Fact) If $z, w \in \Lambda(\Gamma)$ → non-elt, tors free, disc

$\Rightarrow \exists$ sequence $\{\gamma_n\} \subseteq \Gamma$

So $\gamma_n(x_0) \to z$, $\gamma_n^{-1}(x_0) \to w$

fixed pts of hyp

Prop) $\{(\gamma^-, \gamma^+) \mid \gamma \in \Gamma \text{ is hyp}\}$ is dense in $\Lambda(\Gamma) \times \Lambda(\Gamma)$

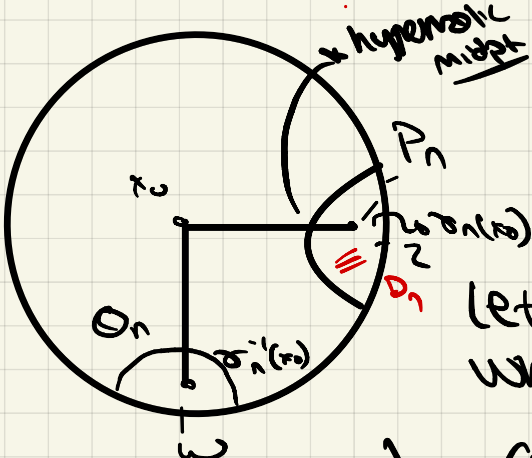
Pf) Spse, $z \neq w \in \Lambda(\Gamma)$

Know, $\exists$ seq $\{\gamma_n\} \subseteq \Gamma$

So $\gamma_n(x_0) \to z$, $\gamma_n^{-1}(x_0) \to w$

Let P_n be $\perp$ bisec of $\overline{x_0 \gamma_n(x_0)}$

let Q_n be $\perp$ bisec of $\overline{x_0 \gamma_n^{-1}(x_0)}$



So. $P_n \to z$ & $Q_n \to w$

& diam P_n, diam $Q_n \to 0$

let D_n be half sp. bdd by P_n which contains $\gamma_n(x_0)$.

$\text{diam}_{euc}(\overline{D_n}) \to 0 \Rightarrow \overline{D_n} \to z$

Similar for E_n half sp bdd by Q_n w/ $\gamma_n^{-1}(x_0)$.

For large n $\overline{D_n} \cap \overline{E_n} = \emptyset$ as $z \neq w$ so $d(z,w) > 0$

So, by constr $\boxed{\gamma_n(Q_n) = P_n}$ $\gamma_n(\mathbb{H}^2 - \overline{E_n}) \subseteq \overline{D_n}$ ↳ it is either in $\overline{D_n}$ or $\mathbb{H}^2 - \overline{D_n}$ but $x_0 \in \mathbb{H}^2 - \overline{E_n}$ & maps to $\overline{D_n}$ so done

In particular, $\gamma_n(\overline{D_n}) \subseteq \overline{D_n} \Rightarrow$ Brouwer

$\Rightarrow \gamma_n$ has fixed pt in $\overline{D_n}$

Similarly γ_n^{-1} has fp in $\overline{E_n}$.

So, for all large n, γ_n is hyperbolic.

$\gamma_n^+ \in \overline{D_n}$, $\gamma_n^- \in \overline{E_n}$

so $\gamma_n^+ \longrightarrow z$, $\gamma_n^- \longrightarrow w$

So, $\{(\gamma^-, \gamma^+) \mid \gamma \text{ hyp}\}$ is dense in $\Lambda(\Gamma)^{(2)}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \parallel$

but $\Lambda(\Gamma)^{(2)}$ dense in $\Lambda(\Gamma) \times \Lambda(\Gamma)$ $\qquad$ $\Lambda(\Gamma) \times \Lambda(\Gamma) - \text{diag}$

So, desired result follows.

$\hookrightarrow$ perfect set so chill.

So, if $G \leq \text{homeo}(M)$ compact, perfect metric sp

G acts as a (discrete) convergence grp on M

if whenever $\{g_n\} \subseteq G$ seq of dist elts

then $\exists$ subseq $\{g_n\}$ & pts $a, b \in M$

$\qquad\qquad\qquad\quad \hookrightarrow$ lol

So, $g_n|_{M \setminus \{a\}} \longrightarrow$ const map with image b

uniformly on cmpt subsets.

(generalization of prev arg) $\qquad$ i.e. any pt in

$M - \{a\}$, c

have $g_n(c) \to b$.

Thm (Bowditch) |

if G acts as a conv on M and acts

co-compactly on $M^{(3)}$ = {triples distinct pts in M}

$\hookrightarrow$ i.e. $M^{(3)}/G$ is cmcpt

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \nwarrow$ bruh

$\Rightarrow$ G is Gromov hyperbolic grp

moreover, M is G-equivariantly homeomorphic to

$\partial_\infty G$ (something abt Cayley gr)

F) Γ acts as a conv. grp on $\overline{\mathbb{H}^2} = \mathbb{H}^2 \cup \partial\mathbb{H}^2$ and on $\partial\mathbb{H}^2$, & on $\Lambda(\Gamma)$.

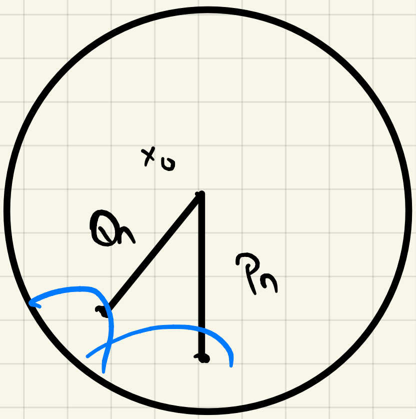
Pf) let $\{\gamma_n\}$ be $\underset{n}{\overset{\text{dist.}}{}}$ seq in Γ, pass to subseq so

$$\gamma_n(x_0) \longrightarrow b \qquad \gamma_n^{-1}(x_0) \longrightarrow a$$

w/ $a, b \in \Lambda(\Gamma)$

let P_n be $\perp$ bisec of $\overline{x_0 \gamma_n(x_0)}$

Q_n ——— $\overline{x_0 \gamma_n^{-1}(x_0)}$



let D_n, E_n halfsp bdd by P_n, Q_n cont $\gamma_n(x_0)$ & $\gamma_n^{-1}(x_0)$ resp

know $\overline{D_n} \to a$ $\overline{E_n} \to a$ by diam.

$\gamma_n(Q_n) = P_n$
$\gamma_n(\mathbb{H}^2 - \overline{E_n}) \subseteq \overline{D_n}$ } same as earlier just might not have disj.

If $K \subseteq M - \{a\}$ is cmpct

$\Rightarrow \gamma_n(K) \longrightarrow b$

For large n, $K \subseteq \overline{\mathbb{H}^2} - \overline{E_n}$ $\sim \overline{E_n} \to \{a\}$

$\Rightarrow \gamma_n(K) \subset \overline{D_n}$

$\Rightarrow \gamma_n|_K$ conv to const map w/ img b unif

Def) $\Omega(\Gamma) = \partial\mathbb{H}^2 - \Lambda(\Gamma)$

↳ domain of discontinuity as Γ acts disc here!

Fact 1 Γ acts freely, prop disc on $\Omega(\Gamma)$ —→ this true

Pf Γ acts freely on $\Omega(\Gamma)$ as all f.p. of nontriv elts lie in limit set.

Spse not prop. disc.

$\Rightarrow \exists K \subset \Omega(\Gamma)$ cmpct & distinct elts $\{\gamma_n\}$ so $\gamma_n(K) \cap K \neq \emptyset$

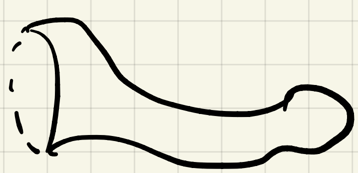
So, earlier, $\{\gamma_n\}$ has subseq (called same)

So, $\exists\, a,b \in \Lambda(\Gamma)$

s.t. $\gamma_n|_{\partial \mathbb{H}^2 - \{a\}} \longrightarrow b$ unif on cmpct

but, $K \subseteq \partial \mathbb{H}^2 - \{a,b\}$ in $\Omega(\Gamma)$ ← so pulled unif to b but actually never close to it $\boxed{\text{oops}}$.

Now $\mathbb{H}^2 \cup \Omega(\Gamma) / \Gamma$ is torus w/ hole.



$G \subseteq \text{homeo}(M)$ acts ergodically wrt a measure μ on M if whenever $A \subseteq M$ is Γ-inv $\Rightarrow \mu(A) = 0$ or $\mu(M - A) = 0$

Fact if $X = \mathbb{H}^2/\Gamma$ is closed surface Then Γ acts ergo on $\partial \mathbb{H}^2$ w/ leb measure!