Last time if c - singular k-chain $\alpha \in \Omega^{k-1}$

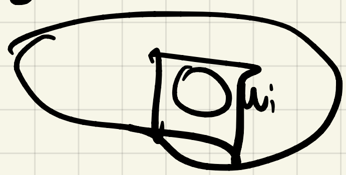
$$\int_{\partial c} \alpha = \int_c \alpha$$

Now, wanna get statement on Mfds! (M oriented compact)

Defn) $\int_M \omega$?

let $\{M_i\}_{1 \le i \le N}$ an open cover of chart image so

$M_i \subseteq C_i$ sing n-cube (or pos)



Take a partition of unity subord to M_i

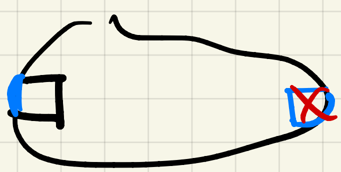
$\omega = \sum f_i \omega$ $\quad f_i \omega$ supp on M_i

$$\sum_i \int_{C_i} f_i \omega =: \int_M \omega$$

Lemma) Above well def!

Pf) C.O.V

Pf of Stokes) (idea)



in defn of $\int_M \omega$ use $C_i \Rightarrow$ only one face of C_i on ∂M.

$$\int_M \partial\alpha = \sum_i \int_{C_i} \underline{f_i \partial \alpha} \overset{\text{Chart}}{=} \sum_i \int_{\partial C_i} \alpha_i = \int_{\partial M} \alpha$$

closed

Degree & $H^n(M)$ $\quad \dim M = n$

Thm) if M cmpct & oriented

$\Rightarrow \mathbb{H}^n_{\partial R}(M) = \mathbb{R}$

More precisely!

$\mathbb{H}^n_{\partial R}(M) \longrightarrow \mathbb{R}$

ω - n form $\longrightarrow \int_M \omega$ $\qquad$ M oriented $\Rightarrow \int_M \omega > 0$

Volume form

~~Spse~~ $\int_M \omega = 0 \xRightarrow{\boxed{\text{exin}}} \omega = d\eta$ for some eta

If M N dim $= n$, or compt

$f : M \to N$ let ν vol frm on N

$$\int_M f^* \nu = \deg f \int_N \nu$$

e.g. | $S^1 \longrightarrow S^1$ has degree 7.
$z \longmapsto z^7$

Claim

if $f : S^1 \to S^1$ a diffeo (or pres)
$\Rightarrow$ degree 1

Prop | $\deg(f \circ g)$ $M \xrightarrow{g} N \xrightarrow{f} P$
$= \deg f \cdot \deg g$

Brouwer Fixed pt thm |

let D^n be closed ball in $\mathbb{R}^n$

$f : D^n \circlearrowleft$ has a fixed pt

i.e $\exists\, p \in D^n$ so $f(p) = p$.

Only pf for f smooth (i.e has any fr (?))

Spse f smooth | spse no fixed pts

Defn $G(x) = \dfrac{x - f(x)}{\| x - f(x) \|}$ $\quad \boxed{G : D^n \to S^{n-1}}$

let $g = G|_{S^{n-1}}$ on S^{n-1}

let $H(t, x) = \dfrac{x - t f(x)}{\| \text{up} \|}$

if $t < 1 \Rightarrow \| t f(x) \| < \| x \|$ so not one!

if $t = 1 \Rightarrow x = f(x)$ ag.

Pf $H(0,x) = x$

$\Rightarrow \deg\left(H(0,-) : S^{n-1} \to S^{n-1}\right) = 1$

but if σ is the vol form on S^{n-1} induced from vol form μ on D^n

$$\int_{S^{n-1}} g^*\sigma = \int_{S^{n-1} = \partial D^n} G^*\sigma = \int_{D^n} G^*(\partial\sigma)$$

$$= \int_{D^n} G^*(0)$$

oops from $\boxed{\text{hw}}$

$$= 0$$

$\neq$