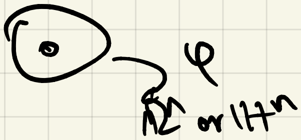
Manifolds with Boundary



& the boundary are points $p \in M$ so that there exists a $\mathbb{H}^n$ chart around it & $p = \varphi^{-1}(v)$ for $v \in \partial \mathbb{H}^n$

E.g.) ① $\mathbb{H}^n$ ③ $\mathbb{R}^n$

② ├──┤ $[a,b]$

Lemma ∂M = the points, which is well def!

def $M, \partial M$ mfd with ∂ no bdry

Suppose M is oriented $\Rightarrow \partial M$ is also oriented!

traverse with normal to the right!

Stokes if M oriented:

$$\int_M d\omega = \int_{\partial M} \omega \qquad \text{for } \omega \in \Omega^{n-1} M$$

E.g.) $M = [0,1]$ ├──→──┤

$\omega = f : [0,1] \to \mathbb{R} \in \Omega^0([0,1])$

Stokes $\int_M d\omega = \int_0^1 d\omega = \int_{\{0,1\}} \omega = \omega(1) - \omega(0)$

$$\int_0^1 d\omega = \int_0^1 \omega'(t)\,dt$$

Cool if ω is $\boxed{\text{exact}}$ & $\partial M = \emptyset$

$\Rightarrow \int_M \omega = 0$ as $\exists\, \nu$ so $\int_M d\nu = \int_{\partial M} \nu = 0$