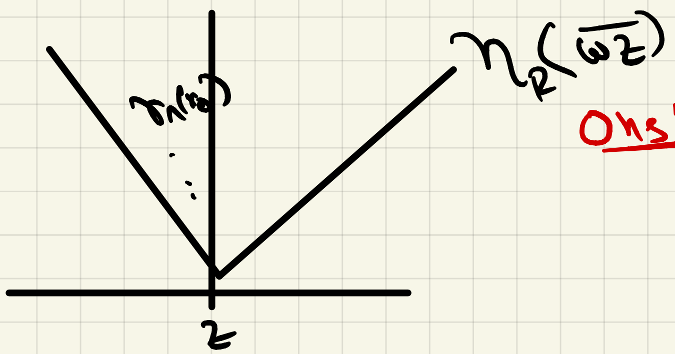
If $\phi : \mathbb{R}\times Y \to Y$ is a flow a point $y \in Y$ is pos divergent if $\nexists\, s_n \to \infty$ st $\phi_{s_n}(y)$ conv in Y

neg div if $\nexists\, s_n \to -\infty$ st $\phi_{s_n}(y)$ conv in Y

We say $z \in \partial \mathbb{H}^2$ is a conical limit pt for $\Gamma \leq PSL(2,\mathbb{R})$ if $\exists$ geodesic $\overline{wz}$ and a seq $\{\gamma_n\}$ in Γ st

$\gamma_n(x_0) \to z$ and $\exists R$ st $d(\gamma_n(x_0), \overline{wz}) \leq R$ $\forall n$



Obs1 If so $\forall\, x \in \mathbb{H}^2$ and any $v \in \partial \mathbb{H}^2 - \{z\}$ $\exists\, \hat{R}$ st $d(\gamma_n(v), \overline{vz}) \leq \hat{R}$

Key fact 1 If x_n is a seq on $\overline{wz}$ st $x_n \to z$
$\exists$ a seq y_n on $\overline{vz}$ $y_n \to z$ $d(x_n, y_n) \to 0$

Fact 2 $\pi(w,z,t) \in \Gamma' X$ is pos div iff it ends in a conical limit pt (z)

Similarly neg div $\iff$ z not a con limit pt.

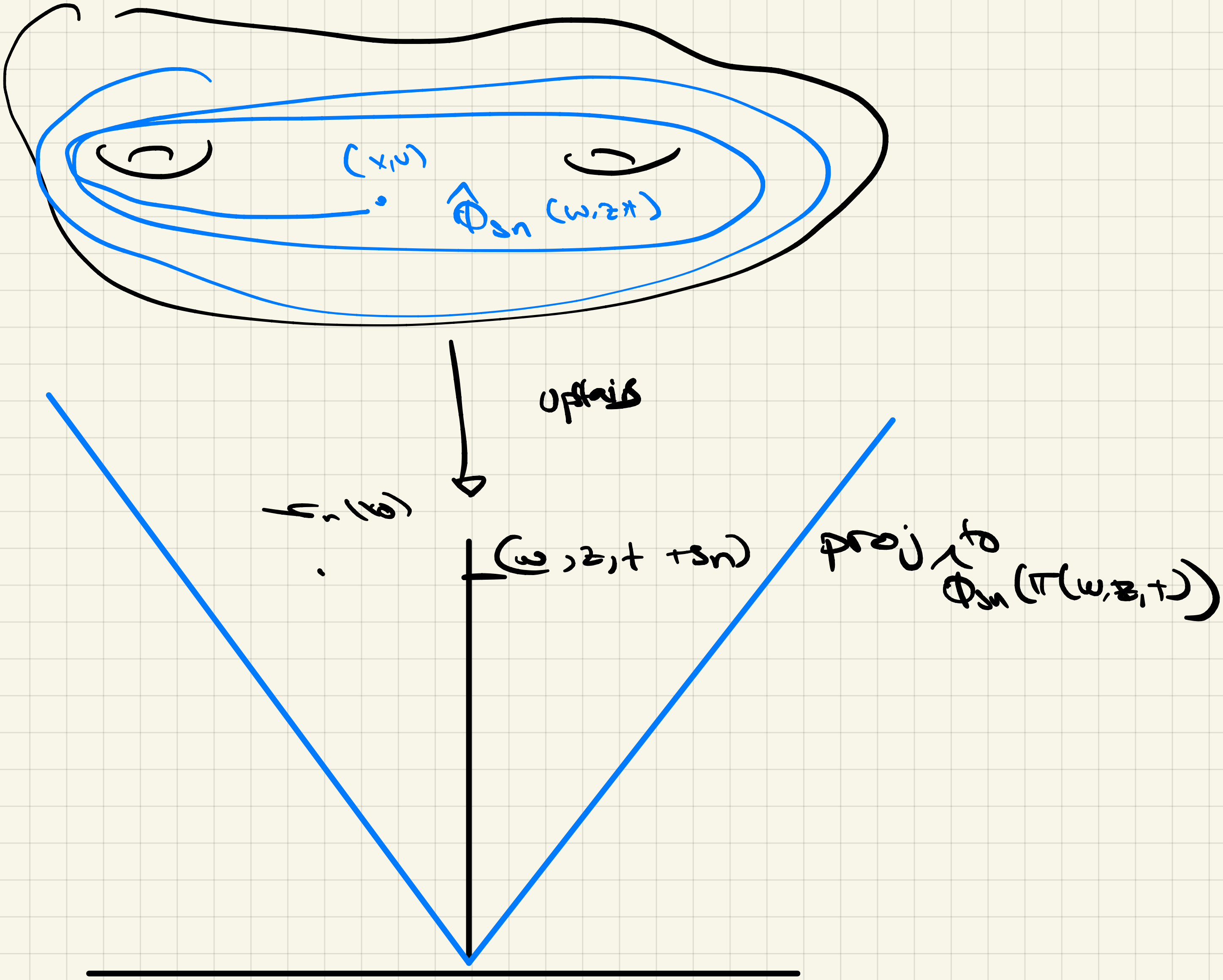
Suppose $\pi(w,z,t)$ is not pos div $(\Leftarrow)$

$\exists\, s_n \to +\infty$ so $\phi_{s_n}(\pi(w,z,t)) \to (x,\vec{v}) \in \Gamma' X$
$$= (x_n, \vec{v}_n)$$

Since $(x_n, v_n) \to (x,v) \Rightarrow \exists R$ so $\forall n$
$d(\pi(x_0), x_n) \leq R$

Upstairs, if $(w, z, s_n + t) - (y_n, \vec{u}_n)$
and $\exists\, \gamma_n \in \Gamma$ st $d(y_n, \gamma_n(x_0)) \leq R$

$\phi_{s_n}(w,z,t) = (w,z,t+s_n)$ $\pi(\phi_{s_n}(w,z,t)) = \phi_{s_n}(w,z,t)$



So $\partial(\gamma_n(x_0), \overline{wz}) \le R$ & moreover

as $s_n \to \infty \Rightarrow \gamma_n(x_0) \to z$

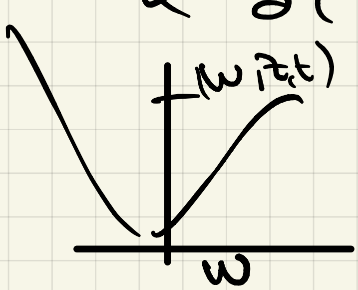
$\Rightarrow z$ is conical lim pt

Consider | $\pi(w,z,t) \in T^1X$ & suppose z is a conical lim pt

(wts not pos div).

$\exists$ seq $\gamma_n \in \Gamma$ so $\gamma_n(x_0) \to z$

& $\partial(\gamma_n(x_0), \overline{wz}) \le R$ $\forall n$ & some R.



Choose $y_n \in \overline{wz}$ so $d(y_n, \gamma_n(x_0)) \leq R$

Since $\gamma_n(x_0) \to z$, $y_n \to z$

Let $v_n \to$ unit tgt vec to $\overline{wz}$ at y_n point towards z

$(y_n, \vec{v_n}) = (w, z, s_n + t)$ and $s_n \to \infty$ so $y_n \to z$

$\varphi_{s_n}(w, z, t) = (w, z, t + s_n)$

So $\hat{\varphi}_{s_n}(\pi(w, z, t)) = \pi(w, z, t + s_n)$

$= (x_n, \vec{v_n})$

$\to$ projection of prev pic to surface $X = \mathbb{H}^2/\Gamma$

$d(x_n, \pi(x_0)) \leq R$ $\forall n$.

But the set of unit tgt vec in $T'X$ whose base pts lie distance $\leq R$ from $\pi(x_0)$ is cmpct set. So, $\hat{\varphi}_{s_n}(\pi(w, z, t))$ has a conv subseq. Cool! $\Rightarrow \pi(w, z, t)$ is not pos div!