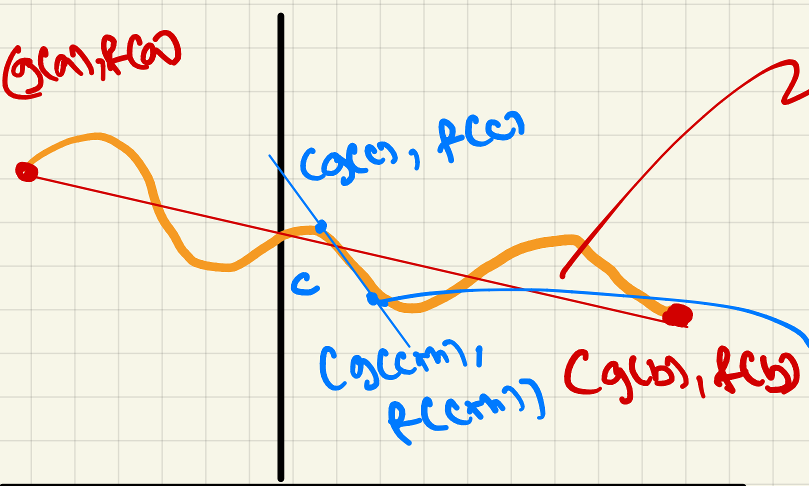
What is the picture for CMVT?

Recall $f, g : [a,b] \to \mathbb{R}$ is cts with $a<b$
$f, g : (a,b) \to \mathbb{R}$ is diff'ble
then $\exists\, c \in (a,b)$ s.t.

$$f'(c)\,(g(b)-g(a)) = g'(c)\,(f(b)-f(a))$$

Consider a parametric path ...

$$[a,b] \to \mathbb{R}^2$$
$$t \longmapsto (g(t), f(t))$$



Slope of this line is

$$\boxed{\frac{f(b)-f(a)}{g(b)-g(a)}}$$

Slope of this is

$$\boxed{\frac{f(c+h)-f(c)}{g(c+h)-g(c)}}$$

rearranging CMVT we have

$$\boxed{\frac{f'(c)}{g'(c)}} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

$$\frac{f'(c)}{g'(c)} = \lim_{h\to 0} \frac{\dfrac{f(c+h)-f(c)}{h}}{\dfrac{g(c+h)-g(c)}{h}}$$

$$= \lim_{h\to 0} \frac{f(c+h)-f(c)}{g(c+h)-g(c)}$$

So $\boxed{M_{Tan} = \dfrac{f'(c)}{g'(c)}}$

CMVT extends MVT to parametric case. If we take $g = \mathrm{Id}_{\mathbb{R}}$

$t \mapsto (t, f(t)) \to$ graph

What is the pic of L'Hopital?

Again consider a param fn

$[a,b] \to \mathbb{R}^2$

$t \mapsto (g(t), f(t))$

we know that,

$(g(a), f(a)) = (0,0)$ as f, g are diffble $\Rightarrow$ cts at a

L'Hopital's Rule

Spse $A \subseteq \mathbb{R}$ is an open intvl & $f,g : A \to \mathbb{R}$ are 2 functions

① Spse f,g are diffble at $a \in A$

② $\lim_{x\to a} f(x) = \lim_{x \to a} g(x) = 0$

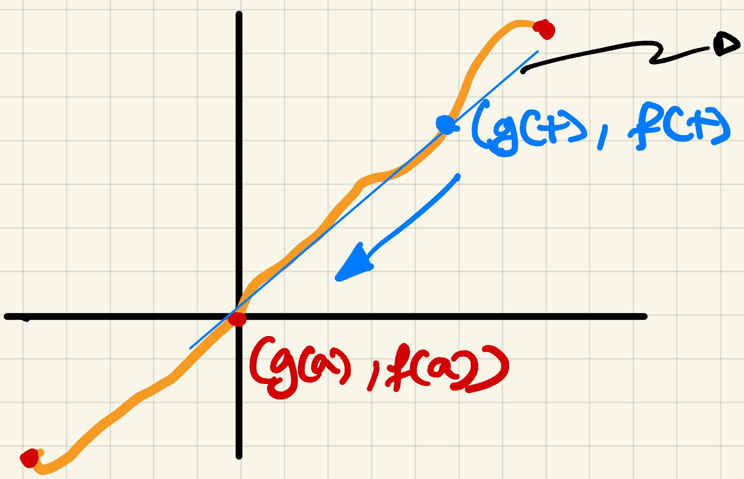
③ $\lim_{x\to a} \frac{f'(a)}{g'(x)} = \ell$

Then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} = \ell$

2 additional Assumptions are made!

★ $\exists\, \mu' > 0$ s.t $f'(x)$ & $g'(x)$ exist on $B_{\mu'}(a)$

★★ $\exists\, \mu'' > 0$ s.t $g'(a)$ non zero on $B_{\mu''}(a) \setminus \{a\}$



Slope is

$$\boxed{\frac{g(t)}{f(t)}}$$

what happens to param slope as $t \to a$?

note: $\lim_{t\to a} \frac{f(t)}{g(t)} \to$ slope of line from $(0,0)$ to $(g(t), f(t))$

$$\square = \lim_{h\to 0} \frac{f(a+h) - \cancel{f(a)}^{\to 0}}{g(a+h) - \cancel{g(a)}^{\to 0}}$$

$$= \lim_{h \to 0} \frac{\dfrac{f(a+h) - f(a)}{h}}{\dfrac{g(a+h) - g(a)}{h}} = \frac{f'(a)}{g'(a)}$$