$$V(w,z,t) = \pm \begin{cases} e^{t/2}\begin{bmatrix} z \\ 1 \end{bmatrix}\frac{1}{1+z^2} & \text{if } z \neq \infty \\ e^{t/2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} & \text{if } z = \infty \end{cases}$$

$$P\left(\frac{e^{t/2}}{\sqrt{1+t^2}}\right) = z \quad \forall t$$

"The ray thru $\begin{bmatrix} z \\ 1 \end{bmatrix}$ is the set of horocycles based at z"

let $\gamma = \begin{bmatrix} c\sqrt{\lambda} & 0 \\ 0 & \frac{1}{c\sqrt{\lambda}} \end{bmatrix} = H_\lambda$

$$H\left(c\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \pm c\sqrt{\lambda}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

If γ is hyp w/ attr f.p γ^+ & $\gamma \sim H_\lambda$

$$\gamma\left(c\begin{bmatrix} \gamma^+ \\ 1 \end{bmatrix}\right) = c\sqrt{\lambda}\begin{bmatrix} \gamma^+ \\ 1 \end{bmatrix}$$

if $\gamma\left(c\begin{bmatrix} z \\ 1 \end{bmatrix}\right) = \partial\begin{bmatrix} \gamma(z) \\ 1 \end{bmatrix}$ for some $\partial \in \mathbb{R}_{\geq 0}$

Fact If Γ is not elementary $\Rightarrow \exists\ (x,v) \in \Omega_h(\Gamma' x)$ so, $\{h_s(x,v) \mid s \in \mathbb{R}\}$ is dense in $\Omega_h(\Gamma' x)$

Recall, $\Omega_h(\Gamma) = \{(w,z,t) \mid z \in \Lambda(\Gamma)\}$

$$\Omega_h(\Gamma' x) = \Omega_h(\Gamma)/\Gamma$$

Fact If Γ is non-elem $z \in \Lambda(\Gamma)$ is a conical lim pt then $\{h_s(w,z,t) \mid s \in \mathbb{R}\}$ is dense $\forall w \in \partial\mathbb{H}^2 - \{z\}$ $\forall t$.

Proof! Spose, $(y, \vec{u}) = \pi(\omega, \gamma^+, t)$ & $a^+ = V(\omega, \gamma^+, t) \in E(\Gamma)$

set of all noro ap pl

$V(\Omega_n(\Gamma))$

Cl! $\Gamma(a^+)$ is dense in $E(\Gamma)$

$\approx$ $\{h_s(y,u) \mid s \in \mathbb{R}\}$ is dense in $\Omega_n(T^1(x))$

We know $\exists\, u \in E(\Gamma)$ s.t. $\Gamma(u)$ dense in $E(\Lambda)$

as Γ acts linearly $\Gamma(u)$ dense $\Rightarrow \forall c > 0$ $\Gamma(cu)$ dense.

So, it suffices to show $cu \in \overline{\Gamma(a^+)}$ for some $c > 0$.

since this implies $\overline{\Gamma(cu)} \subseteq \overline{\Gamma(a^+)}$

As Γ acts minimally on $\Lambda(\Gamma)$ $\to$ lim set

$\exists\, \{\alpha_n\} \subseteq \Gamma$ s.t. $\alpha_n(\gamma^+) \longrightarrow p(u)$.

Let us spse that γ is conj to H_λ w $\lambda > 1$

Choose $\{q_n\} \subseteq \mathbb{Z}$ so $\lambda^{q_n/2} \| \alpha_n(o) \| \to c > 0$

(may req subseq)

$\| \alpha_n \gamma^{-q_n}(a^+) \| \longrightarrow c$

$\Rightarrow \alpha_n \gamma^{-q_n}(a^+) \longrightarrow d$

$p(\alpha_n \gamma^{-q_n}(a^+)) = p(\alpha_n(\gamma^+)) \longrightarrow p(u) = p(d)$

$\Rightarrow \alpha_n \gamma^{-q_n}(a^+) \longrightarrow c \cdot u$ some $c > 0$

$\Rightarrow cu \in \overline{\Gamma(a^+)}$ $\Rightarrow \Gamma(a^+)$ dense in $E(\Gamma)$

$\Rightarrow \{h_s(\pi(\omega, \gamma^+, t)) \mid \forall s \in \mathbb{R}\}$ is dense in $\Omega_n(T^1 x)$.

Now supp z is a conical limit point of

Γ which is not a hyp FP.

pick $\gamma \in \Gamma$ hyperbolic show that $\exists \{\beta_n\}$ s.t

. - - -

- - - . . . .

. . . - - -

$\beta_n(x)$ conv to ∂

so $p(\partial) = \gamma^-$

first part — - - —

Choose $\{\alpha_n\} \subseteq \Gamma$ st $\alpha_n(w, z, t)$ app z conically

(w_n, z_n, t_n)

$z_n \to z$, $t_n \to \infty$.

$\gamma(\alpha_n(w,z,t)) = (\alpha_n(a))$

$\| (\alpha_n(a)) \| \to \infty$

Supp, $\gamma = H_\lambda$ choose q_n s.t. $\lambda^{q_{n/2}} \| \alpha_n(a) \| \to c > 0$

$\Rightarrow \gamma^{q_{n/2}} \alpha_n(a) \to \partial$, $r_n \to -\infty$

$P(\gamma^{q_n}(\alpha_n(a))) = P(\gamma^{q_n}(z)) \to \gamma^-$

So $\gamma^{q_n} \alpha_n(a) \to \partial$ st $p(\partial) = \gamma^-$ so

$\overline{\Gamma(\partial)} = E(\Gamma)$

$\Rightarrow \overline{\Gamma(a)} = E(\Gamma)$

Corollary | if X is a closed hyperbolic surface
$\Rightarrow$ every horocycle traj is dense
in T^1X.

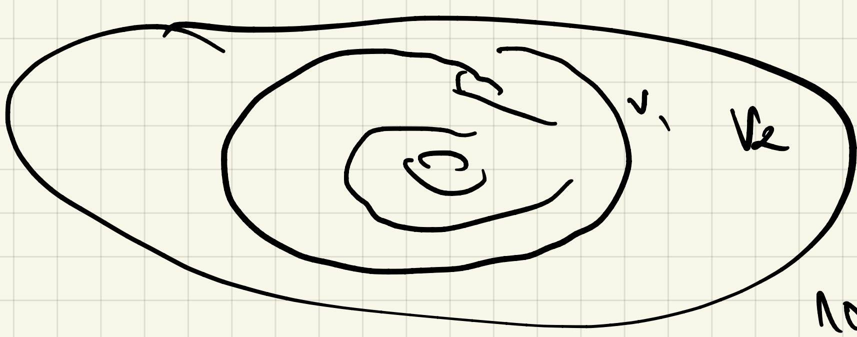
We say Γ is convex co-compact if every point in $\Lambda(\Gamma)$ is conical.

If Γ is disc, tors. free, fin gen & has no hyperbolic elements $\Rightarrow$ convex co-compact.

"Proof" | H^2/Γ is a surf w/ fin gen fundamental grp.

$\Rightarrow$ homeo, to int of cmpct surface.

Consider take solid torus inside solid torus knotted



$\pi_1(V_1) = \mathbb{Z}$
$i_*(\pi_1(V_1)) = \{1\}$ in $\pi_1(V_2)$

Now keep doing inf times
$\Rightarrow$ simply conn mfd not ball

$\pi_1(V_2 \setminus V_1)$ is complicated
$\Rightarrow \pi_1(\partial V_2)$ injects in but doesn't surject.

So, something abt a funny first thing...