


Recall

- Ind RV
- Expectation \rightarrow saw examples of expectation of no of heads in 1000 coin tosses

Properties

- $E(X+Y) = E(X) + E(Y)$
- $E(cX) = c E(X)$
- $E(X) \geq 0$ if $X \geq 0$
- If X takes value 1 with prob 1. $E(X) = 1$

Pr immediate from def

Ex. 1 Revisit 1000 coin toss example.

$X = \#H$ in 1000 toss

$X_i = \#H$ in the i th toss $X_i = \{0, 1\}$

$$\therefore X = \sum_{i=1}^{1000} X_i$$

$$E(X) = E\left(\sum_{i=1}^{1000} X_i\right) = \sum_{i=1}^{1000} E(X_i) = \sum_{i=1}^{1000} \frac{1}{2} = 500$$

let $i \in I$

$$E(X_i) = 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{2}\right) = \frac{1}{2}$$

Note: A rv that takes only 2 values is called Bernoulli

Ex. 1 n people coming to a party w/ their hats.

After the party, they were too drunk so they pick their hat randomly.

Find the expected no of people getting their hat correctly

$X = \# \text{ ppl getting their hat correctly}$

$$x_i = \begin{cases} 1 & \text{if person } i \text{ gets their hat} \\ 0 & \text{otherwise} \end{cases}$$

$$X = \sum_{i=1}^n x_i \Rightarrow E(X) = \sum_{i=1}^n E(x_i)$$

$$P(x_i = 1) = \frac{1}{n}$$

$$E(x_i) = 1$$

e.g. | tossing a fair coin n times.

Find expected no of string of H

$X = \# \text{ of consec heads}$

$x_i = \text{Toss } (i-1 \rightarrow T, i \rightarrow H)$

$$P(x_i = 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$E(X) = \frac{1}{2} + \sum_{i=2}^n \frac{1}{4} = \frac{1}{2} + \frac{n-1}{4} = \frac{2+n-1}{4} = \frac{n+1}{4}$$

e.g. | pay \$100k to play a game.

Game: | a fair coin is tossed repeatedly.

For every T your reward is doubled.

First tail rewards \$2. Game ends when you hit a H.

$X \rightarrow \text{money earned from game.}$

$$E(X) = \frac{1}{2}(2) + \frac{1}{4}(4) + \dots = \sum_{n=1}^{\infty} 1 = \infty$$

Lemma X r.v. g function.

$$E(g(X)) = \sum_{x: P_X(x) > 0} g(x) \underbrace{P_X(x)}_{\text{prob func}}$$

→ Whenever APs converges

Lemma if X, Y independent.

$$E(XY) = E(X) E(Y)$$

(proof, next time)

We want to numerically measure the dep of RV

↳ covariance

↳ correlation