Lec 1

→ bin op

Def) A composition law on a set S is a fnc
$f: S \times S \to S$

e.g. addition, multiplication, ... $(S = \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C})$
multiplication of matrices $(S = GL_n(\mathbb{R}), n \in \mathbb{N})$
comp of func (same domain)

Notation: multiplicative notation → ab or $a \cdot b$
long form " → $m(a,b)$ → $m: S^2 \to S$
additive " → $a+b$

Def) A composition law on S is assoc if $\forall x,y,z \in S$
- $m(m(x,y),z) = m(x, m(y,z))$
- $(x \cdot y) z = x (y \cdot z)$

nonegs) $S = \mathbb{R}$ $\quad m: S^2 \to S \quad m(x,y) = \frac{x+y}{2}$ non assoc

Def) let m CL on S. An identity elt is $e \in S$ so
$m(e,x) = x = m(x,e) \quad \forall x \in S$ → notation 1 or 0

Def) lets assume m CL on S has identity e & is assoc.
Given $a \in S$ an inverse of a is an elt b so that
$ab = ba = e$ $\quad$ usually b is noted as a^{-1} or $-a$

Lemma) If the inverse of a exists its unique. Let b, b' be inv.

$$b(ab') = be = b$$
$$\| \qquad\qquad \|$$
$$(ba)b' = eb' = b'$$
by assoc |

Defn) A grp is a set G equipped with a comp law that:
- is assoc
- has an identity
- each elt has inv!

Def) $GL_n(F) = \{ n \times n$ matrices with entries in F (ring) $\}$

Note $GL_n(\mathbb{Z})$ is integer matrices with integer matrix inv
⟸ det of $n = \pm 1$

e.g. $GL_1(\mathbb{Z}) = \{\pm 1\}$ $GL_2(\mathbb{Z})$ is infinite & shear matrices like $\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ $x \in \mathbb{Z}$ inv $\begin{pmatrix} 1 & -x \\ 0 & 1 \end{pmatrix}$

- $X = \{1, \dots, n\}$ let $G = \{ f : X \to X \mid f \text{ inv} \}$ $\circ$ is comp
  $\hookrightarrow$ permutations

Symmetric grp on n letters S_n notation

Def let $(G, \cdot)$ be a grp. A subgroup is $H \subseteq G$
1) H is closed under $\cdot$
2) $e \in H$ $\longrightarrow$ equiv to $H \neq \emptyset$
3) $a \in H \Rightarrow a^{-1} \in H$

$\hookrightarrow$ notation $H \leq G$

e.g. • $G = (\mathbb{Z}, +)$ let $H = n\mathbb{Z}$

- $GL_n(\mathbb{Q}) \leq GL_n(\mathbb{R})$
- $\overline{\mathbb{Q}} \leq \mathbb{C}$