## Comment on Lie Subgrps

$H \xrightarrow{\varphi} G$ given ^smooth hom bet $H, G$ want $\mathfrak{h} \xrightarrow{\Phi} \mathfrak{g}$.

consider the following (regarding $\mathfrak{h} = T_1 H$, $\mathfrak{g} = T_1 G$)

$$\Phi : \mathfrak{h} \longrightarrow \mathfrak{g}$$
$$X \longmapsto D\varphi(1)(X)$$

↳ Claim: $X, D\varphi(1)(X)$ are $\varphi$ related as left-inv vf.

**Clean up.**

k-form ~~measures area of intersection~~ (of proj) → look at area of a projection.

**Let**

$V$ fin dim $= n$ vect sp $/\mathbb{R}$.

dim $\Lambda^n V = 1$ → either is $-$ or pos.

**Defn** 2 $n$-forms on $V$ have the same orientation if $\beta = c \cdot \alpha$ where $c > 0$.

↳ otherwise opposite orientation.

**Defn** $M$ $n$-dim'l mfd

$\Lambda^k M := \Lambda^k (TM)$ → defn short hand

↳ $(\Lambda^k M)_p = \Lambda^k T_p M$

A $k$-form $\alpha$ is a smooth section of

$$\begin{tikzcd} \Lambda^k M \arrow[d, "\pi"'] \\ M \arrow[u, bend right, "\alpha"'] \end{tikzcd}$$

E.g. 2 form on $\mathbb{D}^2$: $x^2 dx \wedge dy$.

$M$ with dim $n$

dim $\Lambda^n T_p M = 1$

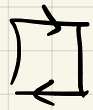
$$\begin{tikzcd} \Lambda^n M \arrow[d, "\pi"'] \\ M \arrow[u, bend right, dashed, "\text{section}"'] \end{tikzcd}$$

**Def** Any ^smooth section $\sigma : M \to \Lambda^n M$ s.t. $\forall p \in M$ $\sigma(p) \neq 0$ is an orientation of $M$.

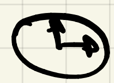
Given an orientation $\tau$ and another $\sigma$, we say they define the same orientation if $\exists$ sm $f: M \to (0,\infty)$ so that $\sigma = f \cdot \tau$.

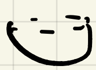
Also if $\sigma$ is orientation $-\sigma$ is the reverse orientation.

$\Rightarrow$ orientations may not exist $\to$ Möbius Band.

D) Möbius Band. 

Defn) $M$ orientable if have orientation.
$\Rightarrow$ also an oriented manifold is a manifold with given orientation

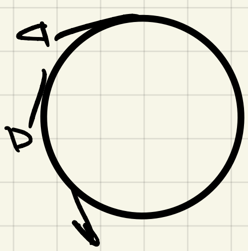
Obs)  is orientable since looks like 

So is  and they agree.

$\Rightarrow$ $\therefore$ gluing these gives $S^2$ which is orientable.

e.g.) $\mathbb{RP}^n = S^n/\mathbb{Z}_2$ $\quad \mathbb{Z}^n \curvearrowright S^n$
$A \neq 1 \quad x \mapsto -x$
$A(x)$

Q) does $A$ pres orientation? on $S^1$ yes!



$\mathbb{RP}^1$ is orientable
$\mathbb{RP}^2$ not
$\mathbb{RP}^3$ is as diffeo to $SO_3$

or, when does $A: S^n \to S^n$ pres orientation

Lie grp

$SU(2)$

double cover

Prop) Any $G$ Lie grp is orientable say $\dim G = n$

Pf) $\Lambda^n G \quad \tau(1) = \Lambda^k T_1 G$ and make left inv.

# Basis Multilinear Algebra

<u>Spose</u> $\alpha \in \Lambda^k V$, $\beta \in \Lambda^\ell V$ <u>wts</u> $\alpha \wedge \beta \in \underline{\Lambda^{k+\ell} V}$.

$$(\alpha \wedge \beta)(v_1, \dots, v_{k+\ell})$$

$$= \frac{1}{k!\,\ell!} \sum_{\sigma \in S(k+\ell)} (-1)^\sigma \alpha(v_{\sigma(1)}, \dots, v_{\sigma(k)}) \cdot \beta(v_{\sigma(k+1)}, \dots, v_{\sigma(k+\ell)})$$

$\rightarrow$ <u>permutations</u>