Lec 1

$f : \mathbb{R}^n \to \mathbb{R}^k$ $\quad n \in \mathbb{N}$

can take $\frac{\partial f_j}{\partial x_k}$ for $j,k \in \mathbb{N}$

can take $Df = f_* = \left(\frac{\partial f_j}{\partial x_k}\right)_{j,k}$

can take $\int_\Omega f$

OH
M 12-1
T 5-6
HW Weekly Wed
M: Oct 19 F: Dec 15th

- John Lee - Mfds
- Loring Tu
- Milnor (Intro to diff)
- Guillemin

Lets say $M \subseteq \mathbb{R}^N$ can we diff on M thinking of $f : M \to \mathbb{R}^k$ as a composition of incl on ambient sp

Mobius $\subseteq \mathbb{R}^3$, Klein bottle $\subseteq \mathbb{R}^4$ not $\mathbb{R}^3$ ✗

Important: make constructions, product of mfds, connected sum?

$S^n \subseteq \mathbb{R}^{n+1}$ $\quad S^n = \{x \in \mathbb{R}^{n+1} \mid \|x\|^2 = 1\}$ → euclidean norm

$\mathbb{RP}^n$ — real projective space (lines through origin in $\mathbb{R}^{n+1}$)

take $S^n / x \sim -x$ ← intersect S^n at 2 pts!

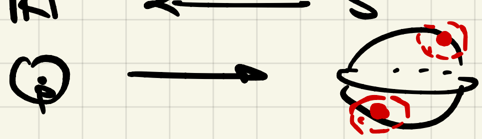
So $\mathbb{RP}^n$ locally looks like S^n

& to differentiate, we just need nice local coverage!

quaternions
$\{a + ib + jc + kd \mid a,b,c,d \in \mathbb{R}\}$
$i^2 = j^2 = k^2 = -1$
$ij = -ji$

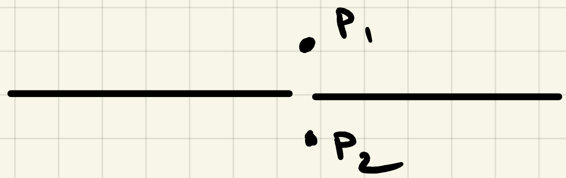
Defn A top sp M, hausdorff & 2nd ctbl → 2D countable basis
is a n dimensional top mfd. If $\forall$ $p \in M$ $\exists$ $p \in U \subseteq M$ which is homeomorphic to $\mathbb{R}^n$.

e.g. $\mathbb{RP}^n \leftarrow S^n$



→ might seem disc so not $\cong \mathbb{R}^n$ but they are glued!

non eg



where nbhds of p_1 is $\{p_1\} \cup (-\delta, \delta) \setminus \{0\}$ and same with $\{p_2\}$

This is non hausdorff as can't separate p_1, p_2