Char Table of S_5

1. Conj Classes

$5 = 5$ — c_7
$4+1$
$3+2$
$3+1+1$
$2+2+1$
$2+1+1+1$ — c_2
$1+1+1+1+1$ — c_1

	#1	#10	#15	#20	#20	#30	#24
	(1)	(12)	(12)(34)	(123)	(123)(45)	(1234)	(12345)
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
χ_1 ≅ triv	1	1	1	1	1	1	1
χ_2 ≅ sgn	1	−1	1	1	−1	−1	1
χ_3 ≅ std	4	2	0	1	−1	0	−1
χ_4 ≅ Hom std sgn	4	−2	0	1	1	0	−1
χ_5	5	1	1	−1	1	−1	0
χ_6 ≅ Hom χ_5 sgn	5	−1	1	−1	−1	1	0
χ_7	6 (?)	0	−2	0	0	0	1

$\underline{\chi_{std}}$ | $P = \mathbb{C}^5$ is a rep of S_5 w/ S_5 perm basis vect

$\mathbb{C}^5 = \overset{1\text{-d}}{\text{triv}} \oplus \overset{4\text{-d}}{\text{std}}$

$\Rightarrow \chi_P = \chi_{triv} + \chi_{std} \Rightarrow \chi_{std} = \chi_P - \chi_{triv}$

trace $\leftarrow \Rightarrow \chi_P(g) \rightarrow$ fixed vect

$\chi_4 = \chi_{Hom(std, sgn)}$

we note : sum of sq is dim

guess χ_5, χ_6 same dim (possibly a Hom pair)

$\Rightarrow \underbrace{d_5^2 + d_6^2} + d_7^2 = 86$

$\Rightarrow 2d_5^2 + d_7^2 = 86$ $(d_1, d_2) = (5, 6)$ ~~was~~

only soln

χ_5 | X = collection of partition of set $\{1, \dots, 5\}$ into subset of size $2, 3$ = subsets of size 2

$\# X = {}^5C_2 = 10$

$S_5 \curvearrowright X$ get a 10 dim permutation rep $\mathbb{C}[X]$.

write $[i,j]$ for basis $\mathbb{C}[X]$ for set $\{i,j\}$

$\exists$ map of S_5 rep $f : \mathbb{C}[X] \to P$

$[i,j] \longmapsto e_i + e_j$

1) $\exists$ map of V sp. as we have shown where basis vect go

2) G-equivariant by checking ! $f(g[i,j]) = g f([i,j])$

$f(g[i,j]) = f(g i, g j)$ $\quad$ $g f([i,j]) = g(e_i + e_j)$

$= e_{gi} + e_{gj}$ (defn f) $\quad$ $= e_{gi} + e_{gj}$ (defn of action)

3) Surj as the img is subrep $\to$ so either 0, P, std rep, triv

surjective ! $\quad$ clearly not any

$\Rightarrow$ triv & std have multiplicity ≥ 1 in $\mathbb{C}[X]$!

Hopefully, $\mathbb{C}[X] = \text{triv} \oplus \text{std} \oplus \dim 5$ rep

can check by schur

1st Char for $\mathbb{C}[X]$!

	C_1 (1)	C_2 (12)	C_3 (12)(34)	C_4 (123)	C_5 (123)(45)	C_6 (1234)	C_7 (12345)
$\chi_{\mathbb{C}[X]}$	10	4	2	1	1	0	0

$\Rightarrow$ fix pts

any ab, 345 fixed $\alpha\{12\}$, $\alpha\{3,4\}$ $2\{4,5\}$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
$\chi_{\mathbb{C}[X]} - \chi_P = \varphi$	5	1	1	-1	1	-1	0

$$\langle \chi_{\mathbb{C}[X]} - \chi_P, \chi_{\mathbb{C}[X]} - \chi_P\rangle = \langle \varphi, \varphi\rangle = \frac{1}{120}(25 + 10 + 15 + 20 + 20 + 30)$$

$$= \frac{1}{120}(120) = 1$$

$\Rightarrow$ by Schur this is irred!

$\Rightarrow$ $\boxed{\text{ker } f \text{ rep!}}$

$f: \mathbb{C}[X] \to P$

since f is surj $\mathbb{C}[X] \cong \ker f \oplus P$

$\Rightarrow \chi_{\mathbb{C}[X]} - \chi_P = \boxed{\chi_{\ker(f)}} \to$ irred by Schur

χ_6 hom of $\mathbb{C}[X]$ & std!

χ_7 | know it is self hom img (otherwise have another irred)

$\Rightarrow$ (sgn)

blw

$$\chi_{reg} = \chi_1 + \chi_2 + 4\chi_3 + 4\chi_4 + 5\chi_5 + 5\chi_6 + 6\chi_7$$

χ_{reg}	120	0	0	0	0	0	0

on C_3 $\quad 0 = 1 + 1 + 4 \cdot 0 + 4 \cdot 0 + 5 \cdot 1 + 5 \cdot 1 + 6 \cdot \chi_7(C_3)$

$$\Rightarrow \chi_7(C_3) = -2$$

on C_4 $\quad 0 = 1 + 1 + 4 + 4 - 5 - 5 + 6\chi_7(C_4)$

$$\Rightarrow \chi_7(C_4) = 0$$

on C_7 $\quad 0 = 1 + 1 - 4 - 4 + 0 + 0 + 6\chi_7(C_7)$

$$\Rightarrow \chi_7(C_7) = 1$$