Lec 2

Def | let $\mathcal{X}$ collection of sets $\subseteq M$

$\mathcal{X}$ is locally finite if $\forall p \in M$ has a nbhd $\mathcal{U}$ so $\mathcal{U}$ intersects finitely many $C \in \mathcal{X}$.

Def | let M top sp is paracompact if $\forall$ open cover $\mathcal{X}$ of M admits a locally finite subcover

↳ slightly different than usual

D | a subcover of $\mathcal{X}$ is a cover $\mathcal{X}^*$ so $\forall V \in \mathcal{X}^*$ $\exists \mathcal{U} \in \mathcal{X}$ so $V \subseteq \mathcal{U}$

Def | A cover $\mathcal{X}$ is open if $\forall \mathcal{U} \in \mathcal{X}$ $\mathcal{U}$ is open

Thm | Topological Manifolds are paracompact

hausdorff, locally euclidean, second ctbl

Def | M is called locally cmpt if $\forall p \in M$ and $\mathcal{U}$ nbhd of p $\exists$ nbhd $V \subseteq \mathcal{U}$ so $\overline{V} \subset \mathcal{U}$ compact.

Lemma | Top mfd locally compact

Pf | locally euclidean $\times$ $\mathbb{R}^n$ is locally compact

Earlier: Top mfd

- $\forall p \in M$ $\exists$ $\mathcal{U}$ nbd of p & homeo $\mathcal{U} \to \mathbb{R}^n$

L | Enough that $\forall p \in M$ $\exists V$ nbhd & homeo $\varphi: V \to$ open set $\mathbb{R}^n$

Pf | look at connected cmp of $\varphi(p)$ and take ball.

Lemma | A second ctbl, loc compact, hausdorff sp M admits an exhaustion by compact sets

↳ increasing seq of sets K_n so $\bigcup K_n = M$

Closure is cmpt

Pf | ① $\exists$ basis of precompact open sets. Since M is loc cmpt.

② 2^{nd} ctbl $\Rightarrow$ $\exists$ countably many precmpt sets $\{U_i\}_{i=1,2,\ldots}$ so $\bigcup U_i = M$

idea: $p \in M$. 2^{nd} ctbl $\Rightarrow$ $\exists$ ctbl basis $\{W_j\}$ let $p \in W_j$

$\exists$ $p \in V$ so $\overline{V} \subseteq W_j$ compact by loc cmptness

let this be contained in $\mathcal{U}$ precpt

Choose basis elt $\{W_\ell\}$ so $\bigcup_\ell W_\ell \supseteq \overline{V}$. Since $\overline{V}$ cmpt choose finite $W_{\ell_1}, \ldots, W_{\ell_k}$ to cover $\overline{V}$

Then W_{ℓ_i} are precompact as $\overline{W_{\ell_i}} \subseteq \overline{U}$ compact as $\overline{U}$ compact (closed subset of cmpct)

take all $\{W_{jx}\}$ (countable union of finite) $\square$

3) Define the exhaustion by cmpct sets.

let $K_k = \bigcup_{i \leq k} \overline{U_i}$

Consider) Competing coord