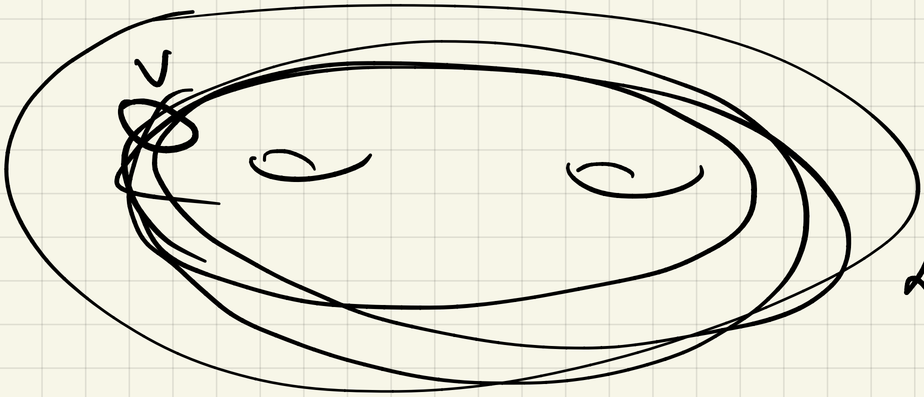
Fact) $\Omega_n(T^1 X)$ is the non-wandering part of horocycle flow

$\Omega(T^1X) = \pi(\{(w,z,t) \mid z \in \Lambda(\Gamma)\})$

We know $\exists\, (x,v^0) \in \Omega_n(T^1X)$ s.t. $\{h_s(x,v^0)\}$ is dense in $\Omega_n(T^1X)$.



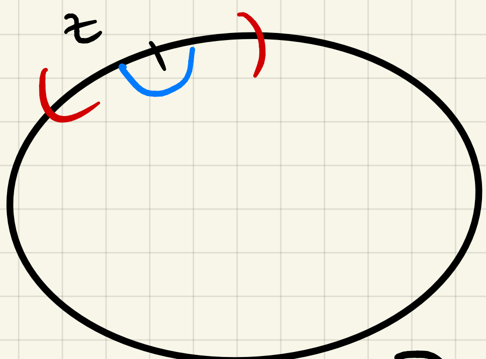
(y,v^0) is non wander if $\forall$ open nbhd V of (y,v) $\{s \mid h_s(V) \cap V \neq \emptyset\}$ is unbdd

since $h_s(x,v)$ is dense
$\Rightarrow \{s \in S \mid h_s(x,v) \in V\}$ is unbdd

so diff $s-s'$ is unbdd & arb sm.

So if $(y,u) \in \Omega_n(T^1X)$ it is non-wandering.

$\pi(x,z,t)$ if $z \notin \Lambda(\Gamma)$ in



Ω_0 downstairs horocycle looks like it is shooting off to ∞ in an embedded half plane.

$H_{z,t}$ hor thru (w,z,t)
$\exists$ half plane D w/ $z \in \partial_\infty D$
so $\gamma(D) \cap D = \emptyset$ $\forall \gamma \neq id$.

So, $\partial(\,b(h_s(x,z,t)),\,\pi(x_0)) \geq \partial((x,z,t),\,\partial D)$

if $(x,z,t) \in D$

But $h_s(x,z,t) \notin D$ if $|t|$ is large.

So, $\partial((x,z,t),\,\partial D) \longrightarrow \infty$ as $t \longrightarrow \pm\infty$

$\Rightarrow$ $h_s(\pi(x,z,t))$ leaves every compact set in T^1X

So, $\pi(x,z,t)$ is wandering.

If z is a parabolic fixed point, fixed by $\gamma \in \Gamma$

$\pi(H_{z,t})$ is a periodic trajectory $\forall t$

On the other hand if

$h_s(\pi(w,z,t)) = \pi(w,z,t)$ for $s > 0$

$\Rightarrow \exists\, \gamma \in \Gamma$ so $\gamma(w,z,t) = h_s(w,z,t)$

$\Rightarrow \gamma(z) = z$ γ pres the horo $H_{z,t}$.

$\Rightarrow \gamma$ is parabolic.

In summary, periodic orbits of horocycle flow in T^1X are exactly quotients of horocycles based at parabolic fixed points of Γ.

Fact: If the horocycle flow on T^1X has a periodic trajectory, then periodic traj of the horocycle flow are dense in $\Omega_h(T^1(X))$

Since $\exists$ period traj, $\exists\ \alpha \in \Gamma$ parabolic

let $a \in \Lambda(\Gamma)$ be a f.p. of α.

Consider $\pi(\omega, z, t) \in \Omega_n(T(x))$, so $z \in \Lambda(\Gamma)$

$\exists$ seq $\{\gamma_n\} \subseteq \Gamma$ so that $\gamma_n(a) \to z$

$(\omega, \gamma_n(a), t) \longrightarrow (\omega, z, t)$

$\pi(\omega, \gamma_n(a), t)$ lies on a periodic trajectory

since $\gamma_n(a)$ is fixed by parabolic elt $\gamma_n \alpha \gamma_n^{-1}$

$\pi(\omega, \gamma_n(a), t) \longrightarrow \pi(\omega, z, t)$

Γ is geom finite if every limit pt of Γ is conical or parabolic f.p.

Fact) if Γ fin gen it is geom finite (and vice versa)

Def) flow $\phi: \mathbb{R} \times Y \to Y$ is top mixing if given any two open set $U, V \subseteq Y$ if $\exists T$ s.t. $\phi_t(U) \cap V \neq \emptyset\ \forall t \geq T$

Fact) The geodesic flow is topologically mixing on $T^1 X^{nw}$ ~~p non wnd~~

Let U be an open set $T^1 X^{nw}$

Since periodic pts are dense in $T^1 X^{nw}$ $\exists (x, \vec{v}) \in U$ lie on period orbit of geodesic flow of par P

So, if we know $\Phi_{np}(x,\bar{v}^{\circ}) = (x,\bar{v}^{\circ})$ $\forall n \in \mathbb{Z}$

So, $\Phi_{np}(\mathcal{U}) \cap (\mathcal{U}) \neq \emptyset$ $\forall n \in \mathbb{N}$

Lets choose $q \in (0,P)$

Cl: $\Phi_{nP+q}(\mathcal{U}) \cap \mathcal{U} \neq \emptyset$ $\forall$ large n

Since $(x,\bar{v}^{\circ}) = \pi(\gamma^-, \gamma^+, t)$ for some hyp $\gamma \in \Gamma$

$\Rightarrow \{h_s(x,\bar{v}^{\circ})\}$ is dense in $\Omega_n(T'(x))$

By def $T' x^{nw} \subseteq \Omega_n(T'(x))$

$\Rightarrow \exists s_0$ s.t. $h_{s_0}(x,\bar{v}^{\circ}) \in \underline{\hat{\Phi}_q(\mathcal{U})}$.

So, $\exists (y,\bar{u}^{\circ}) \in \mathcal{U}$ so $h_{s_0}(x,\bar{v}^{\circ}) = \hat{\Phi}_q(y,u)$

So, $\hat{\Phi}_{nP+q}(y,\bar{u}^{\circ}) = \hat{\Phi}_{np}\, h_s(x,\bar{v}^{\circ})$

$= h_{e^{-nP}s} \circ \hat{\Phi}_{np}(x,v)$

$= h_{e^{-nP}s}(x,v)$

$\downarrow$ conv as goes to $\underline{0}$.

$\underline{(x,v)}$

So $\hat{\Phi}_{nP+q}(y,\bar{u}) \in \mathcal{U}$ for large enough $\underline{n}$.