# Recall

Weak LLN: $X_1 \dots X_n$ ind & identical. $E(X_i) = \mu < +\infty$

$S_n = \sum X_i$

$\frac{S_n}{n} \xrightarrow{P} \mu \Rightarrow \forall \varepsilon > 0 \;\; P\left(\left|\frac{S_n}{n} - \mu\right| > \varepsilon\right) \to 0$

$S_n > (\varepsilon + \mu) n$

$S_n < (\mu - \varepsilon) n$

How quickly is this conv?

Large deviation says this conv$^n$ is exponential

More precisely, suppose

$M(t) = E(e^{tX_i})$ is def in nbhd of 0

$\Rightarrow \frac{1}{n} \log P(S_n \geq (\mu+\varepsilon)n) \to -\lambda_\varepsilon < 0$

where $\lambda_\varepsilon = \sup\left((\mu+\varepsilon)t - \log(M(t))\right) > 0$ $t \in \mathbb{R}$

When $n$ large,

$P(S_n > (\mu+\varepsilon)n) \simeq e^{-\lambda n}$

Proof of the upper bd |

$P(S_n \geq (\mu+\varepsilon)n) = P(e^{tS_n} \geq e^{t(\mu+\varepsilon)n})$ → as

$\leq M_{\underset{\sum_{i=1}^n X_i}{S_n}}(t)\, e^{-t(\mu+\varepsilon)n}$

$= e^{-t(\mu+\varepsilon)n} M(t)^n$ → This is $X_i$'s mmnt

$= e^{-t(\mu+\varepsilon)n} e^{n \log M(t)}$

max the neg'd stuff ← minimize upper bd

$= e^{-((\mu+\varepsilon)t - \log M(t))n}$ → $\boxed{\lambda_\varepsilon}$

$= e^{-\lambda_\varepsilon n}$

So, we get result. But still need $\lambda_\varepsilon > 0$

↳ resolved by graph

~~Step~~ 1 $X_1 \dots X_n$ ind, i.d., $E(X_i) = 0$ & $Var(X_i) = \sigma^2 = 1$

$S_n = \sum_{i=1}^{n} S_n$

CLT $\to$ $\frac{S_n}{\sqrt{n}} \xrightarrow{D} N(0,1)$

$\to$ says $S_n$ Distributed around 0 of order $\sqrt{n}$

Q1 Can we get abs bound on $S_n$?

Law of iterated logarithm

$$P\left( \limsup_{n \to \infty} \frac{S_n}{\sqrt{2n \log\log(n)}} = 1 \right) = 1$$

$\boxed{\begin{array}{l} (x_n) \text{ seq} \\ \limsup x_n \\ = \lim_{n \to \infty} s_n \\ s_n = \sup\{x_m \mid m \geq n\} \end{array}}$