January 7, 2019
CS: DS&A
PROOF SCHOOL

Assignment 12: *Big O, Ω, and Θ* *Due* **in class** *Thursday, 1/10*

Please write up your assignment on a sheet of paper and turn it in in class!

1. For each of the following functions $f(n)$ and $g(n)$, determine **with proof** whether f is $O(g)$ and whether f is $\Omega(g)$. (Your proofs can be quite short. You may assume all the O relationships among polynomials. For example, you may assume that $n^2 + 100n$ is $O(n^3)$, and that n^9 is not $O(n^8)$.)

   (a) $f(n) = \sqrt{n}$, $g(n) = \sqrt[3]{n}$ (You don't have to write this one up; see the next page for my writeup, so you can see an example of this kind of proof.)

   (b) $f(n) = 2^n$, $g(n) = 3^n$

   (c) $f(n) = 2^n$, $g(n) = 3^{n/2}$

   (d) $f(n) = \log^2 n$, $g(n) = \log n^2$

   (e) $f(n) = \log_{10} n$, $g(n) = \log n$ (Here "log" with no subscript means base 2. Remember/relearn your log rules!)

2. Let f and g functions with $f(n) > 0$ and $g(n) > 0$ for all $n \in \mathbb{N}$. Prove carefully that

   $$\max\{f(n), g(n)\} \quad \text{is} \quad \Theta(f(n) + g(n))$$

   (The left side is the function whose value on n is the bigger of $f(n)$ and $g(n)$.)

3. Read the notes from today's class that I posted on Classroom. Here's a new experiment for Block 3. We're going to have *mini-quizzes* every day, for the first 5 minutes of class. Get excited! And bring paper!

   One of the habits that helps immensely in college math and CS classes is looking over notes from the previous class. In this class I provide you with notes; all you have to do is read them. To help instill the habit, we'll start every class with a short one-question quiz on a definition, or computation, or idea from the previous class. *The intention is that if you looked over the previous class notes for 5-10 minutes the night before (or morning of) class, you'll probably find this quiz pretty easy.*

   We'll start next Thursday, 1/10.

Writeup of (1a), so you have some sense of this kind of proof:

Claim: $\sqrt{n}$ is $\Omega(\sqrt[3]{n})$.
Proof: This is the easy direction: We have $\sqrt[3]{n} \leq \sqrt{n}$ for all $n \in \mathbb{N}$ (i.e. take $c = 1$ in the definition of Ω).

Claim: $\sqrt{n}$ is not $O(\sqrt[3]{n})$.
Proof: We use proof by contradiction. Suppose that for some $c > 0$ we had

$$\sqrt{n} \leq c\sqrt[3]{n} \qquad \text{for large enough } n.$$

Raising both sides to the 6th power gives

$$n^3 \leq c^6 n^2 \qquad \text{for large enough } n,$$

which we know is impossible since we know that n^3 is not $O(n^2)$.