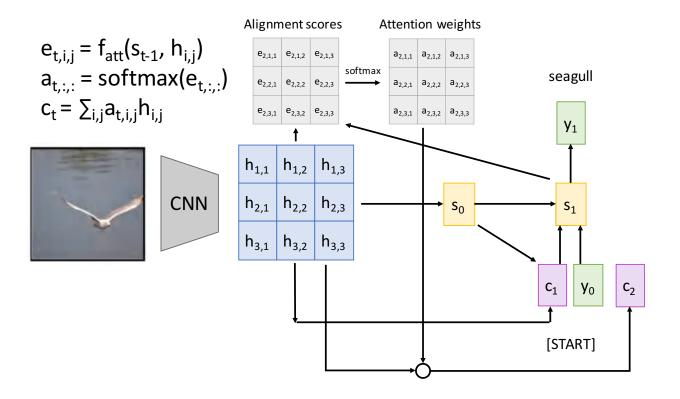
Then  $|a| = (\sum_i a^2)^{1/2} = a \sqrt{D}$ 

## **Computation**:

**Similarities**: e (Shape:  $N_X$ )  $e_i = \mathbf{q} \cdot \mathbf{X}_i / \sqrt{D_Q}$ 

**Attention weights**: a = softmax(e) (Shape:  $N_x$ )

Output vector:  $y = \sum_i a_i X_i$  (Shape:  $D_X$ )



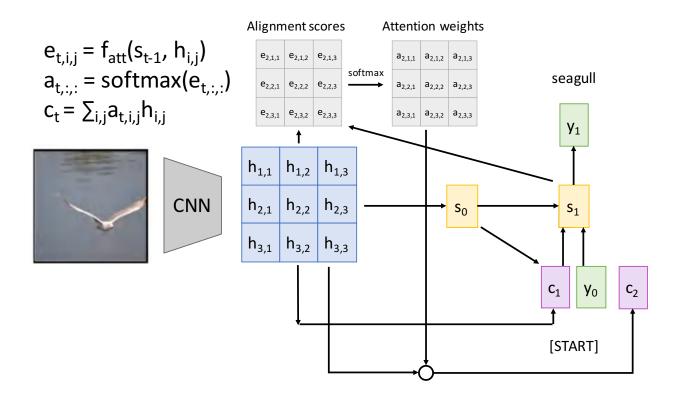
### Changes:

Use scaled dot product for similarity

Inputs:

Query vectors: Q (Shape:  $N_Q \times D_Q$ )

Input vectors: X (Shape:  $N_X \times D_Q$ )



## **Computation**:

Similarities:  $E = QX^T/\sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot X_j)/\sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )

Output vectors: Y = AX (Shape:  $N_Q \times D_X$ )  $Y_i = \sum_j A_{i,j} X_j$ 

# Changes:

- Use scaled dot product for similarity
- Multiple query vectors

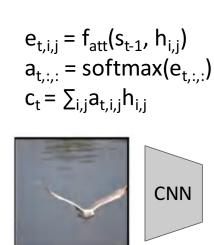
Inputs:

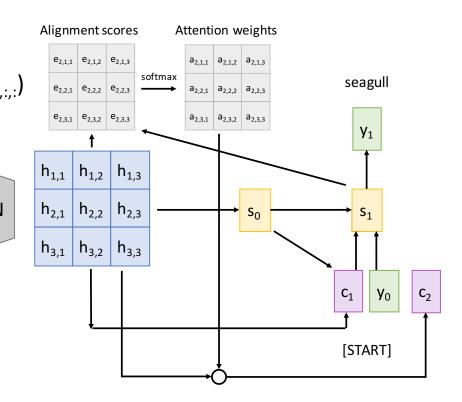
**Query vectors**:  $\mathbf{Q}$  (Shape:  $N_Q \times D_Q$ )

**Input vectors**: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

**Value matrix:**  $W_V$  (Shape:  $D_X \times D_V$ )





# **Computation**:

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{Q}\mathbf{K}^{\mathsf{T}}/\sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (\mathbf{Q}_i \cdot \mathbf{K}_j)/\sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )

Output vectors: Y = AV (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$ 

## Changes:

- Use scaled dot product for similarity
- Multiple query vectors
- Separate key and value

### Inputs:

Query vectors: Q (Shape:  $N_Q \times D_Q$ ) Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

### **Computation**:

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )

Output vectors: Y = AV (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$ 

$$X_1$$





 $Q_1$ 

 $Q_2$ 

 $Q_3$ 

 $Q_4$ 

### Inputs:

Query vectors: Q (Shape:  $N_Q \times D_Q$ ) Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

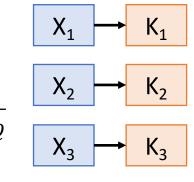
### **Computation**:

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_0 \times N_x$ )

Output vectors: Y = AV (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$ 



 $Q_1$ 

 $Q_2$ 

 $Q_3$ 

 $Q_4$ 

### Inputs:

Query vectors: Q (Shape:  $N_Q \times D_Q$ ) Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

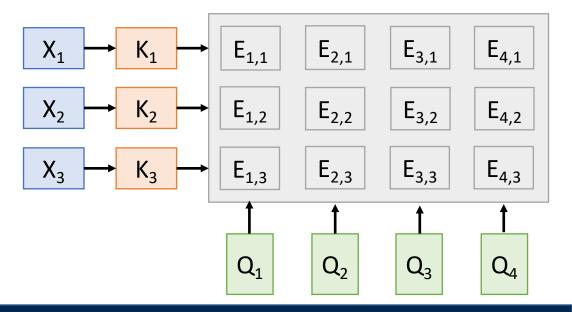
#### **Computation**:

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

**Attention weights**: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )



#### Inputs:

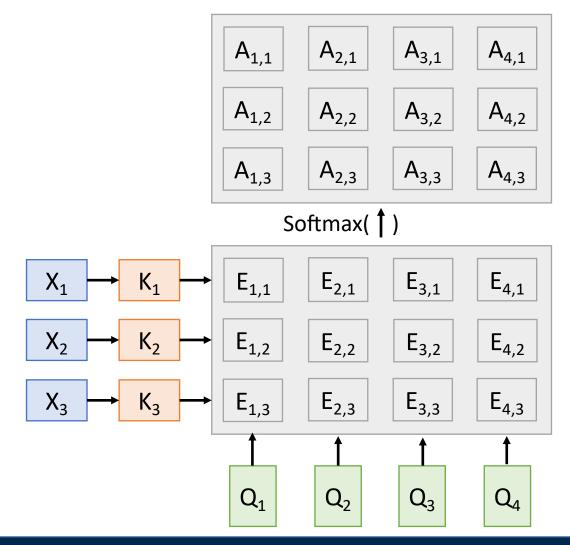
Query vectors: Q (Shape:  $N_Q \times D_Q$ ) Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

#### **Computation**:

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )



#### Inputs:

Query vectors: Q (Shape:  $N_0 \times D_0$ ) **Input vectors**: X (Shape:  $N_x \times D_x$ ) **Key matrix**:  $W_K$  (Shape:  $D_X \times D_O$ ) Value matrix:  $W_v$  (Shape:  $D_x \times D_v$ )

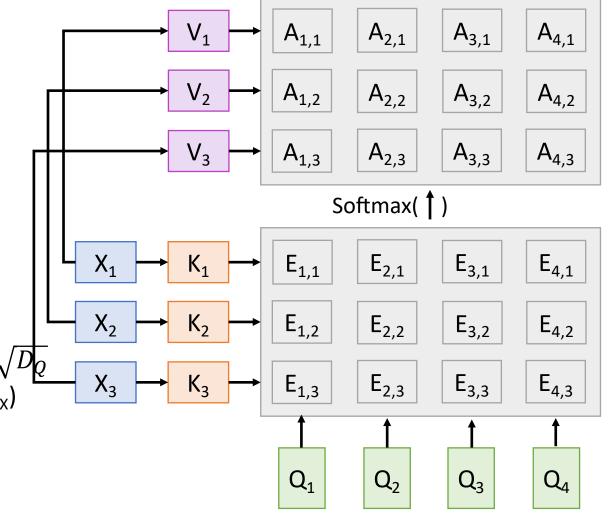
### **Computation:**

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_O$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )



### Inputs:

Query vectors: Q (Shape:  $N_0 \times D_0$ ) Input vectors: X (Shape:  $N_x \times D_x$ ) **Key matrix**:  $W_K$  (Shape:  $D_X \times D_O$ )

Value matrix:  $W_v$  (Shape:  $D_x \times D_v$ )

### **Computation:**

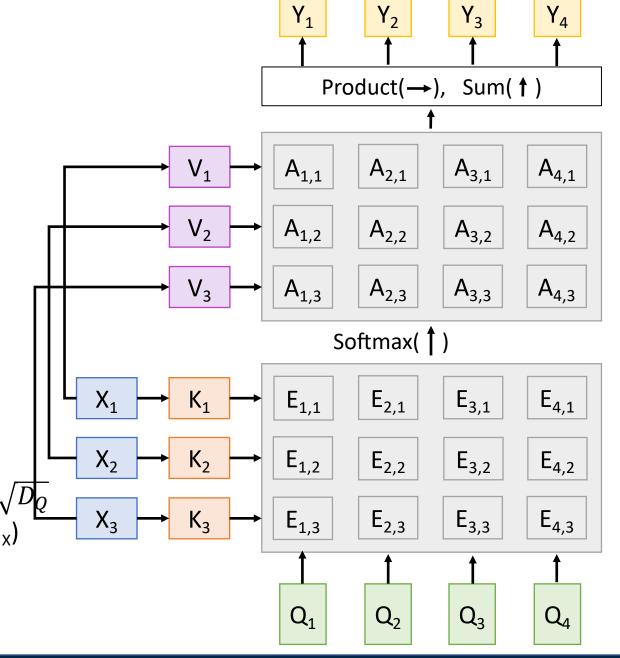
**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_O$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )

Output vectors: Y = AV (Shape:  $N_O \times D_V$ )  $Y_i = \sum_i A_{i,i} V_i$ 



Justin Johnson March 21, 2022 Lecture 17 -

One query per input vector

## Inputs:

Query vectors: Q (Shape:  $N_Q \times D_Q$ ) Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

### **Computation**:

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_Q \times N_X$ )

Output vectors: Y = AV (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$ 

 $X_1$   $X_2$   $X_3$ 

One query per input vector

## Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ ) Query matrix:  $W_O$  (Shape:  $D_X \times D_Q$ )

#### **Computation**:

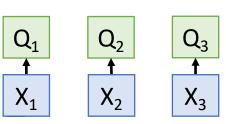
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



One query per input vector

## Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

### **Computation**:

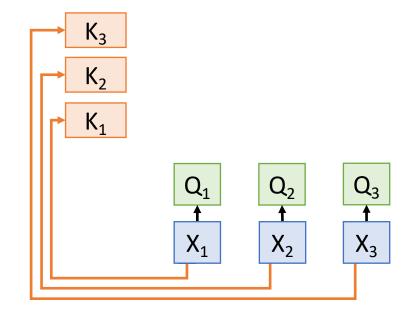
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



One query per input vector

### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ ) Query matrix:  $W_O$  (Shape:  $D_X \times D_O$ )

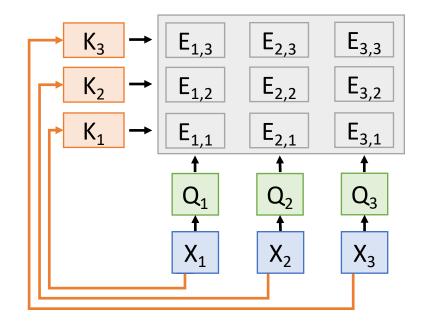
**Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

**Attention weights**: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



One query per input vector

## Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

### **Computation**:

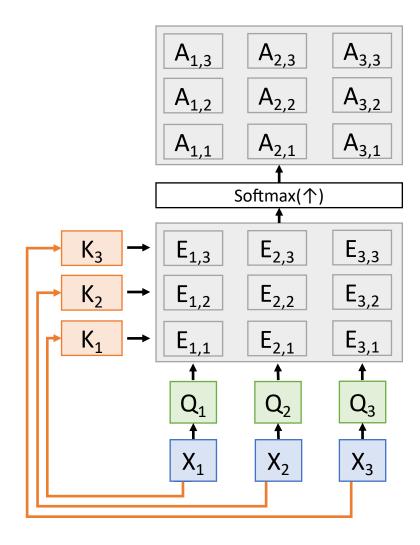
Query vectors:  $Q = XW_Q$ 

Key vectors:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



One query per input vector

### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

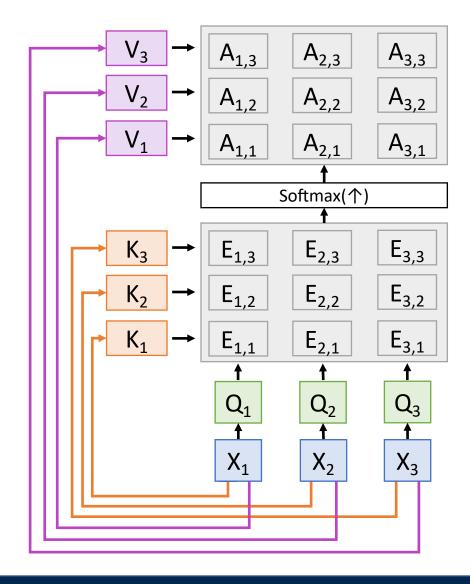
### **Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



One query per input vector

### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_O$ )

Value matrix:  $W_v$  (Shape:  $D_x \times D_v$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

### **Computation**:

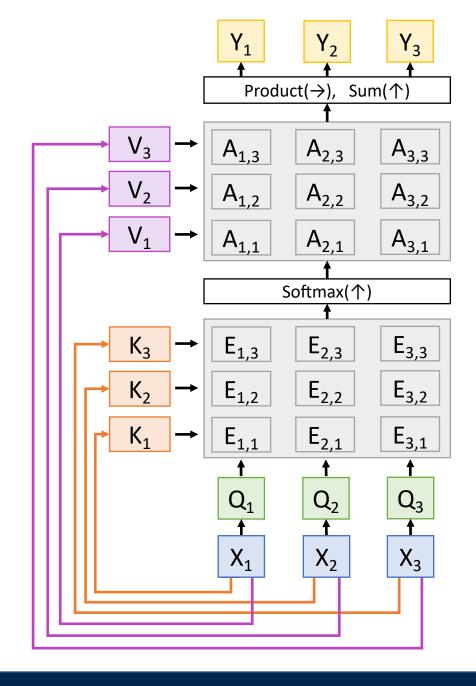
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



Consider **permuting** the input vectors:

### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ ) Query matrix:  $W_O$  (Shape:  $D_X \times D_O$ )

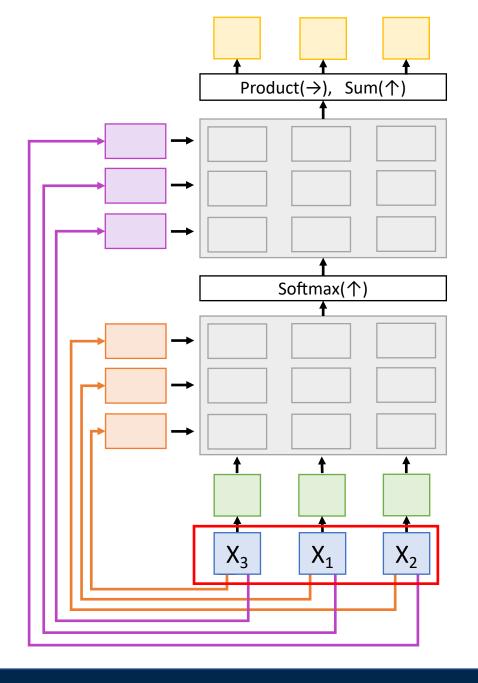
#### **Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



Consider **permuting** the input vectors:

## Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Queries and Keys will be the same, but permuted

### **Computation**:

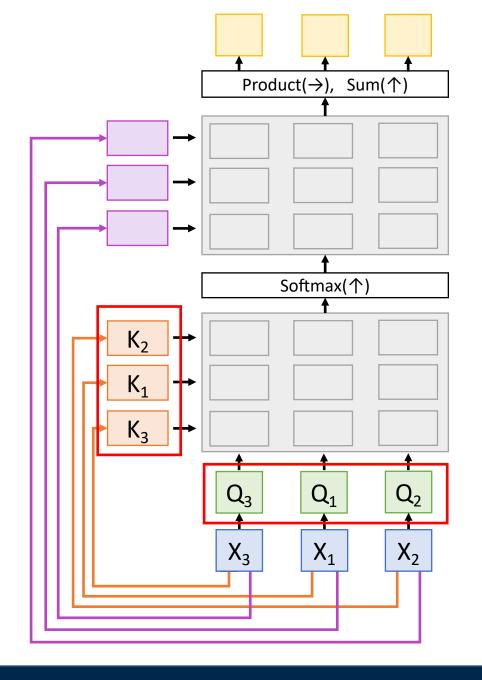
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



Consider **permuting** the input vectors:

### Inputs:

**Input vectors**: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

**Value matrix:**  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Similarities will be the same, but permuted

### **Computation**:

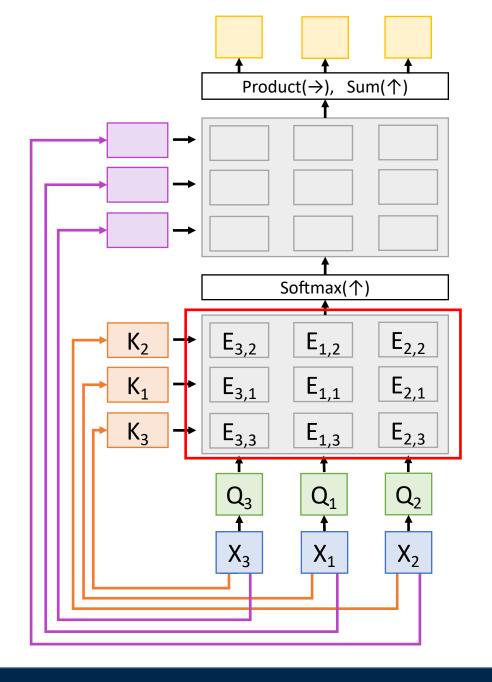
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



Consider **permuting** the input vectors:

### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Attention weights will be the same, but permuted

### **Computation**:

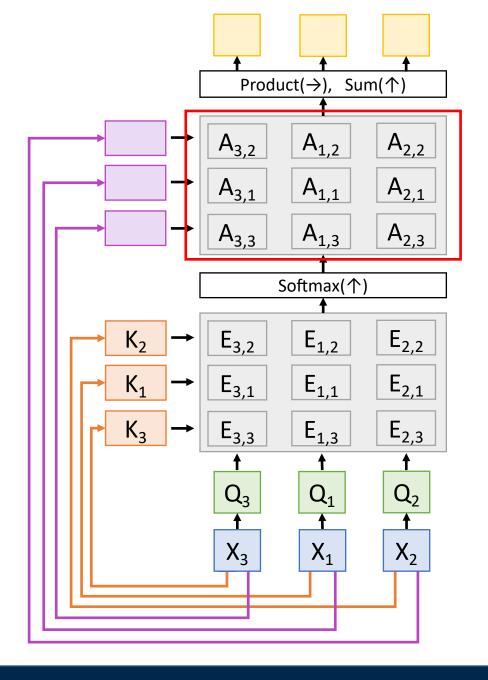
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



Consider **permuting** the input vectors:

### Inputs:

**Input vectors**: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

**Value matrix:**  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Values will be the same, but permuted

### **Computation**:

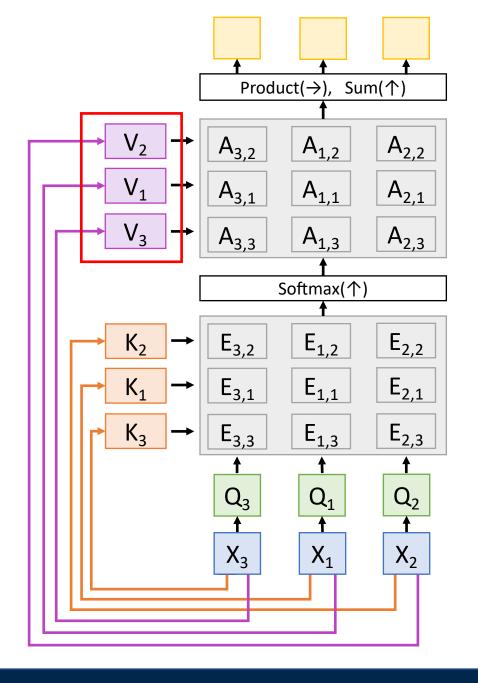
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



Consider **permuting** the input vectors:

### Inputs:

**Input vectors**: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

**Value matrix:**  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Outputs will be the same, but permuted

### **Computation**:

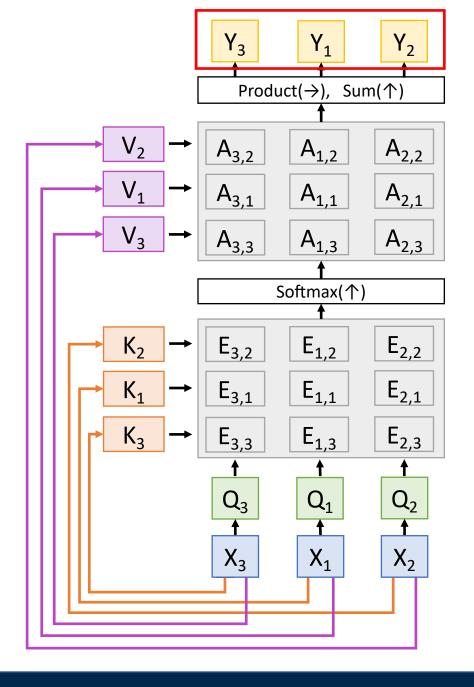
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ ) Query matrix:  $W_O$  (Shape:  $D_X \times D_Q$ )

#### **Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )

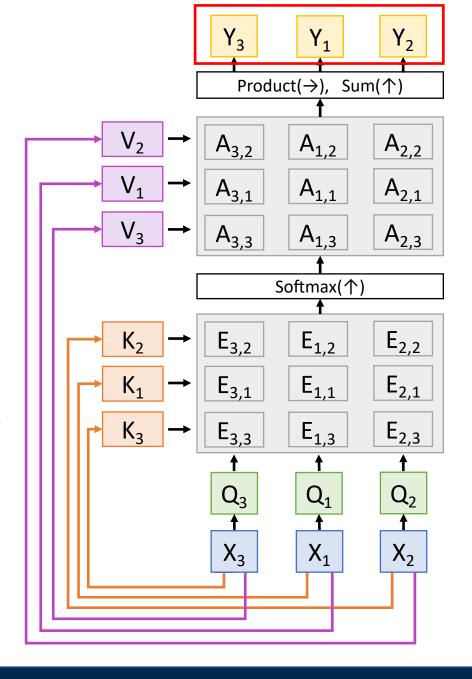
Output vectors: Y = AV (Shape:  $N_X \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$ 

Consider **permuting** the input vectors:

Outputs will be the same, but permuted

Self-attention layer is **Permutation Equivariant** f(s(x)) = s(f(x))

Self-Attention layer works on **sets** of vectors



Self attention doesn't "know" the order of the vectors it is processing!

## Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ ) Query matrix:  $W_O$  (Shape:  $D_X \times D_Q$ )

#### **Computation**:

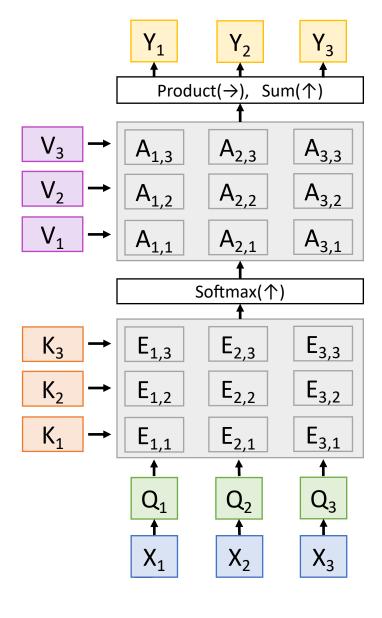
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



### Inputs:

**Input vectors**: X (Shape:  $N_X \times D_X$ ) **Key matrix**:  $W_{\kappa}$  (Shape:  $D_{\chi} \times D_{O}$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ ) Query matrix:  $W_0$  (Shape:  $D_x \times D_0$ ) Self attention doesn't "know" the order of the vectors it is processing!

In order to make processing positionaware, concatenate or add positional encoding to the input

#### **Computation**:

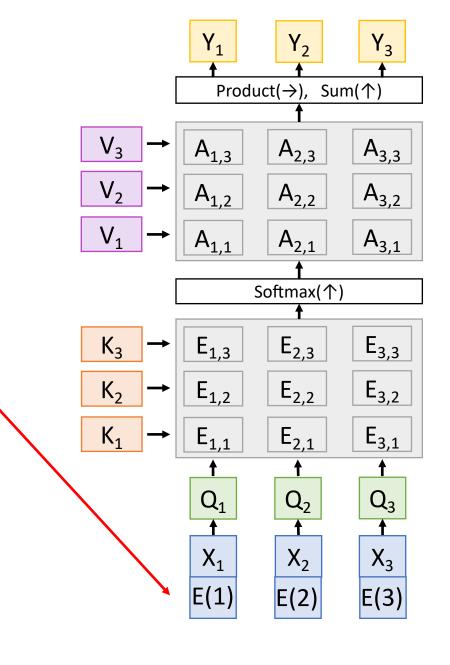
Query vectors:  $Q = XW_0$ 

E can be learned lookup **Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_O$ ) table, or fixed function

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



# Masked Self-Attention Layer

Don't let vectors "look ahead" in the sequence

## Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_O$ )

**Value matrix:**  $W_v$  (Shape:  $D_x \times D_v$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

#### **Computation**:

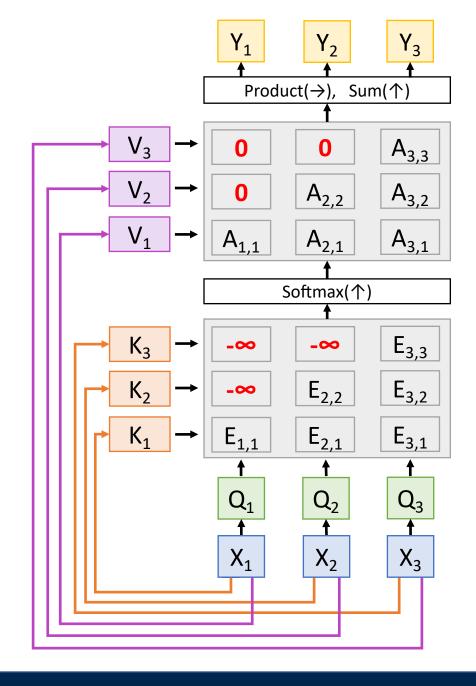
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



# Masked Self-Attention Layer

Don't let vectors "look ahead" in the sequence Used for language modeling (predict next word)

#### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

#### **Computation**:

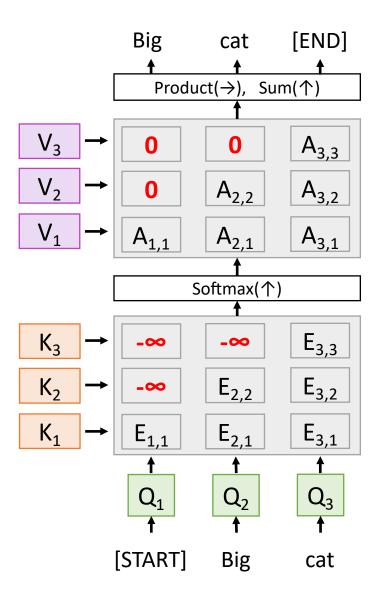
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



#### Inputs:

```
Input vectors: X (Shape: N_X \times D_X)
Key matrix: W_K (Shape: D_X \times D_Q)
Value matrix: W_V (Shape: D_X \times D_V)
```

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Use H independent "Attention Heads" in

parallel

#### **Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )

Output vectors: Y = AV (Shape:  $N_X \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$ 

 $X_1$ 

 $X_2$ 

 $X_3$ 

#### Inputs:

```
Input vectors: X (Shape: N_X \times D_X)
Key matrix: W_K (Shape: D_X \times D_Q)
Value matrix: W_V (Shape: D_X \times D_V)
```

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Use H independent "Attention Heads" in

parallel

#### **Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )

Output vectors: Y = AV (Shape:  $N_X \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$ 

X<sub>1,1</sub>
X<sub>1,2</sub>
X<sub>1,3</sub>

X<sub>2,1</sub>
X<sub>2,2</sub>
X<sub>2,3</sub>

X<sub>3,1</sub>
X<sub>3,2</sub>
X<sub>3,3</sub>

#### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Use H independent "Attention Heads" in

parallel

#### Computation:

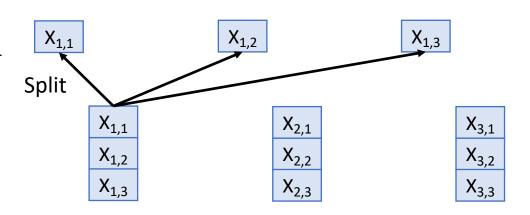
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q}_i \cdot \mathbf{K}_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



#### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ ) Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_o$  (Shape:  $D_x \times D_o$ )

Use H independent "Attention Heads" in

parallel

#### **Computation**:

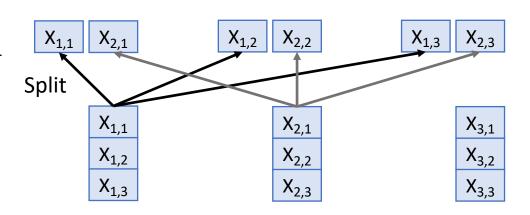
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



#### Inputs:

```
Input vectors: X (Shape: N_X \times D_X)
Key matrix: W_K (Shape: D_X \times D_Q)
Value matrix: W_V (Shape: D_X \times D_V)
```

**Query matrix**:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Use H independent "Attention Heads" in

parallel

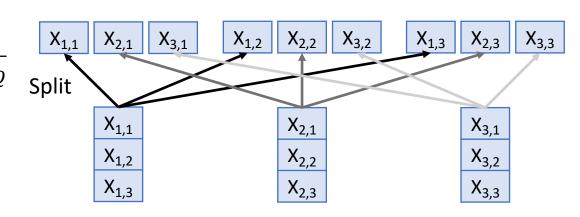
#### **Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ ) **Value Vectors**:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



Run self-attention in parallel on each set of input vectors (different weights per head)

#### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ ) Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ )

**Value matrix:**  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Use H independent "Attention Heads" in parallel

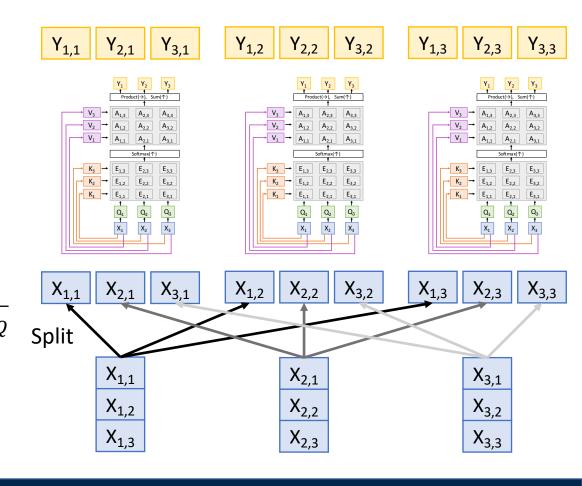
#### **Computation**:

Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ ) Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

Attention weights: A = softmax(E, dim=1) (Shape:  $N_x \times N_x$ )



#### Inputs:

Input vectors: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_Q$ )

**Value matrix:**  $W_V$  (Shape:  $D_X \times D_V$ )

**Query matrix**:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Use H independent "Attention Heads" in

parallel

#### **Computation**:

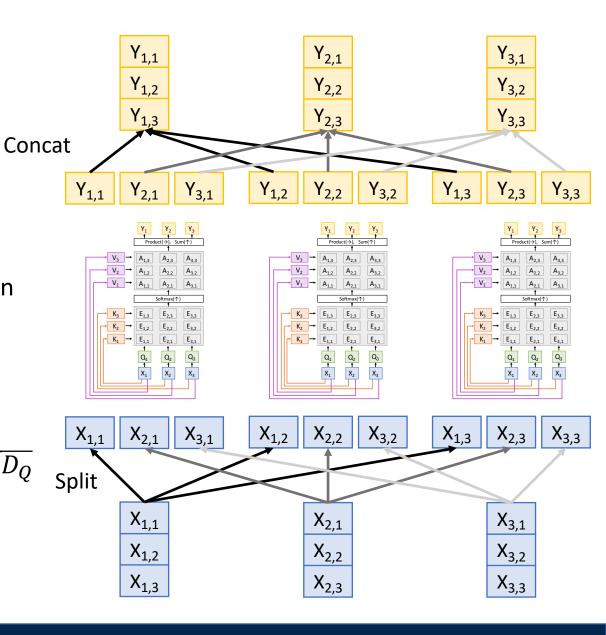
Query vectors:  $Q = XW_Q$ 

**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = \mathbf{QK^T} / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (\mathbf{Q_i} \cdot \mathbf{K_j}) / \sqrt{D_Q}$ 

**Attention weights**: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )



<u>Inputs</u>:

Input vectors: X (Shape:  $N_X \times D_X$ )

**Key matrix**:  $W_K$  (Shape:  $D_X \times D_O$ )

**Value matrix:**  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

Use H independent "Attention Heads" in

parallel

**Computation**:

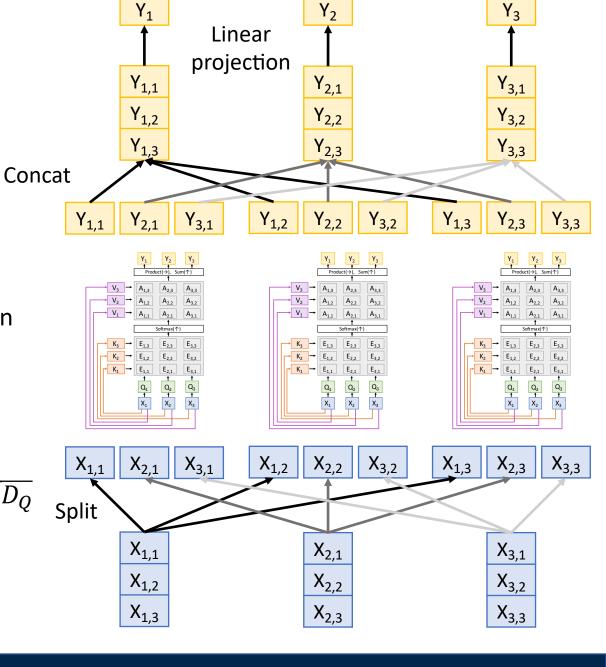
Query vectors:  $Q = XW_Q$ 

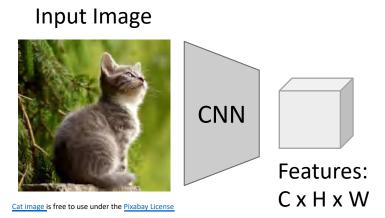
**Key vectors**:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

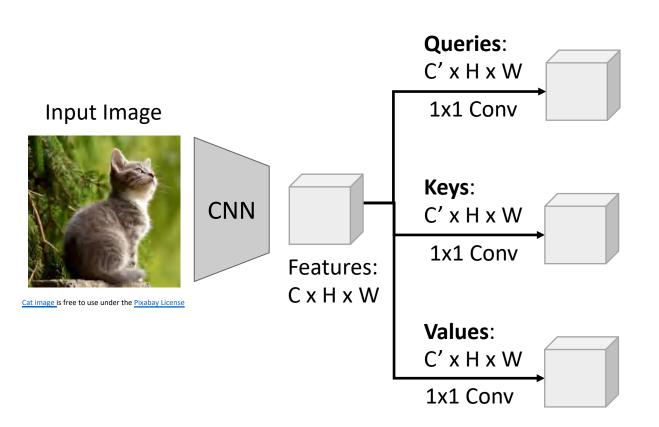
Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

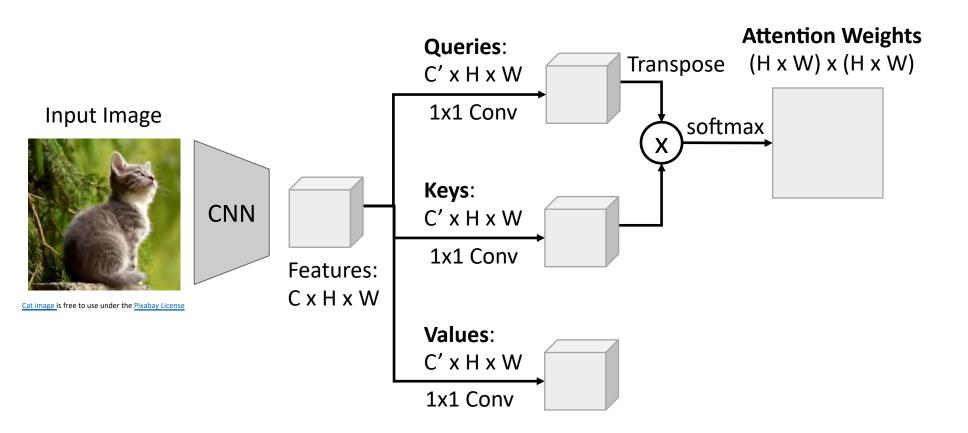
Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$ 

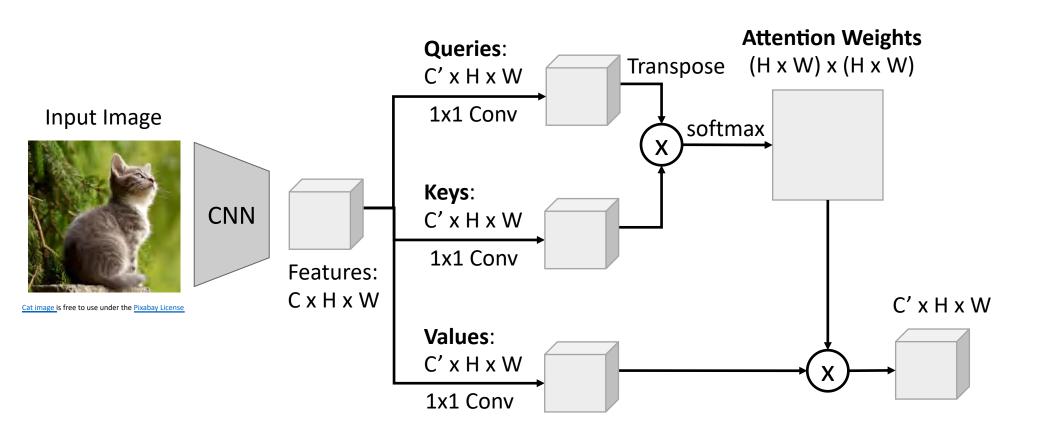
Attention weights: A = softmax(E, dim=1) (Shape:  $N_X \times N_X$ )

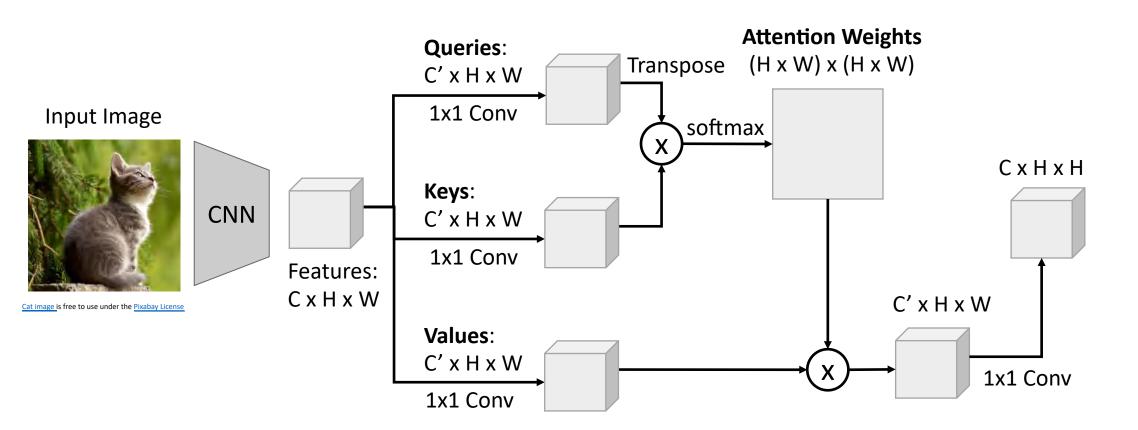


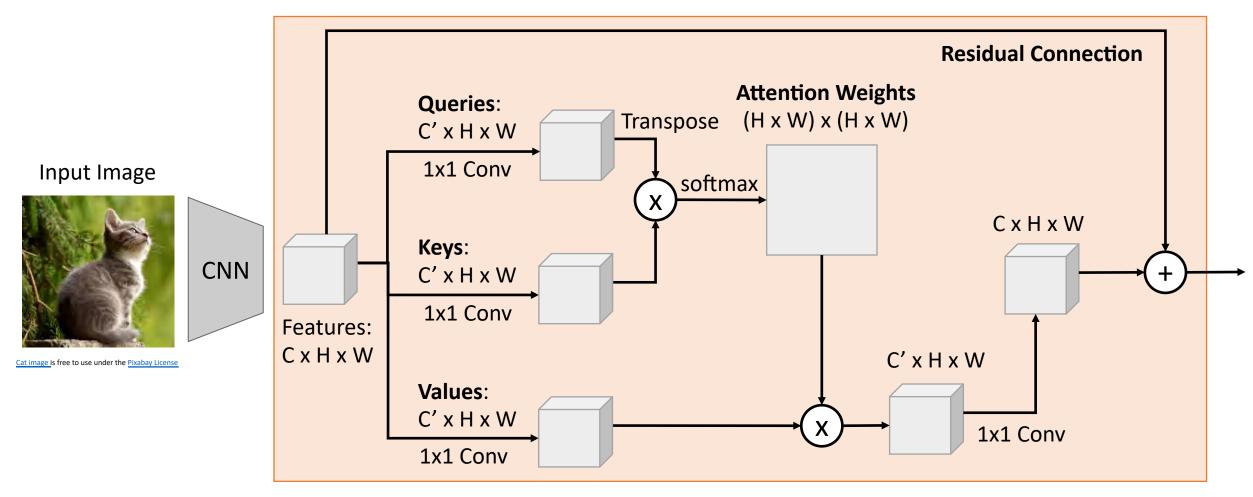






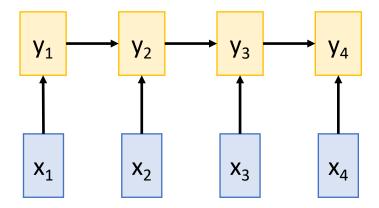






Self-Attention Module

#### **Recurrent Neural Network**



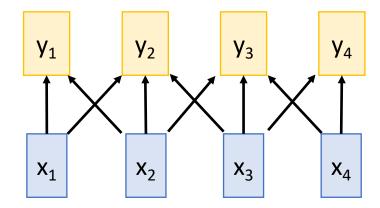
#### Works on **Ordered Sequences**

- (+) Good at long sequences: After one RNN layer, h<sub>T</sub> "sees" the whole sequence
- (-) Not parallelizable: need to compute hidden states sequentially

#### Recurrent Neural Network

#### 

#### 1D Convolution



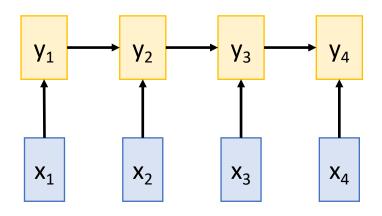
#### Works on **Ordered Sequences**

- (+) Good at long sequences: After one RNN layer, h<sub>T</sub> "sees" the whole sequence
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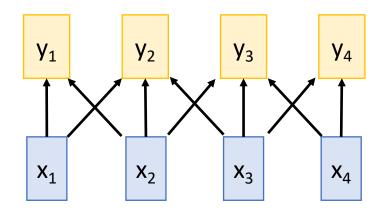
#### Works on Multidimensional Grids

- (-) Bad at long sequences: Need to stack many conv layers for outputs to "see" the whole sequence
- (+) Highly parallel: Each output can be computed in parallel

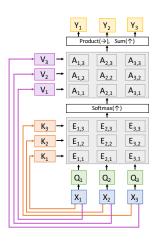
#### Recurrent Neural Network



#### 1D Convolution



#### **Self-Attention**



#### Works on **Ordered Sequences**

- (+) Good at long sequences: After one RNN layer, h<sub>T</sub> "sees" the whole sequence
- (-) Not parallelizable: need to compute hidden states sequentially

#### Works on Multidimensional Grids

- (-) Bad at long sequences: Need to stack many conv layers for outputs to "see" the whole sequence
- (+) Highly parallel: Each output can be computed in parallel

#### Works on **Sets of Vectors**

- (-) Good at long sequences: after one self-attention layer, each output "sees" all inputs!
- (+) Highly parallel: Each output can be computed in parallel
- (-) Very memory intensive

Recurrent Neural Network

1D Convolution

Self-Attention

# Attention is all you need

Vaswani et al, NeurIPS 2017

#### Works on **Ordered Sequences**

- (+) Good at long sequences: After one RNN layer, h<sub>T</sub> "sees" the whole sequence
- (-) Not parallelizable: need to compute hidden states sequentially

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- (-) Very memory intensive

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**X**<sub>1</sub>

 $X_2$ 

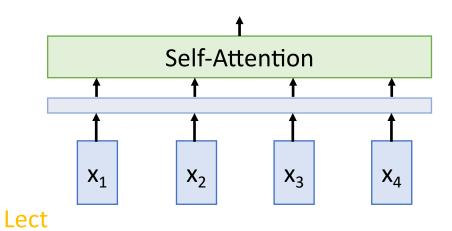
**X**<sub>3</sub>

 $X_4$ 

Vaswani et al, "Attention is all you need", NeurIPS 2017

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All vectors interact with each other

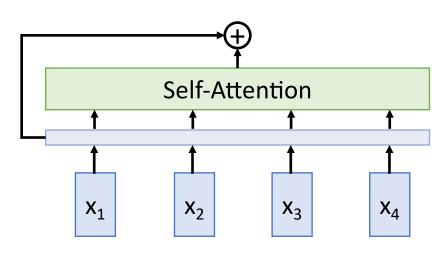


Vaswani et al, "Attention is all you need", NeurIPS 2017

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Residual connection All vectors interact with each other



#### Recall Layer Normalization:

Given  $h_1, ..., h_N$  (Shape: D)

scale:  $\gamma$  (Shape: D)

shift:  $\beta$  (Shape: D)

$$\mu_i = (\sum_j h_{i,j})/D$$
 (scalar)

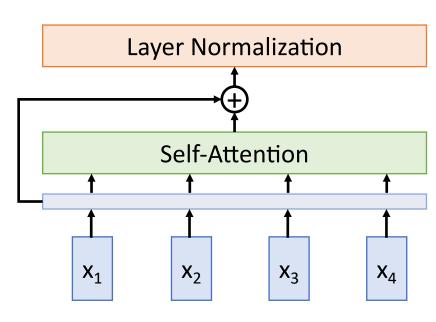
$$\sigma_{i} = (\sum_{i} (h_{i,i} - \mu_{i})^{2}/D)^{1/2}$$
 (scalar)

$$z_i = (h_i - \mu_i) / \sigma_i$$

$$y_i = \gamma * z_i + \beta$$

Ba et al, 2016

Residual connection
All vectors interact
with each other



#### Recall Layer Normalization:

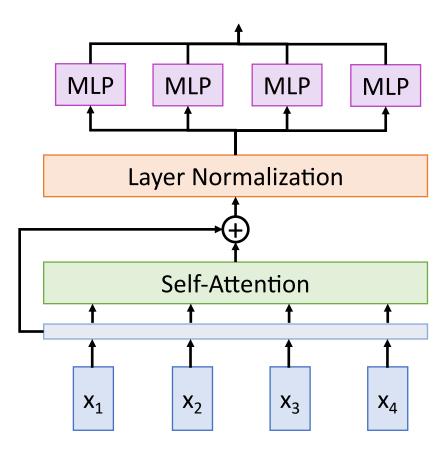
Given  $h_1$ , ...,  $h_N$  (Shape: D) scale:  $\gamma$  (Shape: D) shift:  $\beta$  (Shape: D)  $\mu_i = (\sum_j h_{i,j})/D$  (scalar)  $\sigma_i = (\sum_j (h_{i,j} - \mu_i)^2/D)^{1/2}$  (scalar)  $z_i = (h_i - \mu_i) / \sigma_i$ 

Ba et al, 2016

 $y_i = \gamma * z_i + \beta$ 

MLP independently on each vector

Residual connection
All vectors interact
with each other



#### Recall **Layer Normalization**:

Given  $h_1, ..., h_N$  (Shape: D)

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 $z_i = (h_i - \mu_i) / \sigma_i$ 

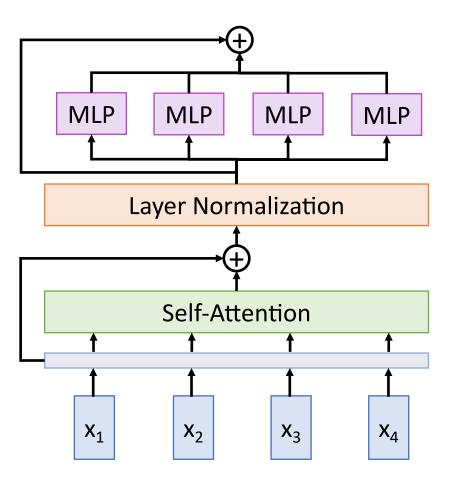
 $y_i = \gamma * z_i + \beta$ 

Ba et al, 2016

Residual connection

MLP independently on each vector

Residual connection
All vectors interact
with each other



#### Recall Layer Normalization:

Given  $h_1, ..., h_N$  (Shape: D)

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shift:  $\beta$  (Shape: D)

 $\mu_i = (\sum_i h_{i,i})/D$  (scalar)

 $\sigma_{i} = (\sum_{i} (h_{i,i} - \mu_{i})^{2}/D)^{1/2}$  (scalar)

 $z_i = (h_i - \mu_i) / \sigma_i$ 

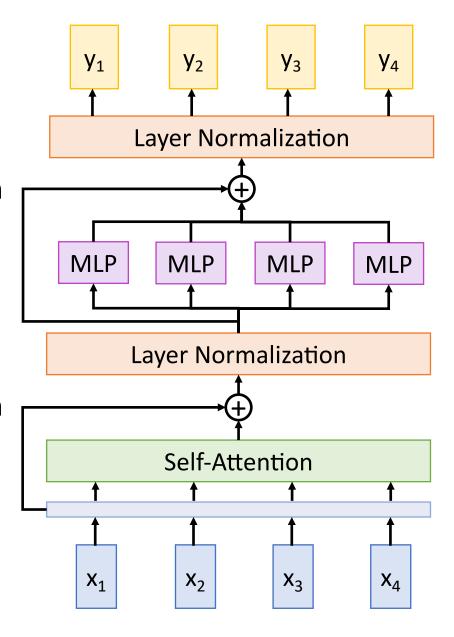
 $y_i = \gamma * z_i + \beta$ 

Ba et al, 2016

Residual connection

MLP independently on each vector

Residual connection
All vectors interact
with each other



#### **Transformer Block:**

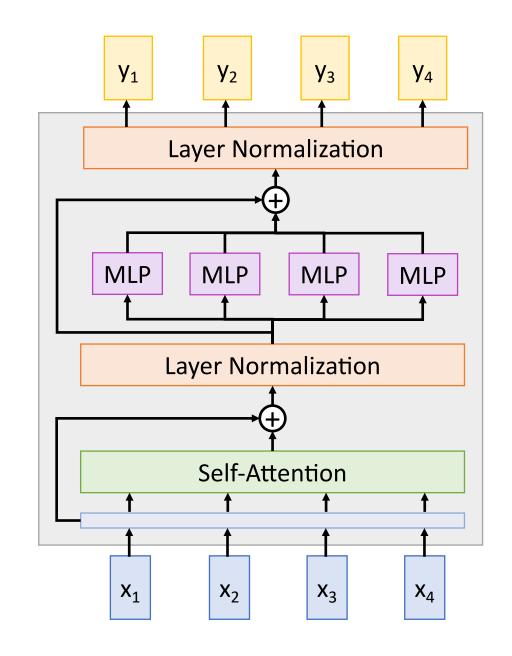
**Input**: Set of vectors x

Output: Set of vectors y

Self-attention is the only interaction between vectors!

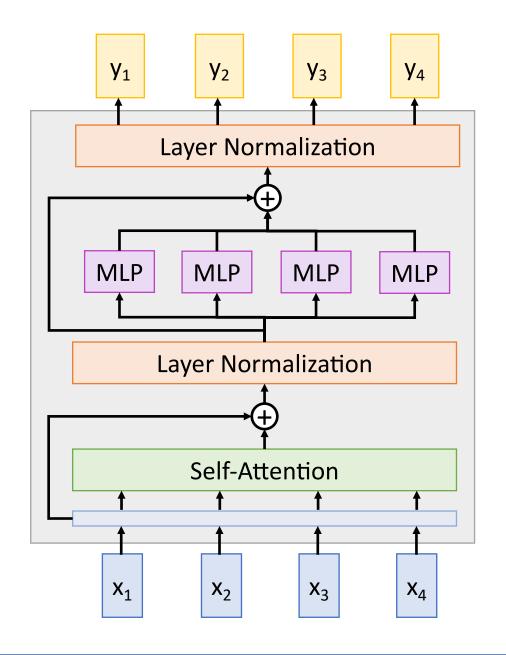
Layer norm and MLP work independently per vector

Highly scalable, highly parallelizable



### Post-Norm Transformer

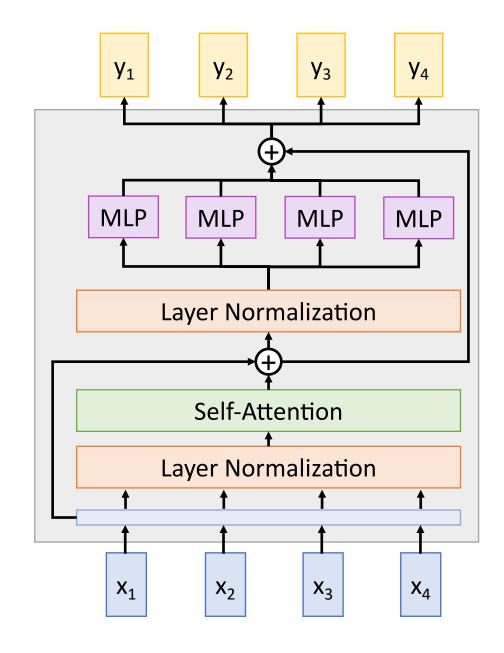
**Layer normalization** is **after** residual connections



### Pre-Norm Transformer

**Layer normalization** is **inside** residual connections

Gives more stable training, commonly used in practice



Baevski & Auli, "Adaptive Input Representations for Neural Language Modeling", arXiv 2018

#### **Transformer Block:**

**Input**: Set of vectors x

Output: Set of vectors y

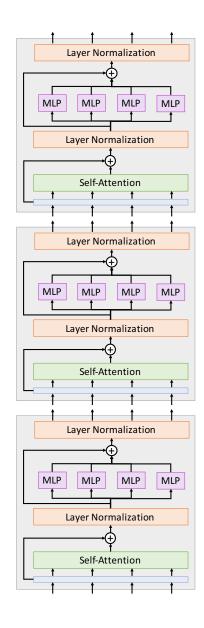
Self-attention is the only interaction between vectors!

Layer norm and MLP work independently per vector

Highly scalable, highly parallelizable

A **Transformer** is a sequence of transformer blocks

Vaswani et al: 12 blocks, D<sub>o</sub>=512, 6 heads



# The Transformer: Transfer Learning

"ImageNet Moment for Natural Language Processing"

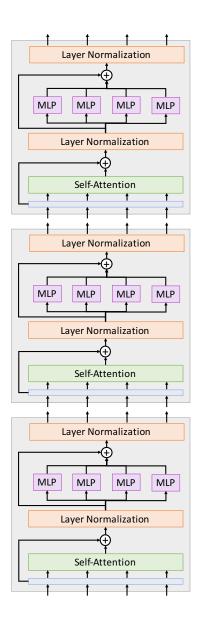
### **Pretraining:**

Download a lot of text from the internet

Train a giant Transformer model for language modeling

### **Finetuning:**

Fine-tune the Transformer on your own NLP task



Devlin et al, "BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding", EMNLP 2018

| Model             | Layers | Width | Heads | Params | Data | Training           |
|-------------------|--------|-------|-------|--------|------|--------------------|
| Transformer-Base  | 12     | 512   | 8     | 65M    |      | 8x P100 (12 hours) |
| Transformer-Large | 12     | 1024  | 16    | 213M   |      | 8x P100 (3.5 days) |

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| BERT-Base         | 12     | 768   | 12    | 110M   | 13 GB |                    |
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Devlin et al, "BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding", EMNLP 2018

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| BERT-Large        | 24     | 1024  | 16    | 340M   | 13 GB  |                        |
| XLNet-Large       | 24     | 1024  | 16    | ~340M  | 126 GB | 512x TPU-v3 (2.5 days) |
| RoBERTa           | 24     | 1024  | 16    | 355M   | 160 GB | 1024x V100 GPU (1 day) |

Yang et al, XLNet: Generalized Autoregressive Pretraining for Language Understanding", 2019 Liu et al, "RoBERTa: A Robustly Optimized BERT Pretraining Approach", 2019

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| Model             | Layers | Width | Heads | Params | Data   | Training               |
|-------------------|--------|-------|-------|--------|--------|------------------------|
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| BERT-Base         | 12     | 768   | 12    | 110M   | 13 GB  |                        |
| BERT-Large        | 24     | 1024  | 16    | 340M   | 13 GB  |                        |
| XLNet-Large       | 24     | 1024  | 16    | ~340M  | 126 GB | 512x TPU-v3 (2.5 days) |
| RoBERTa           | 24     | 1024  | 16    | 355M   | 160 GB | 1024x V100 GPU (1 day) |
| GPT-2             | 48     | 1600  | ?     | 1.5B   | 40 GB  |                        |

Radford et al, "Language models are unsupervised multitask learners", 2019

| Model             | Layers | Width | Heads | Params | Data   | Training               |
|-------------------|--------|-------|-------|--------|--------|------------------------|
| Transformer-Base  | 12     | 512   | 8     | 65M    |        | 8x P100 (12 hours)     |
| Transformer-Large | 12     | 1024  | 16    | 213M   |        | 8x P100 (3.5 days)     |
| BERT-Base         | 12     | 768   | 12    | 110M   | 13 GB  |                        |
| BERT-Large        | 24     | 1024  | 16    | 340M   | 13 GB  |                        |
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| GPT-2             | 48     | 1600  | ?     | 1.5B   | 40 GB  |                        |
| Megatron-LM       | 72     | 3072  | 32    | 8.3B   | 174 GB | 512x V100 GPU (9 days) |

Shoeybi et al, "Megatron-LM: Training Multi-Billion Parameter Language Models using Model Parallelism", 2019

| Model             | Layers | Width | Heads | Params | Data   | Training               |
|-------------------|--------|-------|-------|--------|--------|------------------------|
| Transformer-Base  | 12     | 512   | 8     | 65M    |        | 8x P100 (12 hours)     |
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| BERT-Base         | 12     | 768   | 12    | 110M   | 13 GB  |                        |
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| Megatron-LM       | 72     | 3072  | 32    | 8.3B   | 174 GB | 512x V100 GPU (9 days) |
| Turing-NLG        | 78     | 4256  | 28    | 17B    | ?      | 256x V100 GPU          |

Microsoft, "Turing-NLG: A 17-billion parameter language model by Microsoft", 2020

| Model             | Layers | Width  | Heads    | Params | Data   | Training               |
|-------------------|--------|--------|----------|--------|--------|------------------------|
| Transformer-Base  | 12     | 512    | 8        | 65M    |        | 8x P100 (12 hours)     |
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| GPT-2             | 48     | 1600   | <b>,</b> | 1.5B   | 40 GB  |                        |
| Megatron-LM       | 72     | 3072   | 32       | 8.3B   | 174 GB | 512x V100 GPU (9 days) |
| Turing-NLG        | 78     | 4256   | 28       | 17B    | ?      | 256x V100 GPU          |
| GPT-3             | 96     | 12,288 | 96       | 175B   | 694GB  | ?                      |

| Model             | Layers | Width  | Heads    | Params | Data     | Training               |
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| Turing-NLG        | 78     | 4256   | 28       | 17B    | ?        | 256x V100 GPU          |
| GPT-3             | 96     | 12,288 | 96       | 175B   | 694GB    | ?                      |
| Gopher            | 80     | 16,384 | 128      | 280B   | 10.55 TB | 4096x TPUv3 (38 days)  |

Rae et al, "Scaling Language Models: Methods, Analysis, & Insights from Training Gopher", arXiv 2021

#### \$3,768,320 on Google Cloud (eval price)

| Model             | Layers | Width  | Heads | Params | Data     | Training               |
|-------------------|--------|--------|-------|--------|----------|------------------------|
| Transformer-Base  | 12     | 512    | 8     | 65M    |          | 8x P100 (12 hours)     |
| Transformer-Large | 12     | 1024   | 16    | 213M   |          | 8x P100 (3.5 days)     |
| BERT-Base         | 12     | 768    | 12    | 110M   | 13 GB    |                        |
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| GPT-2             | 48     | 1600   | ?     | 1.5B   | 40 GB    |                        |
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Rae et al, "Scaling Language Models: Methods, Analysis, & Insights from Training Gopher", arXiv 2021

#### Generated Text from GPT-3

**Bold text: Input prompt written by humans** 

*Italics: Completion by GPT-3* 

**Title: United Methodists Agree to Historic Split** 

Subtitle: Those who oppose gay marriage will form their own denomination

**Article:** 

#### Generated Text from GPT-3

**Bold text: Input prompt written by humans** 

Italics: Completion by GPT-3

**Title: United Methodists Agree to Historic Split** 

Subtitle: Those who oppose gay marriage will form their own denomination

**Article:** After two days of intense debate, the United Methodist Church has agreed to a historic split - one that is expected to end in the creation of a new denomination, one that will be "theologically and socially conservative," according to The Washington Post. The majority of delegates attending the church's annual General Conference in May voted to strengthen a ban on the ordination of LGBTQ clergy and to write new rules that will "discipline" clergy who officiate at same-sex weddings. But those who opposed these measures have a new plan: They say they will form a separate denomination by 2020, calling their church the Christian Methodist denomination. The Post notes that the denomination, which claims 12.5 million members, was in the early 20th century the "largest Protestant denomination in the U.S.," but that it has been shrinking in recent decades. The new split will be the second in the church's history. The first occurred in 1968, when roughly 10 percent of the denomination left to form the Evangelical United Brethren Church. The Post notes that the proposed split "comes at a critical time for the church, which has been losing members for years," which has been "pushed toward the brink of a schism over the role of LGBTQ people in the church." Gay marriage is not the only issue that has divided the church. In 2016, the denomination was split over ordination of transgender clergy, with the North Pacific regional conference voting to ban them from serving as clergy, and the South Pacific regional conference voting to allow them.

# GPT-3: Programming by prompt

**Bold text: Input prompt written by humans** 

Italics: Completion by GPT-3

Poor English input: I eated the purple berries.

Good English output: I ate the purple berries.

Poor English input: Thank you for picking me as your designer. I'd appreciate it.

Good English output: Thank you for choosing me as your designer. I appreciate it.

Poor English input: The mentioned changes have done. or I did the alteration that you

requested. or I changed things you wanted and did the modifications.

Good English output: The requested changes have been made. or I made the alteration

that you requested. or I changed things you wanted and made the modifications.

Poor English input: I'd be more than happy to work with you in another project.

**Good English output:** 

# GPT-3: Programming by prompt

Input / Output examples
Test example

Italics: Completion by GPT-3

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**Good English output:** 

# GPT-3: Programming by prompt

Input / Output examples
Test example

*Italics: Completion by GPT-3* 

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**Good English output:** I'd be more than happy to work with you on another project.

Topic: humor: literary parodies and homages, pastiches, style parodies.

Parodies of the fantasy novel series *Harry Potter* in the style of various famous authors:

By Ernest Hemingway:

"It was a cold day on Privet Drive. A child cried. Harry felt nothing.

Topic: humor: literary parodies and homages, pastiches, style parodies.

Parodies of the fantasy novel series *Harry Potter* in the style of various famous authors:

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"It was a cold day on Privet Drive. A child cried. Harry felt nothing. He was dryer than dust. He had been silent too long. He had not felt love. He had scarcely felt hate. Yet the Dementor's Kiss killed nothing. Death didn't leave him less dead than he had been a second before. It wasn't about living or dying really. It wasn't about death. It was about the Dark Lord, his creator, his engenderer. He was not resurrected. He was created again. He was whole again in the courtyard of Malfoy Manor."

Topic: humor: literary parodies and homages, pastiches, style parodies.

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By Jane Austen:

"It is a truth universally acknowledged, that a broken Harry is in want of a book. This he knows to his cost; pressing that wretched nose against the window of a bookstore in Diagon Alley the last thing before he goes into hiding for several years, where upon, he goes straight to Azkaban."

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By Arthur Conan Doyle:

"Harry pushed at the swinging doors of the bookshop hard, and nearly knocked himself unconscious. He staggered in with his ungainly package, his cheeks scarlet with cold and the shame of having chosen the wrong month to go Christmas shopping. The proprietor of the store, however, didn't cast even a cursory glance at him, being far more interested in an enormous hunk of wood lying in the middle of the floor, which certainly looked like a gallows. Yes, the proprietor said to a reedy old man wearing a bowler hat and a forlorn expression that made Harry want to kick him, I can rent you such a gallows for a small fee."

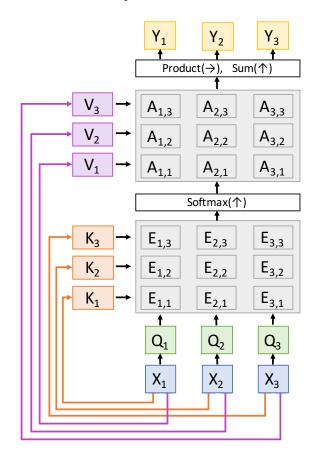
## Summary

Adding **Attention** to RNN models lets them look at different parts of the input at each timestep

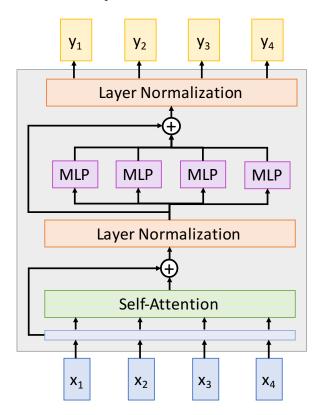


A dog is standing on a hardwood floor.

Generalized **Self-Attention** is new, powerful neural network primitive



**Transformers** are a new neural network model that only uses attention



Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Next Time: Vision Transformers!