Discrete Wavelet Transform (DWT)

Overview

- Introduction to Video/Image Compression
- DWT Concepts
- Compression algorithms using DWT
- DWT vs. DCT
- DWT Drawbacks
- Future image compression standard
- References

Need for Compression

- **Transmission and storage of uncompressed video would be extremely costly and impractical.**
  - **Frame with 352x288 contains 202,752 bytes of information**
  - **Recoding of uncompressed version of some video at 15 frames per second would require 3 MB. One minute→180 MB storage. One 24-hour day→262 GB**
  - **Using compression, 15 frames/second for 24 hour→1.4 GB, 187 days of video could be stored using the same disk space that uncompressed video would use in one day**

Principles of Compression

- Spatial Correlation
  - Redundancy among neighboring pixels
- Spectral Correlation
  - Redundancy among different color planes
- Temporal Correlation
  - Redundancy between adjacent frames in a sequence of image

Classification of Compression

- Lossless vs. Lossy Compression
  - Lossless
    - Digitally identical to the original image
    - Only achieve a modest amount of compression
  - Lossy
    - Discards components of the signal that are known to be redundant
    - Signal is therefore changed from input
    - Achieving much higher compression under normal viewing conditions no visible loss is perceived (visually lossless)
- Predictive vs. Transform coding

Classification of Compression

- **Predictive coding**
  - Information already received (in transmission) is used to predict future values
  - Difference between predicted and actual is stored
  - Easily implemented in spatial (image) domain
  - Example: Differential Pulse Code Modulation(DPCM)

Classification of Compression

- Transform Coding
  - Transform signal from spatial domain to other space using a well-known transform
  - Encode signal in new domain (by string coefficients)
  - Higher compression, in general than predictive, but requires more computation (apply quantization)
- Subband Coding
  - Split the frequency band of a signal in various subbands

Classification of Compression

- Subband Coding (cont.)
  - The filters used in subband coding are known as quadrature mirror filter(QMF)
  - Use octave tree decomposition of an image data into various frequency subbands
  - The output of each decimated subband quantized and encoded separately

Discrete Wavelet Transform

- **The wavelet transform (WT) has gained widespread acceptance in signal processing and image compression.**
- **Because of their inherent multi-resolution nature, wavelet-coding schemes are especially suitable for applications where *scalability* and *tolerable degradation* are important**
- **JPEG committee has released its new image coding standard, JPEG-2000, which has been based upon DWT.**

Discrete Wavelet Transform (Formal)

- **Wavelet transform decomposes a signal into a set of basis functions.**
- **These basis functions are called *wavelets***
- **Wavelets are obtained from a single prototype wavelet ψ(t) called mother *wavelet* by *dilations* and *shifting:***

  - $$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad (1)$$

where *a* is the scaling parameter and *b* is the shifting parameter

Discrete Wavelet Transform

- **Theory of WT**
  - **The wavelet transform is computed separately for different segments of the time-domain signal at different frequencies.**
  - **Multi-resolution analysis: analyzes the signal at different frequencies giving different resolutions**
  - **MRA is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies**
  - **Good for signal having high frequency components for short durations and low frequency components for long duration.e.g. images and video frames**

Discrete Wavelet Transform

- **Theory of WT (cont.)**
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Discrete Wavelet Transform

- **The 1-D wavelet transform is given by :**

$$W_f(a,b) = \int_{-\infty}^{\infty} x(t)\psi_{a,b}(t)\,dt$$

Discrete Wavelet Transform

- **The inverse 1-D wavelet transform is given by:**

$$x(t) = \frac{1}{C}\int_{0}^{\infty}\int_{-\infty}^{\infty} W_f(a,b)\psi_{a,b}(t)\,db\,\frac{da}{a^2}$$

$$\text{where } C = \int_{-\infty}^{\infty}\frac{|\psi(\omega)|^2}{\omega}d\omega < \infty$$

Discrete Wavelet Transform (Formal)

- Discrete wavelet transform (DWT), which transforms a discrete time signal to a discrete wavelet representation.
- it converts an input series $x_0, x_1, ..x_m$, into one high-pass wavelet coefficient series and one low-pass wavelet coefficient series (of length n/2 each) given by:

$$H_i = \sum_{m=0}^{k-1} x_{2i-m} \cdot s_m(z) \qquad (1)$$

$$L_i = \sum_{m=0}^{k-1} x_{2i-m} \cdot t_m(z) \qquad (2)$$

Discrete Wavelet Transform

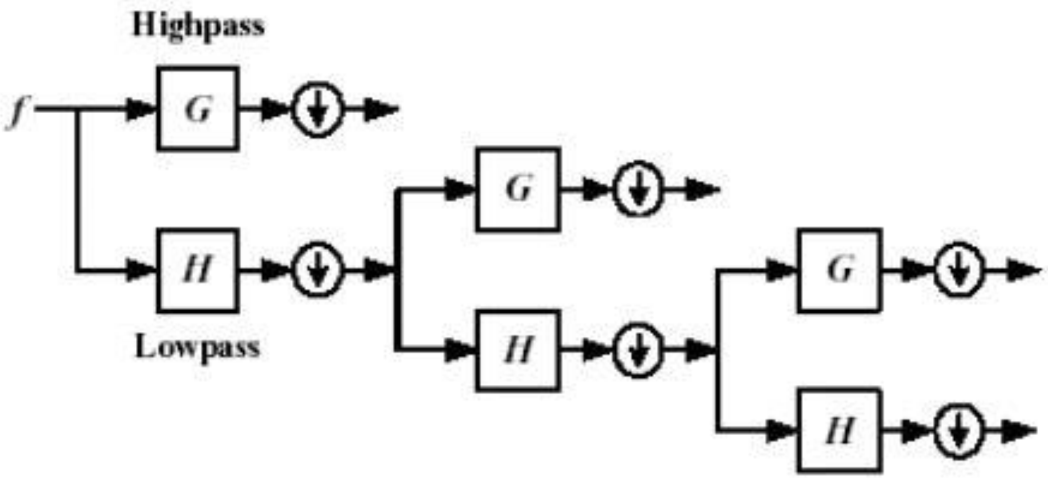
- where $s_m(Z)$ and $t_m(Z)$ are called *wavelet filters*, k is the length of the filter, and i=0, ..., [n/2]-1.
- In practice, such transformation will be applied recursively on the low-pass series until the desired number of iterations is reached.

Discrete Wavelet Transform

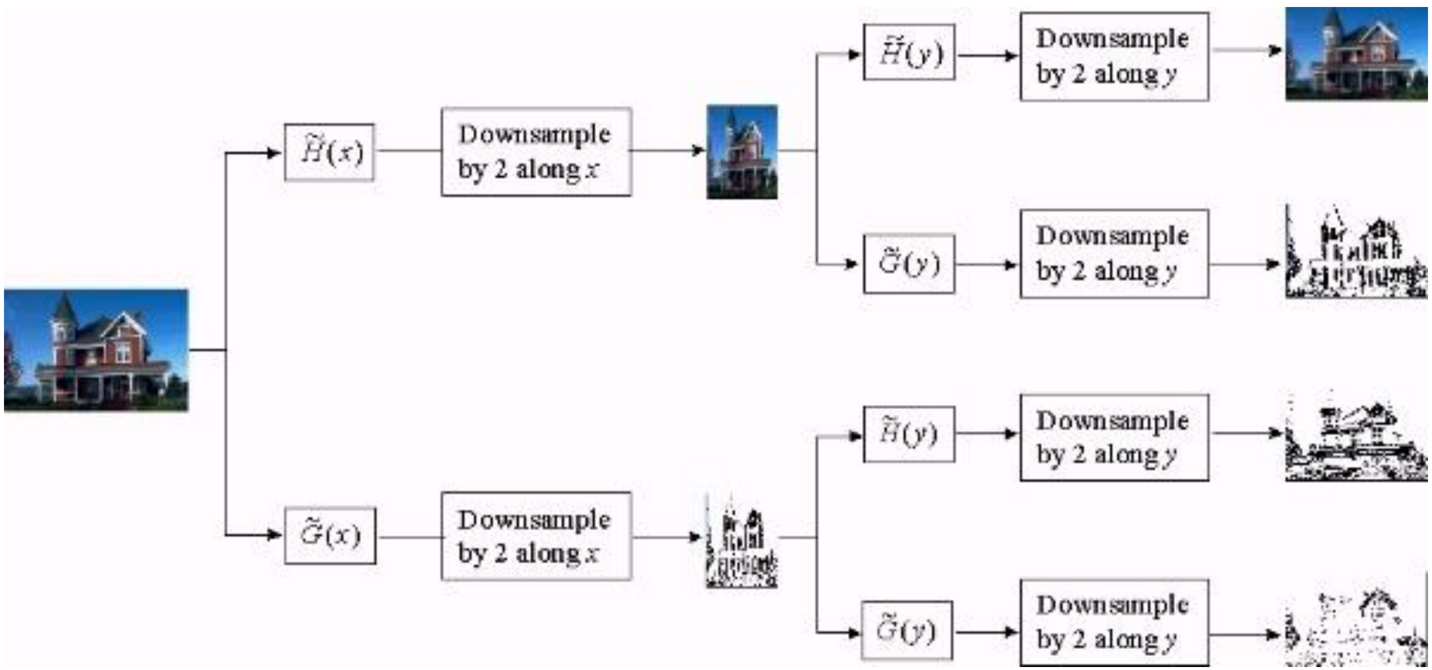
- **Lifting schema of DWT has been recognized as a faster approach**
  - **The basic principle is to factorize the polyphase matrix of a wavelet filter into a sequence of alternating upper and lower triangular matrices and a diagonal matrix .**
  - **This leads to the wavelet implementation by means of banded-matrix multiplications**

Discrete Wavelet Transform

- **2-D DWT for Image**



Discrete Wavelet Transform



Discrete Wavelet Transform

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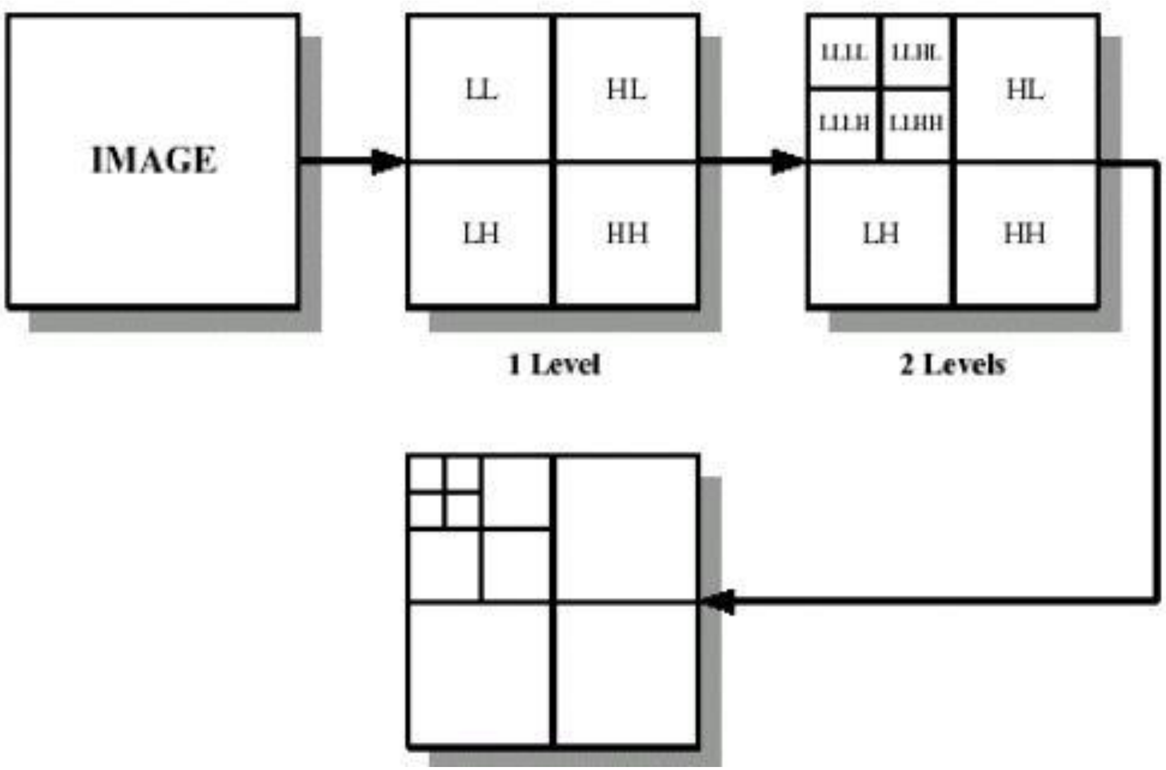

Discrete Wavelet Transform

- Compression algorithms using DWT
  - Embedded zero-tree (EZW)
    - Use DWT for the decomposition of an image at each level
    - Scans wavelet coefficients subband by subband in a zigzag manner
  - Set partitioning in hierarchical trees (SPHIT)
    - Highly refined version of EZW
    - Perform better at higher compression ratio for a wide variety of images than EZW

Discrete Wavelet Transform

- Compression algorithms using DWT (cont.)
  - Zero-tree entropy (ZTE)
    - Quantized wavelet coefficients into wavelet trees to reduce the number of bits required to represent those trees
    - Quantization is explicit instead of implicit, make it possible to adjust the quantization according to where the transform coefficient lies and what it represents in the frame
    - Coefficient scanning, tree growing, and coding are done in one pass
    - Coefficient scanning is a depth first traversal of each tree

Discrete Wavelet Transform

DWT vs. DCT

Discrete Wavelet Transform

- **Disadvantages of DCT**
  - **Only spatial correlation of the pixels inside the single 2-D block is considered and the correlation from the pixels of the neighboring blocks is neglected**
  - **Impossible to completely decorrelate the blocks at their boundaries using DCT**
  - **Undesirable blocking artifacts affect the reconstructed images or video frames. (high compression ratios or very low bit rates)**

Discrete Wavelet Transform

- **Disadvantages of DCT(cont.)**
  - **Scaling as add-on→additional effort**
  - **DCT function is fixed→can not be adapted to source data**
  - **Does not perform efficiently for binary images (fax or pictures of fingerprints) characterized by large periods of constant amplitude (low spatial frequencies), followed by brief periods of sharp transitions**

Discrete Wavelet Transform

- **Advantages of DWT over DCT**
  - **Higher flexibility: Wavelet function can be freely chosen**
    - **No need to divide the input coding into non-overlapping 2-D blocks, it has higher compression ratios avoid blocking artifacts.**
    - **Transformation of the whole image→ introduces inherent scaling**
    - **Better identification of which data is relevant to human perception→ higher compression ratio (64:1 vs. 500:1)**

Discrete Wavelet Transform

- **Performance**
  - **Peak Signal to Noise ratio used to be a measure of image quality**
  - **The PSNR between two images having 8 bits per pixel or sample in terms of decibel (dB) is given by:**
  - **PSNR = 10 $\log_{10}\left(\frac{255^2}{MSE}\right)$**
    - **mean square error (MSE)**
  - **Generally when PSNR is 40 dB or greater, then the original and the reconstructed images are virtually indistinguishable by human observers**

Discrete Wavelet Transform

- Visual Comparison

(a)

(b)

(c)

(a) Original Image256x256Pixels, 24-BitRGB (b) JPEG (DCT) Compressed with compression ratio 43:1(c) JPEG2000 (DWT) Compressed with compression ratio 43:1

Discrete Wavelet Transform

- **Implementation Complexity**
  - The complexity of calculating wavelet transform depends on the length of the wavelet filters, which is at least one multiplication per coefficient.
  - EZW, SPHIT use floating-point demands longer data length which increase the cost of computation
  - Lifting scheme→a new method compute DWT using integer arithmetic
  - DWT has been implemented in hardware such as ASIC and FPGA

Discrete Wavelet Transform

- **Disadvantages of DWT**
  - The cost of computing DWT as compared to DCT may be higher.
  - The use of larger DWT basis functions or wavelet filters produces blurring and ringing noise near edge regions in images or video frames
  - Longer compression time
  - Lower quality than JPEG at low compression rates

Discrete Wavelet Transform

- **Future video/image compression**
  - Improved low bit-rate compression performance
  - Improved lossless and lossy compression
  - Improved continuous-tone and bi-level compression
  - Be able to compress large images
  - Use single decompression architecture
  - Transmission in noisy environments
  - Robustness to bit-errors
  - Progressive transmission by pixel accuracy and resolution
  - Protective image security