1

Lemma If c^*, d^* intersect & c^* is simple!

Then $\sinh\left(\frac{\ell(c^*)}{2}\right)\sinh\left(\frac{\ell(d^*)}{2}\right) > 1$

Talk

Margulis Lemma If G is a Lie Grp

$\Rightarrow \exists$ nbhd $\mathcal{U}$ of identity

s.t. if $\Gamma \leq G$ is disc. & is gen'd by $\langle \Gamma \cap \mathcal{U} \rangle$

$\Rightarrow \Gamma$ is nilpotent.

Idea consider, $C: G \times G \to G$, $(a,b) \mapsto [a,b]$ $\quad C(id, id) = ?$.

So, $X_{thin(\sinh^{-1}(1))}$ = union of collar nbhds of geodesics of length $< 2\sinh^{-1}(1)$

Thm $\forall n\ \exists\ \mu_n > 0$ so if N is closed hyp. n-mfd, & $0 < \varepsilon < \mu_n$, $N_{thin(\varepsilon)} = \{x \in N \mid inj_N(x) < \varepsilon\}$

Then every component of $N_{thin(\varepsilon)}$ is a tubular nbhd of a closed geodesic of length $< 2\varepsilon$.

Thm If $0 < a < 1$ $\exists\ \mu(a,n) > 0$ s.t. if N is a closed Riemannian n-mfd sectional curvature are all in $[-1, -a]$, then every component of $N_{thin(\varepsilon)}$ is a tubular nbhd of a geodesic.

Thm Given $g \geq 2$, $\exists M_g$ s.t. if X is a closed hyp surf. of genus g, then it admits a geodesic pants decomp $\{c_1^*, \ldots, c_{3g-3}^*\}$ so $\ell(c_i^*) \leq M_g$, $\forall i$

Buser (book look at)

take $M_g = \underline{\text{small}} \cdot \underline{26(g-1)}$.

$$\ell(c_i^*) \lesssim 4i \log\left(\frac{8\pi(g-1)}{i}\right)$$

Lemma Given $A, B > 0$ $\exists C > 0$ so that if T is a cmpct hyp surf w/ geodesic ∂ and area$(T) = A$ & if C is a component of ∂T Then $\ell(C) \leq B$ then $\exists$ a simple closed geodesic γ^* s.t. $\ell(\gamma^*) \leq C$ & γ^* is not boundary parallel.

Pf Spse we can find γ^* so $\ell(\gamma^*) \leq 2\sinh^{-1}(1)$ Then stop! $\longrightarrow$ half collar nbhd

If not $\mathcal{H}(\partial T)$

Since the $\ell(\text{comp of } \partial T) \leq B$ $\Rightarrow$ continuity arg + compactness

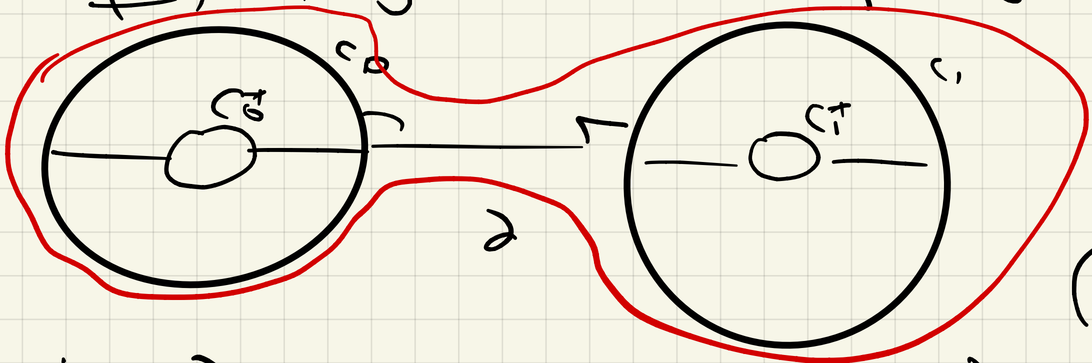
$\exists$ $L(B)$ s.t. $\ell(\partial\mathcal{H}(\text{comp of } \partial T)) \leq L(B)$

If $z \notin \mathcal{H}(\partial T) \Rightarrow r_{j_T}(z) > \sinh^{-1}(1) > \frac{1}{2}$ (abuse).

let p be a shortest closed geodesic joining $\partial\mathcal{H}(\partial T)$ to itself.

Note: P is simple, or we could shorten it (geodesic).

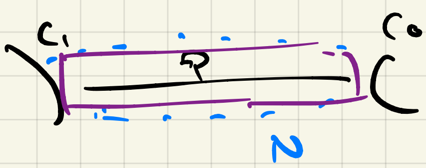
Spse, P joins distinct comp C_0 & C_1 of $\partial^\circ H(\partial T)$



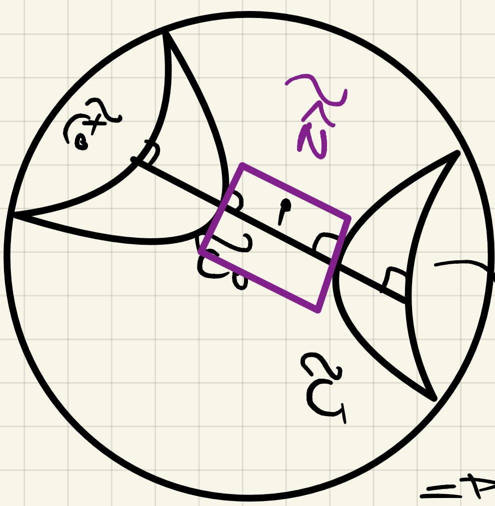
let ∂ be red dumbell $\quad \ell(\partial^*) \leq \ell(C_0) + \ell(C_1) + 2\ell(P)$

$\leq 2L(B) + 2\ell(P)$

let N = nbhd of radius $\frac{1}{2}$ of P



$\hat{N}$ points which proj perp to interior of P



if $\hat{N}$ doesn't embed

$\Rightarrow \exists \tilde{z} \subset \hat{N}$

$\rightarrow \tilde{C}^+$ and $\gamma \in \Gamma$ st $\gamma(z) \in \hat{N}$

If $z, \gamma(z)$ are far apart

$\Rightarrow \gamma(\tilde{C}_0^2)$ is closer to $\tilde{C}_0$ than $\tilde{C}_1$ which is a contradiction! (?)

If $d(z, \tilde{C}_0) < \frac{1}{2}\ell(P)$

Then $d(\tilde{C}_0, \gamma(\tilde{C}_0)) < d(\tilde{z}, \tilde{C}_0) + d(\gamma(z), \gamma(\tilde{C}_0))$