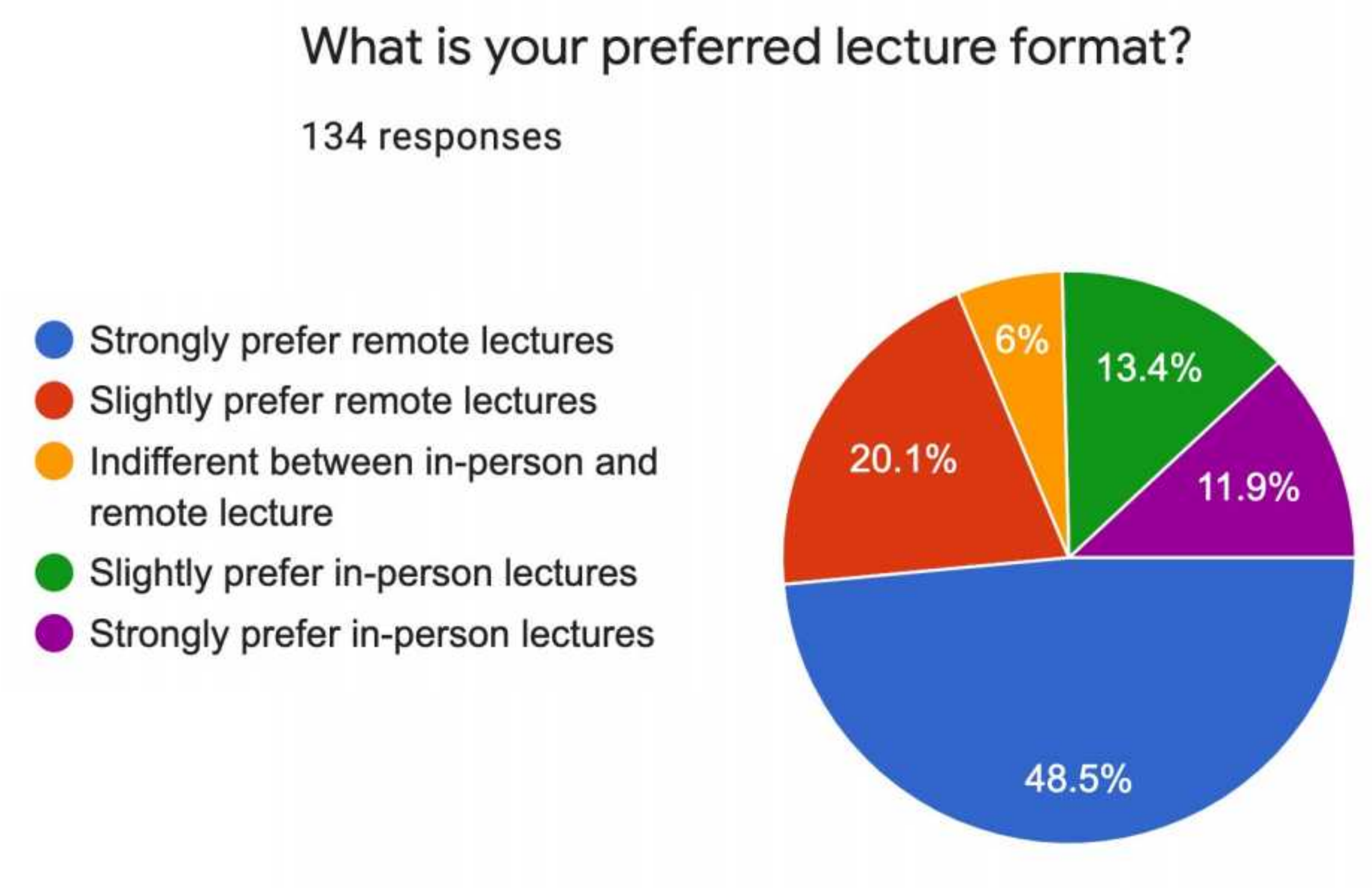
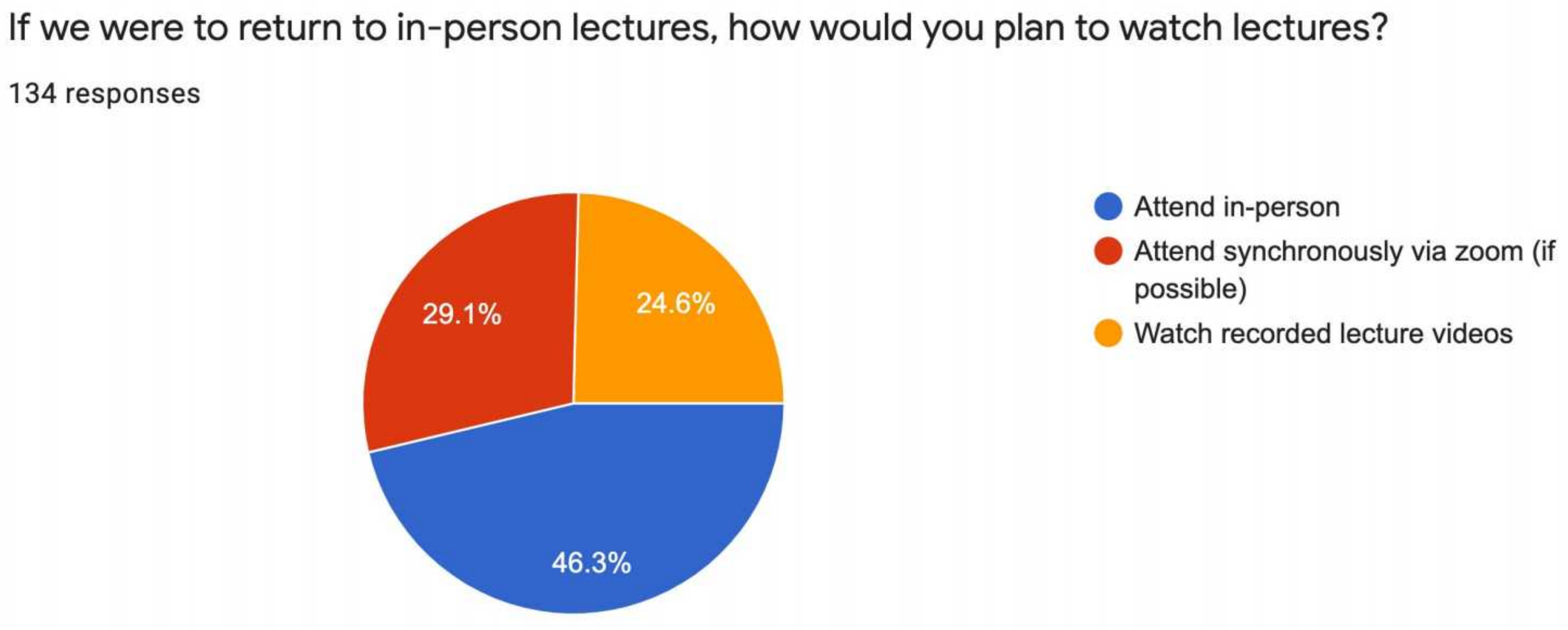
Lecture 7: Convolutional Networks

Lecture Format



Lecture Format



Lecture Format

- We will remain remote for at least another 2-3 weeks
- Idea: book a conference room for "watch parties?"
  Or just use lecture hall
- COVID in MI have (hopefully!) peaked? If they continue to drop we will consider in-person OH in the next 1-2 weeks
- May revisit after Spring Break
- Feel free to raise hand to ask questions in Zoom!
- Midterm will be remote (but still working on exact format)

Reminder: A2

Due last Friday

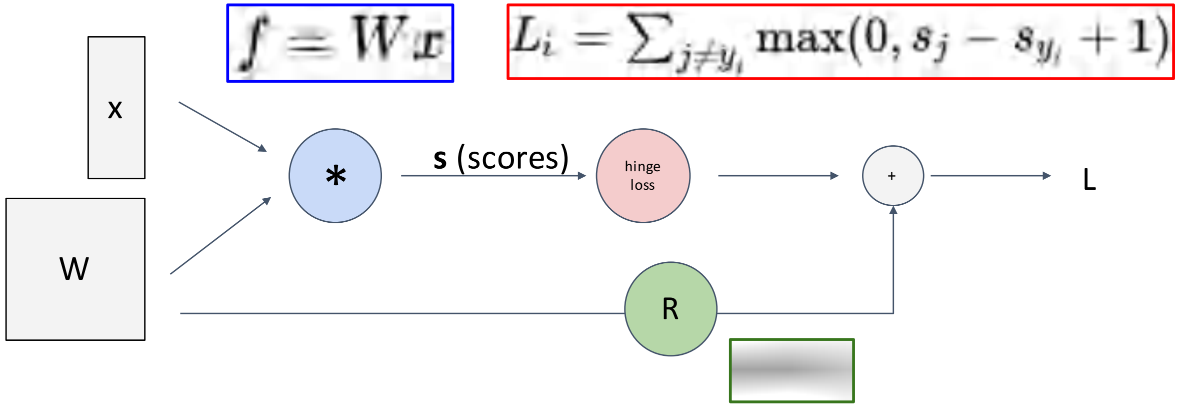
A3

Will be released tonight, covering:

- Backpropagation with modular API
- Different update rules (Momentum, RMSProp, Adam, etc)
- Batch Normalization
- Dropout
- Convolutional Networks

Last Time: Backpropagation

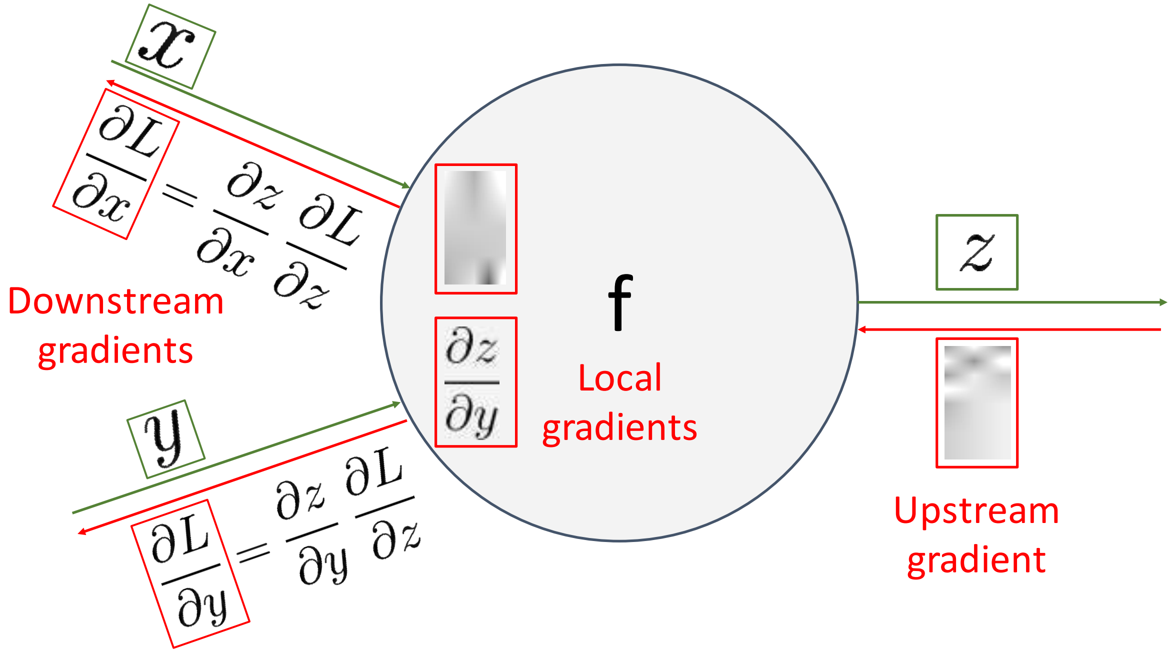
Represent complex expressions as **computational graphs**



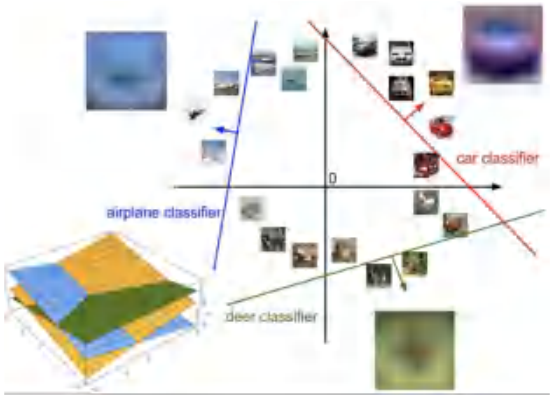
Forward pass computes outputs

Backward pass computes gradients

During the backward pass, each node in the graph receives **upstream gradients** and multiplies them by **local gradients** to compute **downstream gradients**

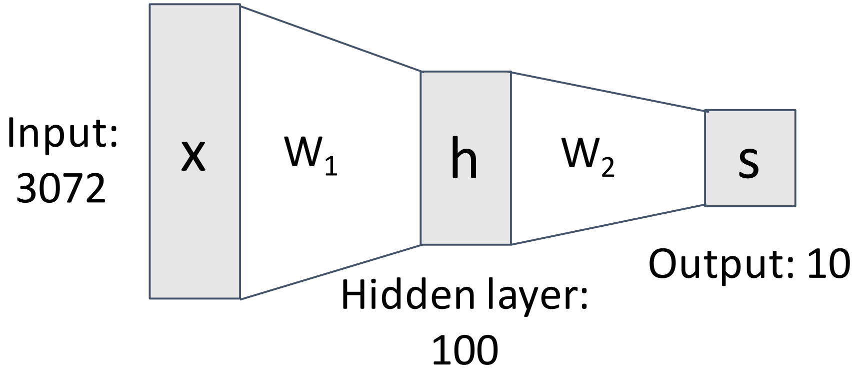


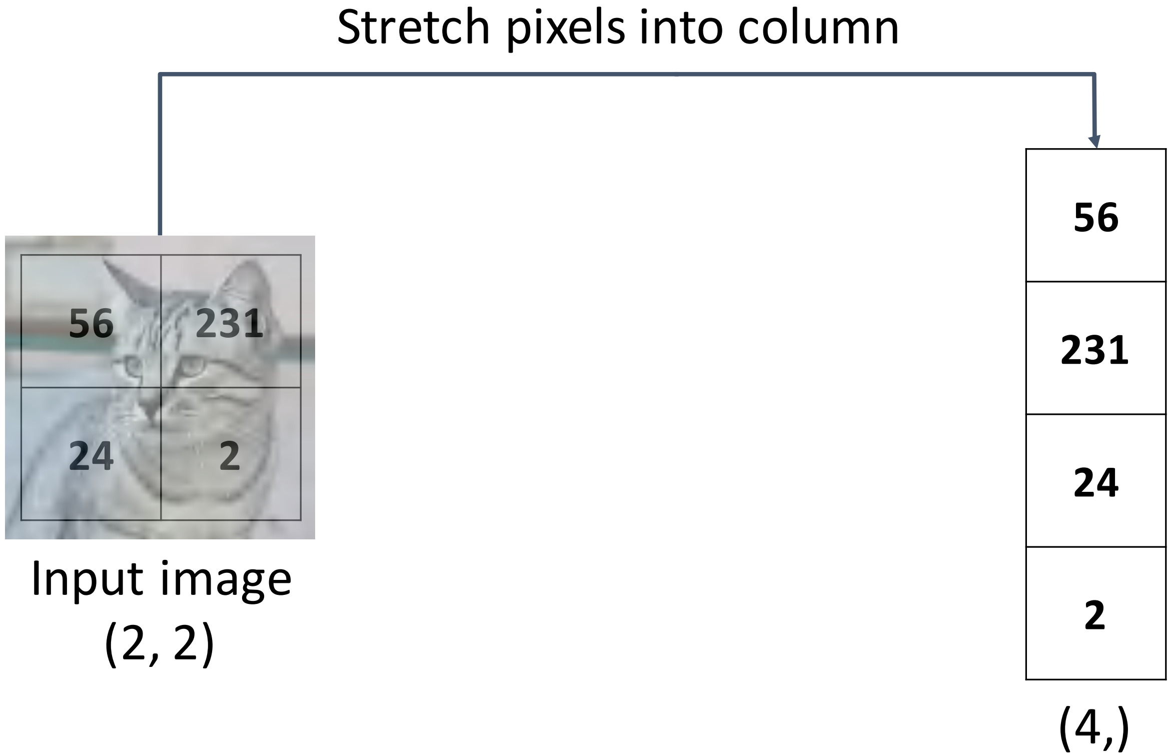
$f(x,W) = Wx$



Problem: So far our classifiers don't respect the spatial structure of images!

$$f = W_2 \max(0, W_1 x)$$





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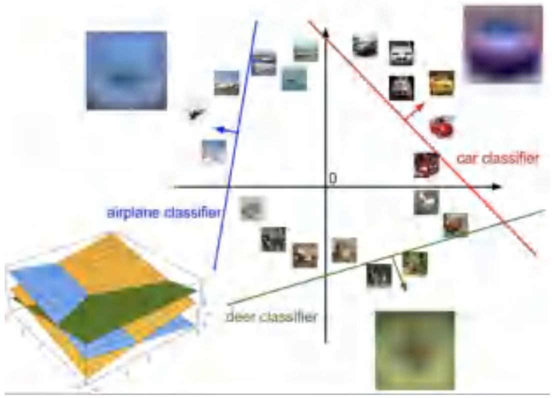

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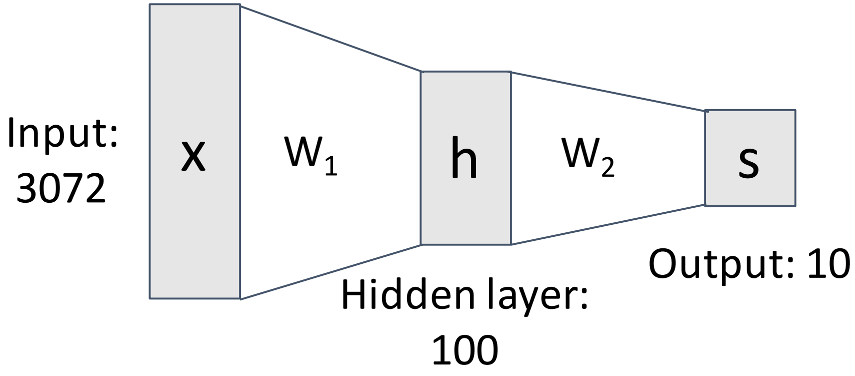
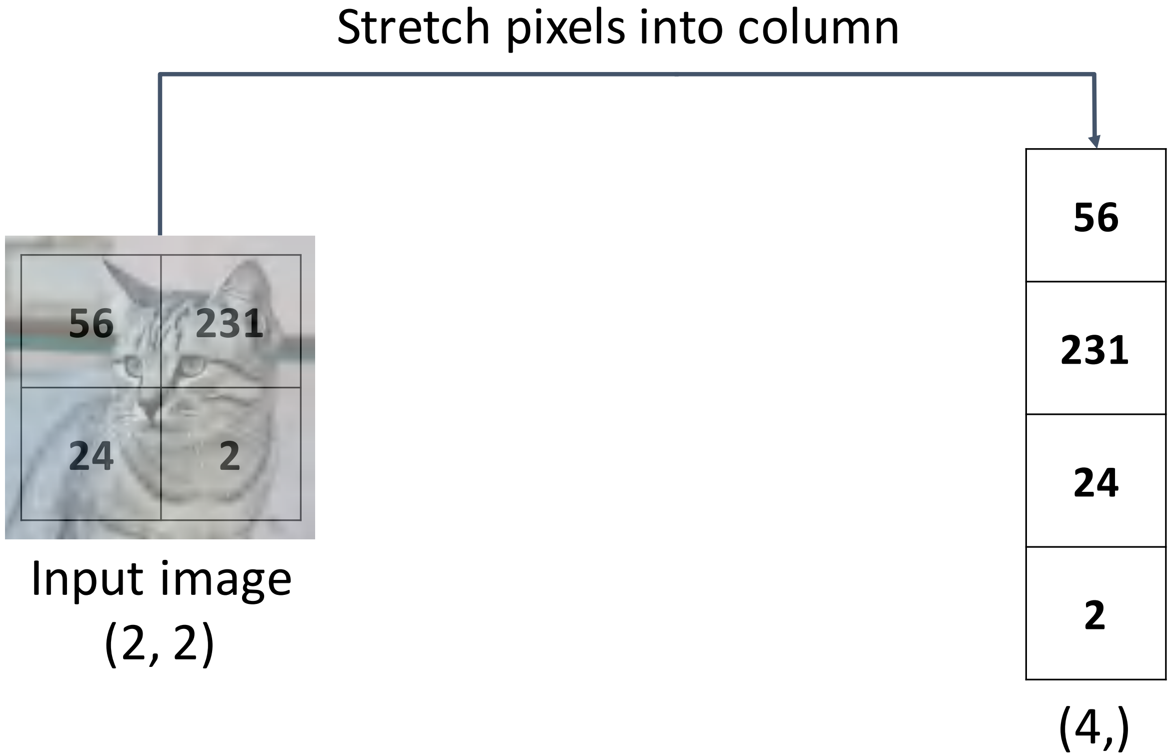


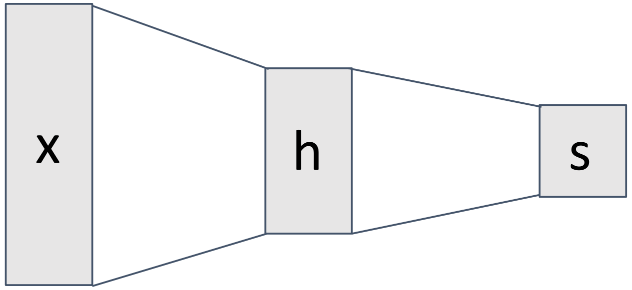
Problem: So far our classifiers don't respect the spatial structure of images!

Solution: Define new computational nodes that operate on images!

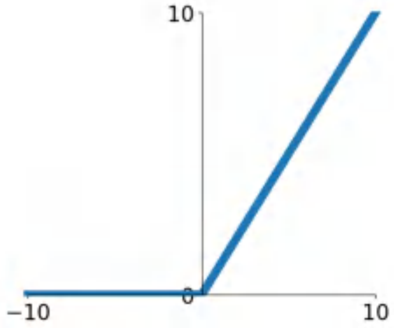


Components of a Fully-Connected Network

Fully-Connected Layers

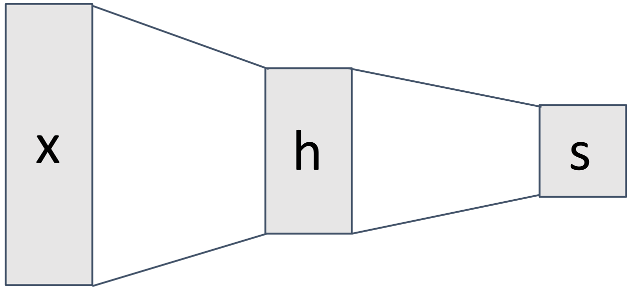


Activation Function



Components of a Convolutional Network

Fully-Connected Layers



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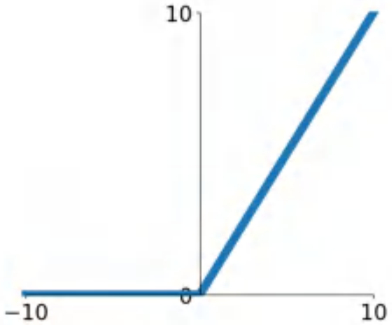
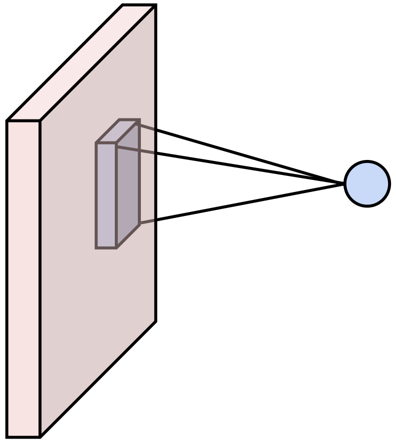
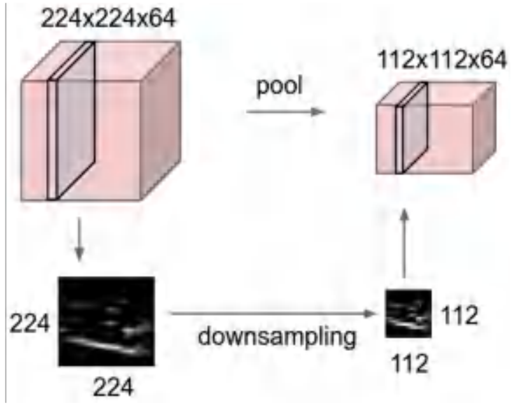


Convolution Layers



Pooling Layers

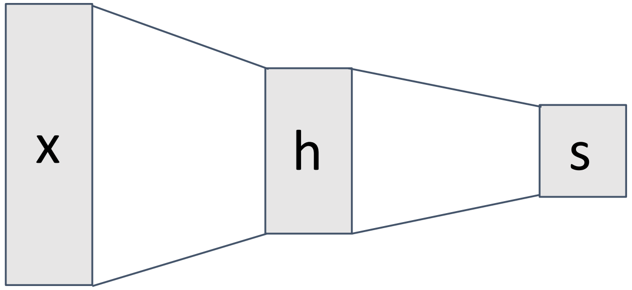


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Components of a Convolutional Network

Fully-Connected Layers



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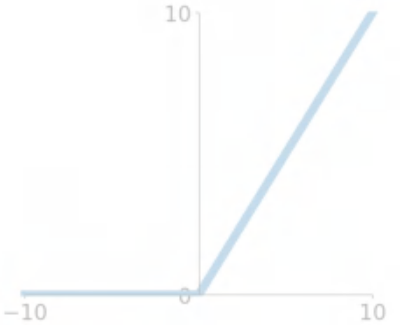
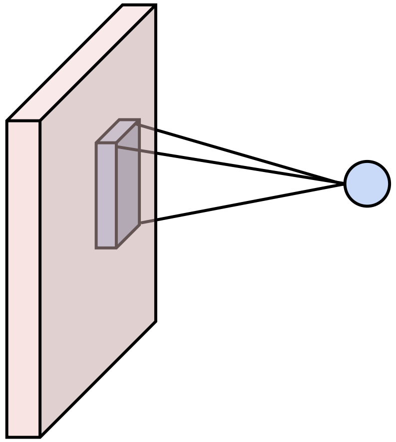


Convolution Layers



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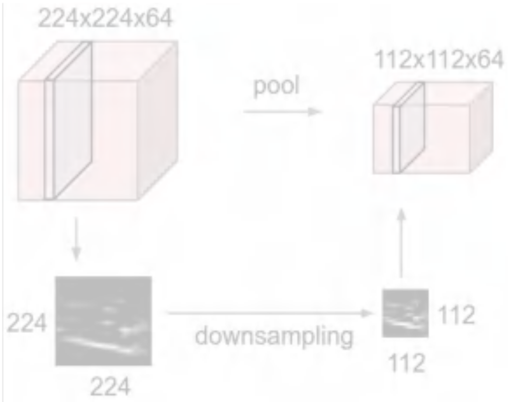
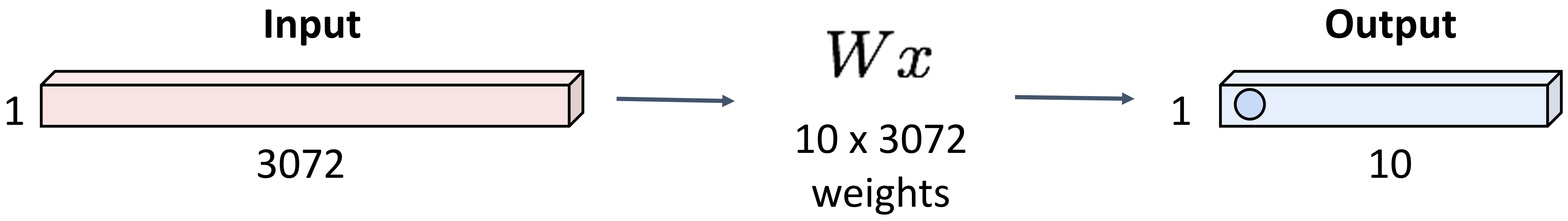


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

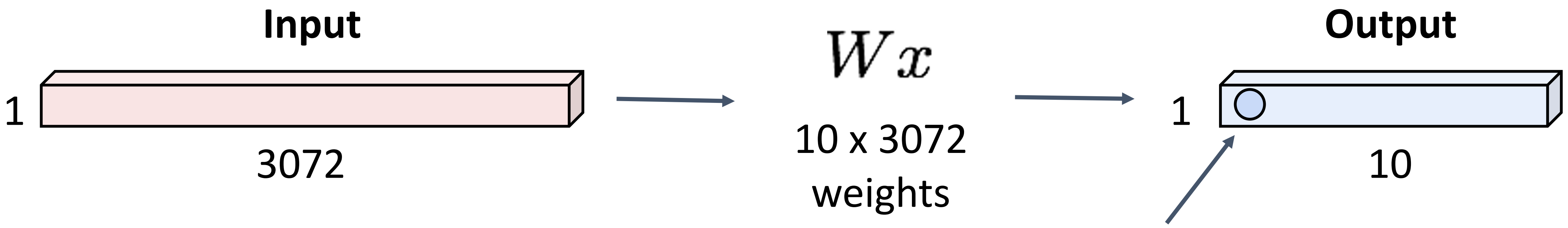
Fully-Connected Layer

32x32x3 image -> stretch to 3072 x 1



Fully-Connected Layer

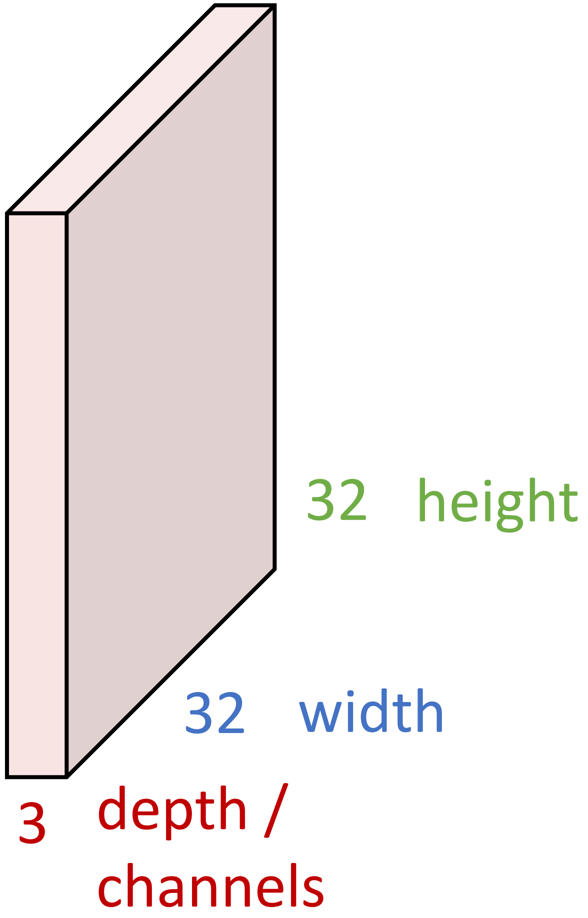
32x32x3 image -> stretch to 3072 x 1



1 number:
the result of taking a dot product between a row of W and the input (a 3072-dimensional dot product)

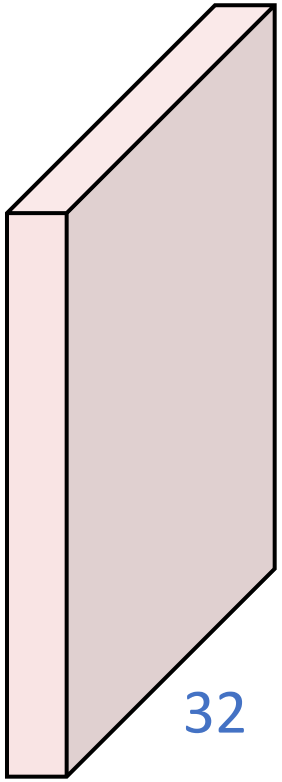
Convolution Layer

3x32x32 image: preserve spatial structure

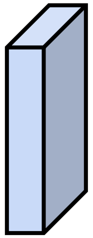


Convolution Layer

3x32x32 image



3x5x5 filter

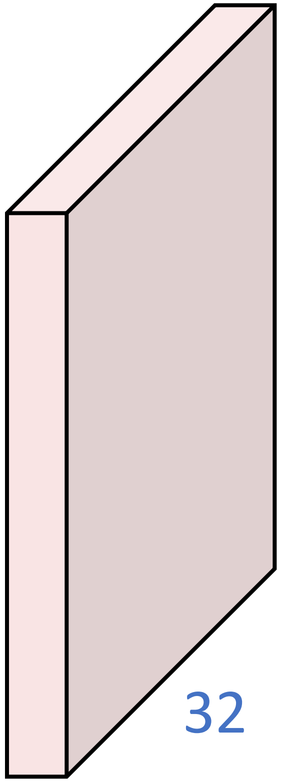


Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolution Layer

Filters always extend the full depth of the input volume

3x32x32 image



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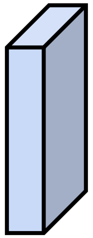

Convolve the filter with the image
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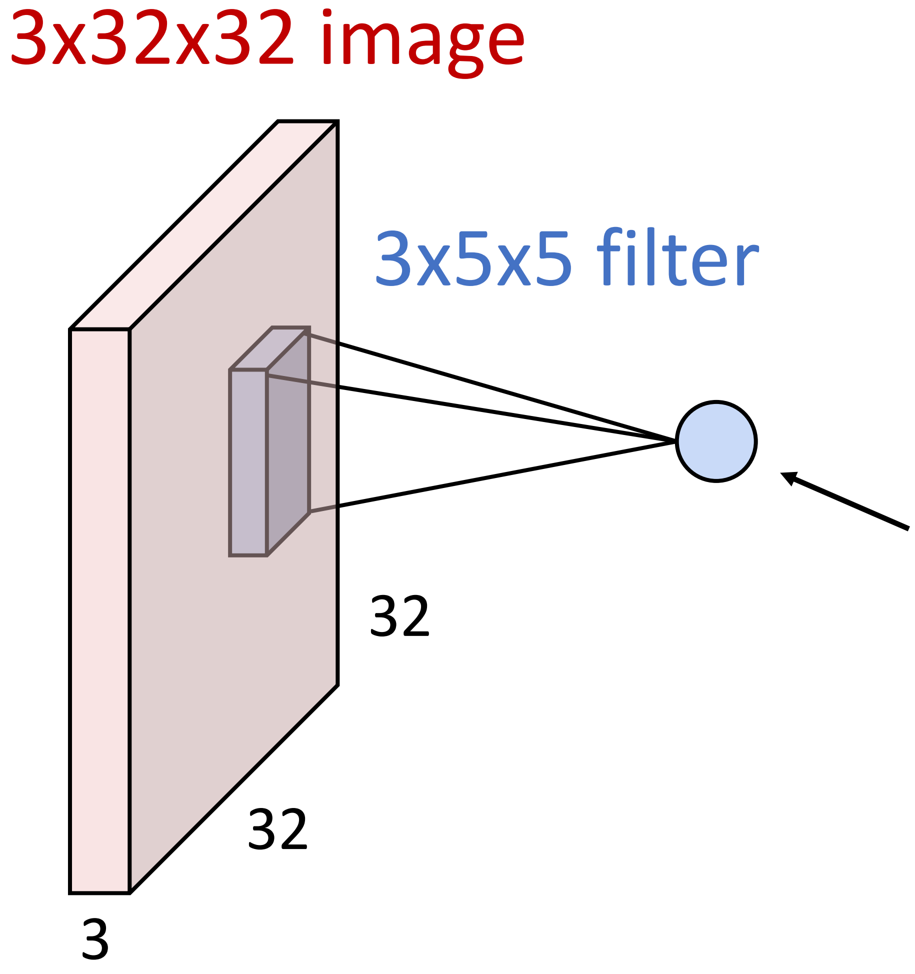
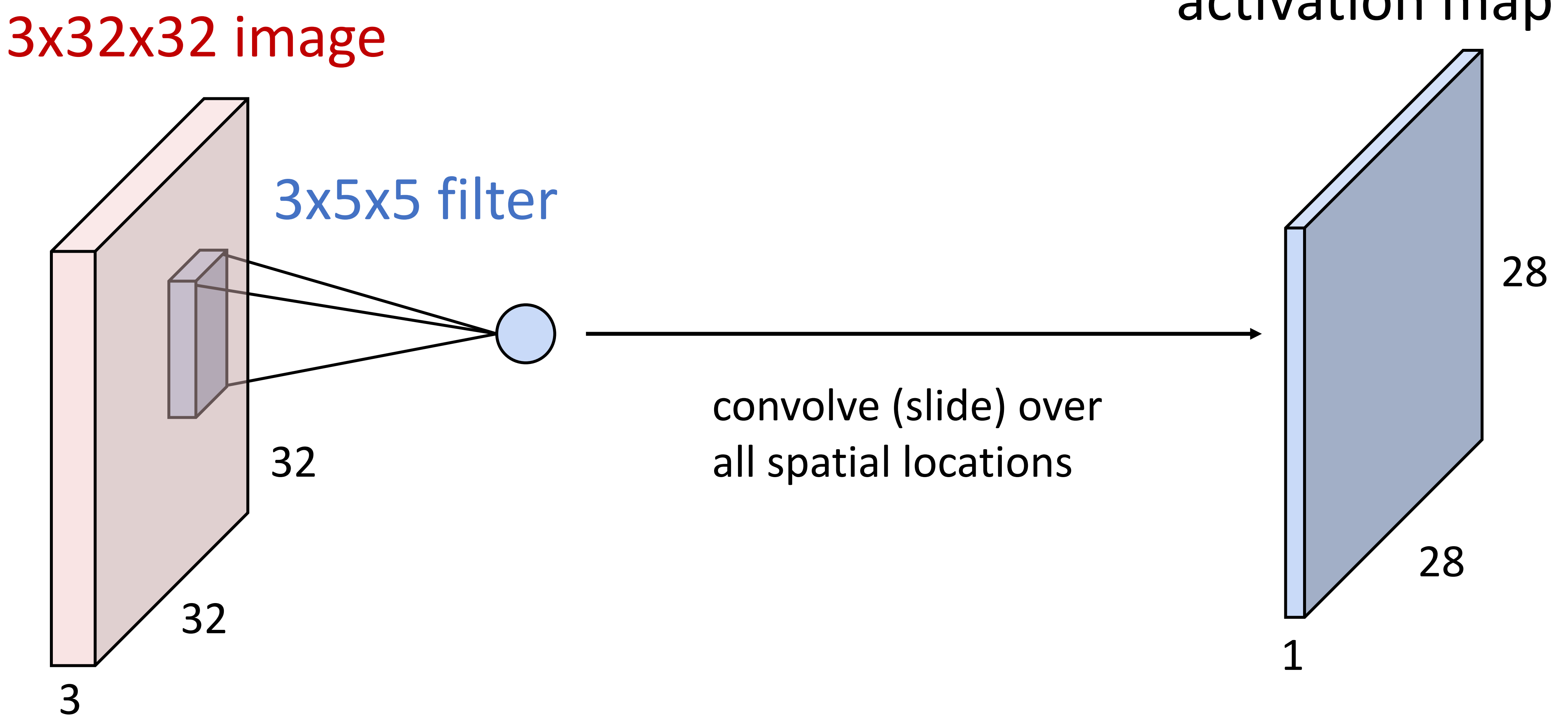
Convolution Layer



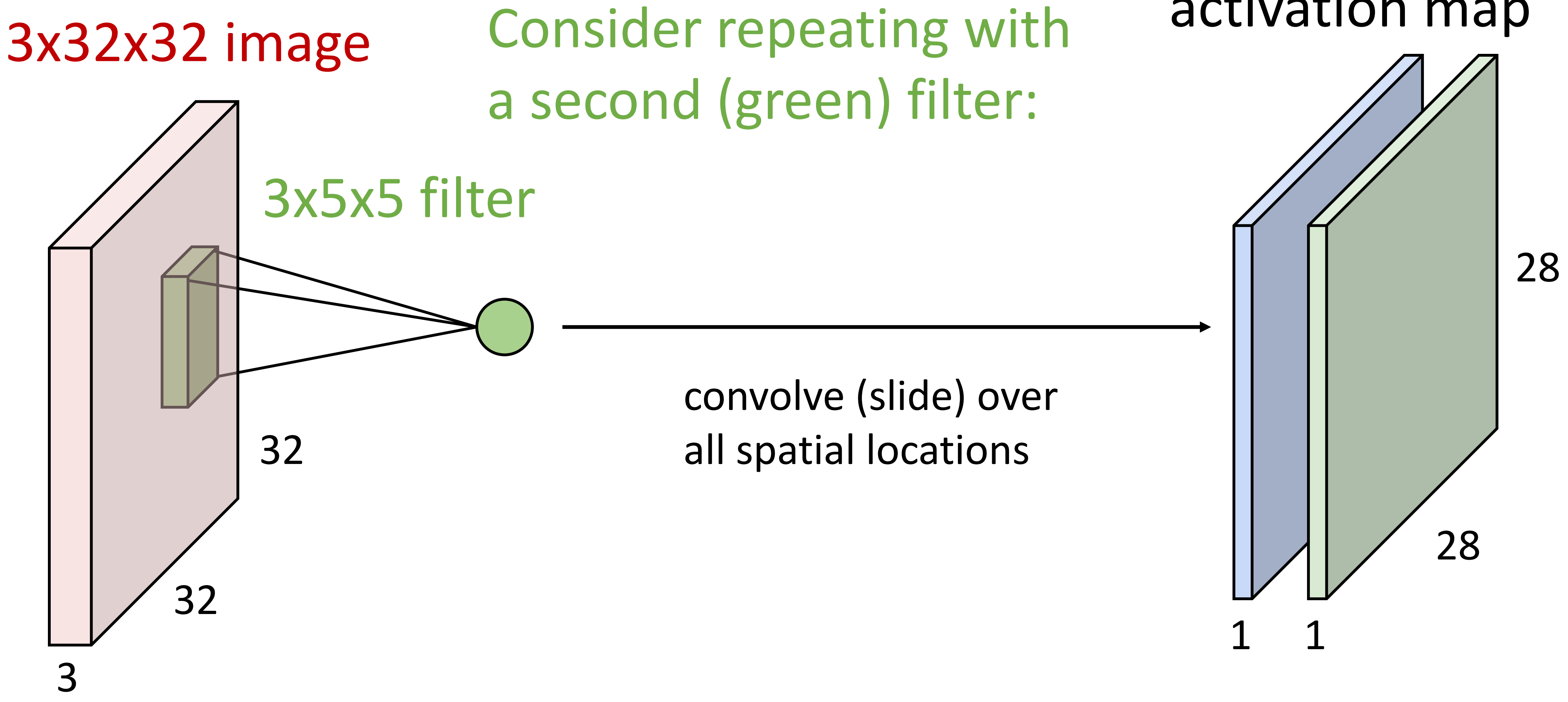
1 number:
the result of taking a dot product between the filter and a small 3x5x5 chunk of the image
(i.e. 3*5*5 = 75-dimensional dot product + bias)

$$w^T x + b$$

Convolution Layer



Convolution Layer



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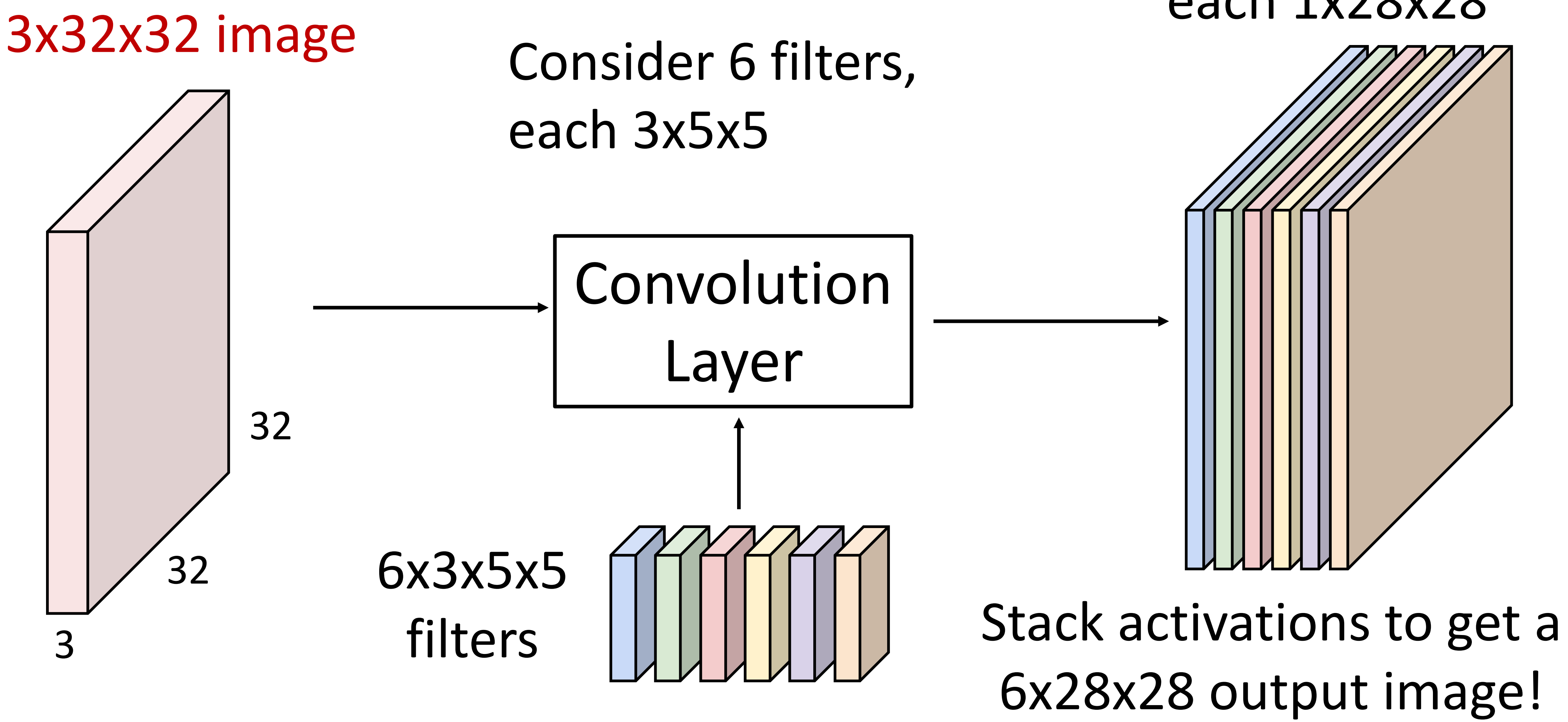
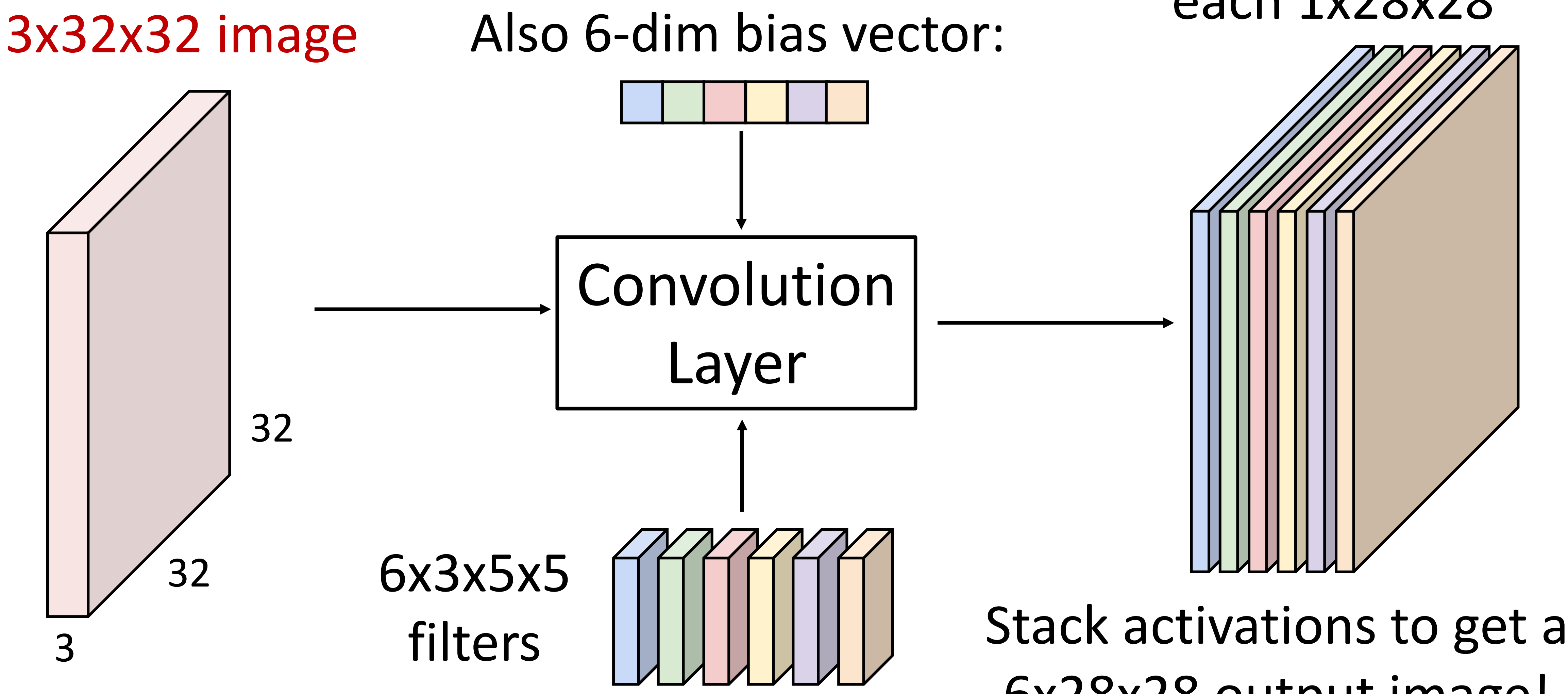


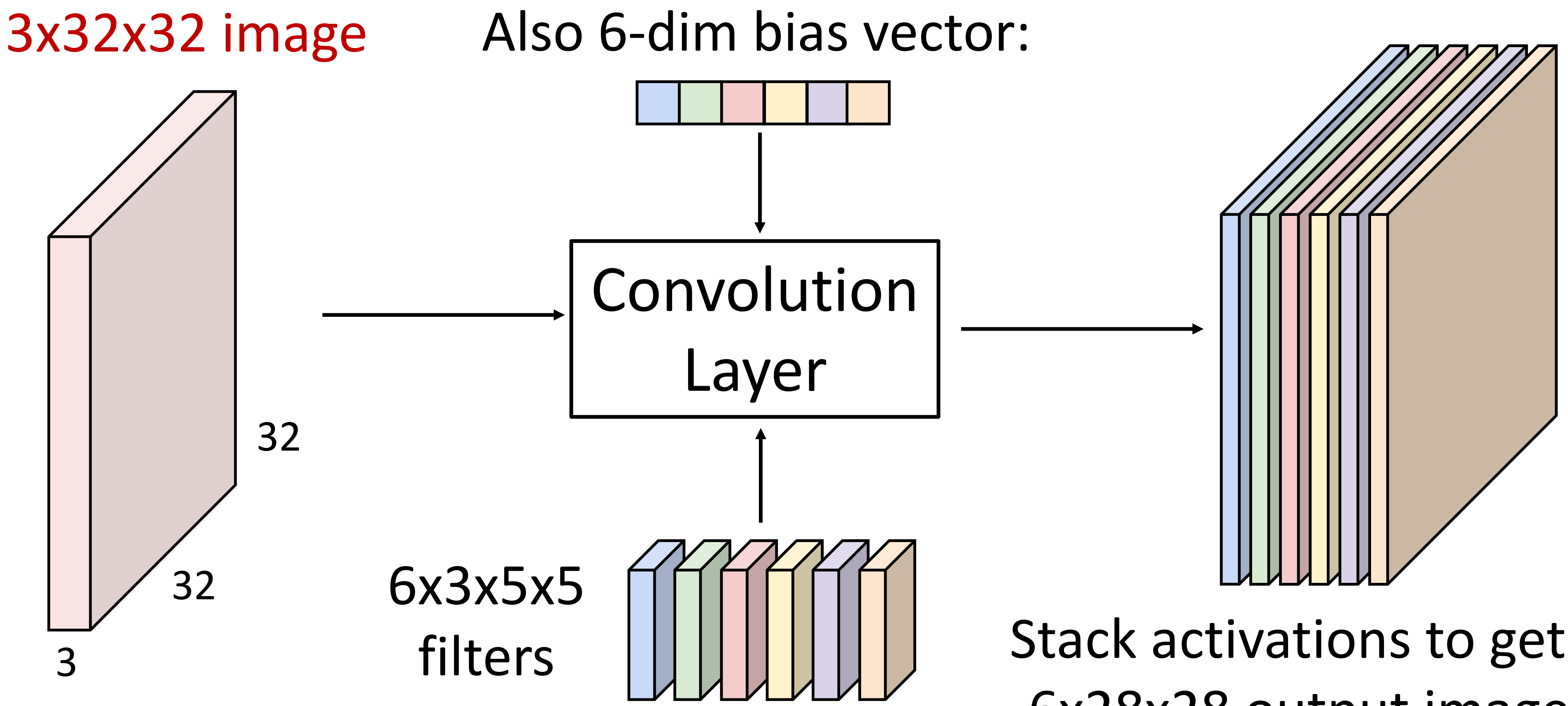
Convolution Layer



Stack activations to get a 6x28x28 output image!

Convolution Layer

28x28 grid, at each point a 6-dim vector



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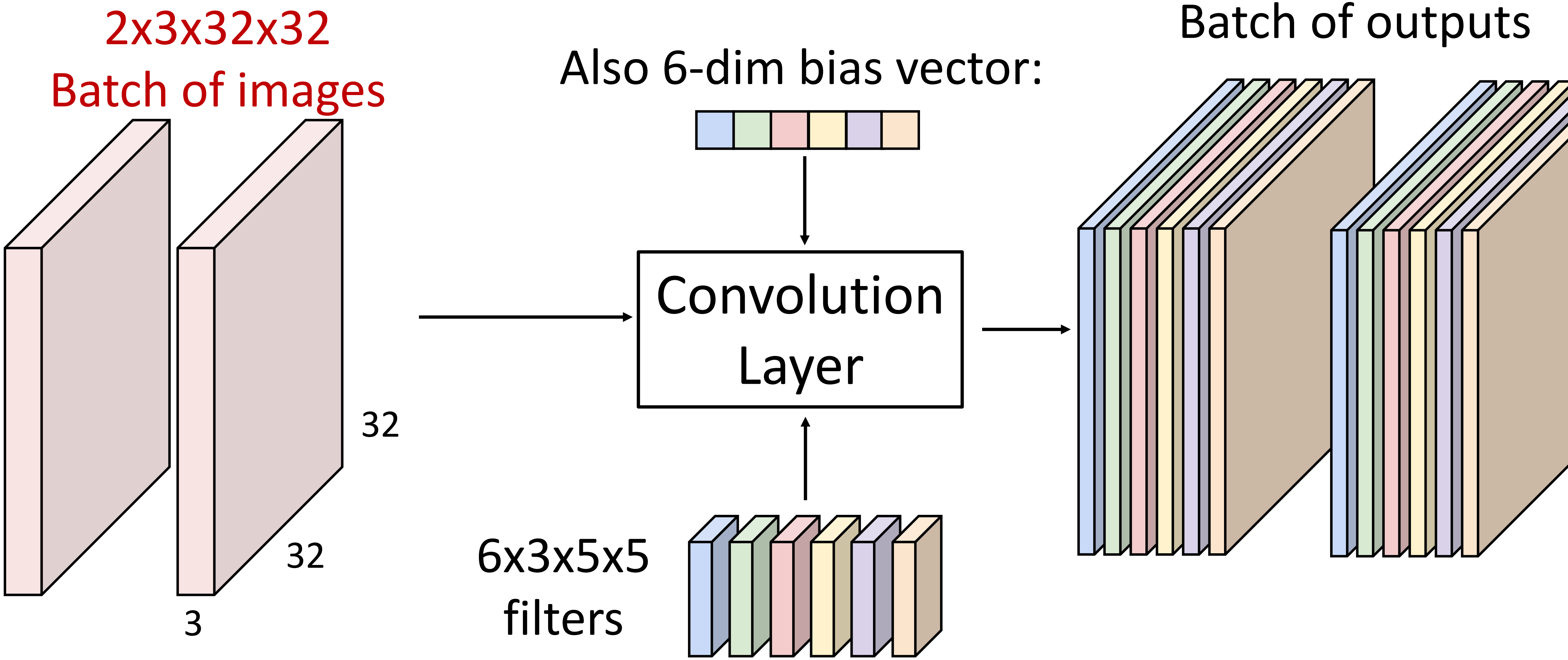
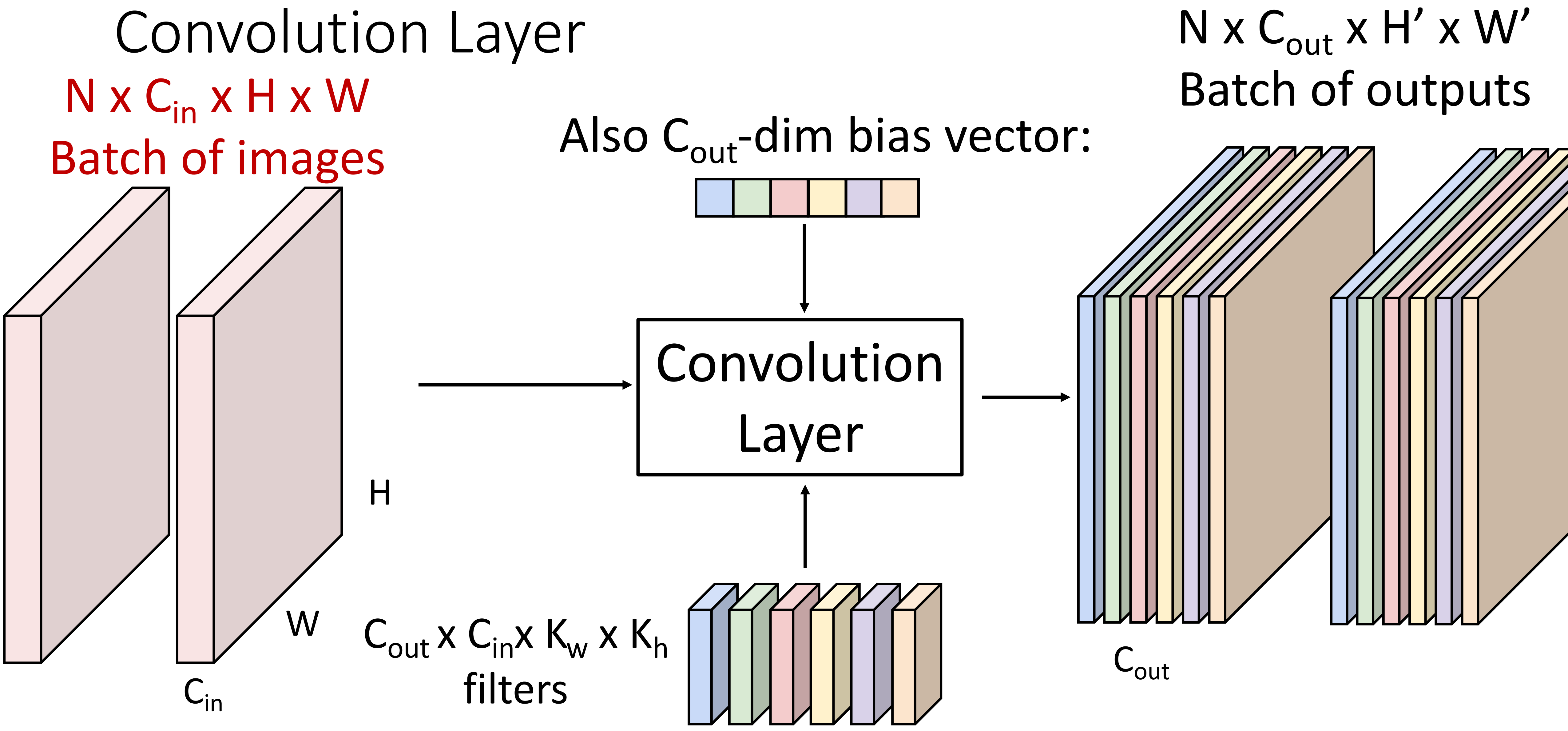
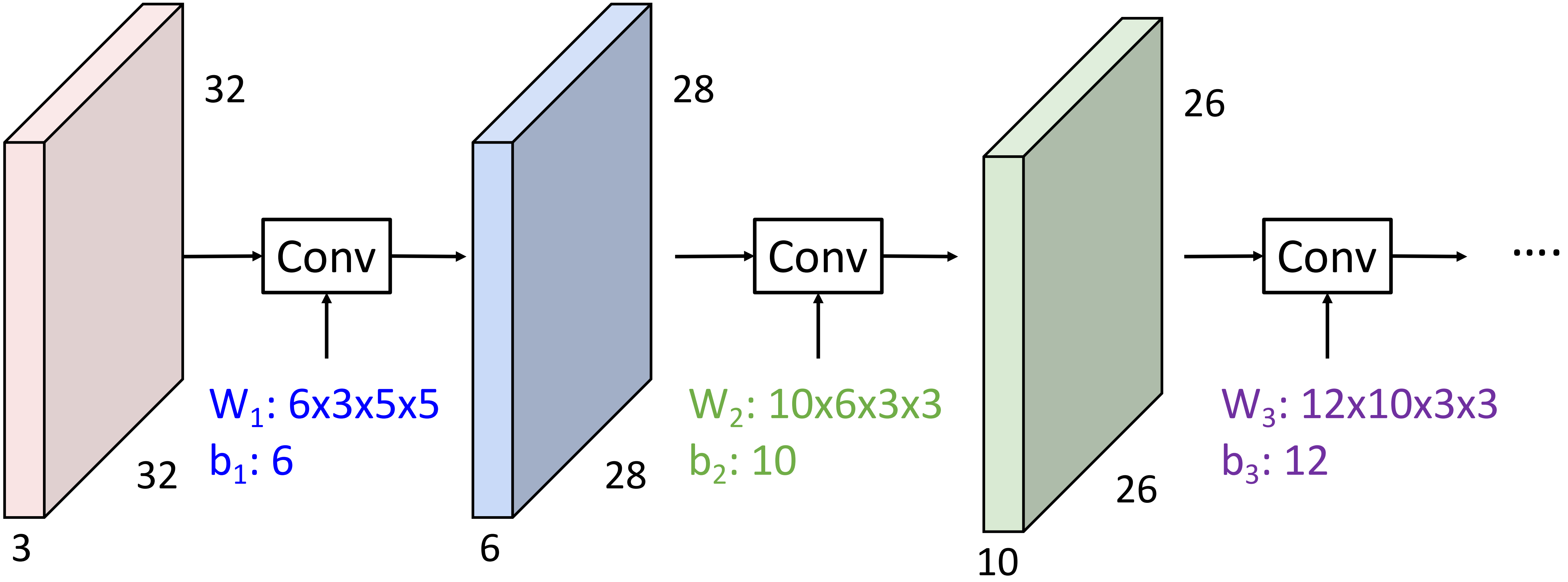
Convolution Layer



Convolution Layer



Stacking Convolutions



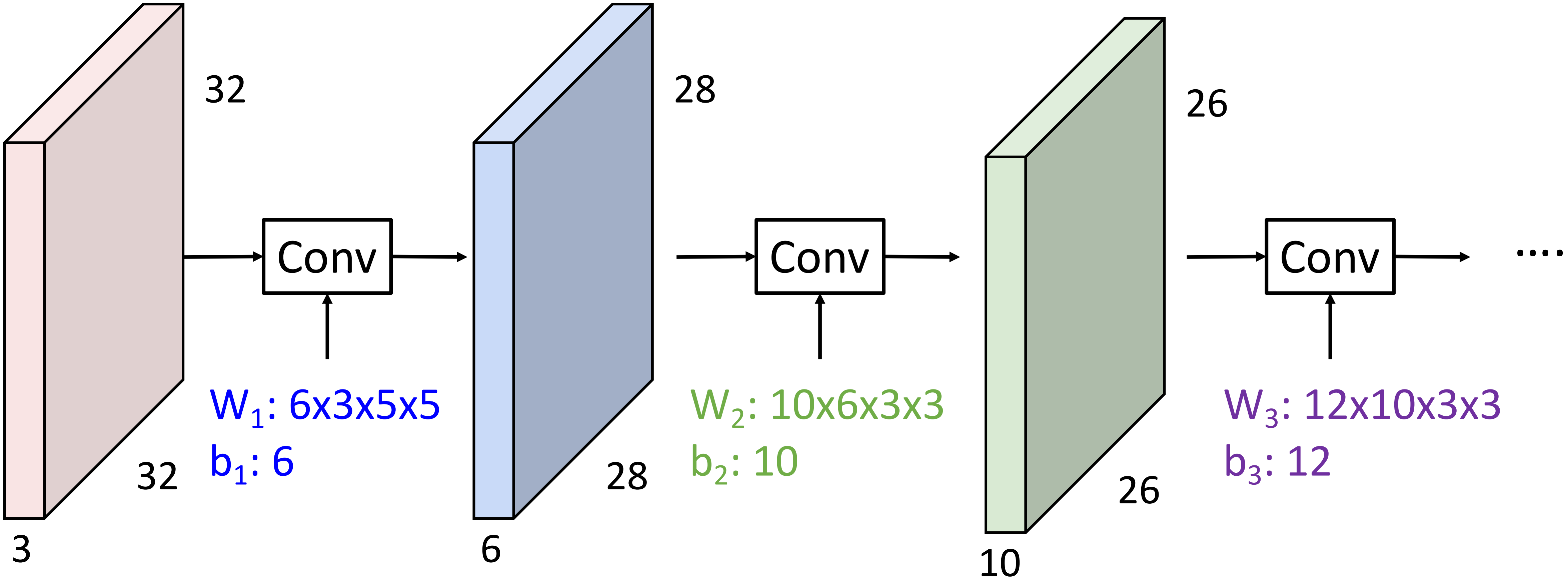
Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

Second hidden layer:
N x 10 x 26 x 26

Stacking Convolutions

Q: What happens if we stack two convolution layers?



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Stacking Convolutions

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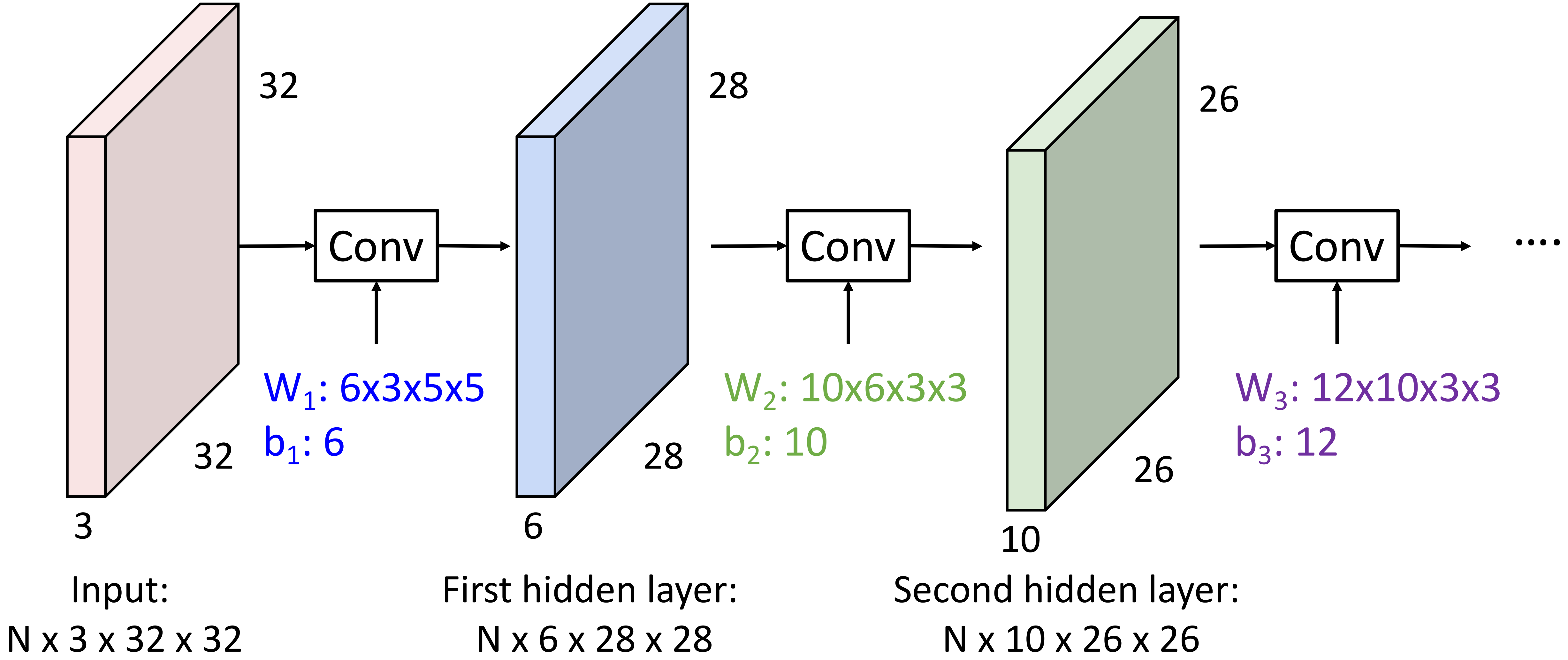
(Recall $y=W_2W_1x$ is a linear classifier)

A: We get another convolution!



Stacking Convolutions

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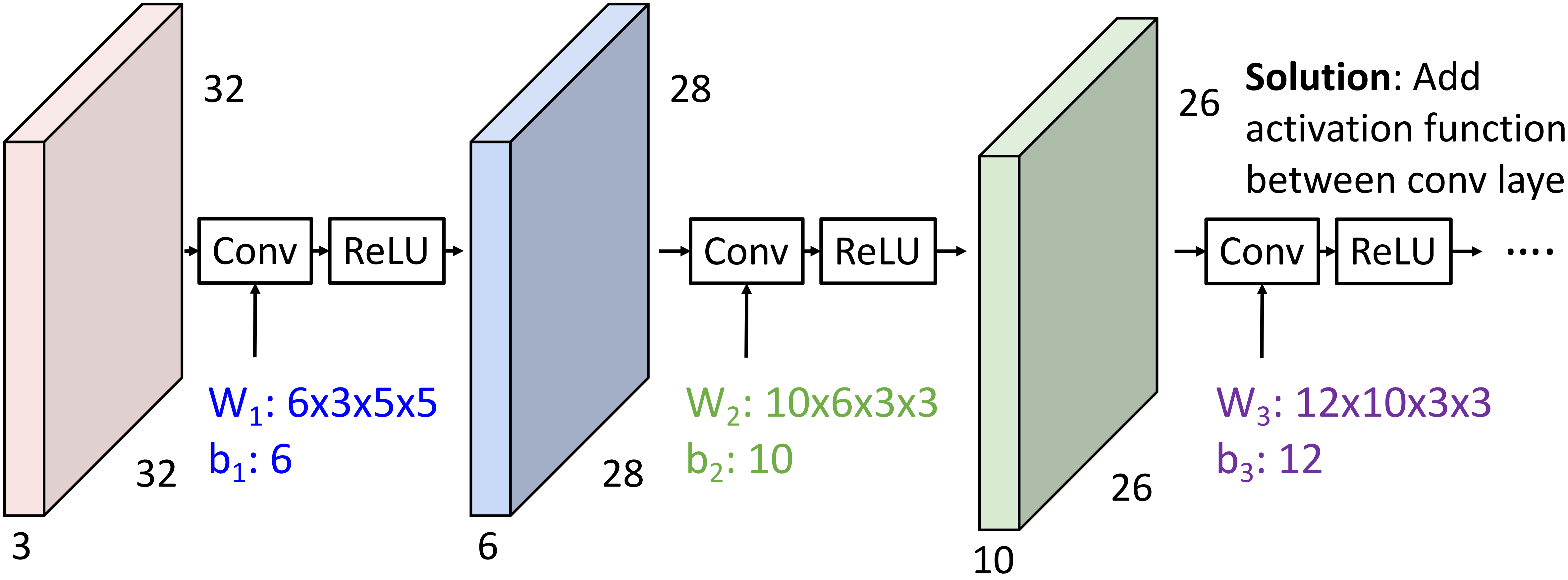

Solution: Add activation function between conv layers

Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

Second hidden layer:
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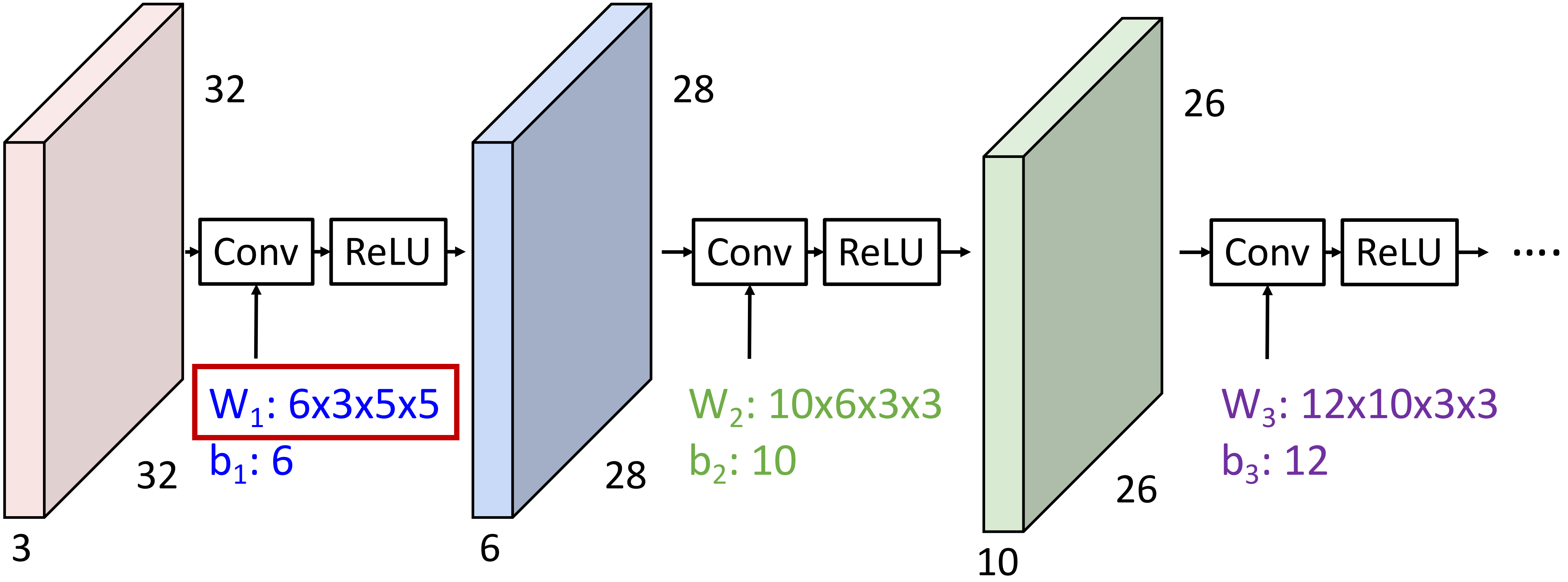
What do convolutional filters learn?



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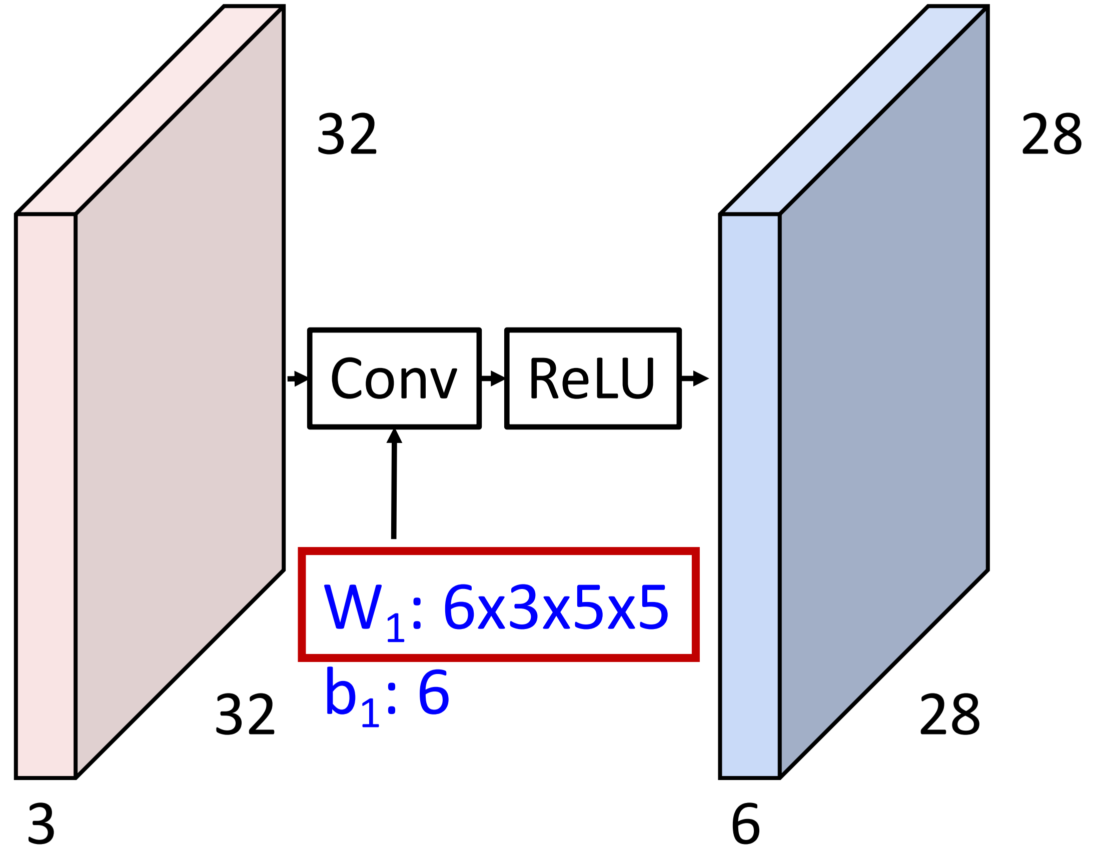

Input:
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First hidden layer:
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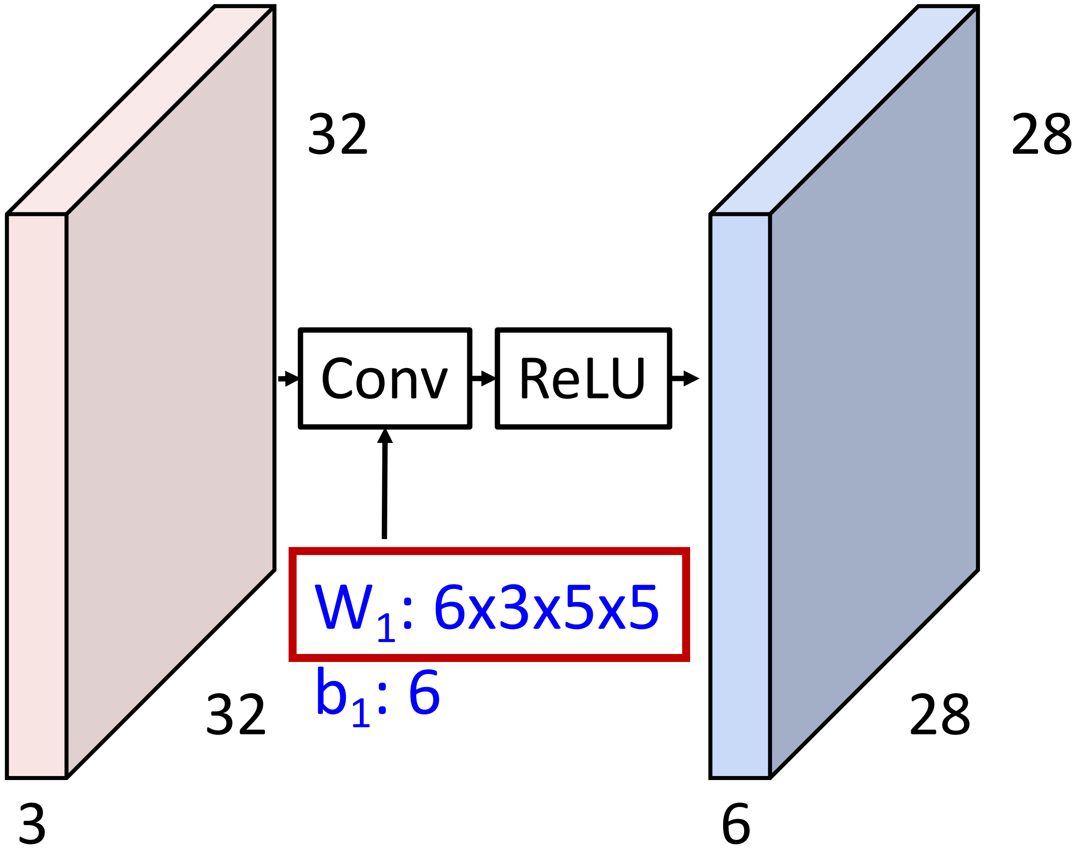
Linear classifier: One template per class

What do convolutional filters learn?



Input:
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First hidden layer:
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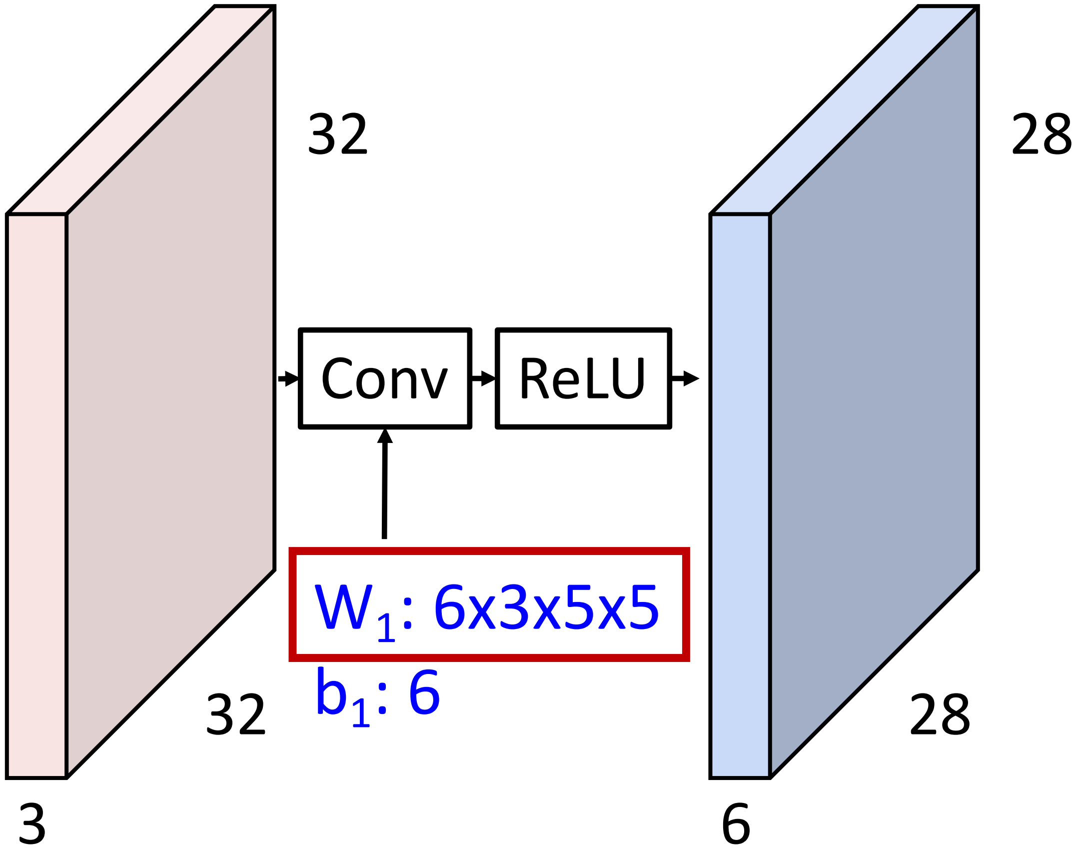
MLP: Bank of whole-image templates

What do convolutional filters learn?



Input:
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First hidden layer:
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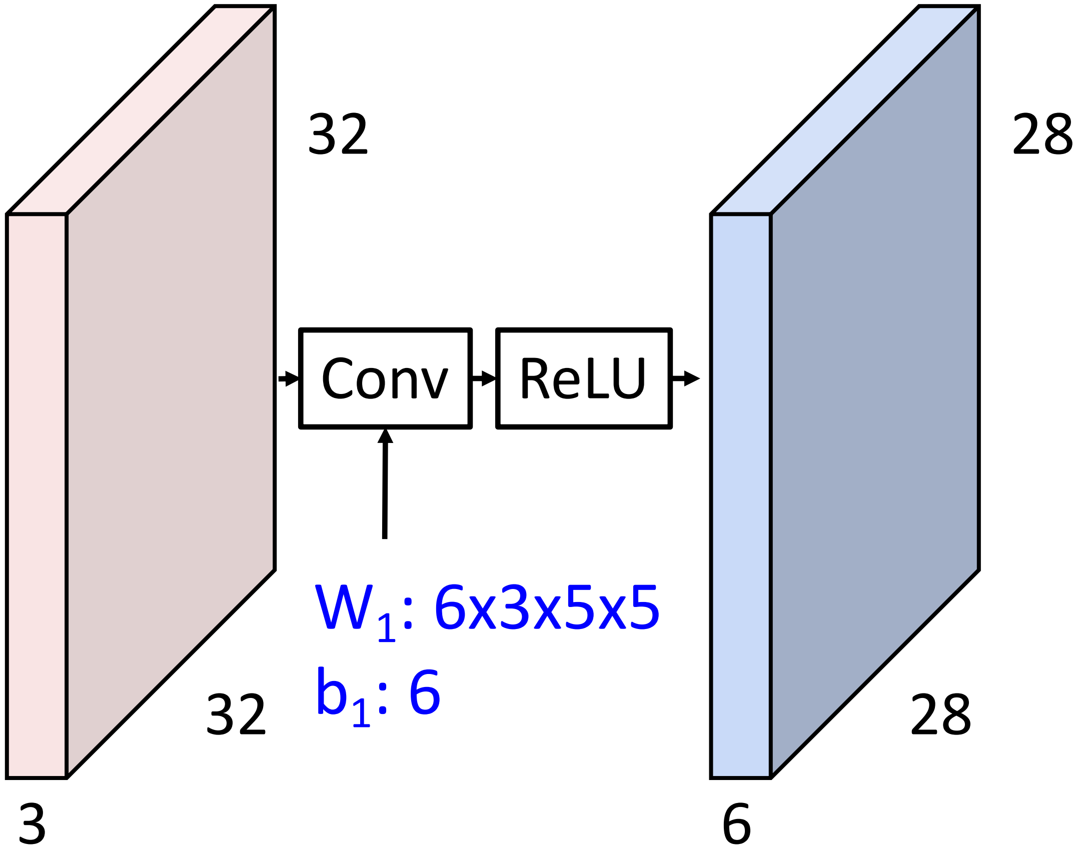
First-layer conv filters: local image templates
(Often learns oriented edges, opposing colors)

AlexNet: 64 filters, each 3x11x11

A closer look at spatial dimensions



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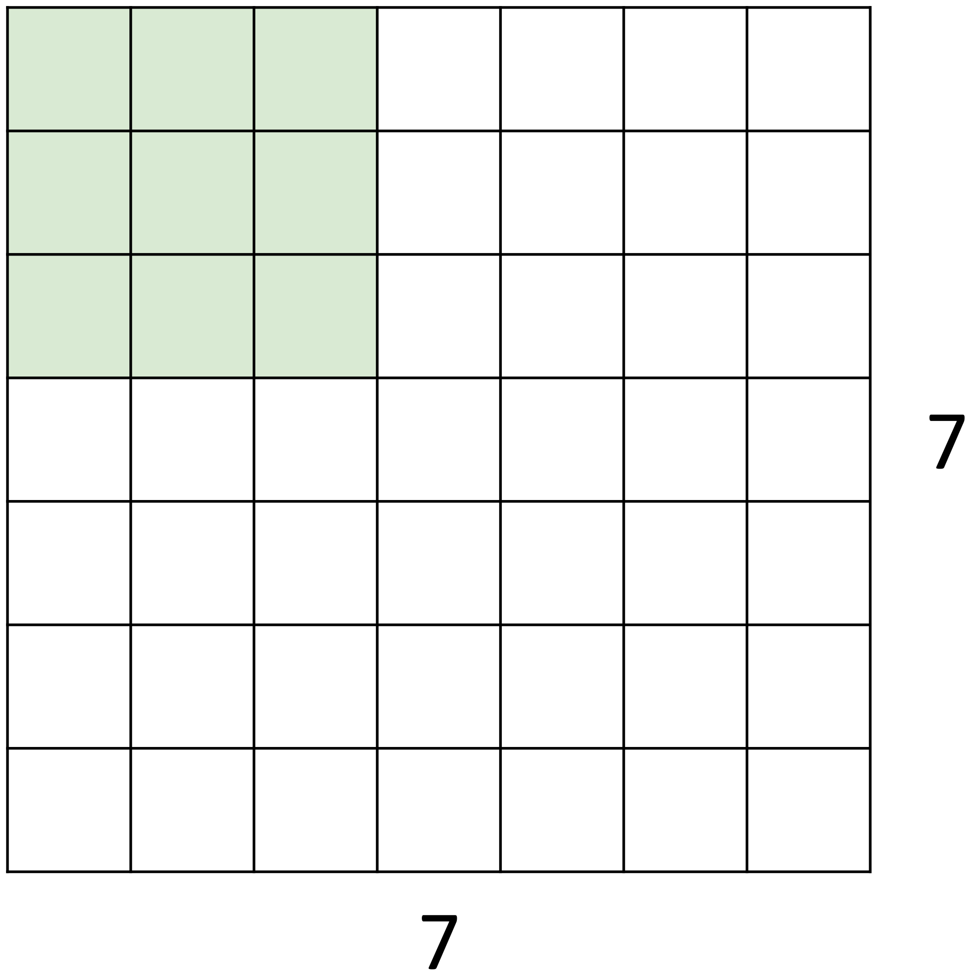
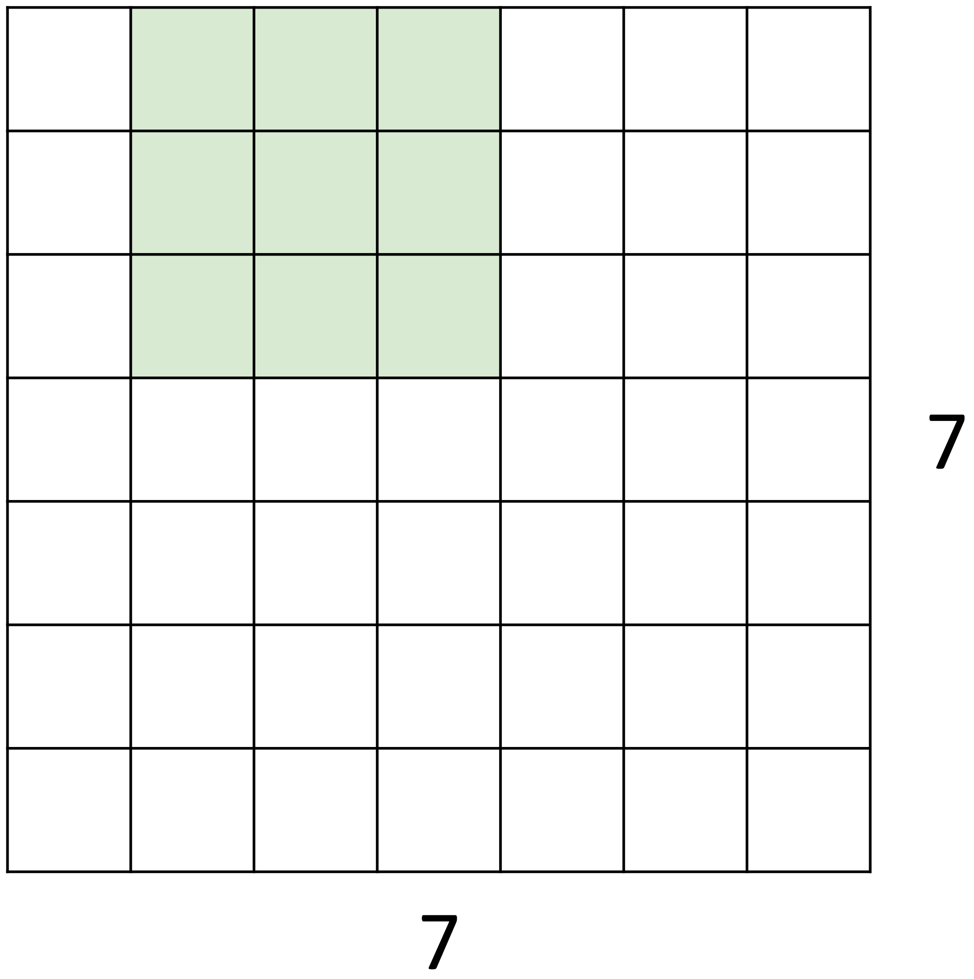
A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

A closer look at spatial dimensions



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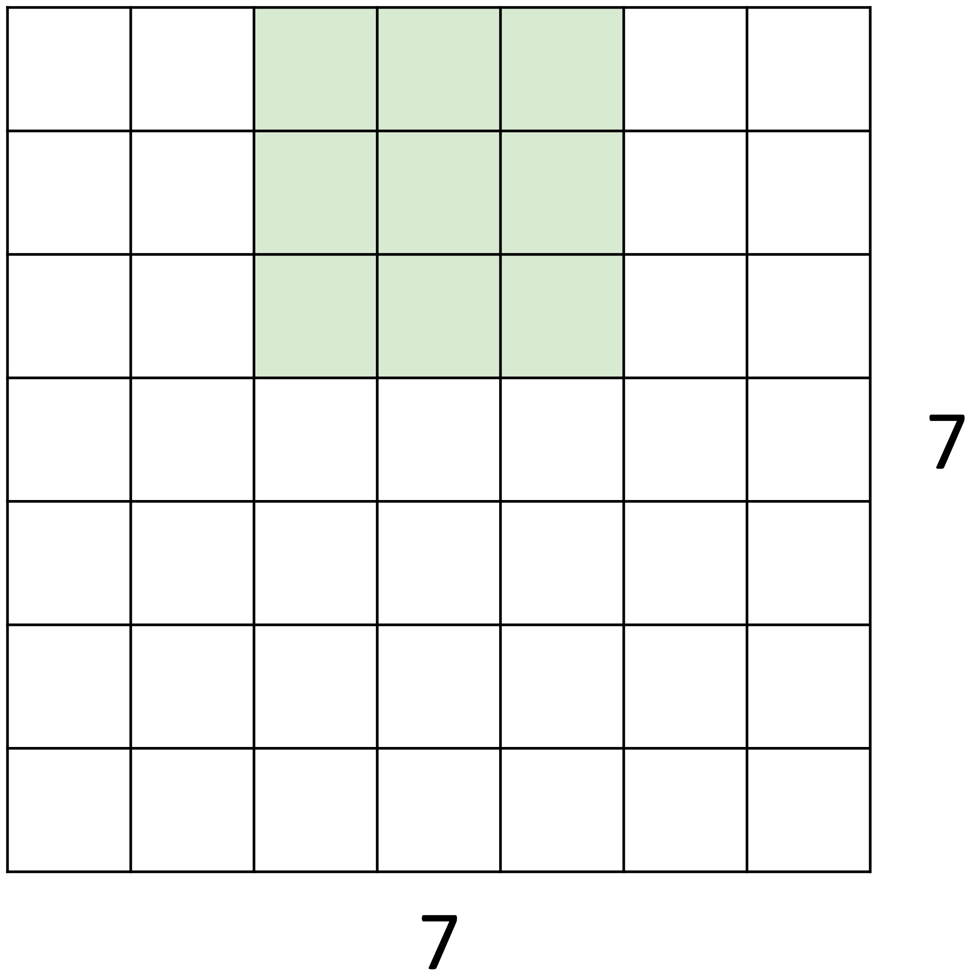

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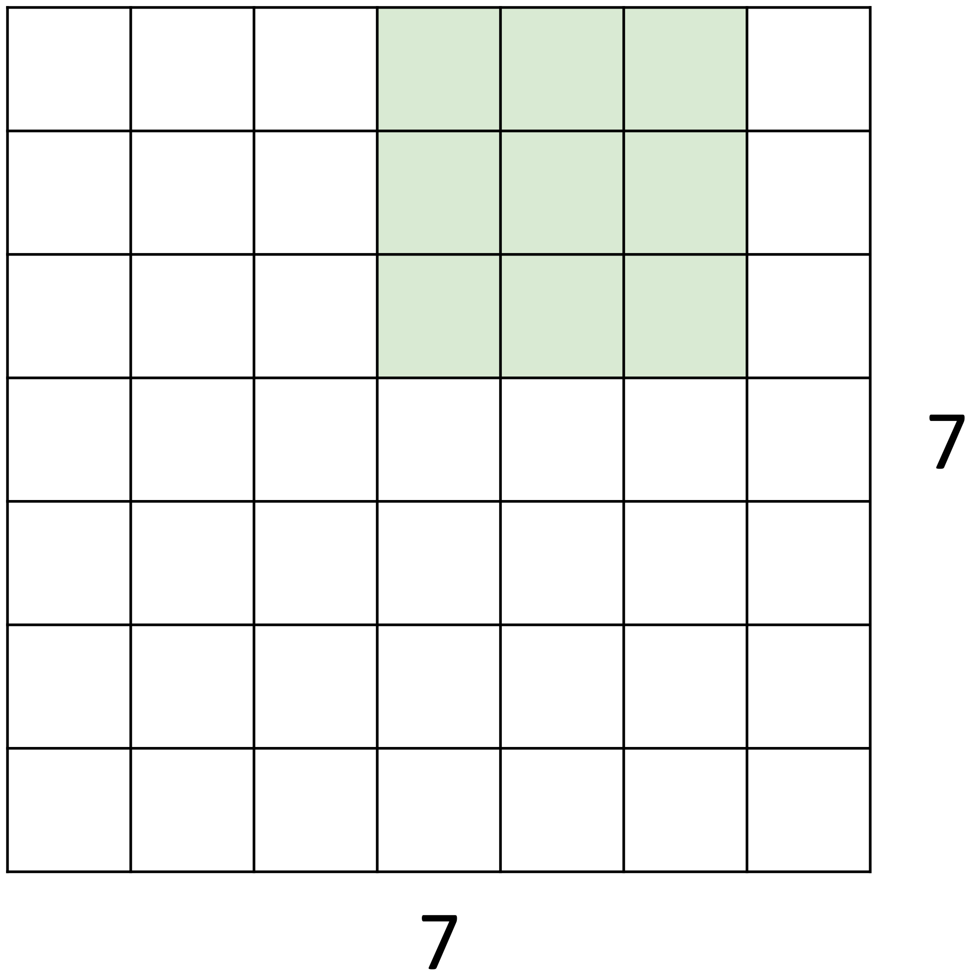

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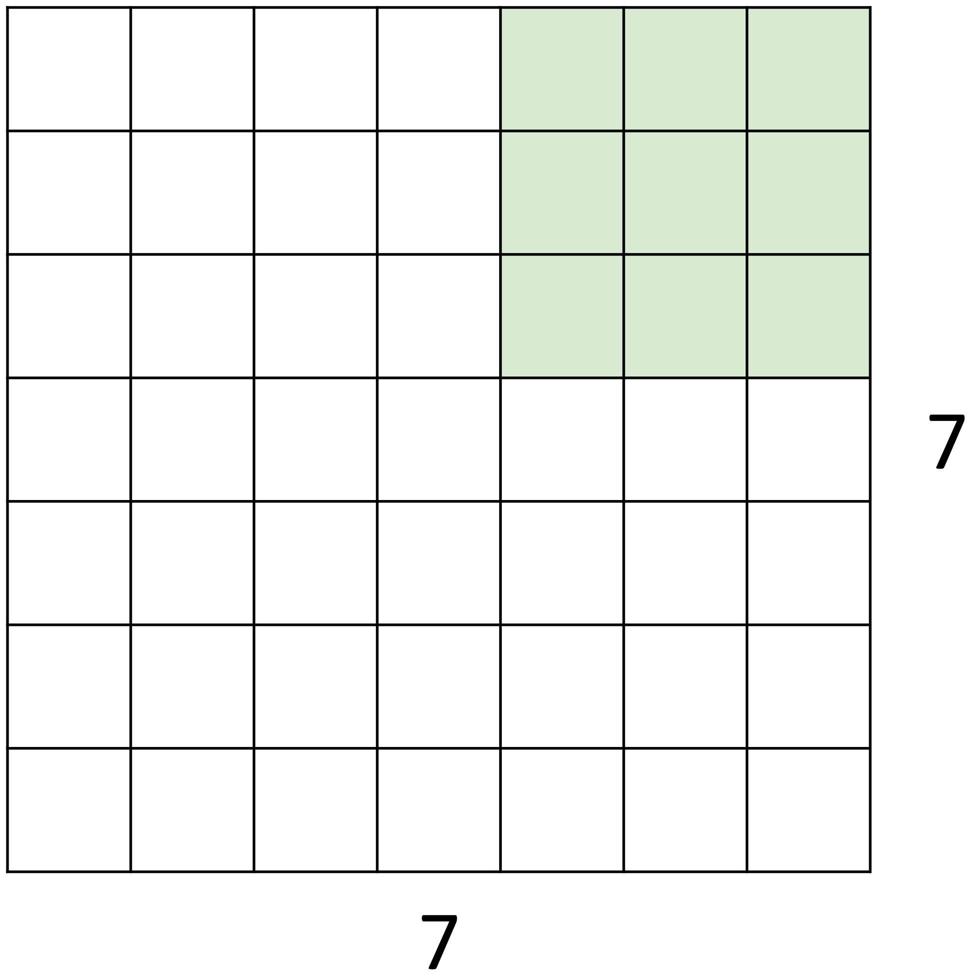
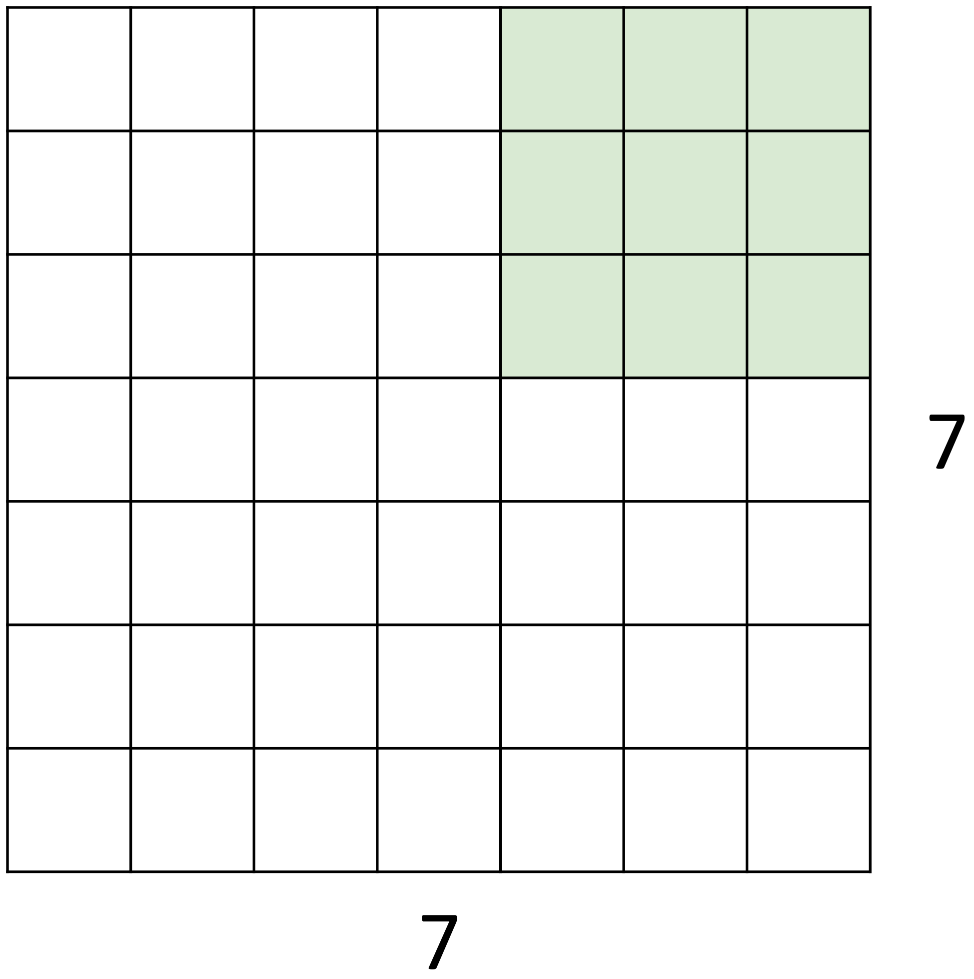
A closer look at spatial dimensions



Input: 7x7
Filter: 3x3
Output: 5x5

A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

Output: 5x5

In general:

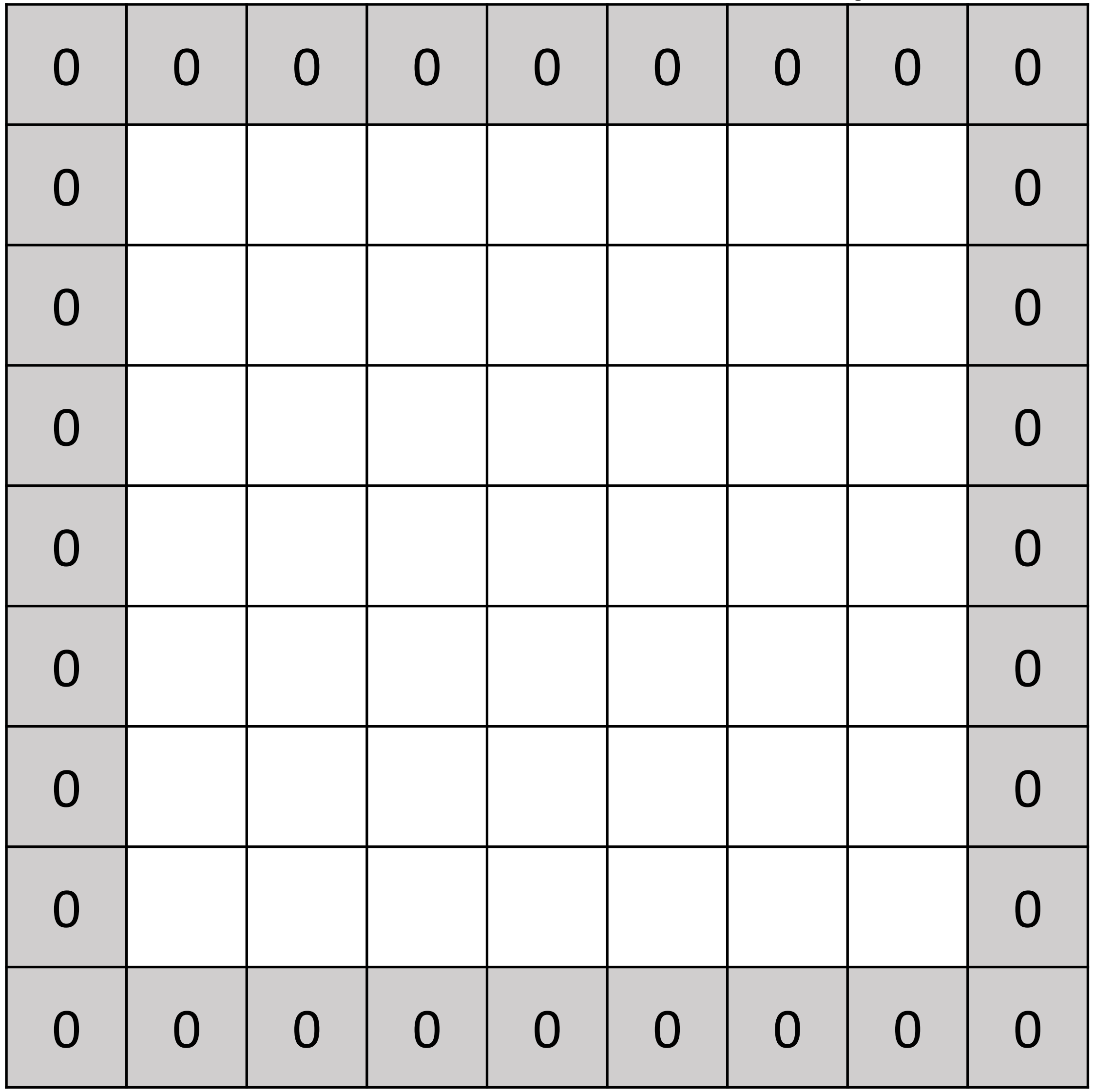
Input: W

Filter: K

Output: W – K + 1

Problem: Feature maps “shrink” with each layer!

A closer look at spatial dimensions



Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Output: W – K + 1

Problem: Feature maps “shrink” with each layer!

Solution: **padding**
Add zeros around the input

A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

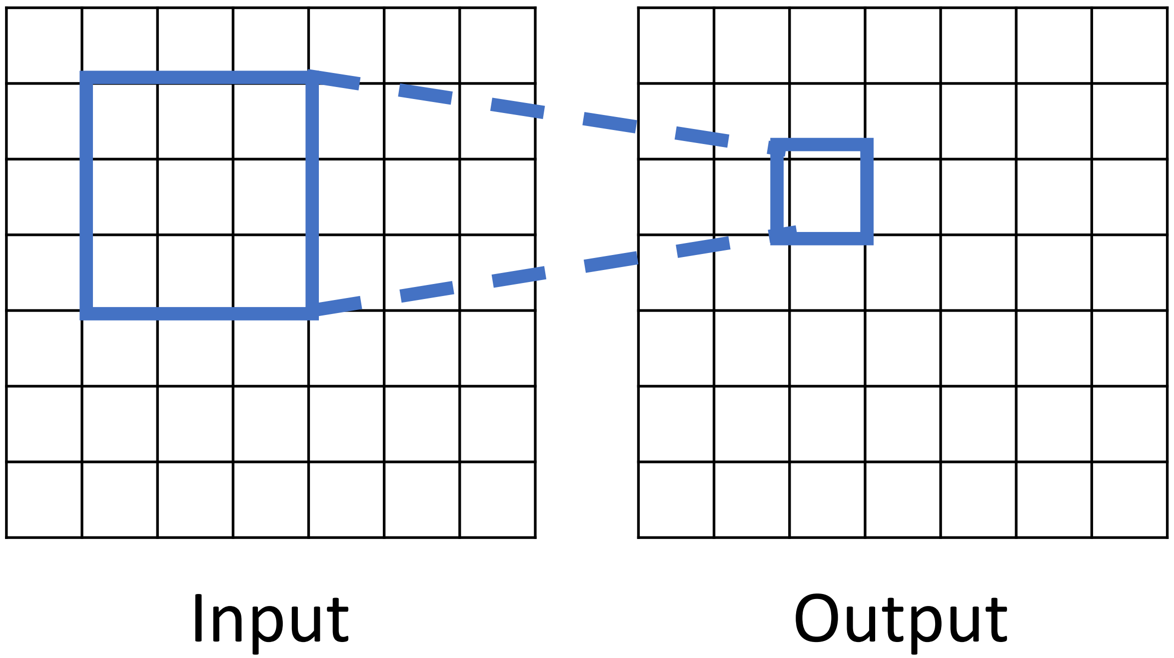
Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Padding: P
Output: W – K + 1 + 2P

Very common:
Set P = (K – 1) / 2 to make output have same size as input!

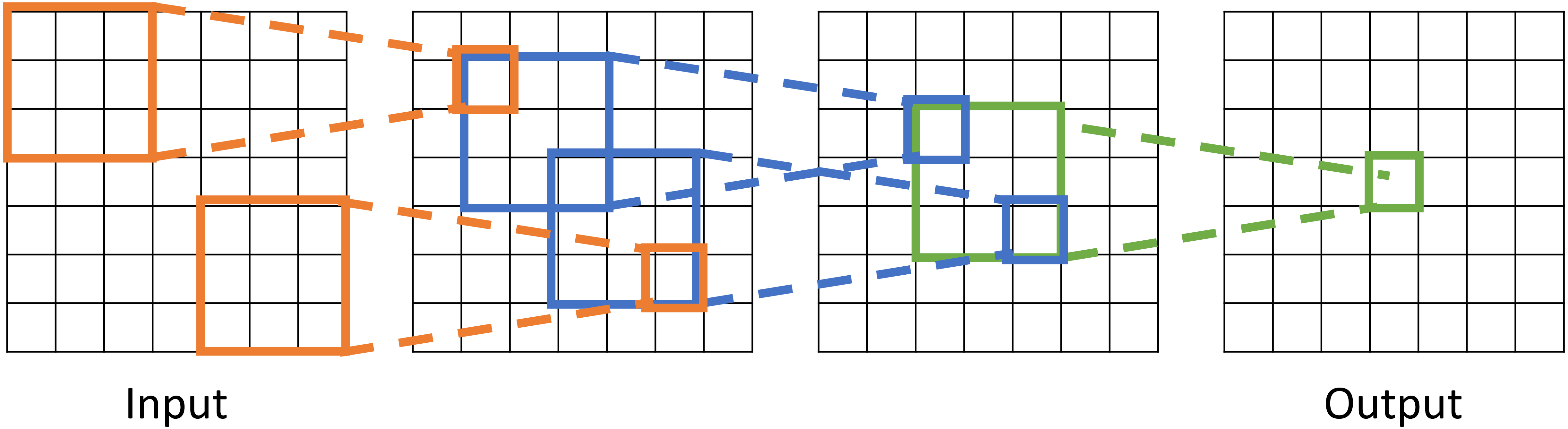
Receptive Fields

For convolution with kernel size K, each element in the output depends on a K x K **receptive field** in the input



Receptive Fields

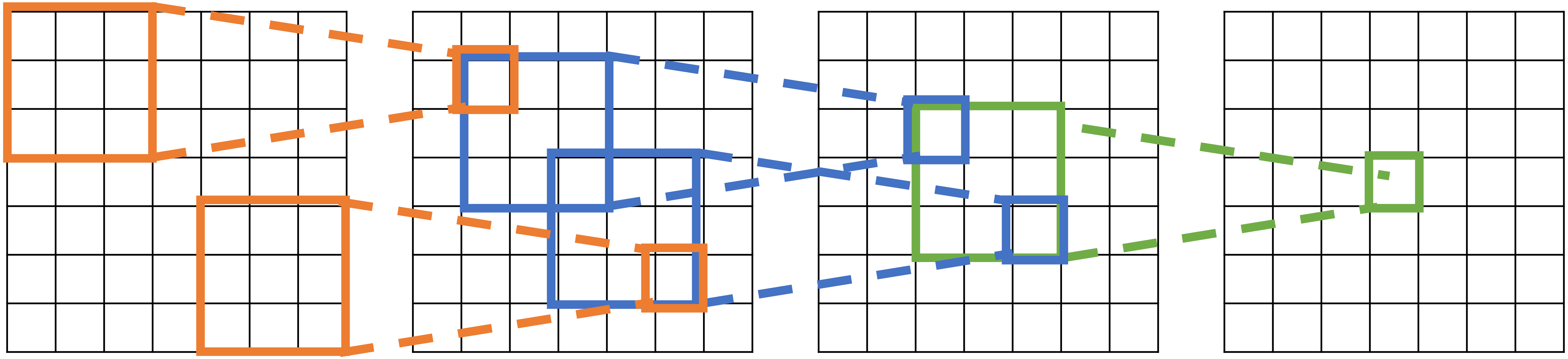
Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



Be careful – ”receptive field in the input” vs “receptive field in the previous layer”
Hopefully clear from context!

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



Input

Output

Problem: For large images we need many layers for each output to “see” the whole image image

Receptive Fields

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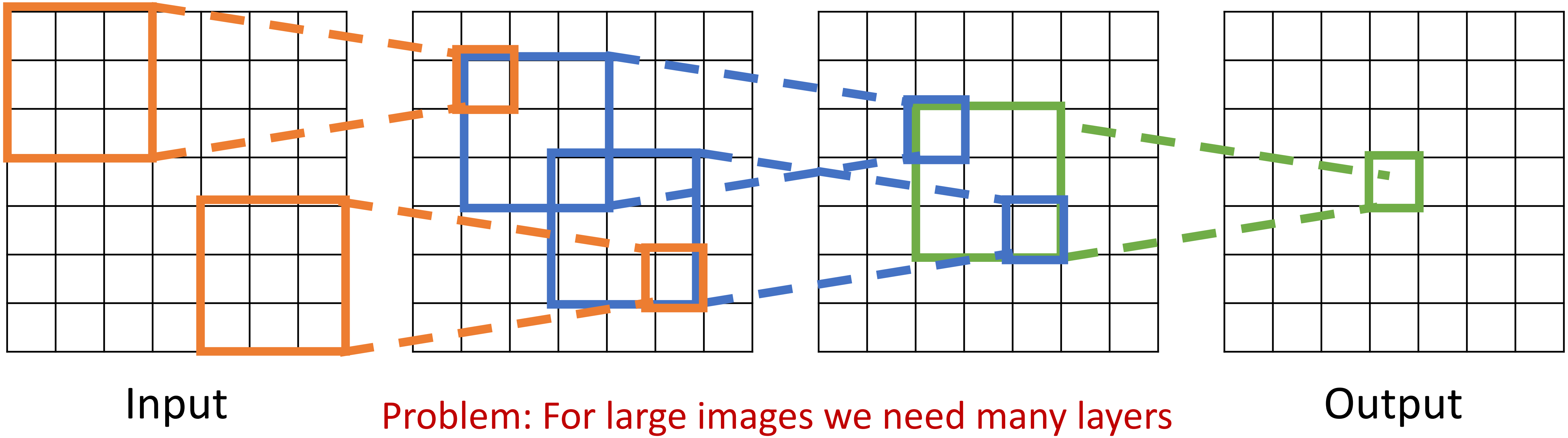
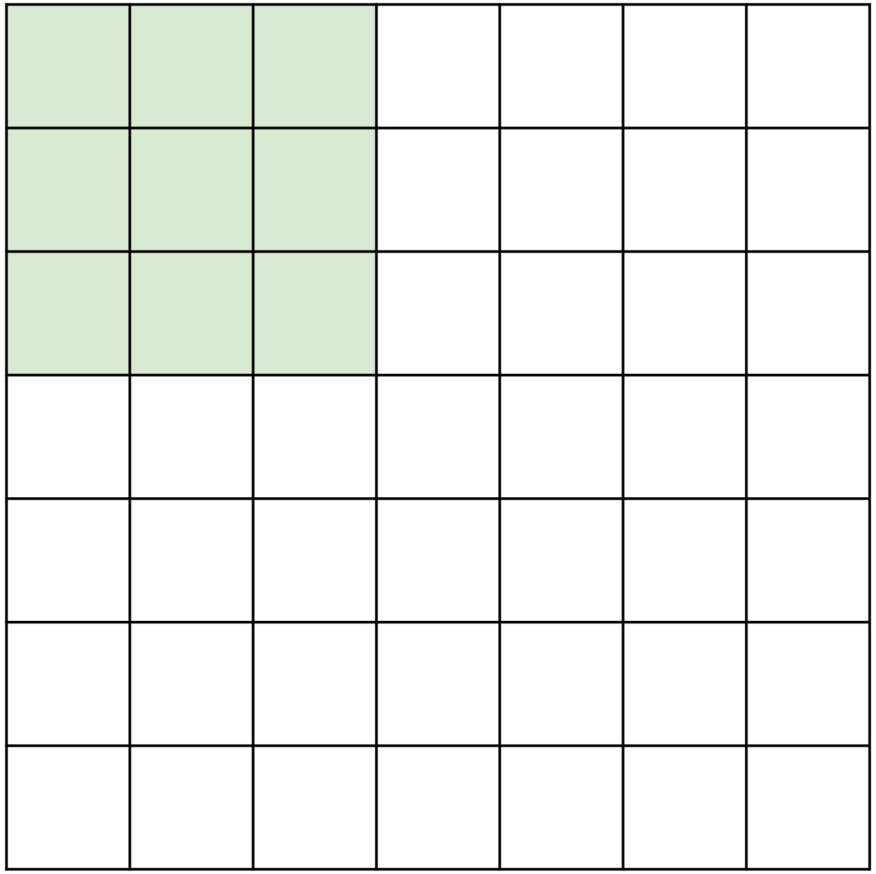
With L layers the receptive field size is 1 + L * (K – 1)



Problem: For large images we need many layers for each output to "see" the whole image image

Solution: Downsample inside the network

Strided Convolution

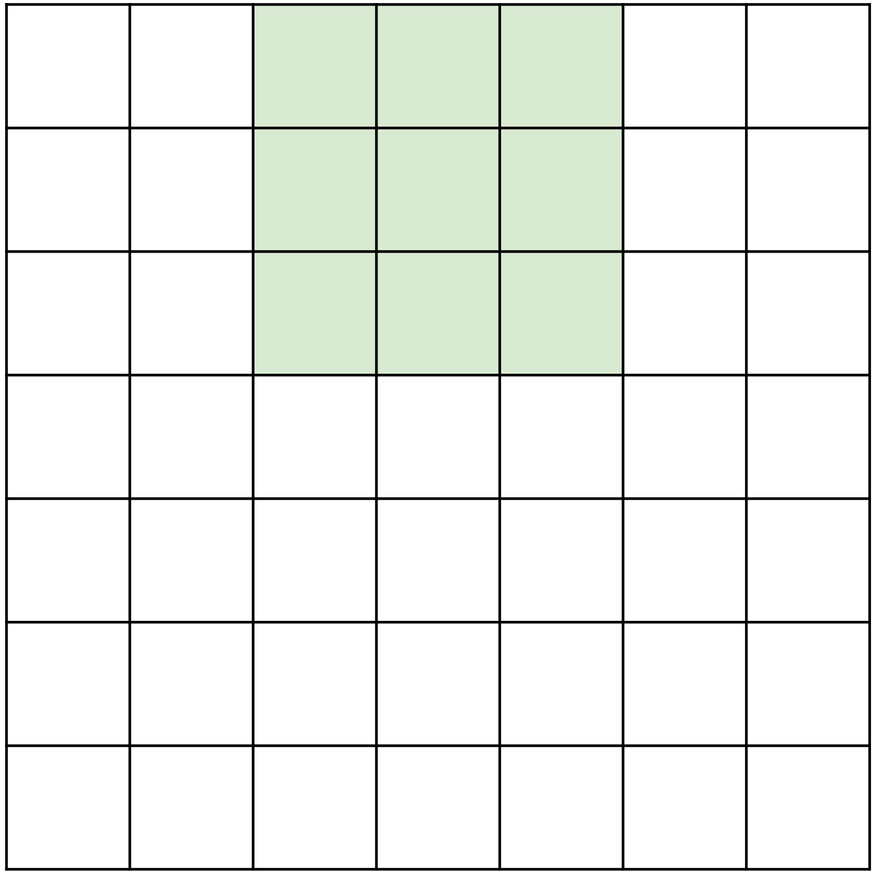


Input: 7x7

Filter: 3x3

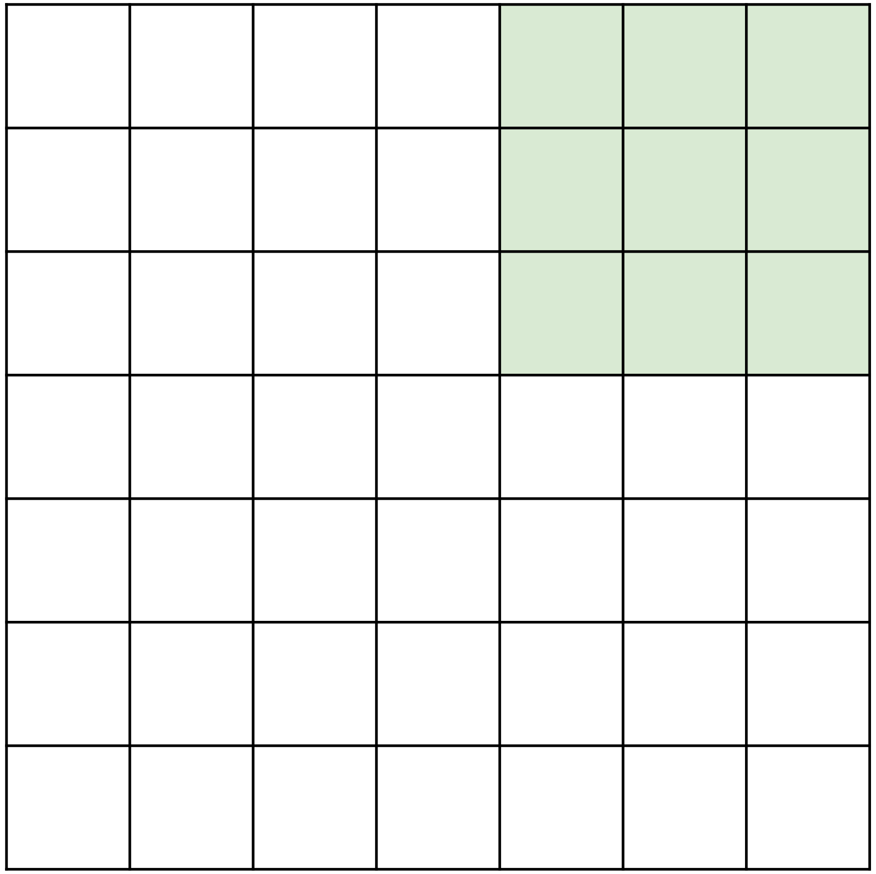
Stride: 2

Strided Convolution



Input: 7x7
Filter: 3x3
Stride: 2

Strided Convolution



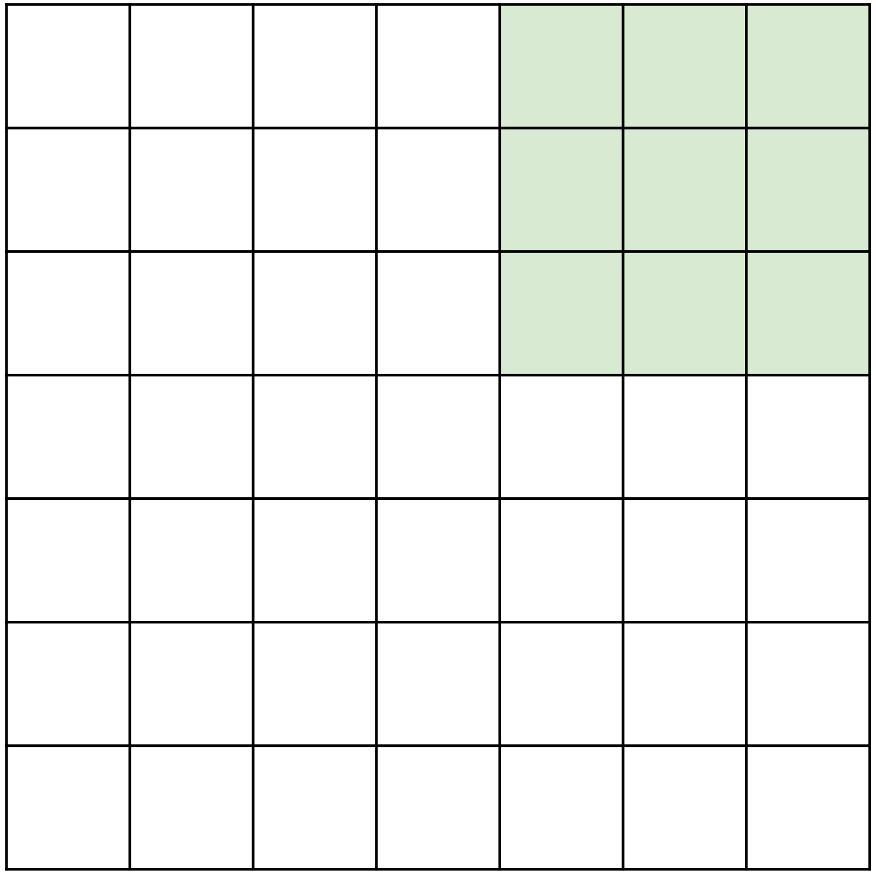
Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

Strided Convolution



Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

In general:

Input: W

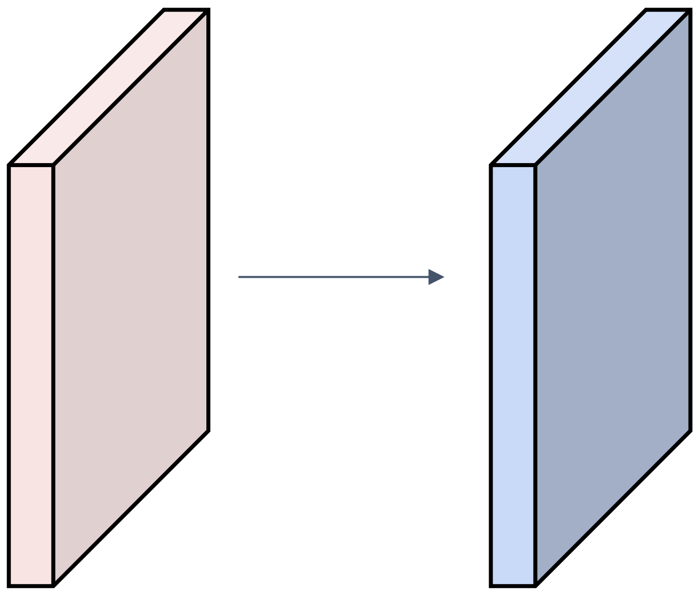
Filter: K

Padding: P

Stride: S

Output: (W – K + 2P) / S + 1

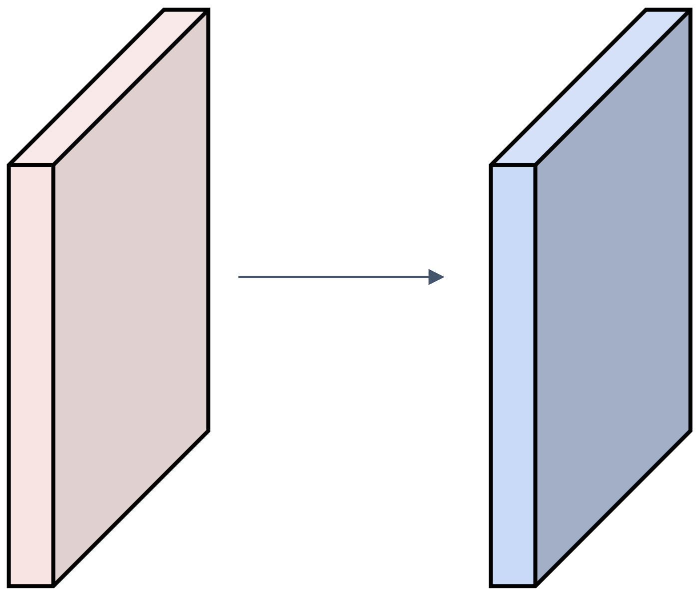
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: ?

Convolution Example



Input volume: 3 x **32** x **32**

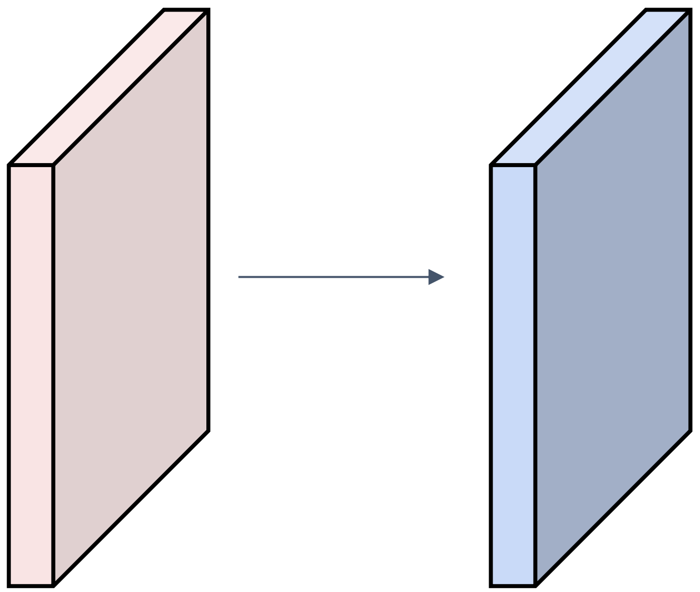
10 5x5 filters with stride **1**, pad **2**

Output volume size:

(**32**+2***2**-**5**)/**1**+1 = 32 spatially, so

10 x 32 x 32

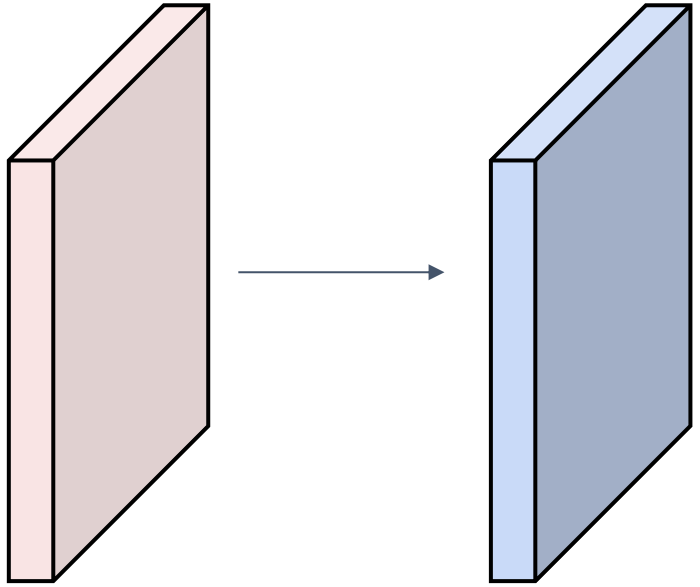
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: ?

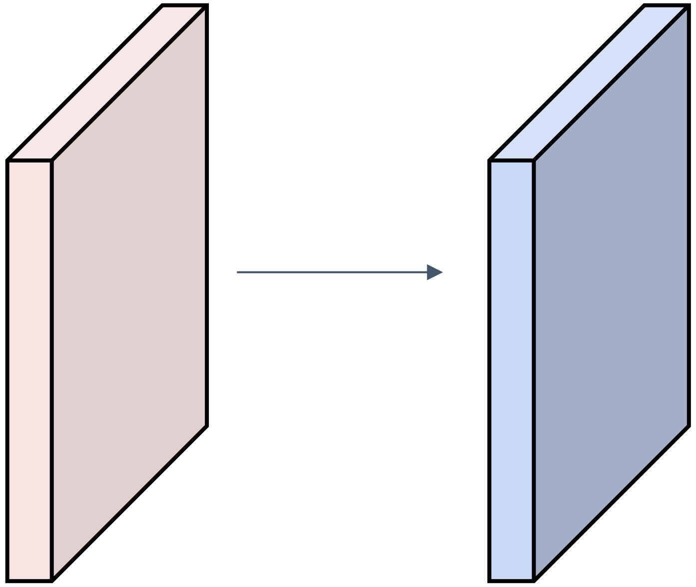
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: **760**
Parameters per filter: 3*5*5 + 1 (for bias) = 76
10 filters, so total is 10 * 76 = **760**

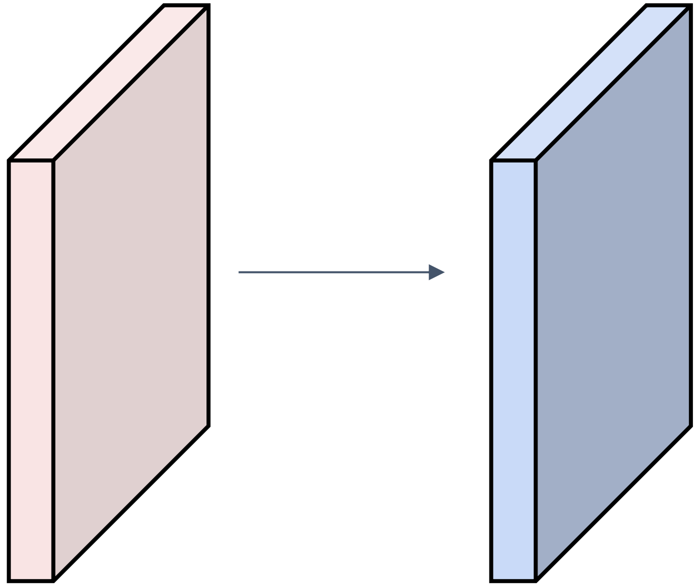
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: 760
Number of multiply-add operations: ?

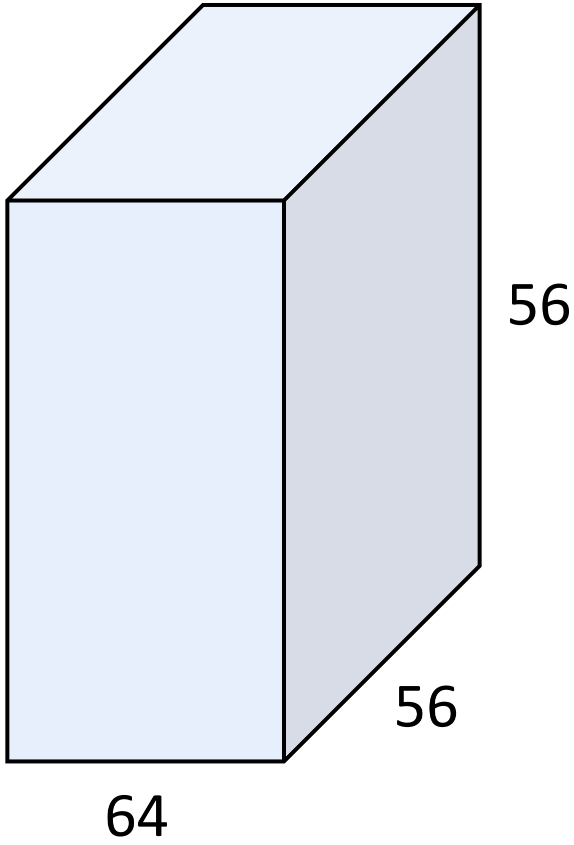
Convolution Example



Input volume: **3** x 32 x 32
10 **5x5** filters with stride 1, pad 2

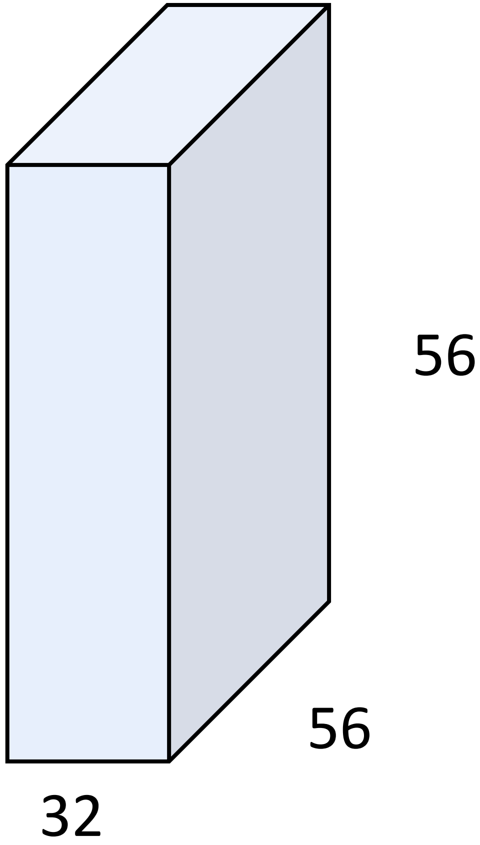
Output volume size: **10 x 32 x 32**
Number of learnable parameters: 760
Number of multiply-add operations: **768,000**
10*32*32 = 10,240 outputs; each output is the inner product of two **3**x**5x5** tensors (75 elems); total = 75*10240 = **768K**

Example: 1x1 Convolution

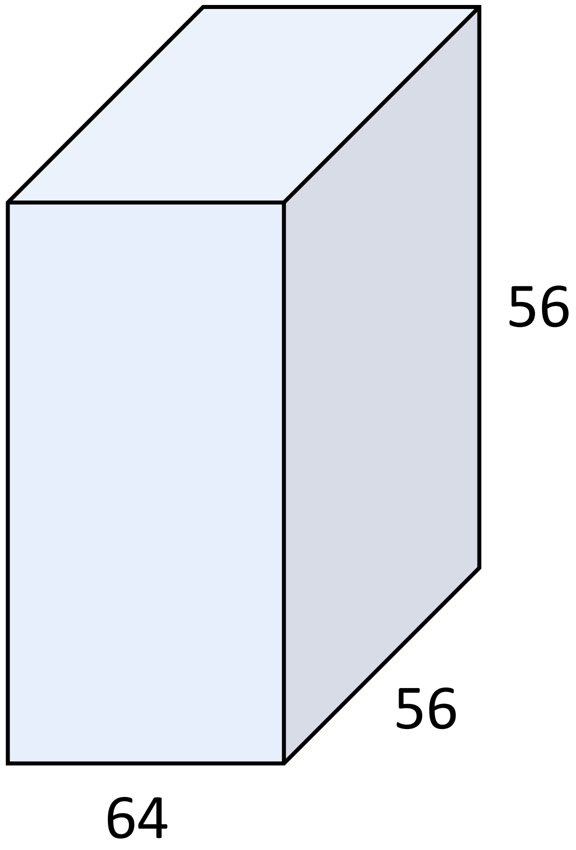


1x1 CONV
with 32 filters

(each filter has size 1x1x64, and performs a 64-dimensional dot product)

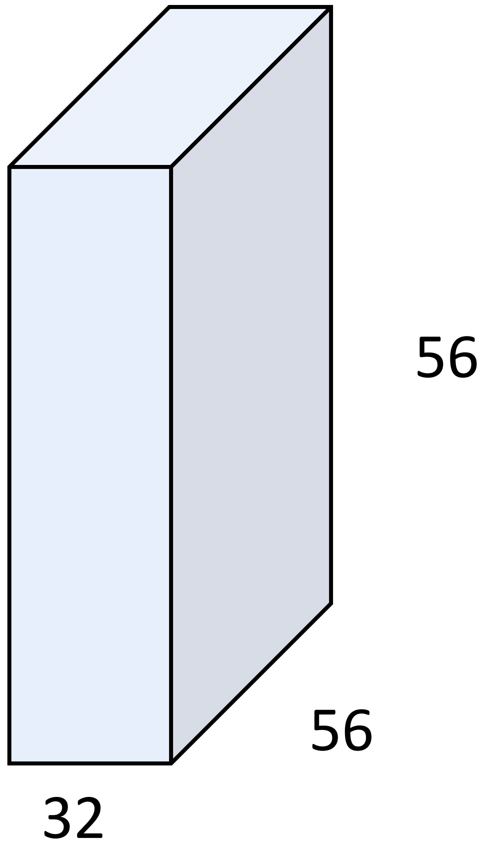


Example: 1x1 Convolution



1x1 CONV
with 32 filters

(each filter has size 1x1x64, and performs a 64-dimensional dot product)



Stacking 1x1 conv layers gives MLP operating on each input position

Lin et al, "Network in Network", ICLR 2014

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W
giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W
giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$

Common settings:

$K_H = K_W$ (Small square filters)

$P = (K - 1) / 2$ ("Same" padding)

C_{in}, C_{out} = 32, 64, 128, 256 (powers of 2)

K = 3, P = 1, S = 1 (3x3 conv)

K = 5, P = 2, S = 1 (5x5 conv)

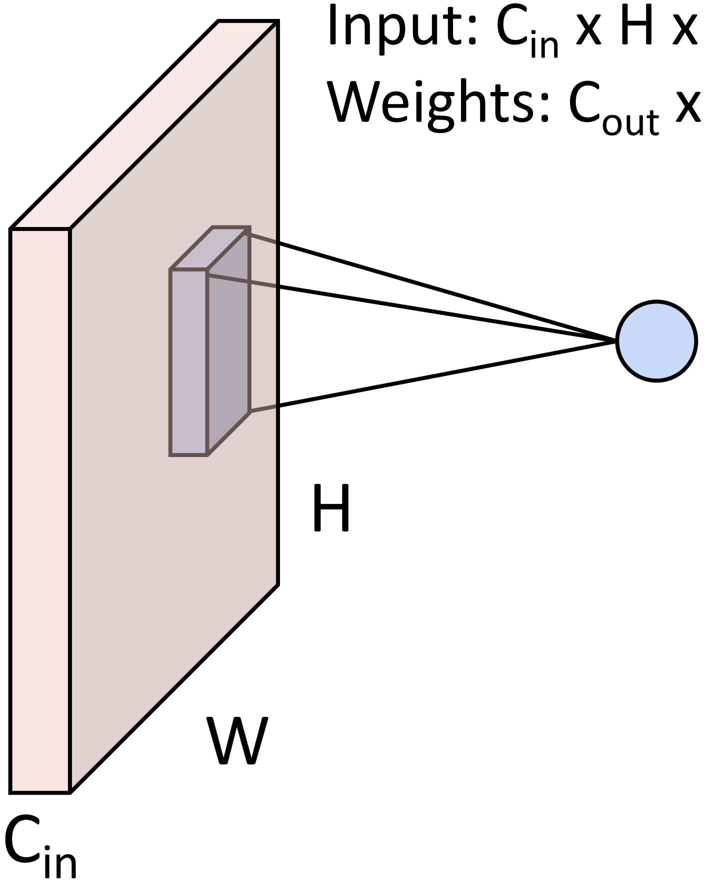
K = 1, P = 0, S = 1 (1x1 conv)

K = 3, P = 1, S = 2 (Downsample by 2)

Other types of convolution

So far: 2D Convolution

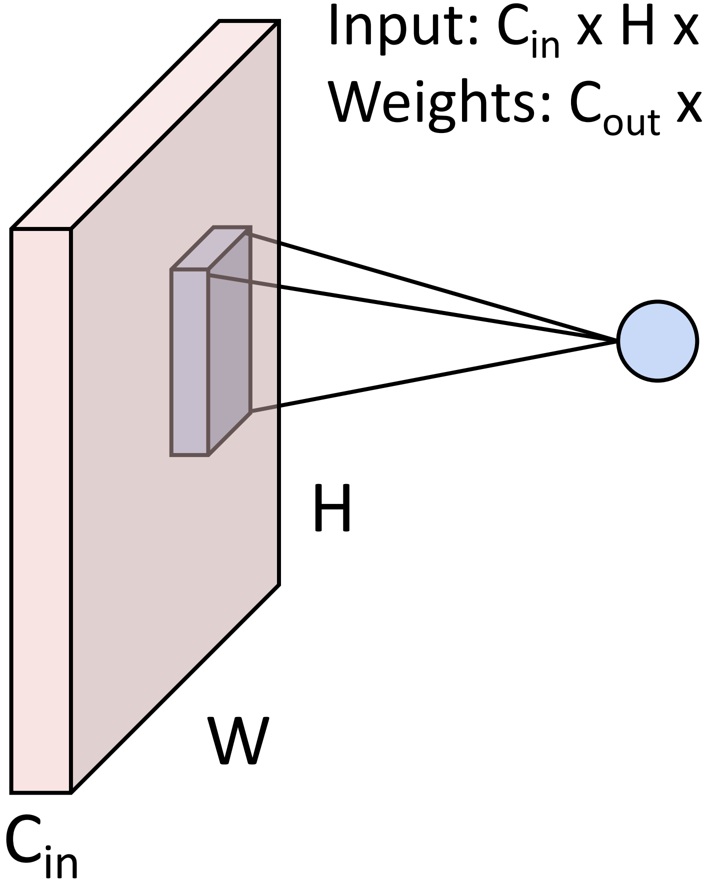
Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



Other types of convolution

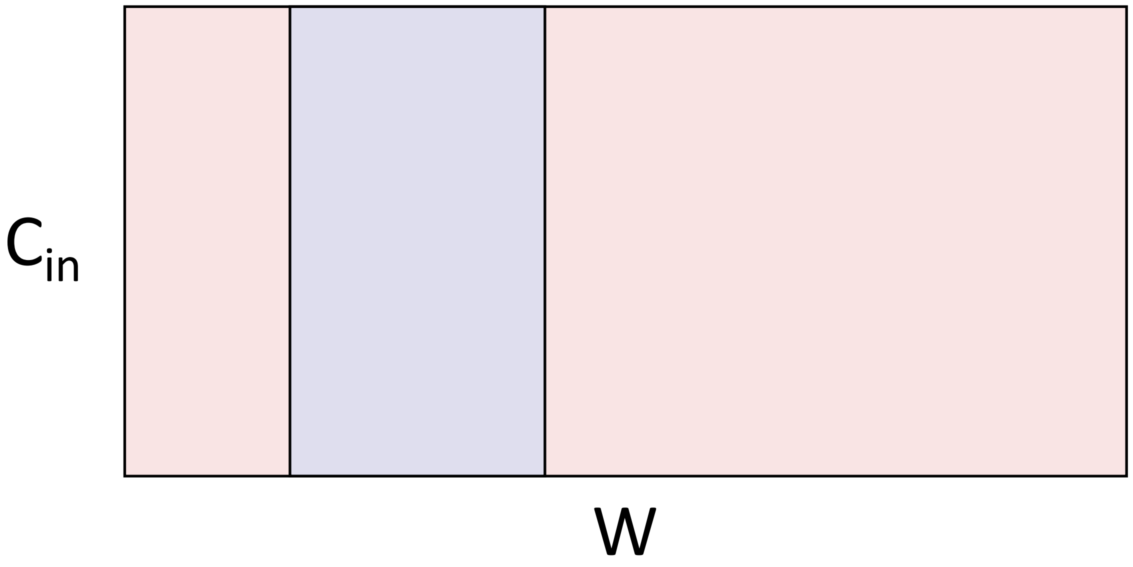
So far: 2D Convolution

Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



1D Convolution

Input: C_{in} x W
Weights: C_{out} x C_{in} x K

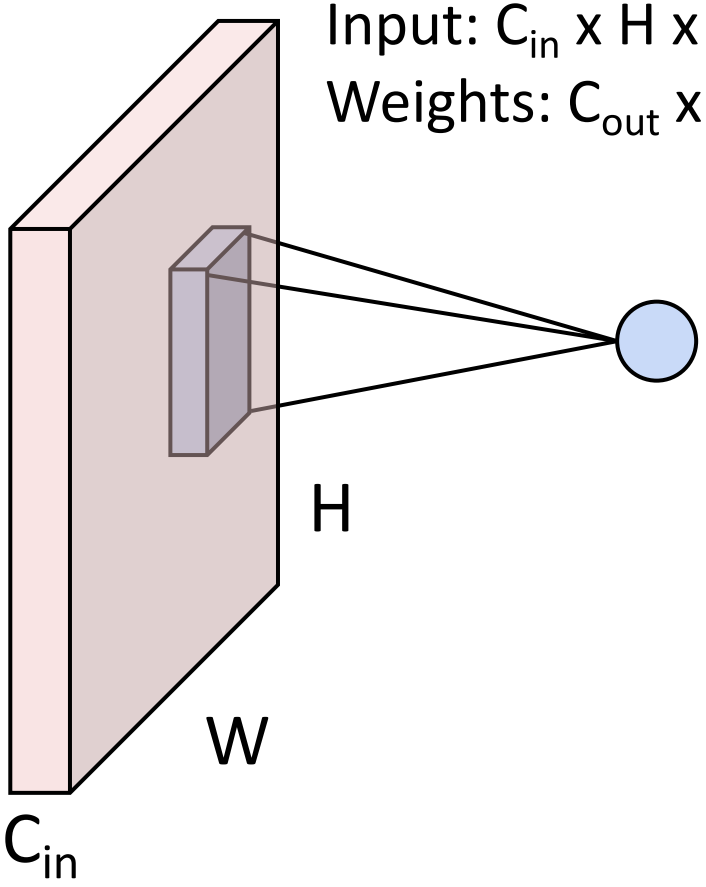


Other types of convolution

So far: 2D Convolution

Input: C_{in} x H x W

Weights: C_{out} x C_{in} x K x K

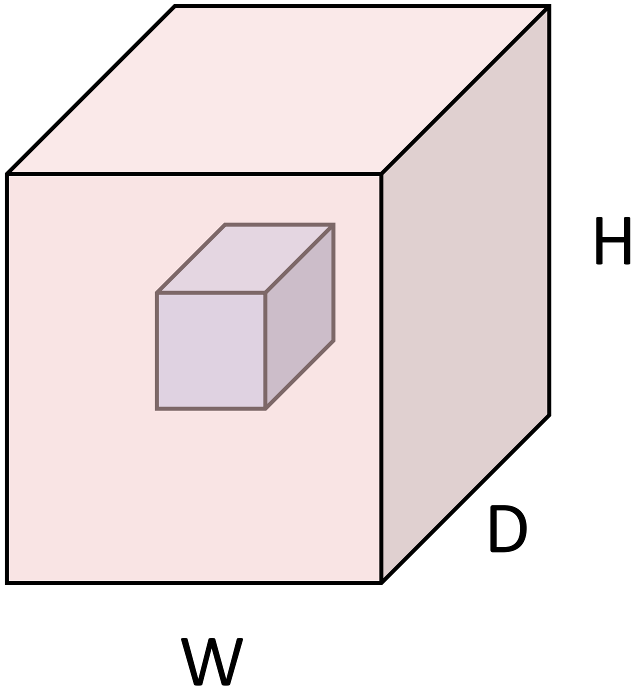


3D Convolution

Input: C_{in} x H x W x D

Weights: C_{out} x C_{in} x K x K x K

C_{in}-dim vector at each point in the volume



PyTorch Convolution Layer

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

PyTorch Convolution Layers

Conv2d

CLASS `torch.nn.Conv2d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

Conv1d

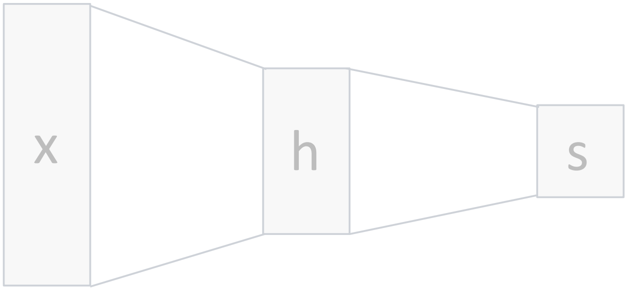
CLASS `torch.nn.Conv1d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

Conv3d

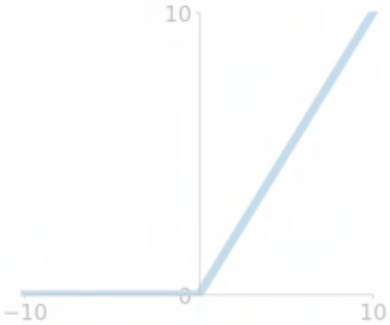
CLASS `torch.nn.Conv3d(`*`in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'`*`)` [SOURCE]

Components of a Convolutional Network

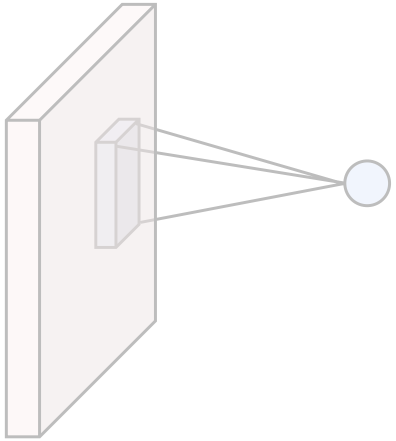
Fully-Connected Layers



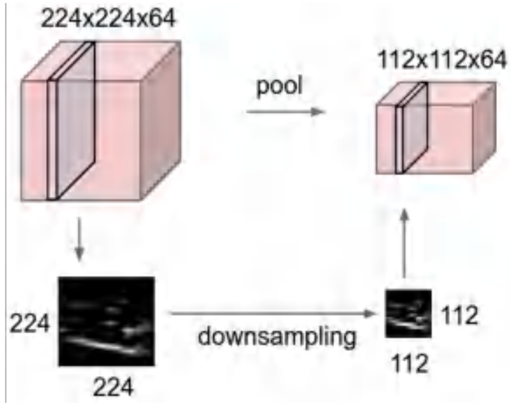
Activation Function



Convolution Layers



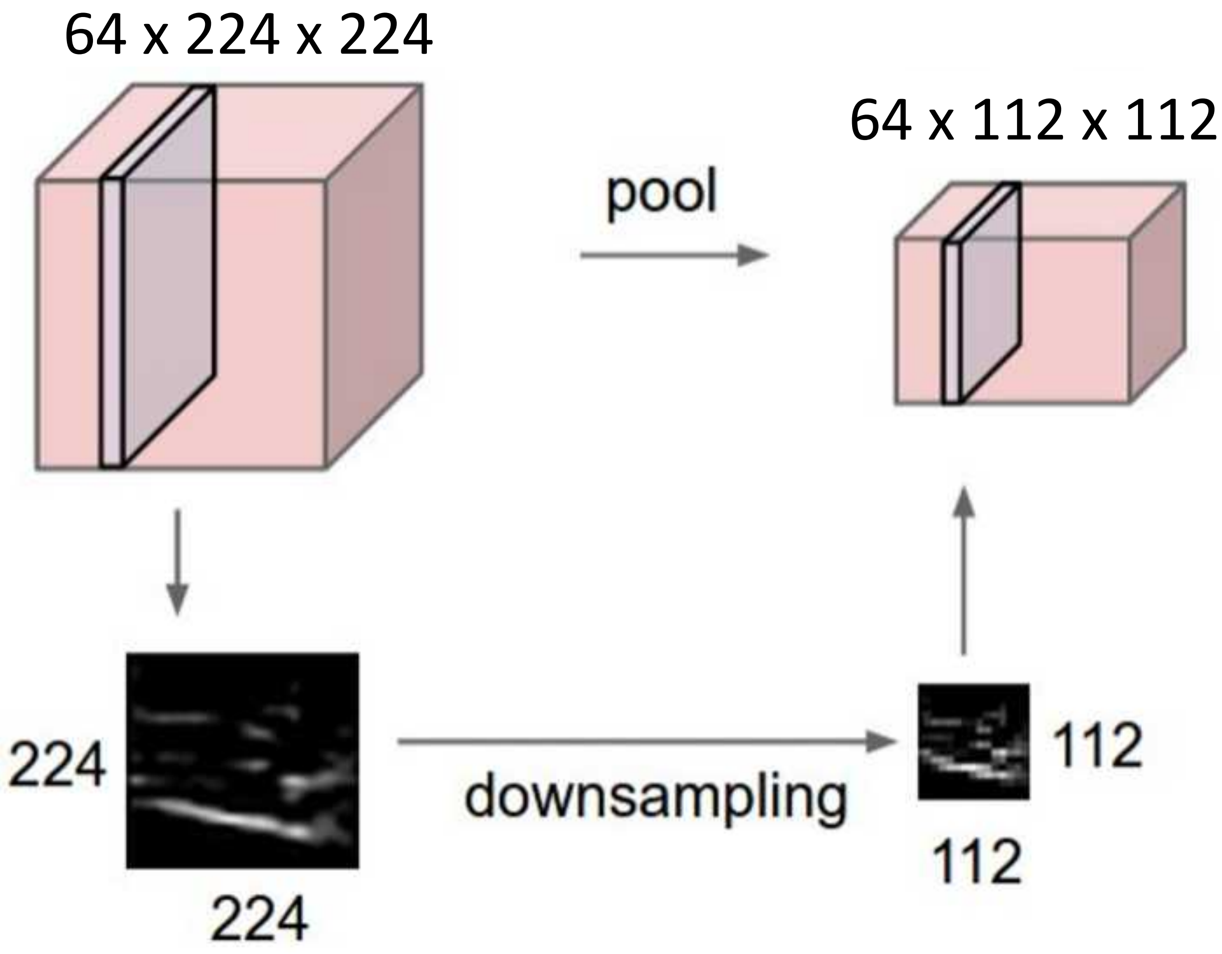
Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

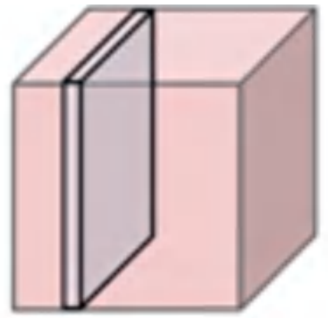
Pooling Layers: Another way to downsample



Hyperparameters:
Kernel Size
Stride
Pooling function

Max Pooling

64 x 224 x 224



Single depth slice

x

1	1	2	4
5	**6**	7	**8**
3	2	1	0
1	2	3	**4**

y

Max pooling with 2x2 kernel size and stride 2

6	8
3	4

Introduces **invariance** to small spatial shifts

No learnable parameters!

Pooling Summary

Input: C x H x W

Hyperparameters:

- Kernel size: K
- Stride: S
- Pooling function (max, avg)

Output: C x H' x W' where

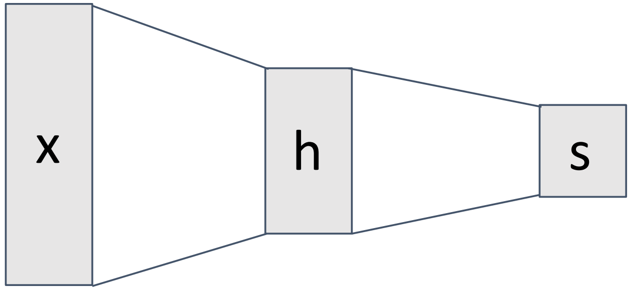
- H' = (H – K) / S + 1
- W' = (W – K) / S + 1

Learnable parameters: None!

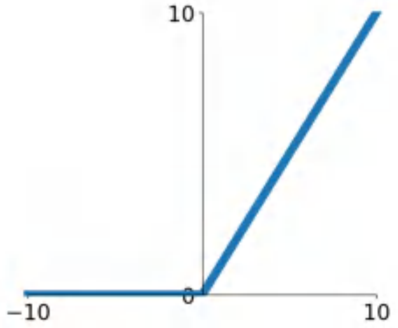
Common settings:
max, K = 2, S = 2
max, K = 3, S = 2 (AlexNet)

Components of a Convolutional Network

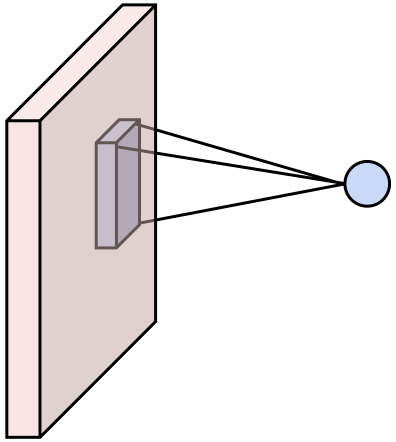
Fully-Connected Layers



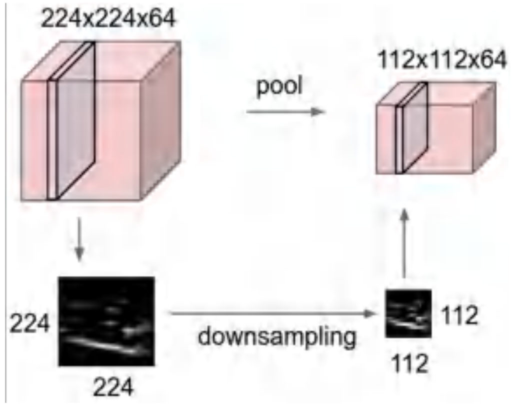
Activation Function



Convolution Layers



Pooling Layers



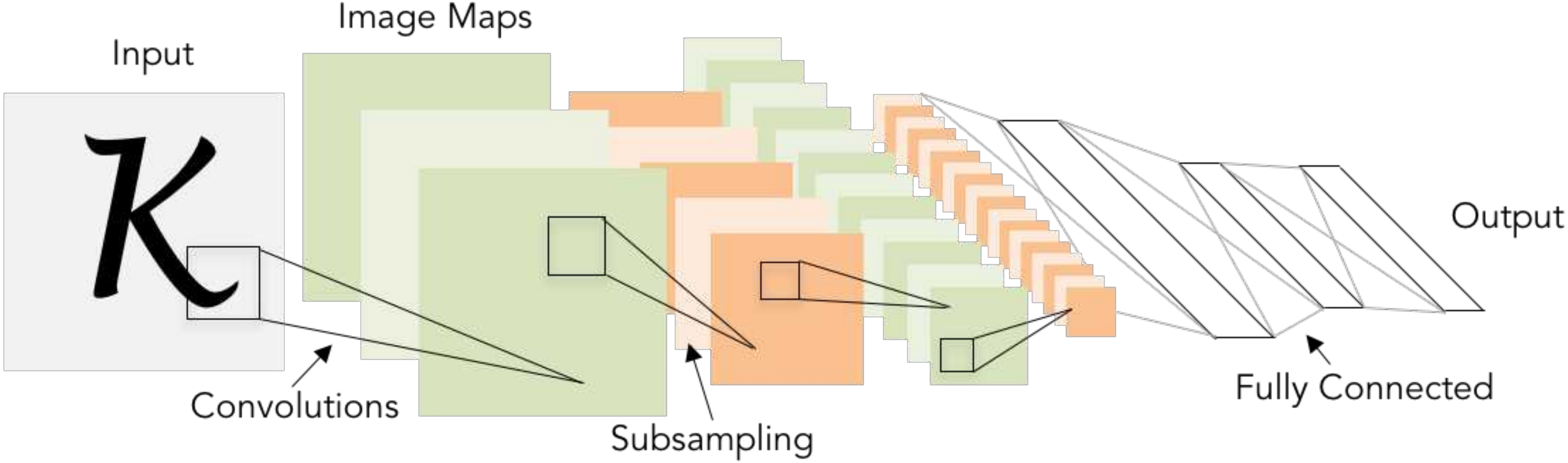
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Convolutional Networks

Classic architecture: [Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

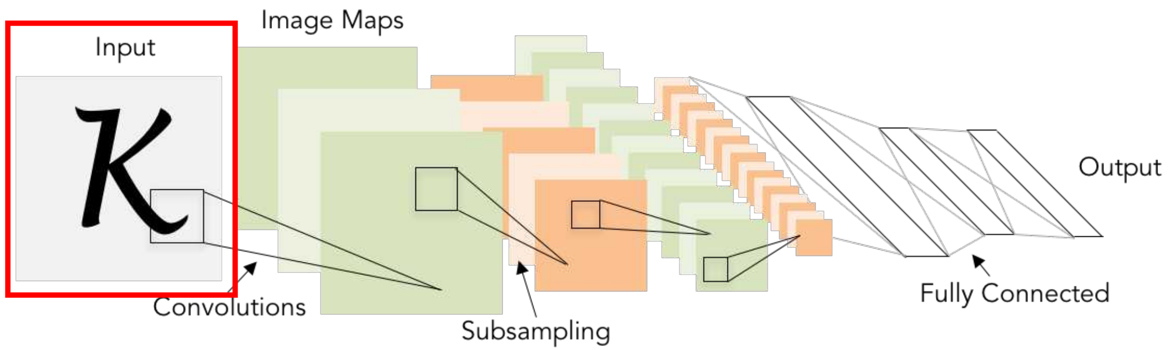
Example: LeNet-5



Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

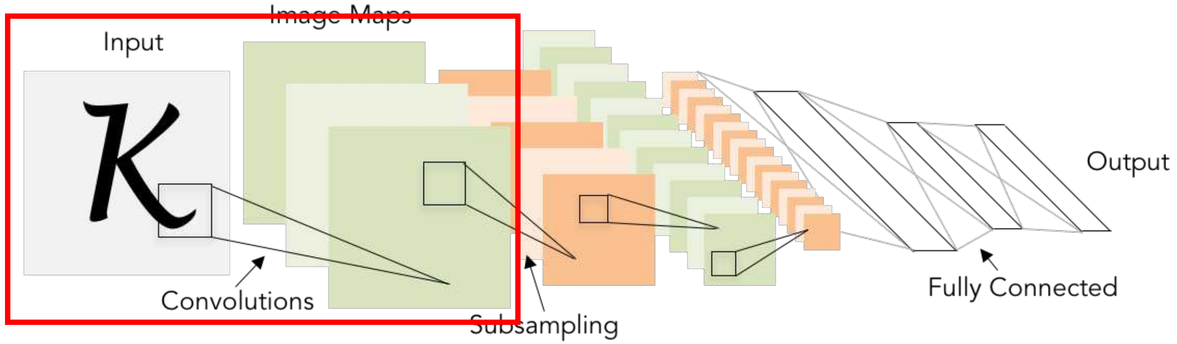
Layer	Output Size	Weight Size
Input	1 x 28 x 28	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

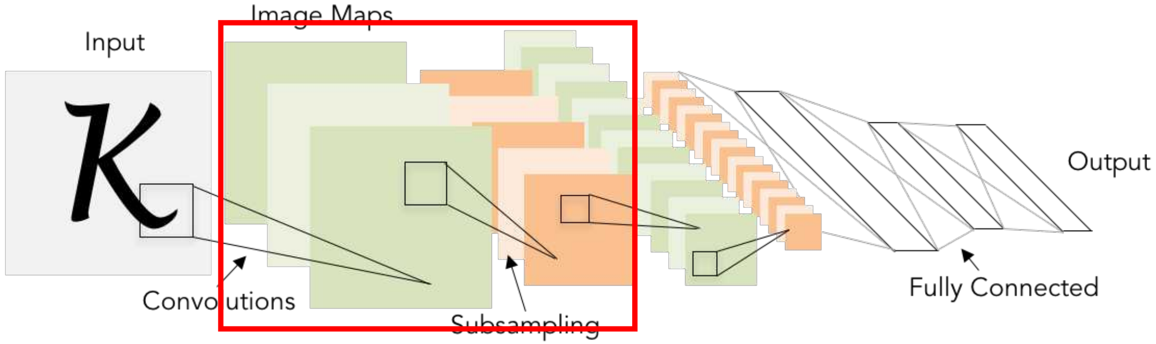
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	



Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

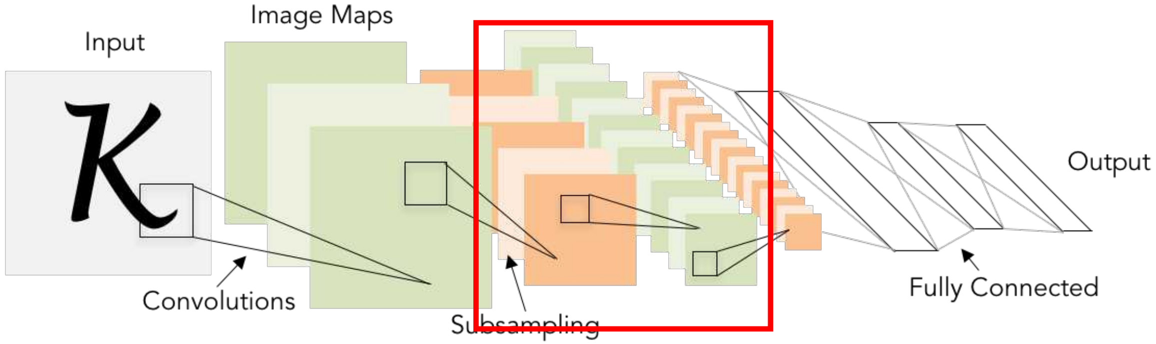
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	



Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

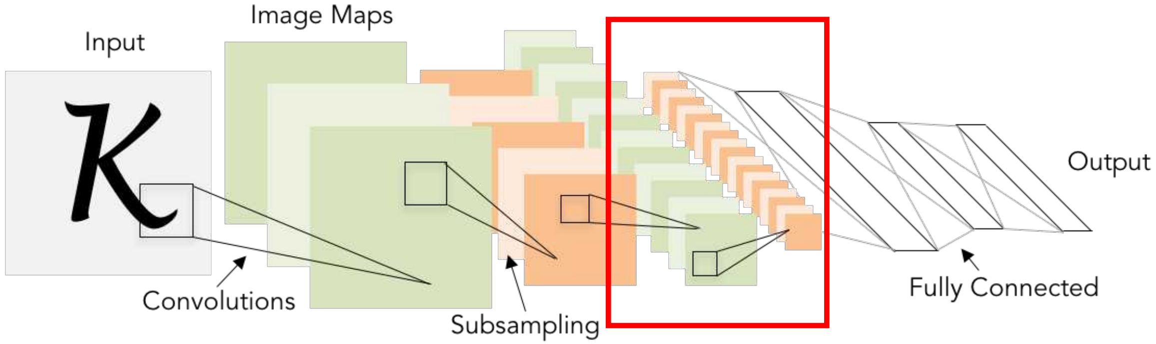
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	



Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	



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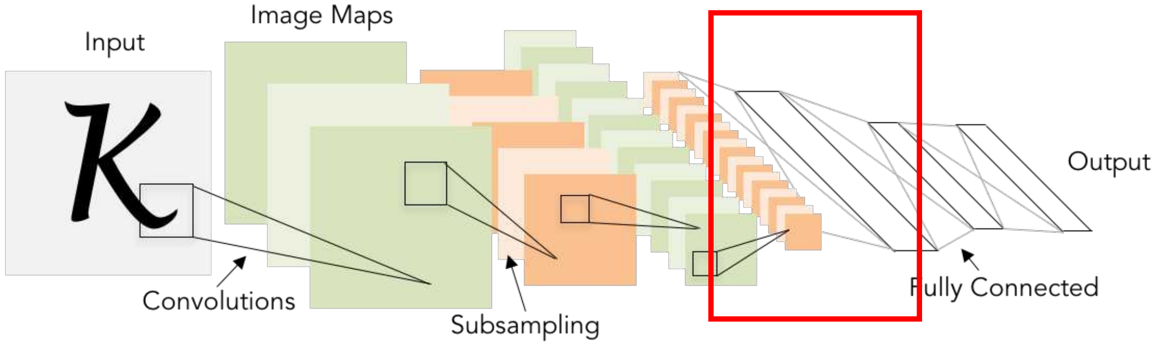
Example: LeNet-5

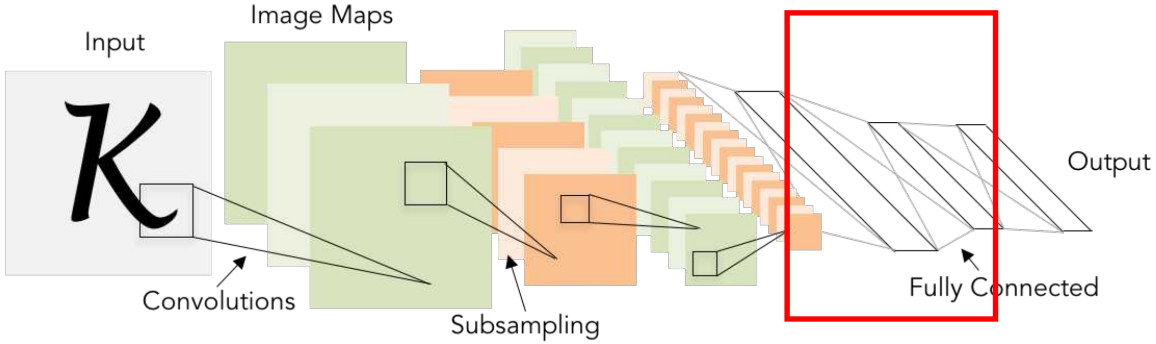


Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	

Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	



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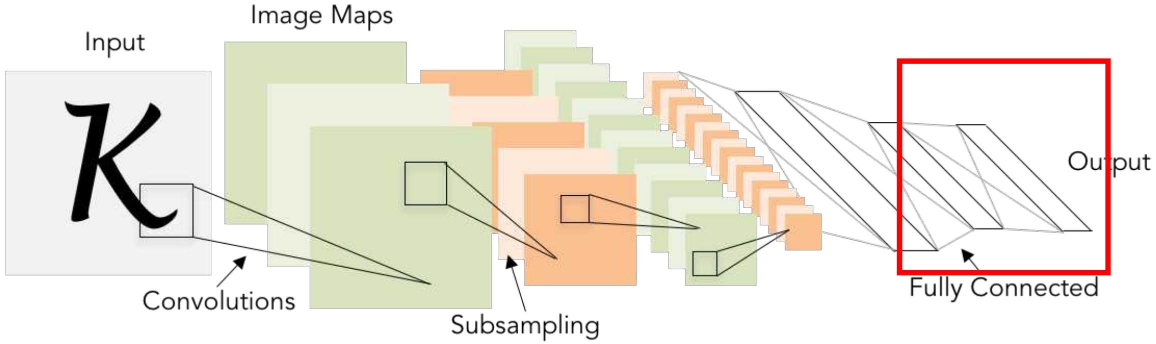
Example: LeNet-5

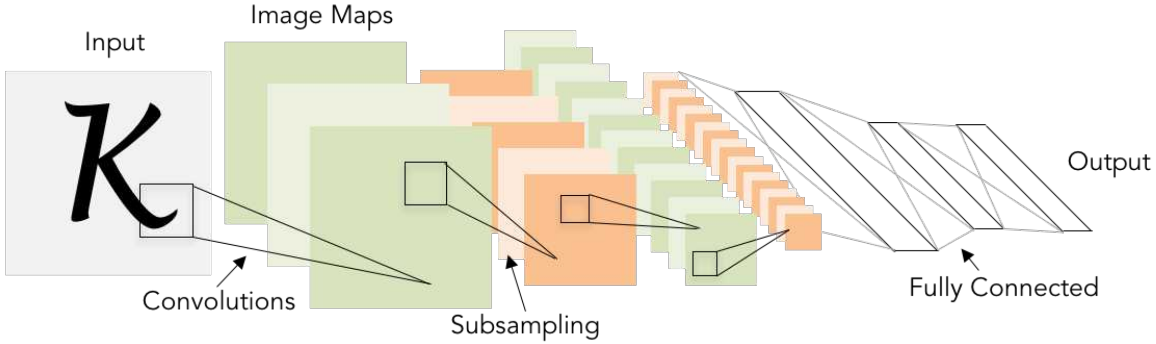


Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10

Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10



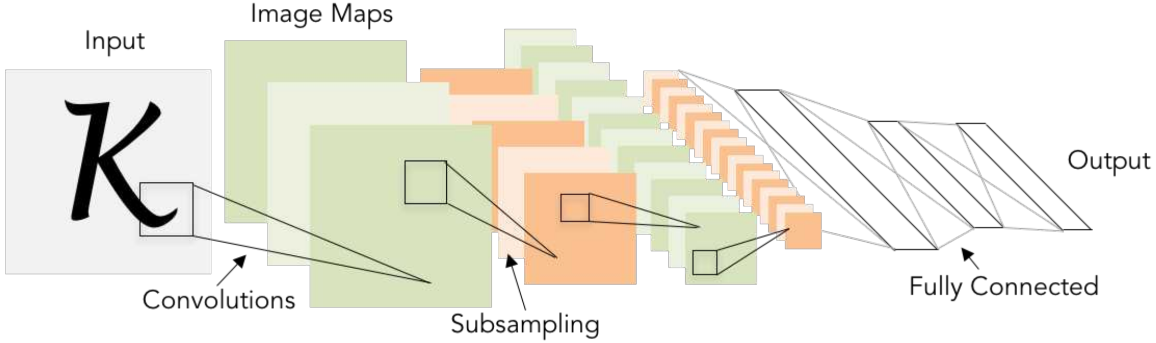
As we go through the network:

Spatial size **decreases**
(using pooling or strided conv)

Number of channels **increases**
(total “volume” is preserved!)

Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5



Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10

As we go through the network:

Spatial size **decreases** (using pooling or strided conv)

Number of channels **increases** (total "volume" is preserved!)

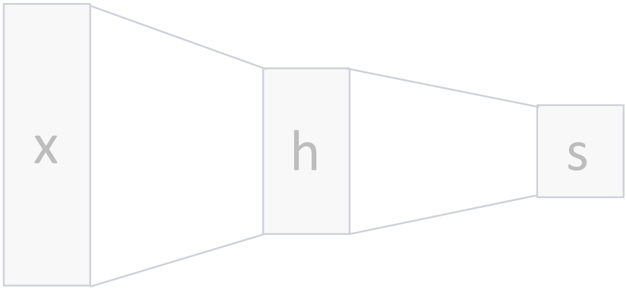
Some modern architectures break this trend -- stay tuned!

Lecun et al, "Gradient-based learning applied to document recognition", 1998

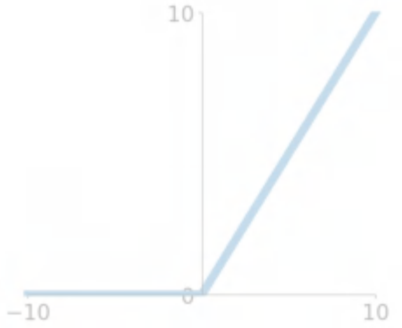
Problem: Deep Networks very hard to train!

Components of a Convolutional Network

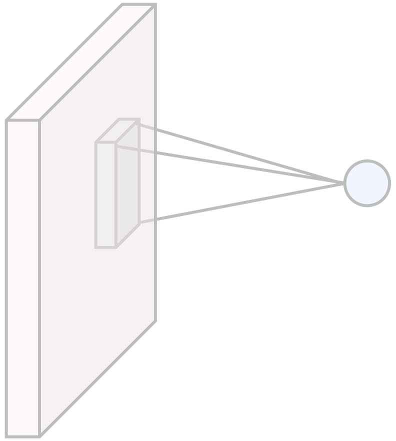
Fully-Connected Layers



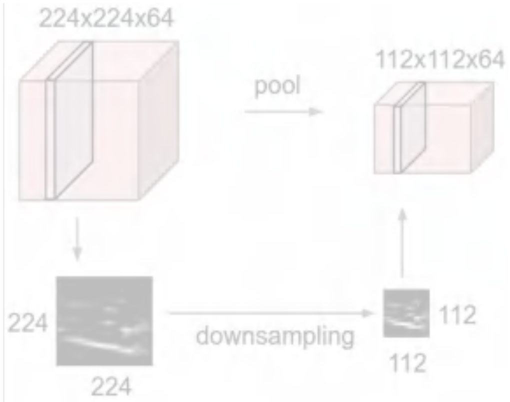
Activation Function



Convolution Layers



Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Batch Normalization

Idea: "Normalize" the outputs of a layer so they have zero mean and unit variance

Why? Helps reduce "internal covariate shift", improves optimization

We can normalize a batch of activations like this:

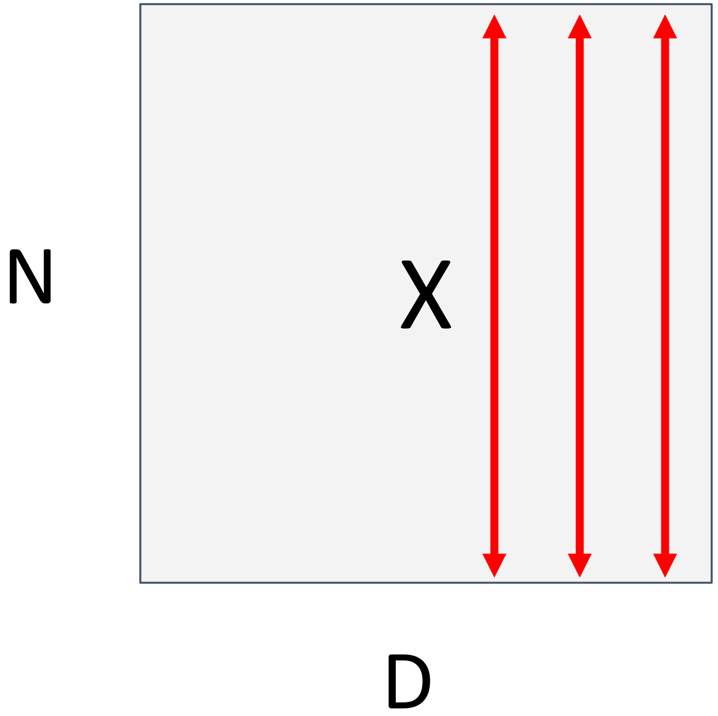
$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

This is a **differentiable function**, so we can use it as an operator in our networks and backprop through it!

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$



$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N} \left(x_{i,j} - \mu_j\right)^2$$

Per-channel std, shape is D

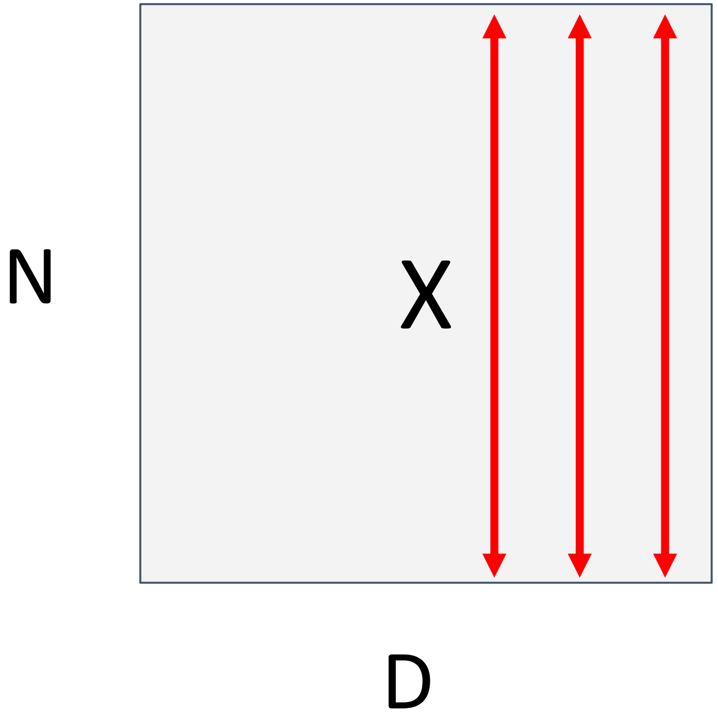
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$



$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

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Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Problem: What if zero-mean, unit variance is too hard of a constraint?

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

Learnable scale and shift parameters:

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N} \left(x_{i,j} - \mu_j\right)^2$$

Per-channel std, shape is D

$$\gamma, \beta \in \mathbb{R}^D$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization

Problem: Estimates depend on minibatch; can't do this at test-time!

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$\gamma, \beta \in \mathbb{R}^{D}$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} \left(x_{i,j} - \mu_j\right)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$\gamma, \beta \in \mathbb{R}^{D}$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$\mu_j =$ (Running) average of values seen during training

Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

$\mu_j =$ (Running) average of values seen during training

Per-channel mean, shape is D

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^{D}$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$\mu_j^{test} = 0$

For each training iteration:

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

$$\mu_j^{test} = 0.99\, \mu_j^{test} + 0.01\, \mu_j$$

(Similar for σ)

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^D$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$\mu_j =$ (Running) average of values seen during training — Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training — Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$\gamma, \beta \in \mathbb{R}^{D}$

During testing batchnorm becomes a linear operator! Can be fused with the previous fully-connected or conv layer

$\mu_j =$ (Running) average of values seen during training

Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization for ConvNets

Batch Normalization for **fully-connected** networks

$$x : N \times D$$

Normalize

$$\mu, \sigma \ : 1 \times D$$
$$\gamma, \beta \ : 1 \times D$$
$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

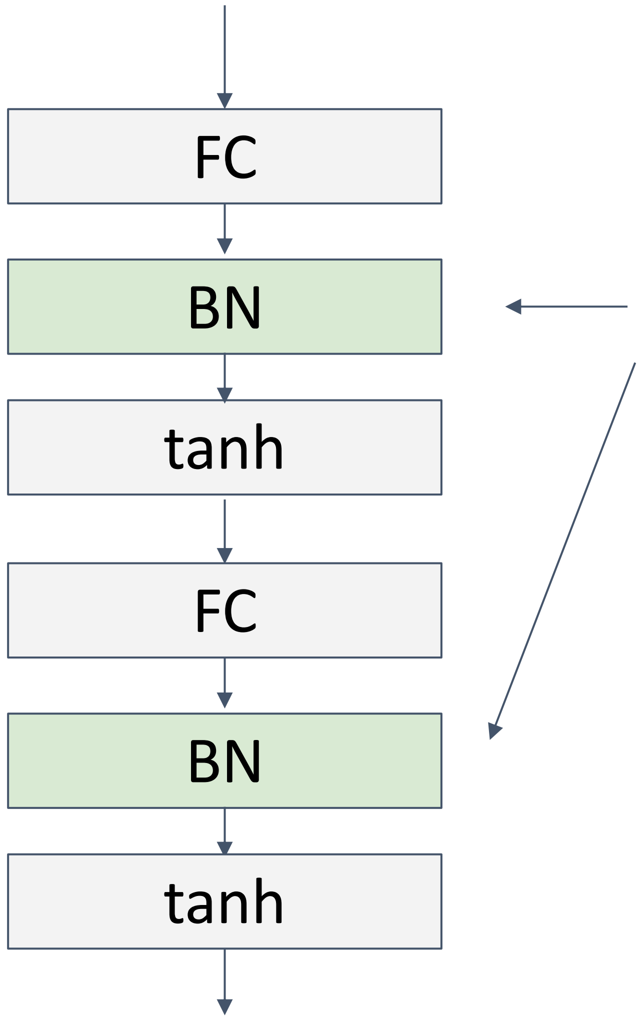
Batch Normalization for **convolutional** networks (Spatial Batchnorm, BatchNorm2D)

$$x : N \times C \times H \times W$$

Normalize

$$\mu, \sigma \ : 1 \times C \times 1 \times 1$$
$$\gamma, \beta \ : 1 \times C \times 1 \times 1$$
$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Batch Normalization

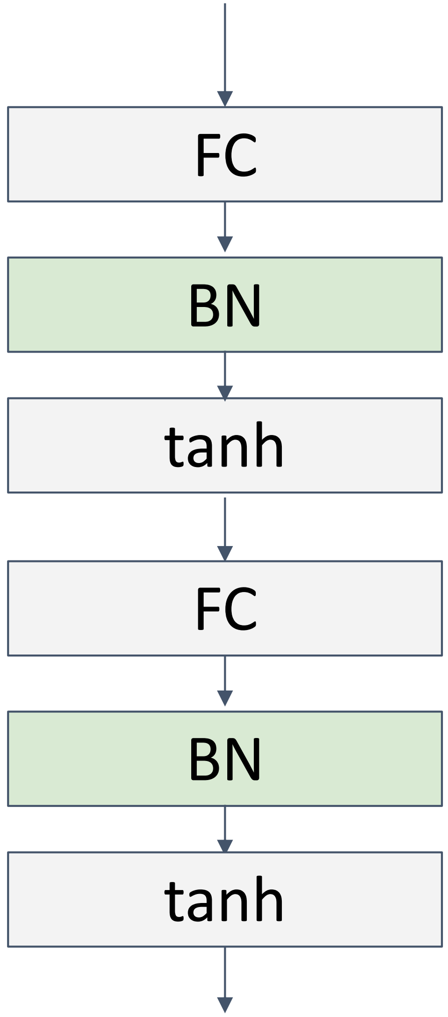


Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

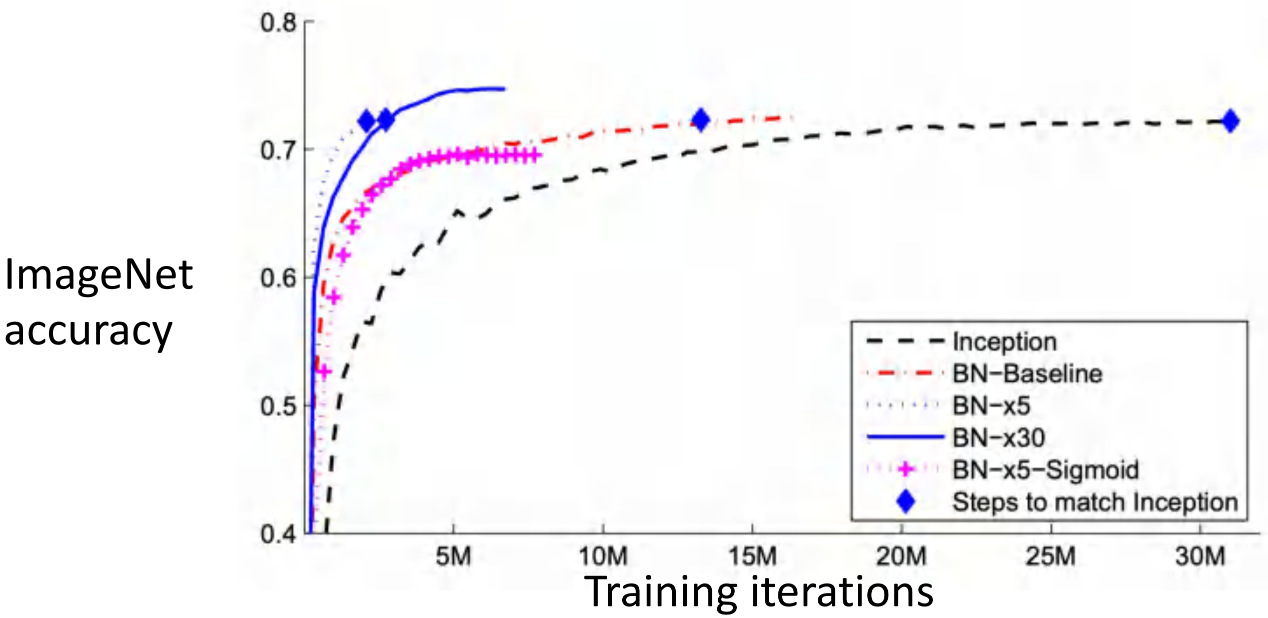
$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Batch Normalization

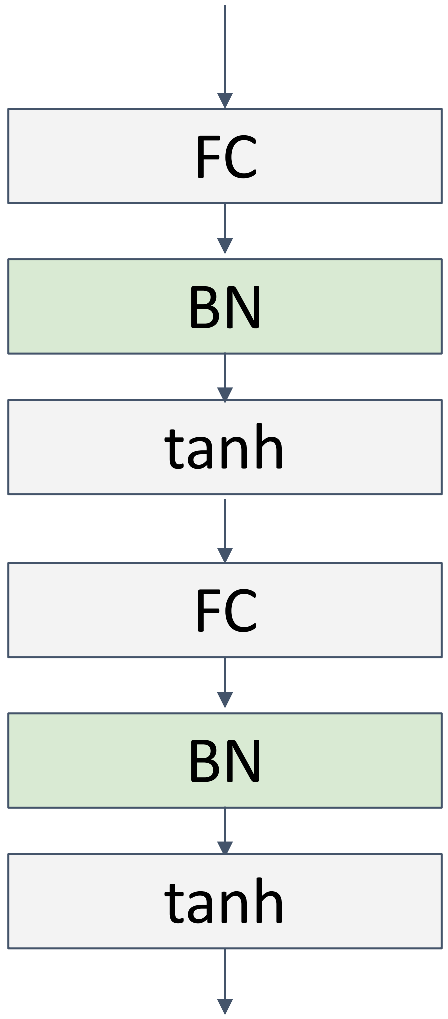


- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!



Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Batch Normalization



- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Not well-understood theoretically (yet)
- Behaves differently during training and testing: this is a very common source of bugs!

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Layer Normalization

Batch Normalization for **fully-connected** networks

$$x : N \times D$$

Normalize

$$\mu, \sigma \; : 1 \times D$$

$$\gamma, \beta \; : 1 \times D$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Layer Normalization for fully-connected networks

Same behavior at train and test!

Used in RNNs, Transformers

$$x : N \times D$$

Normalize

$$\mu, \sigma \; : N \times 1$$

$$\gamma, \beta \; : 1 \times D$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Instance Normalization

Batch Normalization for convolutional networks

$$x : N \times C \times H \times W$$

Normalize

$$\mu, \sigma \ : 1 \times C \times 1 \times 1$$

$$\gamma, \beta \ : 1 \times C \times 1 \times 1$$

$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Instance Normalization for convolutional networks

$$x : N \times C \times H \times W$$

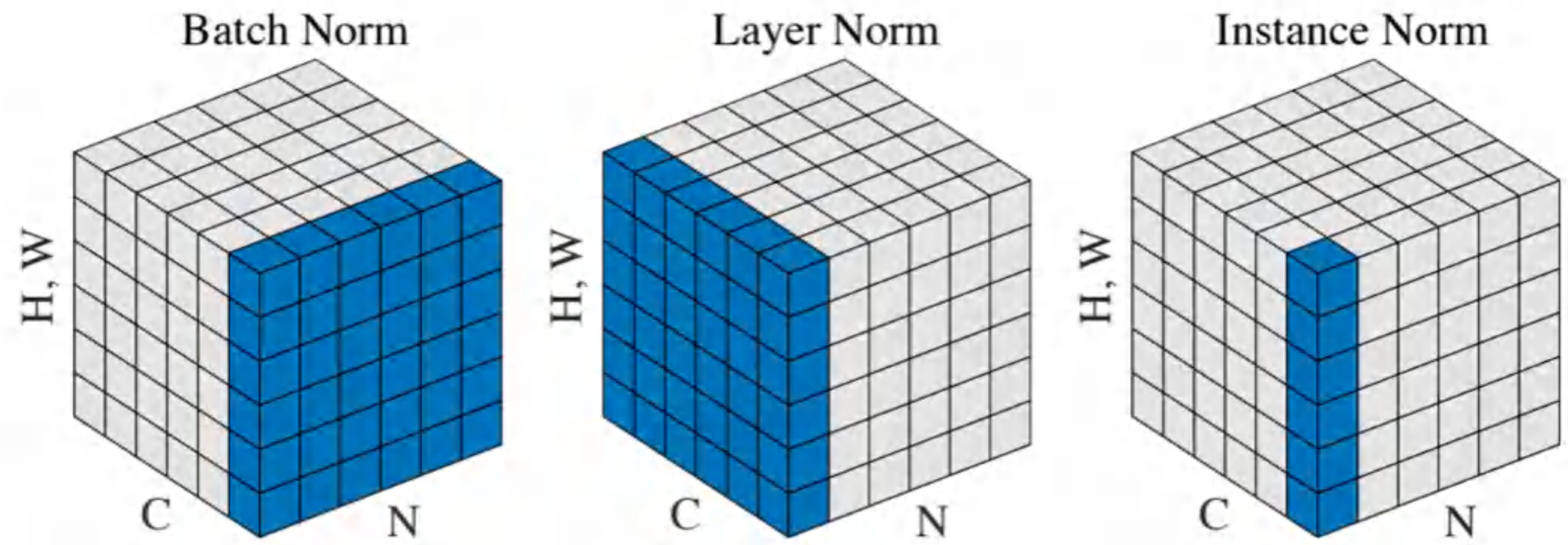
Normalize

$$\mu, \sigma \ : N \times C \times 1 \times 1$$

$$\gamma, \beta \ : 1 \times C \times 1 \times 1$$

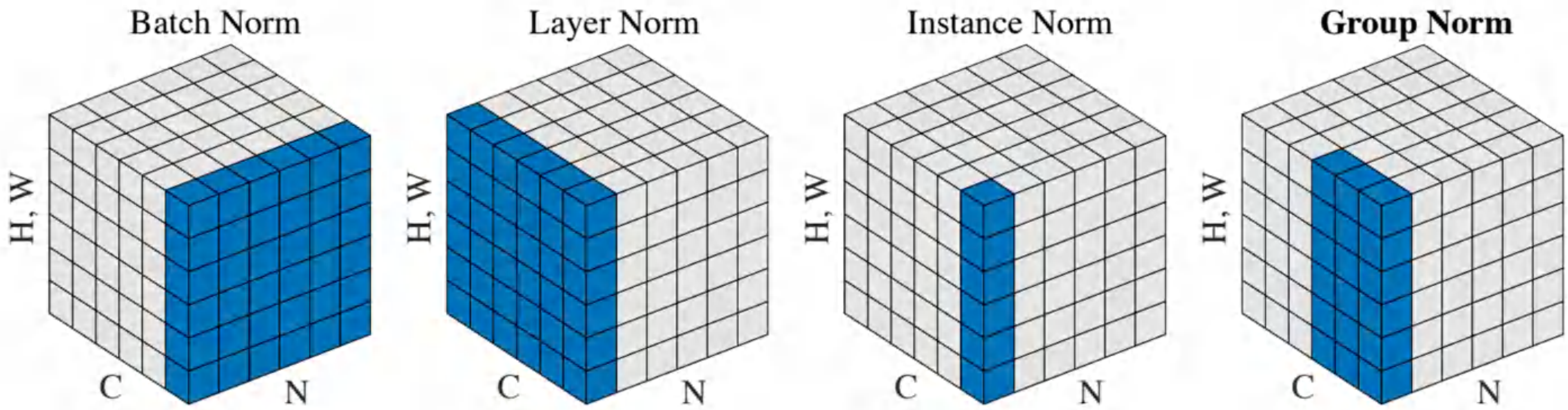
$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Comparison of Normalization Layers



Wu and He, "Group Normalization", ECCV 2018

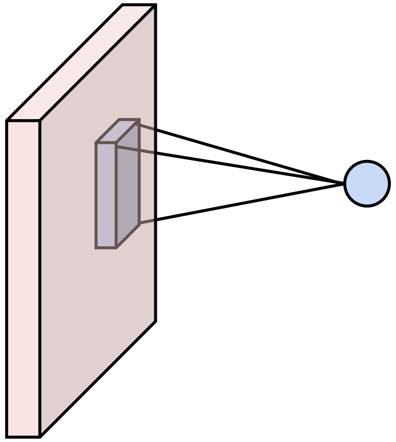
Group Normalization



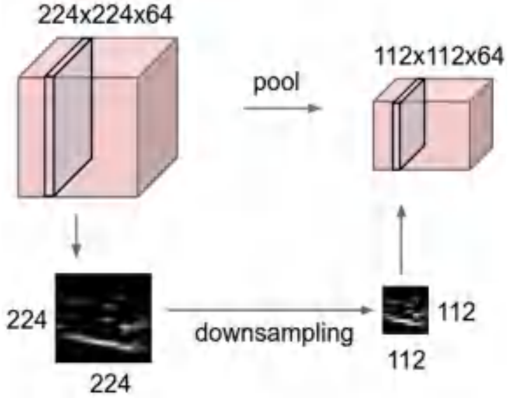
Wu and He, "Group Normalization", ECCV 2018

Components of a Convolutional Network

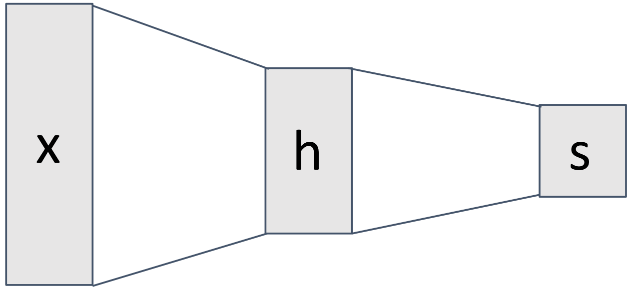
Convolution Layers



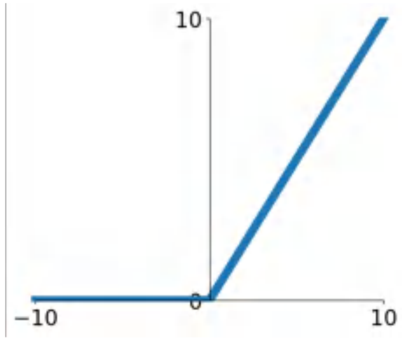
Pooling Layers



Fully-Connected Layers



Activation Function

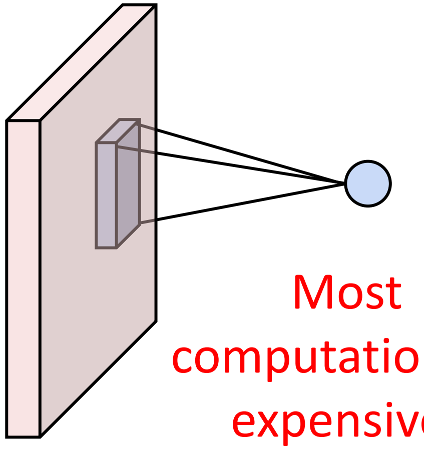


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

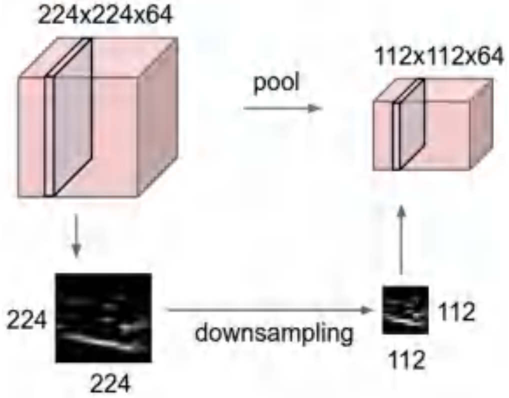
Components of a Convolutional Network

Convolution Layers

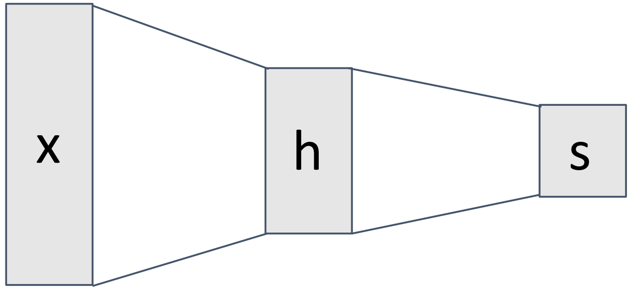


Most computationally expensive!

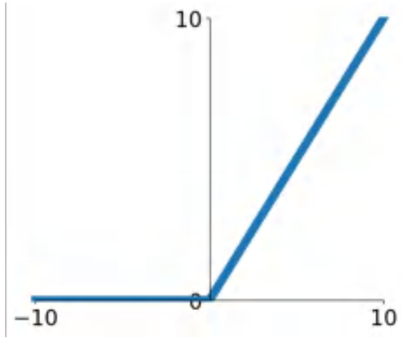
Pooling Layers



Fully-Connected Layers



Activation Function

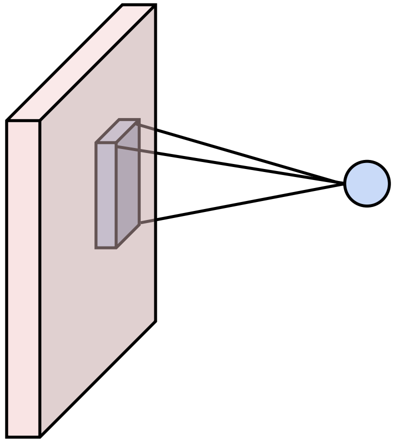


Normalization

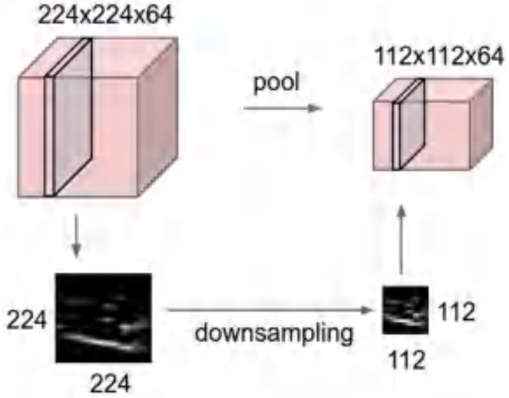
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Summary: Components of a Convolutional Network

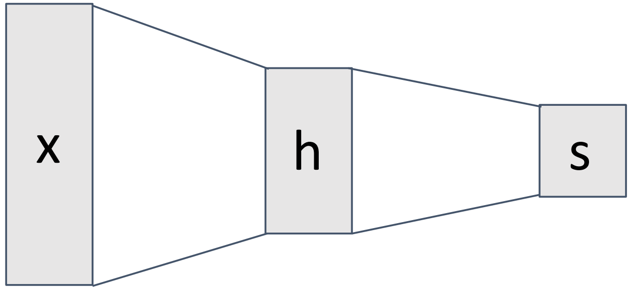
Convolution Layers



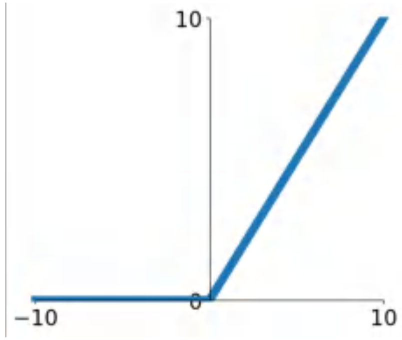
Pooling Layers



Fully-Connected Layers



Activation Function

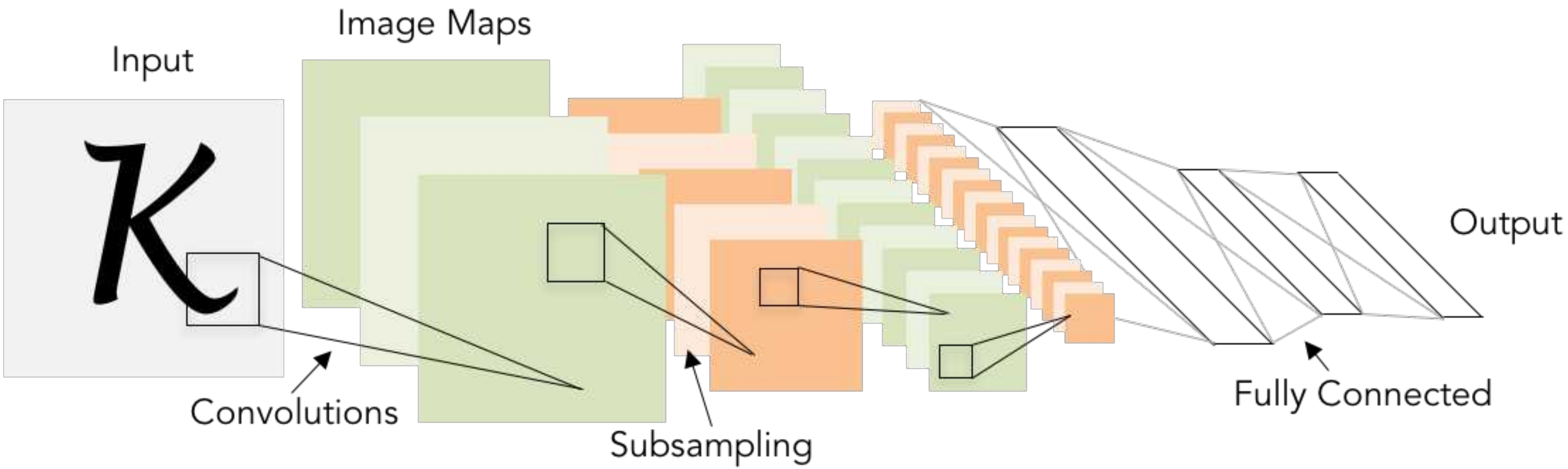


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Summary: Components of a Convolutional Network

Problem: What is the right way to combine all these components?



Next time:
CNN Architectures