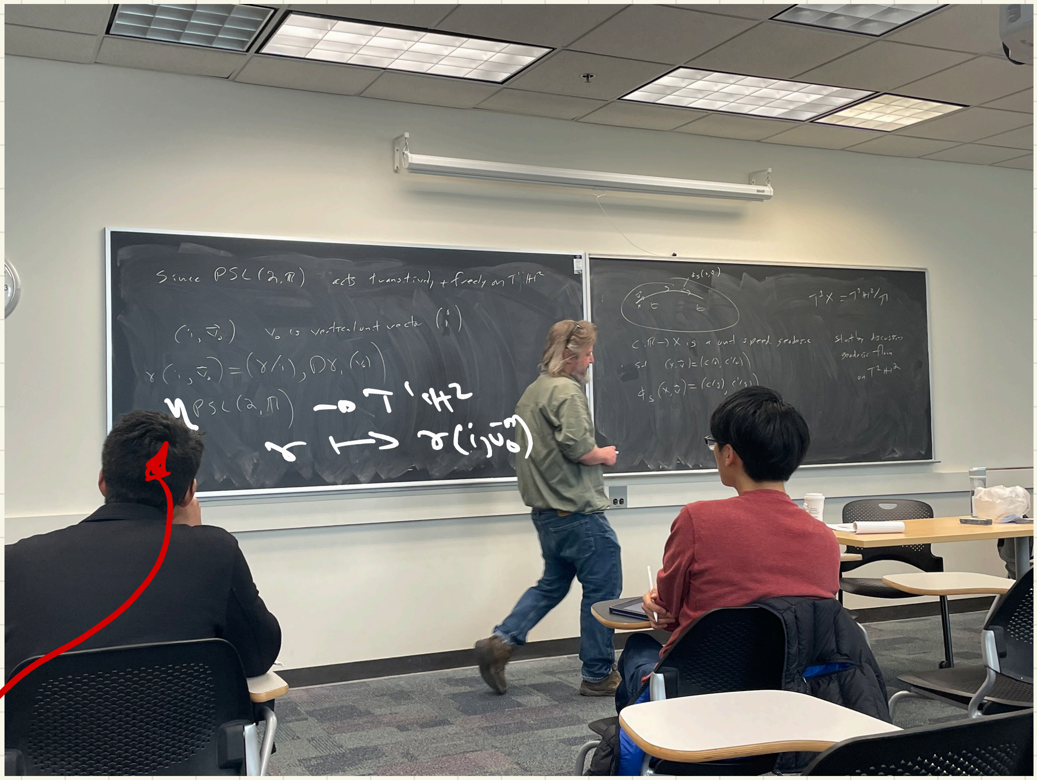


$\alpha, \gamma \in PSL(2,\mathbb{R})$

$$\gamma(\eta(\alpha)) = \gamma(\alpha(i), D\alpha_i(v_0))$$
$$= (\gamma \circ \alpha(i), D\gamma_{\alpha(i)}(D\alpha_i(v_0)))$$
$$= (\gamma \circ \alpha(i), D(\gamma \circ \alpha)_i(v_0))$$
$$= \eta(\gamma \circ \alpha)$$

$X = \mathbb{H}^2/\Gamma$ $\boxed{\text{A left action}}$

$$T^1X = \Gamma \backslash PSL(2,\mathbb{R})$$

(a Rep thy of PSL(2,R).

If $(x, \vec{v}) \in T^1\mathbb{H}^2$, $\exists$ unique unit speed geo $c : \mathbb{R} \to \mathbb{H}^2$

s.t. $c(0) = x$, $c'(0) = \vec{v}$

$c(\mathbb{R}) = \overleftrightarrow{wz}$ $\quad w, z \in \partial\mathbb{H}^2$ $\quad \lim_{t \to +\infty} c(t) = z$

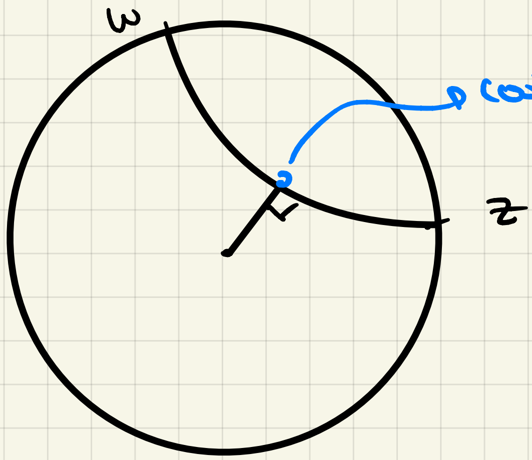
$$P : T^1 \mathbb{H}^2 \xrightarrow{} (\partial \mathbb{H}^2)^{(2)}$$

↳ fibre bundle & princ. $\mathbb{R}$-now

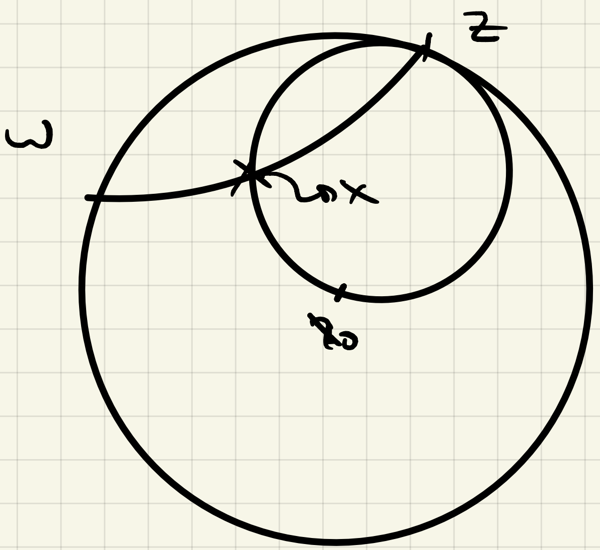
↳ fibre over (ω, z)

↳ $\{(c_{\omega z}(t), c_{\omega z}'(t)) \mid t \in \mathbb{R}\}$

Question: how to choose $c_{\omega z}(0)$



Choose $c_{\omega z}$ s.t $c_{\omega z}(0)$ lies on the horocircle based at z passing through x_0 ← origin



if $x_n \to z$ along $\overline{x_0 z}$

$S(d(x_0, x_n), x_n) \to H(x_0, z)$

↳ sphere ↳ horosp.

$$T^1 \mathbb{H}^2 \longleftrightarrow (\partial \mathbb{H}^2)^{(2)} \times \mathbb{R}$$

$$(c_{\omega z}(t), c'_{\omega z}(t)) \longleftrightarrow (\omega, z, t)$$

Geodesic flow $\Phi_s(\omega, z, t) = \Phi_s(c_{\omega z}(t), c_{\omega z}'(t))$

$= (\omega, z, s+t)$.

What is $\gamma(w,z,t)$?

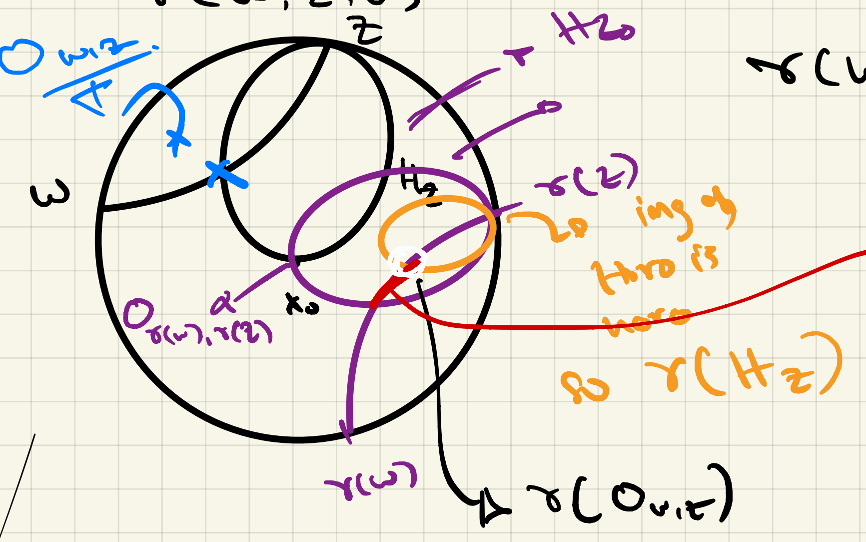
$= (\gamma(w), \gamma(z), t + \sigma)$ — smth!

↳ σ dep on w, z, γ

$\sigma(\gamma, w, z)$ ↳ no dep on t.

need to figure out where 0 goes.

Suffices to understand $\gamma(w,z,0)$



$\gamma(w,z,0)$

so no actual w dep.

$|\sigma(\gamma,z)| = d(H_{\gamma(z)}, \gamma H_z)$

then positive if γH_z inside $H_{\gamma(z)}$

negative if outside.

Busemann cocycle,

$$\sigma : PSL(2,\mathbb{R}) \times \partial\mathbb{H}^2 \longrightarrow \mathbb{R}$$

$$(\gamma, z) \longmapsto \log \gamma'(\gamma(z))$$

$$= -\log(\gamma^{-1})'(z)$$

↳ in Poincaré disc model

if $\gamma(z) = \lambda z$

<u>Now</u> $T^1\mathbb{H}^2 = PSL(2,\mathbb{R})$

$$T^1 X = {}_{\Gamma}\backslash PSL(2,\mathbb{R})$$

$$\phi_s(i, v_0) = (e^s i, e^s v_0)$$
$$= \eta\left(\begin{bmatrix} e^s & 0 \\ 0 & e^{-s} \end{bmatrix}\right)$$

$$\phi_s(\eta(\gamma)) = \phi_s(\gamma(i), D\gamma_i(v_0)) \quad \& \ \underline{\gamma, \phi \text{ com}}$$
$$= (\gamma\phi_s(i), D\gamma_{\phi_s(i)}(v_0))$$
$$= \left(\gamma \circ \begin{bmatrix} e^s & 0 \\ 0 & e^{-s} \end{bmatrix}(i), D\gamma \circ \begin{bmatrix} e^s & 0 \\ 0 & e^{-s} \end{bmatrix} v_0\right)$$

$$= \eta\left(\gamma \circ \begin{bmatrix} e^s & 0 \\ & e^{-s} \end{bmatrix}\right.$$