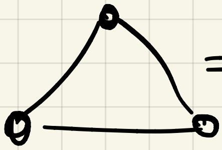
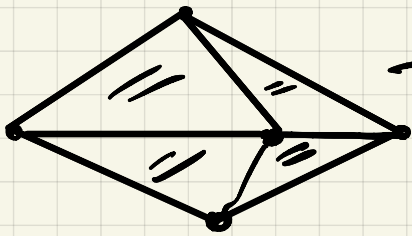
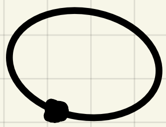
Eg simplicial cmplx.

1)  simplicial cmplx. 2 zero simplices & 1 - 1 simplex

 $= \Delta^2$ simplicial cmplx 3 0-simpl, 3 1-simpl, 1 2-simpl

Δ^1 simplicial cmplx w/ simplices given by subsimpl.

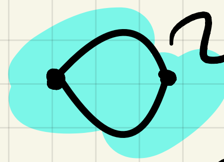
2)  ⟶ simplicial cmplx

3) $S^1 \cong$ 

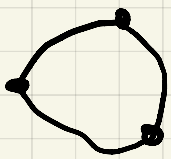
Δ-simplx tho.

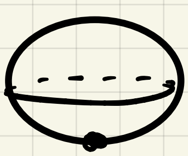
↳ not simpl cmplx

↳ not inj at point

$\cong$  ↳ this is not simpl cmplx as intersection is 2 points! not

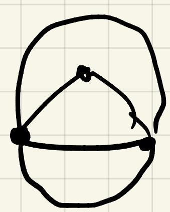
↳ not a subsimplex

$\cong$  ✓

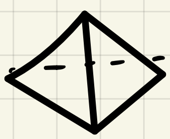
4) S_2 

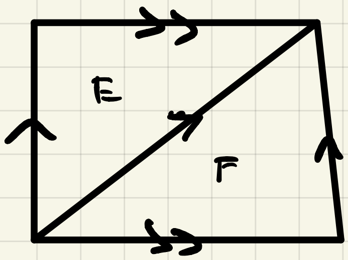
not simpl cmplx

↳ not Δ cmplx.

$\cong$  Δ-cmplx

not a simpl cmplx

$S^2 \cong \partial \Delta^3$  simpl cmplx.

5) $T =$ 

Δ-cmplx.

not simpl complex (injecting on bdry of 1-simpl)

Thm (Hatcher A.7)

Any finite CW complex or compact manifold

$\implies$ is a retract of a finite simpl. complex.

(Almost saying htpy equiv)

Thm (Lefschetz for simpl. complexes)

X retract of a finite simpl. complx.

$f : X \to X$ w/ $\Lambda(f) \neq 0$

$\Rightarrow$ f has a fixed point!

Pf sk 1) Want to reduce from retract of finite simpl cmpx to finite simpl cmpx.

$$X \xrightarrow{i} Y \hookrightarrow \text{finite simp cmpx}$$

$r: Y \to X$ $\to$ retr so $r \circ i \equiv id_X$

$g = i \circ f \circ r : Y \to Y$

Cl. fixed pt of g is fixed pt of f

let $y \in Y$ so $g(y) = y$

$\Rightarrow y \in X$ as it is in img of g which is X.

$g(y) = f(r(y)) = f(y)$

id on X

suppress i

$\square$

$$0 \to H_n(X) \xrightarrow{i_*} H_n(Y) \to Q \to 0$$

r_*

SES

$Q = H_n(Y)/i_* H_n(X)$

$\to$ i_* inj as $r \circ i = id$ $\Rightarrow r_* \circ i_* = id_*$

$\to$ This splits by left inverse

$$(r_*, q) : H_n(Y) \xrightarrow{\cong} H_n(X) \oplus Q$$

$$g_* = i_* \circ f_* \circ r_* : H_n(Y) \xrightarrow[r_*]{\text{proj on 1st summand}} H_n(X) \xrightarrow{f_*} H_n(X) \xrightarrow{i_*} H_n(Y) \text{ incl sum}$$

So, Matrix for g_* and f_* is

$$\begin{array}{cc} H_n(X) & Q \\ \left(\begin{array}{c|c} \text{Matrix for } f_* & 0 \\ \hline 0 & 0 \end{array}\right) \end{array} \Rightarrow \operatorname{tr} g_* = \operatorname{tr} f_*$$

$\Rightarrow \Lambda(g) = \Lambda(f)$ $\square$

2) Now lets prove for X = finite simpl. cmplx. Σ set of simpl for struct

Suppose, f doesn't have fixed pt WTS, $\Delta(f) = 0$.

Key input (Simpl approx thm Hatcher 2.C.1)

$\exists$ simpl cmplx struct $\Sigma' \supseteq \Sigma$ (refinement)

& $g : X \to X$ so that

a) $f \simeq g$

b) g is cellular wrt simpl cmplx struct Σ' s.t

$$g(\underbrace{(X, \Sigma')^n}_{n\text{-skel wrt } \Sigma'}) \subseteq (X, \Sigma')^n$$

c) $\forall \sigma \in \Sigma'$ $g(\sigma) \cap \sigma = \emptyset$

abuse of notation meaning σ not σ as inj map