Assignment 18: Proof of the Master Theorem Due in class Thursday, 1/31

January 29, 2019
CS: DS&A
Proof School

In class we stated the so-called "master theorem", the generalization of the running times from recurrence relations we've been looking at. This can actually be generalized even further, but this is a form good enough for us:

Theorem *(Master Theorem for Divide-and-Conquer Running Times):* Given a running time T satisfying the recurrence relation

$$T(n) = aT(n/b) + \Theta(n^c)$$

(with $a, c > 0$ and $b > 1$), the actual order of T is given as follows:

1. $\Theta(n^{\log_b a})$ if $\log_b a > c$
2. $\Theta(n^c \log_b n)$ if $\log_b a = c$
3. $\Theta(n^c)$ if $\log_b a < c$

For this homework, prove this result. Prove it carefully. Turn in something you're proud of. If you get stuck, ask me about it. It doesn't require a lot of creativity; just rephrasing class calculations in a more general setting, so if you find yourself going out on a limb, question what you're doing!

Again: turn in something you're proud of. ☺

You may assume:

- that input n is a power of b: $n = b^k$.
- that $T(n)$ is in fact equal to $aT(n/b) + \alpha n^c$ for some constant α.

Have fun!