

1. Let T be a binary tree, where each node is assigned an integer value. Design efficient algorithms to compute, for every node $v \in T$, the following quantities:
 - (a) The sum of the values of the nodes on the unique path from the root of T to v .
 - (b) The maximum value among the nodes on the unique path from the root of T to v .
 - (c) The sum of the values of the nodes in the subtree rooted at v .
 - (d) The maximum value among the nodes in the subtree rooted at v .
 - (e) Given an integer k , the element of rank k among the values of the nodes on the unique path from the root of T to v .
 - (f) Given an integer k , the element of rank k among the values of the nodes in the subtree rooted at v .
 - (g) The weight of the shortest path from v to any other node in T (where the weight of a path is the sum of node values).
 - (h) The weight of the shortest path from v to any leaf node in T .
 - (i) If the weight of a path is defined as the maximum node value on that path, compute the weight of the shortest path from v to any other node in T .
 - (j) If the weight of a path is defined as the maximum node value on that path, compute the weight of the shortest path from v to any leaf node in T .
2. Let T be a binary tree, where each node stores an integer value. After performing preprocessing in $O(n)$ time, design algorithms that can answer the following queries in $O(\log n)$ time:
 - (a) Given two nodes $u, v \in T$, determine their least common ancestor (LCA).
 - (b) Given two nodes $u, v \in T$, compute the weight of the shortest path between u and v (where path weight is the sum of node values).