L.P. & Graph Algorithms

Max flow as L.P

$G = (V, E)$, $c : E \to \mathbb{R}^+$

get extra edge $t \to s$ ∞ cap

$x(u,v) \to$ flow along u,v

Capacity constr

$x(u,v) \le \underline{c(u,v)}$ — b

Flow cons

$\forall\, v \in V$ — holds only

$$\sum_z x(v,z) - \sum_z x(z,v) = \underline{0}$$ — b

$x(u,v) \ge 0$

Maximize $\underline{x(t,s)}$

$\to y(u,v)$ are true dual var — $y(v)$ are true

$\to y(u,v) \ge 0$ to pres — $y(v)$ unconstraint

$b^T y \to$ CS and OS

min $\sum_{(u,v)} y(u,v) \cdot c(u,v)$

Dual const — each edge u,v in cap constr. once & flow const twice (as in edge & out edge)

$x(u,v) \xrightarrow{\text{gives}} y(u,v) + y(u) - y(v) \ge 0$ if $(u,v) \in E$

if $(t,s) \in E \to y(t) - y(s) \ge 1$ as $\boxed{c(t,s) = \infty}$

invariant to translation so can take $y(s) = 0$

$\underline{y(u)} + y(u,v) \ge \underline{y(v)}$ → if $y(v)$ is len $y(u,v)$ is dist

$y(t) \ge 1$

→ Solution looks like (assign lengths & assign consistent distances)

if y is 0 or 1 this is min cut ! → partition (ϕ dist zero & dist 1)

obj is now cut val!

Shortest path $s \longrightarrow t$

$$\min \sum x(u,v)\, \ell(u,v)$$

0 or 1 dep on whether we trade edge.

$$\forall v \rightarrow \sum_{s} x(s,v) - \sum_{z} x(v,z) = \begin{cases} 0 & \text{else} \\ -1 & v=s \\ 1 & v=t \end{cases}$$

$$x(u,v) \geq 0$$

even $x(u,v)$ ...

Dual

Max $y(t) - y(s)$

coeff

$$y(v) - y(u) \leq \ell(u,v)$$

dist from s

y(v) unconstr.