Lec 6

Recall

If $F: \mathbb{R} \to [0,1]$ is a cdf of a r.v X then.

(a) if $x \le y$ then $F(x) \le F(y)$

(b) $\lim_{x \to -\infty} F(x) = 0$ $\quad \lim_{x \to +\infty} F(x) = 1$

(c) right cts. $F(y) = \lim_{x \to y^+} F(x)$

+ 4 more

Thm If $F: \mathbb{R} \to [0,1]$ satisfies a, b, c then $\exists$ $(\Omega, \mathcal{F}, P)$ prob sp & $X: \Omega \to \mathbb{R}$ r.v st

- $F = F_X$

Pf **Idea**

Special case F is cts and strictly increasing

$G = F^{-1}(0,1) \to \mathbb{R}$

with prop

$G^{-1}((-\infty, a]) = (0, F(a)]$

Choose $\Omega = (0,1)$. let G be r.v.

verify F is cdf of G.

$\Omega = (0,1)$

$\mathcal{F}$ = Borel σ-field

P = Le s.t $P([a,b]) = b-a$

1) Fill in details

2) extend to general case.

Def) Discrete R.V.

A r.v X is called discrete p.t X takes values in some countable subset of $\mathbb{R}$
(image is countable)

In this case, the prob mass func of X is

$F : \mathbb{R} \to [0,1]$

$f(x) = P(X = x)$

Def) Cts R.V.

A rv X is cts p.t its cdf F can be expressed as

$$F(x) = \int_{-\infty}^{x} f(t)\, dt$$

for some integrable func f.

In this case f is the prob density func

Rmk) Discrete rv & cts rv are not negations of each other
(e.g. sums of 1 discrete & 1 cts one)

Rmk) if X is a cts RV then, F_X is cts.
But the converse is not True.

e.g.) $\Omega = [0,1]$ $\mathcal{F} = \text{Borel}$, $P([a,b]) = (b-a)$

$X : \Omega \to \mathbb{R}$ $\qquad X(y) = x$

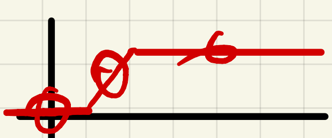
$u \mapsto \frac{1}{2}(u+1)$ $\qquad y = 2x - 1$

Q) find F_X and determine its nature

$F(x) = P(X \le x) = P(\{y \in [0,1] : X(y) \le x\})$

$= P([0, 2x-1])$

$= 2x-1 \quad$ if $\frac{1}{2} \le x \le 1$



$$f(t) = \begin{cases} 2 & \text{pt } t \in [0.5, 1] \\ 0 & \text{pt } t \in (-\infty, 0.5) \\ 0 & \text{otherwise} \end{cases}$$