Recall

C is a singular k-chain $\quad C = \sum \alpha_i c_i \rightsquigarrow c_i$ singular k-cubes

$\alpha \in \Omega^{k-1} C \quad$ Wts $\boxed{\int_c d\alpha = \int_{\partial c} \alpha}$

Cor reg Stokes, divergence, curl, Green's thm!

Pf of Stokes on Chains!

Good enough to check on basis $\to$ singular k cube

$c : [0,1]^k \to M$

<u>Note</u> $\int_c d\alpha \overset{\partial\alpha}{=} \int_{[0,1]^k} c^* d\alpha \quad = \int_{[0,1]} dc^*\alpha$

& $\int_{\partial c} \alpha = \int_{\sum_{\beta=0,1} (-1)^{\beta+1} c_{(i,\beta)}} \alpha = \sum_{\beta=0,1} (-1)^{\beta+i} \int_{I^{k-1}} c^*_{(i,\beta)}(\alpha)$

Good enough to show on $[0,1]^k = I^k$

$$\alpha = \sum_{i=1}^{k} f_i \, dx_1 \wedge \dots \wedge \widehat{dx_i} \wedge \dots \wedge dx_k$$

Good enough for $\alpha = f \cdot dx_1 \wedge \dots \wedge \widehat{dx_i} \wedge \dots \wedge dx_n$

Wts $\int_{I^k} d\alpha = \int_{\partial I^k} \alpha$

$$\int_{\partial I^k} \alpha = \sum_{\substack{\beta=\{0,1\} \\ j=1\cdots k}} (-1)^{j+\beta} \int_{[0,1]^{k-1}} I^*_{(j,\beta)}(\alpha) = (-1)^{i+1} \int_{[0,1]^{k-1}} f(x_1, \dots, 1, x_k) \, dx_1 \dots \widehat{dx_i} \dots dx_k + (-1)^i \int_{[0,1]^{k-1}} f(x_1, \dots 0, x_k) \, dx_1 \dots \widehat{dx_i} \dots dx_k$$

but $I^*_{(j,\beta)} \alpha = \begin{cases} 0 & \text{if } i \neq k \\ \int_{[0,1]^{k-1}} f(x_1, \dots \beta, x_n) & \end{cases}$

on the other hand

$$\int_{I^k} \partial (f \, dx_1 \wedge \dots \wedge \widehat{dx_i} \wedge \dots \wedge dx_n) = \int_{I^k} (-1)^{i-1} \frac{\partial f}{\partial x_i} \, dx_1 \wedge \dots \wedge dx_n$$

$$= (-1)^{i-1} \int_0^1 \int_0^1 \cdots \left(\int_0^1 \frac{\partial f}{\partial x_i} \, dx_i \, dx_1 \dots dx_n \right) \qquad \boxed{\text{Fubini}}$$

$$\parallel \ \boxed{\text{FTC}}$$

$$(-1)^{i-1} \int_0^1 \cdots \int_0^1 \left(f(x_1, \dots, 1, \dots x_k) - f(x_1 \dots, 0, \dots x_k) \right)$$

$$= \boxed{\text{RHS}} \ !$$