

Machine Learning Assignment 1

③ (i) Given, $P(y=1|x, w) = g(w_0 + w_1 x)$

where $g(z)$ is a logistic function.

Also, it is an increasing function

i.e. $g(z) = \frac{1}{1 + e^{-z}}$

As it is a function of x
 $w_0 + w_1 x$ acts as a linear
 equation, so it extends from
 $-\infty$ to $+\infty$ when x extends from
 $-\infty$ to $+\infty$ and $w_1 \neq 0$

so $g(z) = \frac{1}{1 + e^{-z}}$

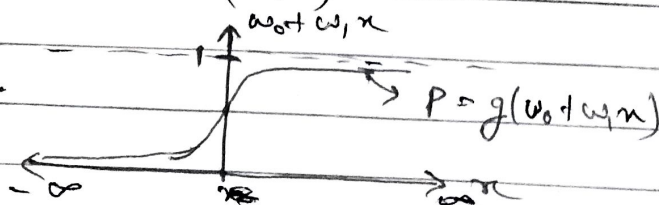
~~also it is a function~~

$-\infty < z = w_0 + w_1 x < \infty$

$g(z)$ at $z = -\infty = \frac{1}{1 + e^{\infty}} \rightarrow 0$

$g(z)$ at $z = \infty = \frac{1}{1 + e^{-\infty}} \rightarrow 1$

So, for the range of $P(y=1|x, w)$
 is $(0, 1)$



(ii) logit function: $l(n) = \log\left(\frac{x}{1-x}\right)$

$$\frac{d}{dx} l(n) = \frac{\left(\frac{1-x}{x}\right)}{\left(\frac{1-x}{x}\right)^2} \frac{(1-x) - x(1)}{(1-x)^2}$$

$$= \frac{1}{x(1-x)}$$

which is always positive for $0 < x < 1$
 Hence, it is an increasing function.

For 0^+ & 1^- , logit tends to $-\infty$ and $+\infty$ respectively.

So, logit goes from $(-\infty$ to $\infty)$ for $0 < x < 1$

(4) (a) As, RMSE takes the square of error, hence the higher errors are given more weights and are ~~easier~~ easier to reduce using the gradient descent as higher errors are undesirable and reduced significantly due to higher weights.

(b) In a data, where noise is very far away from the actual data, RMSE would give higher weight to reduce the loss, which is in fact not good for the performance of the actual data.

RMSE would increase the error in the actual data. In such cases MAE is considered as a better option, as it is more robust to the noise and ~~focus~~ focuses more on actual data.

(C) Quantile loss function is more useful than MAE when we need to predict intervals rather than discrete points which have variable variance. It is basically a modification of MAE

$$L_{\tau}(y, y^p) = \sum_{i: y_i < y_i^p} (\tau - 1) \cdot |y_i - y_i^p| + \sum_{i: y_i \geq y_i^p} \tau |y_i - y_i^p|$$

Where τ is a parameter which is called quantile. By changing it we can adjust the weight of errors which can help us to adjust the ~~amount~~ amount of overestimation or underestimation. For $\tau = 0.5$, it turns into MAE.