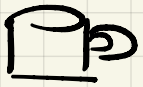
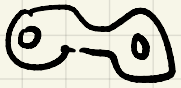
Introduction

Topology is the study of topological spaces, the maps between them & properties of the spaces preserved by →

Basic Question: When are 2 spaces the "same"?

Eg.  same as  but not same  or 

Q: when are 2 spaces different? what tools? (ex holes? → homology)

Goal: Understand what properties are preserved under cts maps?

Define topological invariants!

How to defn a top sp? → should indeed euclidean space $\mathbb{R}^n$, pt, n-dim euclidean space $\mathbb{R}^n$, function spaces ex $\{f: \mathbb{R} \to \mathbb{R}\}$, $S^1, S^2 \dots$, products of circles (and top sp)

Defn Ch 2.12

A **topology** $\mathcal{T}$ on X is a collection of subsets of X st.

(1) $\emptyset, X \in \mathcal{T}$

(2) $\mathcal{T}$ is closed under arbitrary unions

(3) $\mathcal{T}$ is closed under finite intersection

$(X, \mathcal{T})$ is a **top space**

(2) $\iff$ If $A_\alpha \in \mathcal{T}$ $\forall$ $\alpha \in I$ indexing $\Rightarrow \bigcup_{\alpha \in I} A_\alpha \in \mathcal{T}$

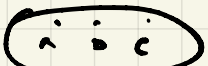
(3) $\iff$ If $A_1 \dots A_n \in \mathcal{T} \Rightarrow \bigcap_{i \in \mathbb{N}_n} A_i \in \mathcal{T}$

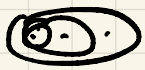
$U \subseteq X$ is **open** $\iff U \in \mathcal{T}$

Eg. A set X, let $\mathcal{T} = \{\emptyset, X\}$ trivial / indiscrete topology

$\mathcal{T} = P(X)$ discrete topology

$X = \{a, b, c\}$ Which work?

1) $\mathcal{T} = \{X, \emptyset\}$ →  ✓

2) $\mathcal{T} = \{X, \{a\}, \{a, b\}, \emptyset\}$ →  ✓

3) $\mathcal{T} = \{X, \{a\}, \{b, c\}, \emptyset\}$ ✓

4) $\mathcal{T} = \{X, \{a\}, \{b\}, \emptyset\}$ ✗

5) $\tau = \{X, \{a,b\}, \{b,c\}, \{b\}, \emptyset\}$ ✓

6) $\tau = \{X, \{a,b\}, \{b,c\}, \emptyset\}$ ✗

Ex The standard top on $\mathbb{R}$

$$\tau_{\text{standard}} = \{\text{open subsets of } \mathbb{R}\} = \{U \subseteq \mathbb{R} \mid \forall x \in U \ \exists \varepsilon > 0 \text{ s.t. } B_\varepsilon(x) \subseteq U\}$$

includes (a,b), $[a,b]$, $(-\infty, b)$, $\mathbb{R}$, $\emptyset$ ....