Let $\Gamma \leq PSL(2,\mathbb{C}) = \mathrm{Isom}^+(\mathbb{H}^3)$ discr, tors free

$CH(\Lambda(\Gamma))$ = convex hull of $\Lambda(\Gamma)$

$N = \mathbb{H}^3/\Gamma$, $C(N) = CH(\Lambda(\Gamma))/\Gamma$ conv core

Γ (or N) is conv cocompact iff $C(N)$ is cmpct

Prop Γ is conv cocompact $\iff$ all lim pts is conical

Milnor-Schwarz Lemma

If Γ acts prop disc by isom on a "nice" metric space X & X/Γ is cmpct.

$\Rightarrow \Gamma$ is fg

Orbit map $\tau : \Gamma \to X$, $\gamma \mapsto \gamma(x_0)$ is a quasi isometry $\forall x_0 \in X$

$\tau : \Gamma \to X$ is a (K, C)-quasi isometry if

$$\frac{1}{K} d(\alpha,\beta) - C \leq d(\tau(\alpha), \tau(\beta)) \leq K\, d(\alpha,\beta) + C$$

$d(\alpha,\beta)$ = word length of $\alpha^{-1}\beta$

(dep on choice of fin gen)

and coarse-surjectivity

$\iff X \subseteq N_C(\Gamma(x_0))$

Upshot Γ convex cocompact

$\tau : \Gamma \hookrightarrow CH(\Lambda(\Gamma))$ is a quasi isometry.

but $i : CH(\Lambda(\Gamma)) \hookrightarrow \mathbb{H}^3$ is isom embedding

So $J:\Gamma \longrightarrow \mathbb{H}^3$ is a QI embedding.

On the other hand suppose $\Gamma \hookrightarrow \mathbb{H}^3$ is a quasi isom embedding

Geometric grp theory says

Γ is a hyp grp, C_Γ is a gromov hyperbolic space,

$\exists\ \partial_\infty \Gamma$ gromov bdry at ∞,

and J extends to a map

$$\hat{J} : \partial_\infty(\Gamma) \longrightarrow \Lambda(\Gamma) \subseteq \mathbb{H}^3$$

If $z, w \in \partial_\infty(\Gamma)$ $J : \overline{zw} \longrightarrow \mathbb{H}^3$ is a quasi-isom embedding

Fellow Traveller Prop

$\exists\ R(K,C)$ so $J(\overline{zw}) \subseteq \mathcal{N}_R(\overline{\hat{J}(z)\,\hat{J}(w)})$

To never move too far for geodesics

$\Rightarrow J(C_\Gamma) \subseteq \mathcal{N}_R$(set of all geodesics w endpts in lim set)

$\boxed{\text{extends to } C_\Gamma}$

$\subseteq \mathcal{N}_R(CH(\Lambda(\Gamma)))$

need reason

but also, $CH(\Lambda(\Gamma)) \subseteq \mathcal{N}_{R+A}(J(C_\Gamma))$

$\Rightarrow C(M) \subseteq \mathcal{N}_{R+1}(\mathcal{J}(C(\Gamma))/\Gamma)$

conv core

$\hookrightarrow$ finite bouquet of O

hence, $C(M)$ is comp

$\Rightarrow \Gamma$ is conv cocompact.

Upshot Γ conv cocompact

$\Updownarrow$

$\mathcal{J}$ is QI embedding.

Stability

Any small deformation of a convex cocompact grp is convex cocompact.

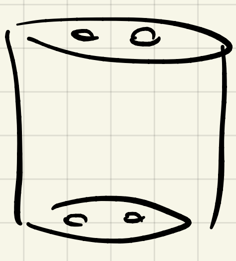
i.e $\exists$ nbhd of id in $\mathrm{Hom}(\Gamma, PSL(2,\mathbb{C}))$ s.t. if $\rho \in \mathcal{U}$ then $\rho(\Gamma)$ is conv cocompact

In $PSL(2,\mathbb{R})$, Γ is conv cocomp

$\Updownarrow$

Γ is f.g and has no parab elts.

This is false in $PSL(2,\mathbb{C})$.



$\phi : S_g \to S_g$ is a homeo

$M_\phi \simeq (S_g \times [0,1])_{(0,x)\sim(1,\phi(x))}$

$\hookrightarrow$ make cylinder & glue via homeo.

Thurston) if ϕ is "irreducible", then
M_ϕ is hyperbolic.
I.e. $M_\phi \cong \mathbb{H}^3/\Gamma$

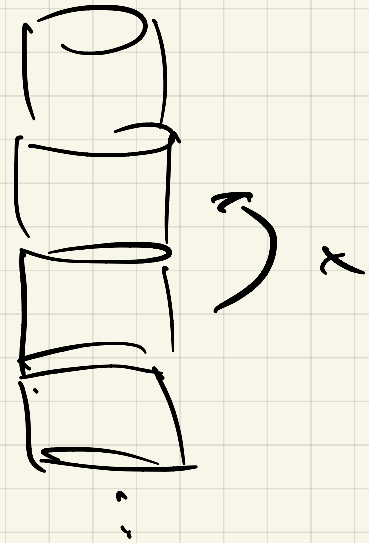
Let Γ_S = subgrp of Γ asso to
cover $\pi_1(S_g) \le \pi_1(M)$;

$N_S = \mathbb{H}^3/\Gamma_S \to$

$\cong S_g \times \mathbb{R}$

$f : N_S \to N_S$

$(x,t) \mapsto (\phi(x), t+1)$



so thurston initially thought this was hyperbolic.

So, finitely gen'd and no parabolics

$C(N_S) = N_S$ conv core

$\Rightarrow N_S$ is not conv cocompact

$\Lambda(\Gamma_S) = \partial\mathbb{H}^3$.

Pf that $\Lambda(\Gamma_S) = \partial\mathbb{H}^3$)

Spse not, then since Γ_S is normal $\le \Gamma$
we see $\Lambda(\Gamma_S)$ is Γ-inv
But, $\Lambda(\Gamma)$ is smallest closed nonempty

Γ - invariant s.s.o. $\partial \mathbb{H}^3$.

$\Longrightarrow$ contradiction.

Agol's Thm)

Every closed hyperbolic 3-manifold has a finite cover which fibres over the circle.

I.e. has the form M_φ