Aayush Arora
EE17BTECH11003

HW0

Q1

for undirected graphs :-

$$p(x_1 \ldots x_n) = \frac{1}{Z} \prod_{C \in \mathcal{C}} \psi_c(x_c)$$

ψ_c = Compatibility function : $x_1 \times x_2 \times \cdots x_{|c|} \rightarrow R_+$
Z = normalization factor
$\mathcal{C}$ = Set of maximal clique of G (graph)

For a Tree;

$$p(x_1 \ldots x_n) = \frac{1}{Z} \prod_{u \in V} \psi_u(x_u) \prod_{(s,t) \in E} \psi_{st}(x_s, x_t)$$

$$\mu_s(x_s) = \sum_{V/x_s} p(x_1 - x_n) \rightarrow \text{marginal where } s \text{ is root node}$$

To Show : Sum-Product Algorithm

$$\mu_s(x_s) = K \psi_s(x_s) \prod_{t \in N(s)} M_{ts}(x_s)$$

$$M_{ts}(x_s) = \sum_{x_{V_t}} \psi_{st}(x_s, x_t)\, p(x_{V_t}; T_t)$$

$$V = \{s\} \cup \{\bigcup_{t \in N(s)} V_t\} \quad \text{and} \quad E = \{\bigcup_{t \in N(s)} (s,t)\} \cup \{\bigcup_{t \in N(s)} E_t\}$$

$$p(x_1 - x_n) = \frac{1}{Z} \prod_{u \in V} \psi_u(x_u) \prod_{s,t \in E} \psi_{st}(x_s, x_t)$$

$$= \frac{1}{Z} \psi_s(x_s) \prod_{\substack{u \in \cup V_t \\ t \in N(s)}} \psi_u(x_u) \prod_{\substack{s,t \in \cup E_t \\ t \in N(s)}} \psi_{st}(x_s, x_t) \prod_{\substack{s,t \\ t \in N(s)}} \psi_{st}(x_s, x_t)$$

$$\mu_s(x_s) = \sum_{\sim x_s} p(x_1 \text{—} x_n)$$

$$= \sum_{V\setminus x_s} \frac{1}{Z} \psi_s(x_s) \prod_{\substack{u \in U_{Vt} \\ t\in N(s)}} \psi_u(x_u) \prod_{\substack{(s,t)\in \mathcal{V}_{Et} \\ t\in N(s)}} \psi_{st}(x_s, x_t) \prod_{\substack{s,t \\ t\in N(s)}} \psi_{s,t}(x_s, x_t)$$

But $\left(p(x_{V_t}; T_t) \propto \prod_{u\in V_t} \psi_u(x_u) \prod_{(v,w)\in E_t} \psi_{uw}(x_v, x_w) \right)$

$$\mu_s(x_s) = K\psi_s(x_s) \sum_{V\setminus x_s} \left(\prod_{t\in N(s)} p(x_{V_t}; T_t)\right)\left(\prod_{t\in N(s)} \psi_{st}(x_s, x_t)\right)$$

$$= K\psi_s(x_s) \sum_{x_1} \dots \sum_{x_{s-1}} \dots \sum_{x_{s+1}} \dots \sum_{x_n} \prod_{t\in N(s)} \psi_{st}(x_s, x_t)\, p(x_{V_t}; T_t)$$

$$= K\psi_s(x_s) \sum_{V_1} \sum_{V_2} \dots \sum_{\substack{V_{|N(s)|}}} \prod_{t\in N(s)} \psi_{st}(x_s, x_t)\, p(x_{V_t}; T_t)$$

Since they are disconnected parts of a tree each summation term are independent of each other

i.e V_i indep of V_j $\forall\, i,j$ $i \neq j$

~~independent~~

$$\mu_s(x_s) = K\psi_s(x_s) \prod_{t\in N(s)} \left(\sum_{x_{V_t}} \psi_{st}(x_s, x_t)\, p(x_{V_t}; T_t)\right)$$

$$\mu_s(x_s) = K\psi_s(x_s) \prod_{t\in N(s)} M_{ts}(x_s)$$

where $M_{ts}(x_s) = \sum_{x_{V_t}} \psi_{st}(x_s, x_t)\, p(x_{V_t}; T_t)$