Midterm

→ Take home:
→ Open everything → $\binom{6}{5}$

Basic Grp Thry

- Grps, subgrps
- Homomorphisms, isomorphism, automorphism
- Kernel, image → good guy
- cosets → Lagrange Thm
  - ↳ number of cosets → index
    - ↳ coset representative
- normal subgrp → left & right cosets same
  - ↳ quotient grp → closed under conj
  - → first isomorphism Thm.
- Products (internal vs external)
- Egs → S_n, $\mathbb{Z}/n\mathbb{Z}$, D_n, GL_n, SL_n, O_n, SO_n
- conjugation (lowkey goated) → defines an automorphism

$$\gamma_g : G \to G$$
$$h \mapsto ghg^{-1}$$

↳ this is called an inner auto
↳ eg of non-inner aut
$(\mathbb{R},+)$ thm $\mathbb{R} \to \mathbb{R}$
$x \mapsto 2x$

all inner aut of abelian trivial

trivial S_n
has a non-trivial outer automorphism iff $n=6$
↳ something called $Out(G) = Aut(G)/_{inner\ Aut(G)}$

$Out(S_6) \cong \mathbb{Z}/2\mathbb{Z}$ $\quad (12) \mapsto (12)(34)(56)$

$$1 \to Z(G) \to G \xrightarrow{\gamma} Aut(G)$$

this is kernel

im of γ normal in $Aut(G)$
$Out(G)$ = quotient!

→ Another eg $GL_n(\mathbb{Q}) \to GL_n(\mathbb{Q})$
$g \mapsto (g^T)^{-1}$

Fields

$\boxed{n+1}$

- Def of it
- e.g. $\mathbb{R}$, $\mathbb{C}$, $\mathbb{R}[\sqrt{2}]$, $\mathbb{F}_p$, k a field get
  ↳ rational fn with coeff in k $k(t)$
- K is a field → K grp under + & $K \setminus \{0\}$ grp under mult K^*

Linear Alg

- Elem Lin Algebra..
- Abstract Vector Space → Subspace → quotient sp
  ↳ subtract the dimen
  $\dim V/W = \dim V - \dim W$
  ↳ produce basis
- Span lin ind, basis, dim
- if V & W subsp
  Then $V \supseteq W$ ~~as~~ $\dim V = W$
  consider map $T: V \to V/W$ quotient map
  $\dim V = \dim V/W + \dim W$ (rank nullity)
- direct sum of vect sp (internal vs external)
- lin trans / operator
- kernel, image, rank-nullity (coker is $W/\mathrm{im}(T)$)
- get matrix of lin trans w.r.t. basis & change of basis
- eigen val & char poly.
- diagonalization $\xleftrightarrow{\text{nec \& suff}}$ eigen basis

e.g. $\mathbb{F}^n$, $\mathbb{F}[x]$, ... $\mathbb{C}$ is an $\mathbb{R}$ vect sp $\mathbb{R}^2$
$\mathbb{R}$ is a $\mathbb{Q}$ vec sp (inf dim)

Rigid Motions

- M = gp of rigid motions
- special elts t_a, ρ_θ, r
- get normal form; $m = t_a \rho_\theta r \quad \forall m \in M$.

Geom Classification

- trans
- rotation } or pres
- glide
- refl } or reversing

$\exists$ homo $M \to \{\pm 1\}$ orientation indicator

- subgrp $T = t_a$ is normal

$M/T \cong O(2)$

O_2 = stab of the origin in M

- finite subgps of $M = \mathbb{Z}/n\mathbb{Z}$ or D_n

↳ key idea every finite subgrp has a fixed pt

↳ find by averaging orbit at any p.t.

- Discrete subgrps

↳ $L_G = \{a \in \mathbb{R}^2 \mid t_a \in G\} \to$ discrt subgp of $\mathbb{R}^2$

$\cong 0, \mathbb{Z}/n\mathbb{Z}, \mathbb{Z}^2$

↳ lattice!

$\overline{G}$ = img of G in O_2

$O_2 \cong M/T \to$ pt grp

$\overline{G}$ pres L_G.

- Crystallographic. If L_G lattice

$\Rightarrow \overline{G}$ is D_n or $\mathbb{Z}/n\mathbb{Z}$ for $n \in \{1,2,3,4,6\}$

Grp Actions

- Def of grp action
- action as homo to the sym grp.
- orbit stab
- transitive & faithful action.
- Classification of transitive G set $\cong G/H$
- counting orbit stab thm.

$$\# O_x \cdot \# G_x = \# G$$

- E.g. $G \curvearrowright G$ left mult, right by inv, conj