Recall: LLN, CLT & Levy Continuity Thm

↳ we learn about the conv of seq by conv of Char func!

@ last lec

- estimate $P(|S_n - \frac{7n}{2}| < \varepsilon n)$ & $P(|S_n - \frac{7n}{2}| < 5\sqrt{n})$ using LLN, CLT, Cheby, Bern

From quiz

$c = 1/35$

Cheby — $1 - \frac{7}{60} = \frac{53}{60}$ & $1 - \frac{35/12\, n}{\varepsilon^2 n^2}$ bern'l $1 - \frac{2}{e^2}$

$P(\dots) \geq 1 - \frac{35}{12\varepsilon^2 n}$ $P(\dots) \geq 1 - e^{-\frac{\varepsilon^2 n^2}{2n}} = 1 - e^{-\frac{\varepsilon^2 n}{2}}$

By WLLN

$$P(|S_n - \tfrac{7n}{2}| < \varepsilon n) = P\left(\left|\tfrac{S_n}{n} - \tfrac{7}{2}\right| < \varepsilon\right) \xrightarrow{n\to\infty} 1$$

By CLT

$P(|S_n - \frac{7n}{2}| < 5\sqrt{n})$ norm var → $\frac{S_n - n\mu}{\sqrt{n\sigma^2}}$

$$= P\left(\left|\frac{S_n - \frac{7n}{2}}{\sqrt{n \cdot \frac{35}{12}}}\right| < \frac{5\sqrt{n}}{\sqrt{n\frac{35}{12}}}\right) \xrightarrow[\text{conv}]{\text{CLT}} P\left(|X| < 5\cdot\sqrt{\tfrac{12}{35}}\right)$$

$X \sim N(0,1)$

$$\int_{-5\cdot\sqrt{\frac{12}{35}}}^{5\cdot\sqrt{\frac{12}{35}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt$$

wolfram

$= 0.99656$ ← pretty good

but only asymptotically

E.g. SSRW S_n pos after n, X_i is each step.

$S_n = \sum_{i=1}^{n} X_i$ by CLT $\frac{S_n}{\sqrt{n}} \xrightarrow{D} N(0,1) \stackrel{\text{def}}{=} X$

$E(X_i) = \mu = 0$ $\underline{\text{var}(X_i) = 1 = \sigma^2}$ consider $E(|S_n|)$

So 1) $\left|\frac{S_n}{\sqrt{n}}\right| \to |X|$ 2) bounded integrals $E\left(\frac{|S_n|}{\sqrt{n}}\right) \to E(|X|)$

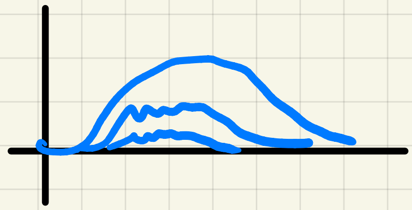
$$E(|X|) = \int_{\mathbb{R}} |x| \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx = 2\int_0^\infty x \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = \sqrt{\frac{2}{\pi}}$$

n large $E(|S_n|) = \sqrt{\frac{2n}{\pi}}$

Ipoe $F_n: \mathbb{R} \to \mathbb{R}$ cts st $F_n \to F$ unif

& F_n & F are C^1. No. You need unif conv of F_n'

Consider



CLT

$$\frac{S_n - n\mu}{\sqrt{n\sigma^2}} \xrightarrow{D} N(0,1)$$ but can't say that density func conv

local CLT → says density conv!

local CLT

Assume CLT holds and same assumptions too

$\operatorname{var}(X_i) = \sigma^2 < +\infty$, $\sigma > 0$ too

also, $\int_{\mathbb{R}} |\phi_{X_1}(t)|^r < +\infty$ for some $r \in \mathbb{N}$

$\Rightarrow$ density of $\frac{S_n - n\mu}{\sqrt{n\sigma^2}} \to \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

Now

$P(|S_n - \mu n| > \varepsilon n) \to 0$ — can we control the rate this goes to 0. Exponentially fast apparently