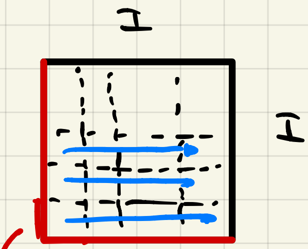
Lemma let $p: E \to B$ covering map with $p(e_0) = b_0$. $F: I \times I \to B$ be cts with $F(0,0) = b_0$

There exists a unique lift $\tilde{F}: I \times I \to E$ with $\tilde{F}(0,0) = e_0$ and the diagram commutes.

If F homotopy, so is $\tilde{F}$.



Pf Same idea as path lifting lemma

Def $\tilde{F}(0,0) = e_0$

Use path lifting lemma to get $\tilde{F}$ on $\{0\} \times I$ & $I \times 0$

both steps restrict F to path

"previous" rectangles illustrated above

Supp $\tilde{F}$ is defined on $A = \{0\} \times I \cup I \times \{0\} \cup \text{prev}(I_{i_0} \times I_{j_0})$

Define $\tilde{F}$ on $I_{i_0} \times I_{j_0}$. By cov map $F(I_{i_0} \times I_{j_0}) \subseteq U$ evenly cov by p.

$p^{-1}(U) = \bigcup_\alpha V_\alpha$ $\quad p|_{V_\alpha}: V_\alpha \to U$ homeo

$\tilde{F}$ def on left bottom edge of $I_{i_0} \times I_{j_0}$ which is $C := A \cap (I_{i_0} \times I_{j_0})$, connected!

Lift exists in exactly one V_α. Ie $\tilde{F}(C) \subset V_0$. Rec $p|_{V_0}$ homeo!

Define $\tilde{F}(x) = p|_{V_0}^{-1} \circ F(x)$ $\quad \forall x \in I_{i_0} \times I_{j_0}$

Check uniqueness & path homotopy!

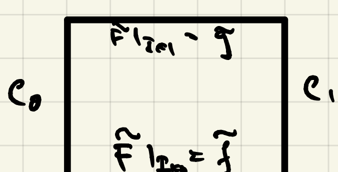
Thm let $p: E \to B$ be a covering map. let $p(e_0) = b_0$
let f, g be two paths from b_0 to b_1 in B.
let $\tilde{f}, \tilde{g}$ be the lifts in E starting at e_0.

If f, g path homotopic, they end at same pt. And are homotopic

Pf $F: I^2 \to B$ path homotopy between f & g.

(lift) $\tilde{F}: I^2 \to E$ path homotopy.

$\tilde{F}(\{0\} \times I) = e_0$ $\quad \tilde{F}(\{1\} \times I) = \{e_1\}$ (singleton as path homo)



$\tilde{F}|_{I \times 0}$ is lift of $F|_{I \times 0} = f$. By uniqueness of path lift (with specified endpt). $\tilde{F}|_{I \times 0} = \tilde{f}$. (also agreed end pts)

Similarly $\tilde{F}|_{I \times 1} = \tilde{g}$ $\quad \therefore \tilde{F}$ p homo bet $\tilde{f}, \tilde{g}$ $\square$

Def) let $p: E \to B$ covering map. $p(e_0) = b_0$. Can def

$$\varphi_{e_0} : \pi_1(B, b_0) \to p^{-1}(b_0)$$
$$[f] \to \tilde{f}(1)$$

loop at b_0 — path $\boxed{\text{lift}}$

called lifting corr (well def by prev bit)

Thm) If E is path connected. Then the lifting corr $\varphi = \varphi_{e_0}$ is surject. Recall $\varphi : \pi_1(B, b_0) \to p^{-1}(b_0)$. Rec $p: E \to B$ covering map

If E simply conn $\Rightarrow$ bijective.

Pf) ① E path connected. let some $e_1 \in p^{-1}(b_0)$. Since E path conn $\Rightarrow$ path from e_0 to e_1. $\tilde{f}$

Consider path $f = p \circ \tilde{f}$. This is a loop! $\in \pi_1(B, b_0)$

$\Rightarrow \varphi([f]) = \tilde{f}(1) = e_1 \Rightarrow$ $\boxed{\text{surjective}}$!

$\boxed{\text{uniqueness}}$

② E simply connected. Suppose $\varphi([f]) = \varphi([g]) = e_1$

let $\tilde{f}, \tilde{g}$ be the lifts. By simply conn $\tilde{f} \underset{pth}{\simeq} \tilde{g}$ (begin at e_0)

Furthermore by same endpt e_1.

By simply conn $\tilde{f} \underset{p}{\simeq} \tilde{g} \Rightarrow \exists$ path homo $\tilde{F}$

$\Rightarrow f \underset{p}{\simeq} g$ with homotopy $F = p \circ \tilde{F}$

So $[f * \tilde{g}^{-1}] = [e_{e_0}]$; mult both sides by $\tilde{g}$

const

$\Rightarrow [\tilde{f}] = [\tilde{g}]$