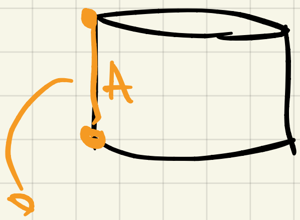
Homotopy ext prop

If CW complex X, subcomplex A

Then $(X \times I)$ deform retr. on $(X \times \{0\}) \cup (A \times I)$

e.g. (htpy ext prop)



$\xrightarrow{\tilde{F}_0} Y$

$\exists$ htpy F_t with $F_0 = \tilde{F}_0|_A$

$\Rightarrow \exists \tilde{F}_t : X \to Y$ extending F_t & $\tilde{F}_0$ in the sense that

$F_t = \tilde{F}_t|_A$

$X \qquad A \subseteq X \qquad h : Y \xrightarrow{\simeq} Y' \qquad \sim$

$f : A \to Y$

$f' : A \to Y' \qquad f \simeq h \circ f$

$$\begin{tikzcd} & Y \arrow[d, "h"] \\ A \arrow[ur, "f"] \arrow[r, "f'"'] \arrow[ur, phantom, shift right=3, "\circlearrowleft"] & Y' \end{tikzcd}$$

$H : X \cup_f Y$

$\downarrow$

$X \cup_{f'} Y'$

Context] + Remarks   note $SS^{n-1} = S^n$.

Corollary 9] If X CW cmpx w/ A sub so $i : A \hookrightarrow X$ null homo.

$\Rightarrow X/A \simeq X \vee SA$

if X is contr. then, all i will be null homo,

$\Rightarrow X/A \simeq X \vee SA \simeq SA$.

e.g] so quotient $\mathbb{R}^n$ by cmpct submanifold A

so $\mathbb{R}^n/A \simeq SA$.

$\hookrightarrow$ observe, $S^n \simeq D^n/\partial D^n \simeq S(\partial D^n)$ → disc contractive.

$= SS^{n-1}$

→ Later, $X \simeq \vee S^n$, $A \simeq \vee S^{n-1}$

$A^{\text{subcompl.}} \subseteq X$ w/ $i : A \to X$ null homo, → works well with $\simeq$.

$\hookrightarrow X \cup_i CA \underset{8}{\simeq} X/A \underset{9}{\simeq} X \vee SA \simeq X \vee (S(\vee S^{n-1}))$

think $\simeq X \vee (\vee S^n)$