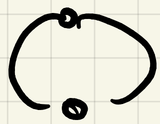
Defn) generalized simp. complex

(Δ-cmplx where every simplex is embedded)

e.g  so vertices allowed to span multiple simpl.

pure - all max'll simpl. have same dim

Finite cmplx is shellable if $\exists$ ordering of max'll simpl. $F_1, \ldots, F_t$

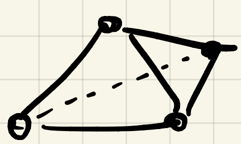
s.t. $\left(\bigcup_{i=1}^{k-1} F_i \right) \cap F_k$ ~ a pure sub of ∂F_k $\subseteq X$ ~> nice

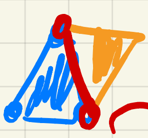
pure of dimension $(\dim(F_k)) - 1$

gluing F_k along pure subcomplex of bdry.

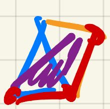
Defn) shellable cmplx ↗

max'll simpl. admit a shelling order.

e.g)  $\partial \Delta^3$ any ordering is a shelling ord

 ~> pure of dim 1

g any contractible along contr subsp ⇒ no change htpy type

 ~> pure of dim 1

but not the last face. Glued along $\sim S^1$

2 cases

① → F_k glued along proper subcomplex of bdry → is included

↳ prove, proper subcomplex is contractible

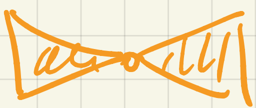
↳ no change htpy type

② → F_k is glued along entire ∂F_k → spanning simplices

$$F_k/\partial F_k = S^{\dim(F_k)}$$

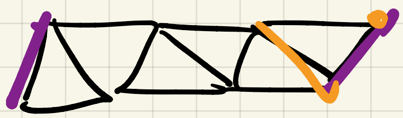
Thm If X is shellable

$$X \simeq \bigvee_{F_i\ sp} S^{\dim(F_i)}$$

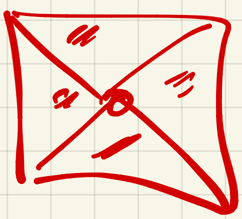


→ not shellable,

not shellable



→ bracket triangulated



→ shellable in only clockwise/counter clockwise order

ex6 $\Delta^{n,k} = (\Delta^n)^{(k)}$ is shellable

↳ vertex set $[n+1]$ → subsets of size $\leq k+1$

Max'l simplices – subsets of $[n+1]$ of size $k+1$

Shelling order, order subsets from within (can make precise) then order lexicographically.

So $\{1,2,\ldots,k+1\}, \{1,2,\ldots,\widehat{k+1},k+2\}, \ldots$

Let us show that this is a shelling order

$F_0 = 1 \; 2 — k{+}1 \qquad F_1 = 1 \; 2 — \widehat{k{+}1} \; k{+}2$

$F_0 \cap F_1 = 1 — k$ | codim 1 ☺

⋮

$F_j = a_0 — a_k$ → span some subsimplex $b_0 — b_p$ is in

intersect → subsimplex

↳ was contained in something of dim $(k-1)$ in intersection.

$\exists \; a_0' — a_k'$ so $b_0 , — , b_p \in$

$\{a_0', —, a_k'\} \cap \{a_0 — a_k\}$.

earlier

consider components of $b_0 — b_p$ in the set

$\underbrace{a'_{i_1} — a'_{i_{k-p}} \qquad a_{i_1} — a_{i_{k-p}}}$

Something ... make smaller.

Spanning Simpl. $\iff$ simplices that don't contain 1

↳ $\forall i \; \{a_0, .. \hat{a}_i, ..., a_k\} \cup \{1\}$
is lex-earlier face.
If it has 1 $\{\hat{1}, a_1 —, a_k\}$
this $(k-1)$ face