

Assignment 16: Coding Karatsuba Due in class Thursday, 1/24

January 22, 2019 CS: DS&A PROOF SCHOOL

Problem 1. Write a function karatsuba(x,y) that takes two positive integers x and y with the same number of digits, and multiplies them together according to Karatsuba's multiplication algorithm.

- You may assume that string to integer conversions functions (i.e. str() and int()) are fast: use them as much as you want.
- You may assume that Python can multiply together single digits in constant time. This is the *only* kind of native multiplication you can use in your function, although you can use native addition, as required by the algorithm.
- When you're done, try timing your results and comparing with regular Python multiplication. What happens?

Problem 2. A review/primer of matrix multiplication. This is optional, and you don't have to write it up. But if you're not familiar with matrix multiplication, do it! We'll be looking at some matrix multiplication algorithms starting Thursday.

- a) If you need to, review the definition of matrix multiplication by reading the link provided in the Classroom announcement for this assignment.
- b) Find a 2×2 matrix I such that AI = I for every 2×2 matrix A. In other words, we want

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot I = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

c) What is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}?$$

d) Calculate the following product. Then, see if you can understand what's going on by breaking the matrices down into 2×2 submatrices and looking at part (c).

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$