"Badness Arg"

Y is an abstract simpl. complex $X \overset{\text{subcmplx}}{\subseteq} Y$. Fix $d \geq 0$.

Thm 1 We have a coll of bad simpl. in $Y \setminus X$ sat.

(i) σ is simplex so that no subsimplex $\rho \subseteq \sigma$ bad $\Rightarrow \sigma$ in X

(ii) if σ, ρ are bad & $\sigma \cup \rho$ is simplex $\Rightarrow \sigma \cup \rho$ bad
union vertex

(iii) For each bad simplex the subcomplex $G_\sigma \subseteq Lk_Y(\sigma)$

good simplices

$G_\sigma := \{ \rho \in Lk_Y(\sigma) \mid$ all bad sub simp of $\rho \cup \sigma$ is in $\sigma \}$

is $(d - \dim(\sigma) - 1)$-conn

Then $(|Y|, |X|)$ - is d connected

↳ inclusion $X \hookrightarrow Y$ isom on π_k $k < d$
surj on π_d.