Probability & Sample Space

Experiment

- flipping a coin, toss a dice
- covid testing
- weather
- companies stock behavior
- height of student
- viral load of coronavir in patient's body, etc

Outcome

- head / tail . 1 – 6
- positive, negative
- sunny, rainy ....
- ↑ n% or ↓ n% or –
- $[0, 9]$ (ft) .
- $[0, 10^6]$ (μg)

Def Sample sp:

The set of all possible outcomes of an experiment.

Denoted by Ω

e.g. $\Omega = \{h, t\}$, $\Omega = [1, 6] \cap \mathbb{N}$ ......

Def Event:

An event is a subset of Ω

e.g. for flipping a coin & getting head $= \{h\}$

$\emptyset$ corr to no flip, no result → impossible if $E = \emptyset$

$\{h, t\}$ is head or tail → certain as $\{h, t\} = \Omega$

e.g. Viral load → There's a threshold for the virus to be threatening

Event → viral load of virus is at most 0.01 μg

note $\Omega = [0, 10^6]$ (4p)

$A = [0, 0.01] \subseteq \Omega$

BUT: not every subset of Ω is an event.

(especially when Ω is uncountable)

Def) 2.0 each event is a subset of Ω that belongs to a special collection of subsets of Ω

Def) σ - field

Let F be a coll of subset of Ω.

F is a σ - field if:

(a) $\emptyset \in F$,

(b) If $A_1, A_2 \dots \in F$, $\bigcup_{i \in \mathbb{N}} A_i \in F$ (countable union)

(c) If $A \in F$, $\Omega / A \in F$

Q: Why countable union?

↳ e.g. → we mark today as day 0

A_1 = event # covid cases drop on day 1

A_2 = event # covid " " " " 2

⋮

A_n = " " " " " " " n

Event: no of cases ever drops. $= \bigcup_{i=1}^{\infty} A_n$

motivates countable unions

Usually countable → describe events if "something ever/never happens"

E.g of σ-field

1) flipping coin. $\Omega = \{H, T\}$

$F_0 = \{\emptyset, \{H, T\}\}$

$F_1 = \{\emptyset, \{T\}, \{H\}, \{H, T\}\} = P(\{H,T\})$

In general for any sample sp Ω.

$\exists$ 2 σ-fields. $F_0 = \{\emptyset, \Omega\}$

$F_1 = P(\Omega)$

2) Borel σ-field on $[0, 1]$

$F = B =$ smallest σ field that has all closed subinterval

curly

lemma: Let F be a σ-field.

a) If $A, B \in F$ then ① $A \cup B \in F$ → $A \cup B \cup \emptyset \cup \emptyset \ldots$ → cuz ∞

$A \cap B \in F$ → $(A^c \cup B^c)^c$ → demorgan

b) $\Omega \in F$ → $\Omega = \emptyset^c$

c) If $A, B \in F$, $A - B \in F$ $= A \cap B^c$