Simplicial Cmplx

Geom $\xleftarrow{\text{realization}}$ **Absr**

- Top sp, simpl. + gluing
- vertex set $V(X)$, simplices $S(X)$. Cow/ axioms

Simplicial Grp Act

X - abstract simpl. cmplx

$G \curvearrowright X$ simplicially (so acts on $V(X)$ so simpl $\to$ simpl)

$\rightsquigarrow$ $G \curvearrowright |X|$ cts (on realization)

Defn) X/G abstract simplicial cmplx

vertices $V(X)/G =: V(X/G)$.

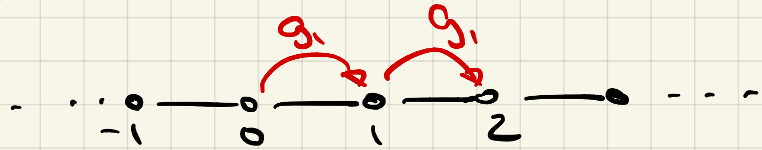
$$\begin{array}{c} V(X)/G \\ \cup \\ v_0^* \end{array} \longleftarrow v_0\text{'s orbit under } G$$

simplex $\{v_0^*, v_1^*, \ldots, v_p^*\} \in S(X/G)$

iff $\exists$ choice of rep in orbit $\{\tilde{v}_0, \tilde{v}_1, \ldots, \tilde{v}_p\} \in S(X)$

Q1 What abt $|X|/G$ & X/G ?

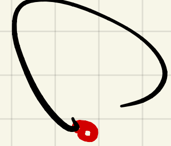
$|X/G|$ may not be in general. $|X|/G$ $\to$ May not be simpl. complex.

e.g.) $\mathbb{Z} \curvearrowright \mathbb{R}$ with 

$\hookrightarrow$ abs simp cmplx $V(\mathbb{R}) = \mathbb{Z}$

$S(\mathbb{R}) = \{\{n\}, \{n, n+1\}\}_{n \in \mathbb{Z}}$

$G \cong \mathbb{Z} = \{g_n\}_{n \in \mathbb{Z}}$ $\quad G \curvearrowright \mathbb{R}$ by trans

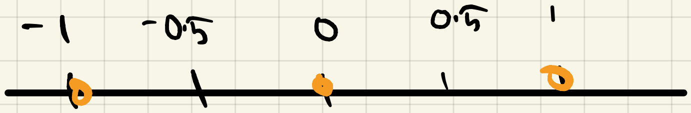
We know Quotient $|X|/G$ is  $\cong S^1$

$= V(X/G)$

not simpl cmplx

But! $V(X)/G = \{*\}$ act trans on vertex set $V(X)$.

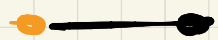
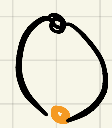
$\therefore \longrightarrow |X/G| = *$ bruh.....

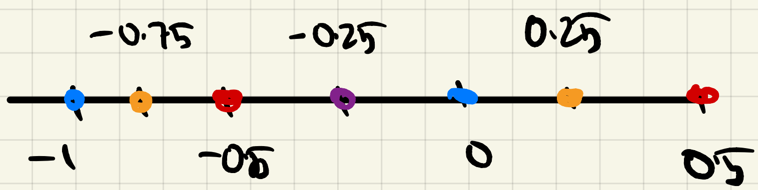
Now $X' =$ 

X' (also like $2\mathbb{Z} \subset \mathbb{R}$)

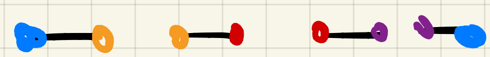
Now $X'/G \longrightarrow V(X'/G) = \{*, *\}$

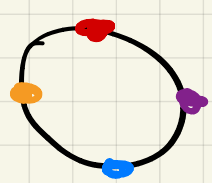
$S(X'/G) = \{\{*\}, \{*\}, \{*, *\}\}$

$\Rightarrow |X'/G| =$  again $|X'|/G =$ 

Now $X'' =$ 

Now $V(X''/G) = \{*, *, *, *\}$

$S(X''/G) = \{$ singletons  $\}$

$\Rightarrow |X''/G| =$  $= |X''|/G$