Math Lab Assignment 03

Course Code: **AMTH 350**   Course Title: **Math Lab III**

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Consider the following nonlinear third order ODE

$$f''' + f f'' = 0, \tag{1}$$

with initial conditions $f(0) = f'(0) = 0$ and $f''(0) = 0.4696$. The independent variable is η. Eq. 1 is associated with a developing laminar boundary past a flat plate.

(a) Convert Eq. 1 and the boundary conditions to a system of three first-order ODEs using substitutions $X = f$, $Y = f'$ and $Z = f''$.

(b) Implement the system of ODEs formulated in (a) in MATLAB, and solve it with ode45. Reproduce the results shown in Fig. 1.
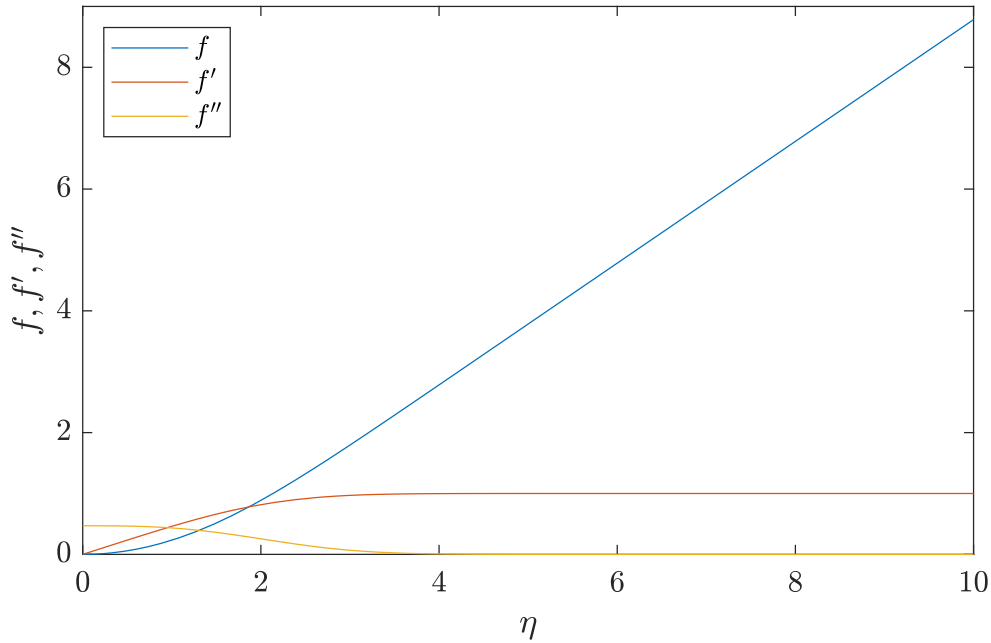

Figure 1: Numerical solution of equation 1.

(c) The self similarity variable η is related to x and z by

$$\eta = z \sqrt{\frac{U_\infty}{2\nu x}} \tag{2}$$

where, U_∞ and ν are the free-stream velocity and kinematic viscosity, respectively. Taking $U_\infty = 0.1 \quad m/s$ and $\nu = 10^{-6} \quad m^2/s$, create a plot of the velocity

$u(x, z) = U_\infty f'(\eta)$, shown below (Fig. 2). Create a grid with *meshgrid* and use 1-D interpolation function *interp1* to obtain the value of η at the grid nodes.
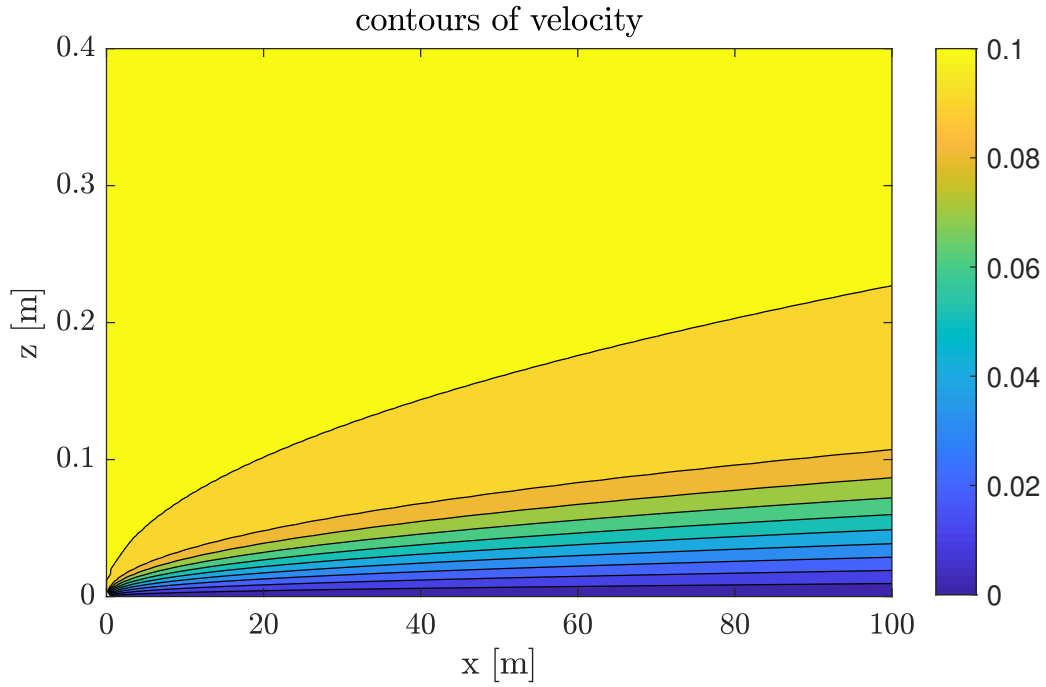

Figure 2: Contours of horizontal velocity.

(d) Change the previous value of U_∞ as $U_\infty = 0.2 \quad m/s$ and reproduce the figure below. Compare and contrast the height of the boundary layers of the figures 2 and 3.
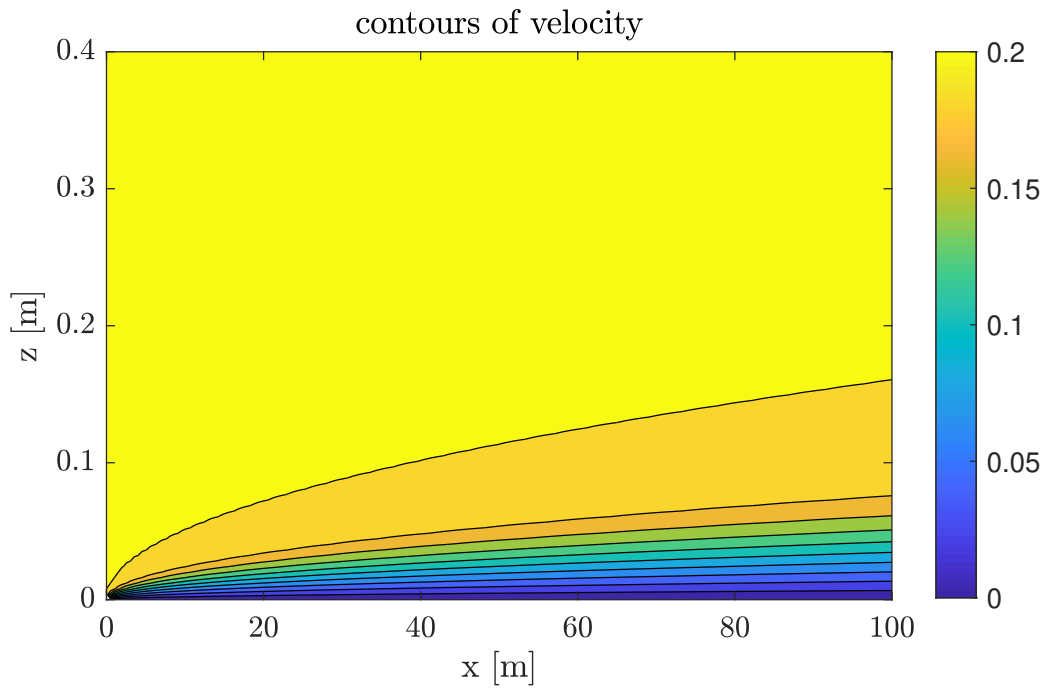

Figure 3: Contours of horizontal velocity. Here, $U_\infty = 0.2 \quad m/s$