Collar lemma 1 |

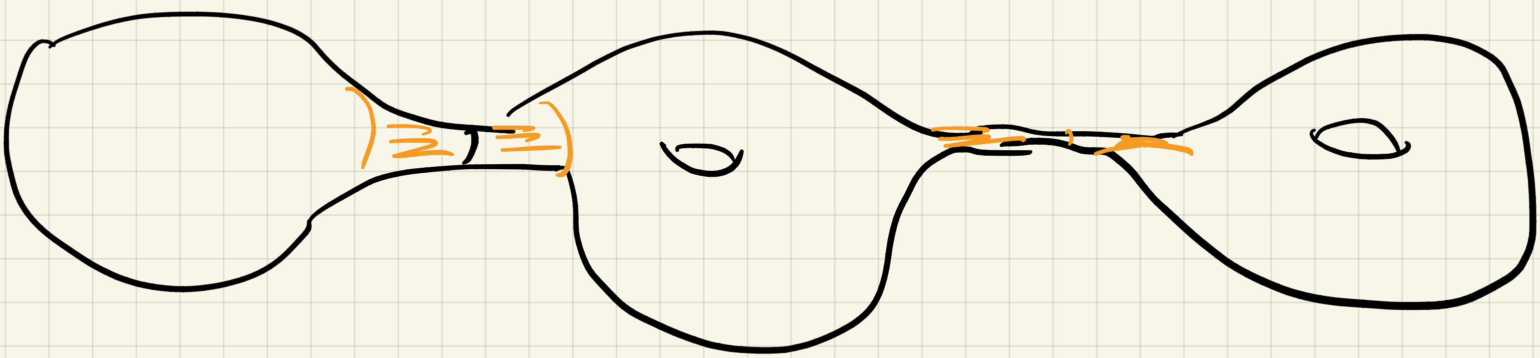
If $\{c_1^*, \dots, c_n^*\}$ is a collection of disj, simple, closed geodesics on a closed hyp surface X. $\rightsquigarrow$ width.

let $C(c_i^*) = \{ z \in X \mid \sinh d(z, c^*) \sinh \frac{\ell(c^*)}{2} \le 1 \}$

then $\{C(c_i^*)\}$ is a collection of annular nbhds of c_i^*. "collar"

$$X_{thin(\sinh^{-1}(1))} = \{ z \in X \mid inj_X(z) < \sinh^{-1}(1) \}$$

Upshot: This is a "collection of subcollars" of collars of short geodesics.



lemma | c^*, d^* are closed geodesics in X which intersect transversally & c^* is simple

$$\sinh\left(\frac{\ell(c^*)}{2}\right) \sinh\left(\frac{\ell(d^*)}{2}\right) > 1$$

Cor | If c^*, d^* are both simple (not equal) & $\ell(c^*)\ell(d^*) \le 2\sinh^{-1}(1)$

Then they are disj. (intersect transversally if they do as it is the case in $\mathbb{H}^2$)

Pf | If not, quantity ≤ 1 oops.

Pf of lemma 1 Notice, → geodesic is shortest curve in htpy class.

δ^* cannot be contained in $\mathcal{C}(c^*)$

since any curve in $\mathcal{C}(c^*)$ is htpc to multiple of c^*, so it but the closed geodesic htpc to multiple of c^* is itself, mult of c^* $\Rightarrow$ no transversal int!

$$\ell(\delta^*) > 2\,\underbrace{w(\mathcal{C}(c^*))}_{\text{width of collar}} = 2\sinh^{-1}\left(\frac{1}{\sinh\left(\frac{\ell(c^*)}{2}\right)}\right)$$

↳ rearrange for result as sinh is increasing!

$$\Rightarrow \sinh\left(\frac{\ell(\delta^*)}{2}\right)\sinh\left(\frac{\ell(c^*)}{2}\right) > 1$$

Cor) The set of simple closed geodesic of length $< 2\sinh^{-1}(1)$ is disj

$\Rightarrow$ there is at most $3g-3$ simple closed geodesics of length $< 2\sinh^{-1}(1)$.

Prop) if $z \in X$ and $\mathrm{inj}_X(z) < \sinh^{-1}(1)$

$\Rightarrow \exists$ simple closed geodesic c^* so

$\ell(c^*) < 2\sin^{-1}(1)$

& $x \in \mathcal{C}(c^*)$

& if $D = d(z, c^*)$

$$\Rightarrow \sinh \mathrm{inj}_X(z) = \sinh\left(\frac{\ell(c^*)}{2}\right)\cosh(D)$$

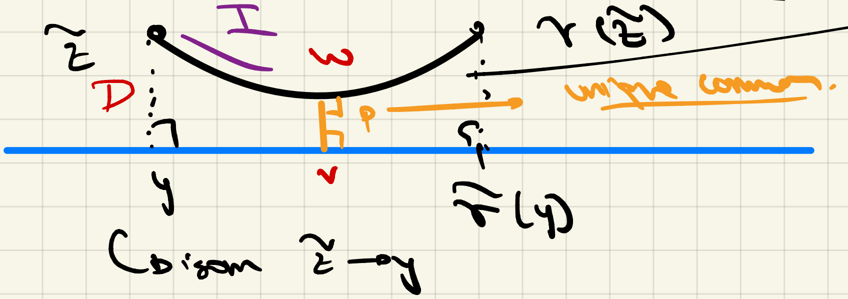
Pf) $I = \mathrm{inj}_X(z)$ & c be a simple, htpically nontriv, closed curve based at z

$X = \mathbb{H}^2/\Gamma$ so $\ell(c) = I \cdot 2$.

c lifts to a geodesic segment $\tilde{c} \subseteq \mathbb{H}^2$

If $\tilde{z}$ is the lift of z which $\tilde{c}$ starts at

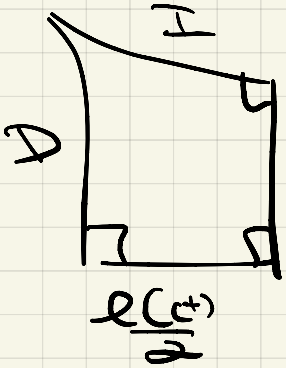
$\Rightarrow$ $\tilde{c}$ ends at $\underline{\gamma(\tilde{c})}$. $\gamma(\tilde{z})$ $\to$ 2 "trirectangles"



A = Axis(γ)

$c^* = \pi\left(\overline{y\,\gamma(y)}\right)$ so $\ell(c^*) < \ell(c) = 2\sinh^{-1}($

Need to show, $D < w(c^*) = \sinh^{-1}\left(\frac{1}{\sinh\left(\frac{\ell(c^*)}{2}\right)}\right)$



$\to$ formula for trirect $\to$ $1 \geq \sinh I = \sinh\left(\frac{\ell(c^*)}{2}\right)\cosh D$

$> \sinh\left(\frac{\ell(c^*)}{2}\right)\sinh D$

$\Rightarrow \sinh D < \frac{1}{\sinh\left(\frac{\ell(c^*)}{2}\right)}$

$\Rightarrow D < \dots\dots\dots$

so $z \in \mathcal{C}(c^*)$

no self loop → not a multiple of another geodesic.

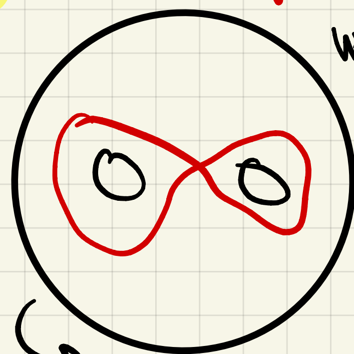
Lemma | If γ^* is a primitive, non-simple closed geodesic, then $\ell(\gamma^*) > 1$

Truth | $\ell(\gamma^*) > 4\sinh^{-1}(1)$ "Talk open" → htpy class

Spse | $\ell(\gamma^*) \le 1$

let $z \in \gamma^*$

$$\Rightarrow I = \text{inj}_x(z) \le \frac{1}{2}$$



↳ make length of all curves → 1 & go to 1

$\exists\, c^*$ so $\ell(c^*) < 2\sinh^{-1}(1)$

& $z \in \text{int}\, \mathcal{C}(c^*)$

Moreover $\sinh I = \sinh \frac{\ell(c^*)}{2} \cosh D$, $D = d(z, c^*)$

$$R = \overbrace{w(c^*)}^{w} - D$$

↳ dist from $z \longrightarrow \partial\mathcal{C}(c^*)$

↳ $d(z, \partial\mathcal{C}(c^*))$

So $\sinh I = \sinh \frac{\ell(c^*)}{2} \cosh(w - R)$ ← compound angle

$\sinh\frac{1}{2} >$

$$= \sinh \frac{\ell(c^*)}{2} \left(\cosh w \cosh R - \sinh w \sinh R\right)$$

$$= \underbrace{\sinh \frac{\ell(c^*)}{2} \cosh w}_{\text{notes}} \cosh R - \sinh R$$

$$= \underbrace{\cosh^2 \frac{\ell(c^*)}{2}}_{\ge 1} \cosh R - \sinh R$$

0.521

$\ell(c^*) \ge 2R \ge 1$

$$> \cosh R - \sinh R = e^{-R}$$

So $\sinh\frac{1}{2} > e^{-R}$, $e^{-\frac{1}{2}} \sim 0.60 \Rightarrow \boxed{R > \frac{1}{2}}$