Defn X, Y top-spaces.

$X * Y$ ← quotient of $X \times Y \times I$

where $X \times Y \times \{0\}$ is collapsed to X

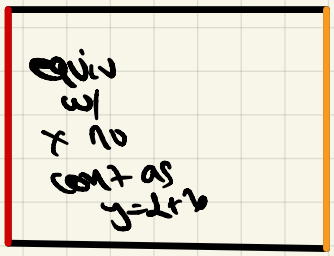
$X \times Y \times \{1\}$ is collapsed to Y

$(x, y_1, 0) \sim (x, y_2, 0)$ $\quad \forall\, x \in X,\ y_1, y_2 \in Y$

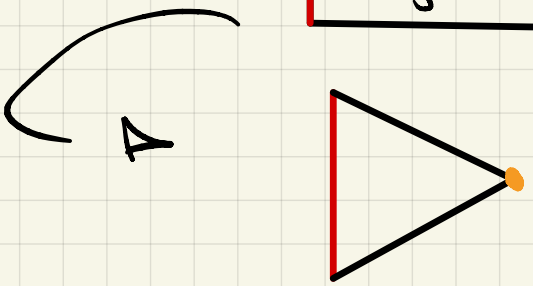
$(x_1, y, 1) \sim (x_2, y, 1)$ $\quad \forall\, y \in Y,\ x_1, x_2 \in Y$.

Eg n simplex iterated join of n pts.

$X =$  , $Y =$ 

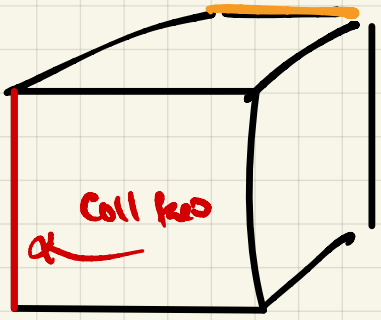
$X \times Y \times I \longrightarrow$ 

coll to pt



Now, if $X =$

$Y =$ 

$X \times Y \times I =$ 

join of anything w/ pt is cone

join w/ S^0-many pts is suspension

Given X, Y abst. simp cmplx

Then $X * Y$ is abstr simp cmplx def as

$V(X*Y) = V(X) \cup V(Y)$

$S(X*Y) = S(X) \cup S(Y) \cup \{\tau \cup \gamma \mid \tau \in S(X), \gamma \in S(Y)\}$

Thm $|X*Y| = |X| * |Y|$ as — geom realizations.