Control Systems

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Contents

1 Stability Problem : Example

1.1. An op amp designed to have a low-frequency gain of 10^5 and a high-frequency response dominated by a single pole at 100 rad/s, acquires, through a manufacturing error, a pair of additional poles at 10,000 rad/s.

a) At what frequency does the total phase shift reach 180° ?
b) At this frequency, for what value of β, assumed to be frequency independent, does the loop gain reach a value of unity?
c) What is the corresponding value of closed-loop gain at low frequencies?

Solution:

$$G(s) = \frac{A}{1+\frac{s}{p}} \tag{1.1.1}$$

Considering manufacturing error

$$G(s) = \frac{A}{\left(1+\frac{s}{p}\right)\left(1+\frac{s}{p_{error}}\right)} \tag{1.1.2}$$

$$A = \text{Low Frequency Gain} = 10^5 \tag{1.1.3}$$

$$p = 100 \tag{1.1.4}$$

$$p_{error} = 10^4 \tag{1.1.5}$$

$$G(s) = \frac{10^5}{\left(1+\frac{s}{100}\right)\left(1+\frac{s}{10^4}\right)^2} \tag{1.1.6}$$

$$\angle G(j\omega) = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{10^4} \tag{1.1.7}$$

1.2. Calculating the frequency at which the total phase shift reach 180°

At ω_{180}, $\angle G(j\omega_{180}) = -180°$
Also $\omega_{180} >> 100$

$$180° = 90° + 2\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) \tag{1.2.1}$$

$$\tan^{-1}\frac{\omega_{180}}{10^4} = 45° \tag{1.2.2}$$

$$\frac{\omega_{180}}{10^4} = \tan 45° = 1 \tag{1.2.3}$$

$$\omega_{180} = 10^4 rad/s \tag{1.2.4}$$

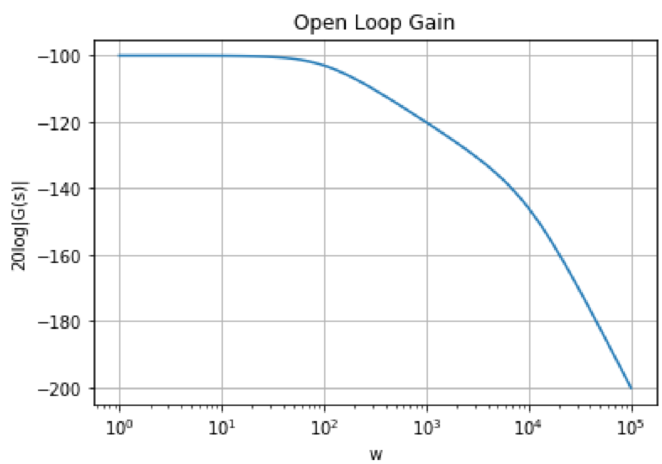

Fig. 1.2: Open Loop Gain

1.3. Calculating feedback factor β for which loop gain at ω_{180} is unity

$$H(s) = G(s)\beta = 1 \tag{1.3.1}$$

$$\frac{10^5\beta}{\sqrt{1^2+\left(\frac{\omega_{180}}{10^2}\right)^2}\sqrt{\left(1+\frac{\omega_{180}}{10^4}\right)^2}} = 1 \tag{1.3.2}$$

$$\beta = 0.002 \tag{1.3.3}$$

1.4. Calculating the closed loop gain at low frequency Let $T(s)$ be the closed loop Transfer Function.

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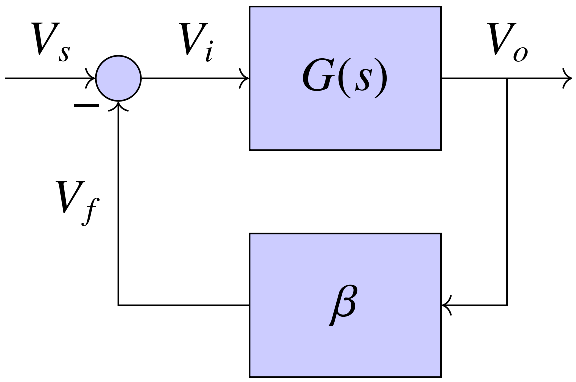

Fig. 1.4: Closed loop circuit

$$T(s) = \frac{G(s)}{1 + \beta G(s)} \tag{1.4.1}$$

$$T(s) = \frac{10^5}{1 + \beta 10^5 + \frac{s}{100}} \tag{1.4.2}$$

$$|T(s)| = \frac{10^5}{\sqrt{(200)^2 + \left(\frac{s}{100}\right)^2}} \tag{1.4.3}$$

At low frequencies

$$|T(s)| = 500V/V \tag{1.4.4}$$

Parameter	**Value**
ω_{180}	$10^4 rad/s$
β	0.002
$\|H(0)\|$	$500V/V$

TABLE 1.4: Obtained Parameters

The following code performs all the calculations of above equations

```
codes/code1.py
```

The following code plots the open loop gains, closed loop gains and step response to the system

```
codes/code2.py
```

1.5. Designing the circuit for transfer function $T(s)$

1) Designing $G(s)$
Let us assume Op-Amp to be ideal. So this means $V_1 = 0$ Applying KCL at node V_1

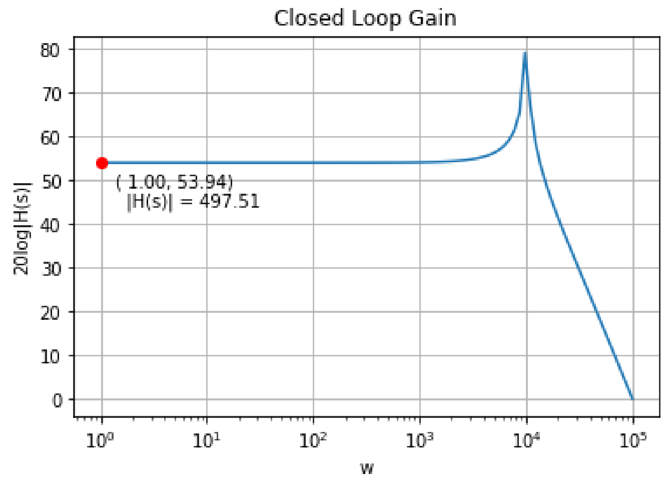

Fig. 1.4: Closed Loop Gain

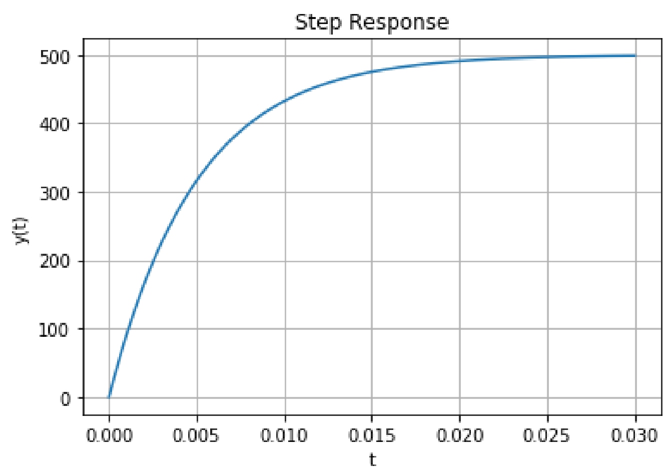

Fig. 1.4: Step Response

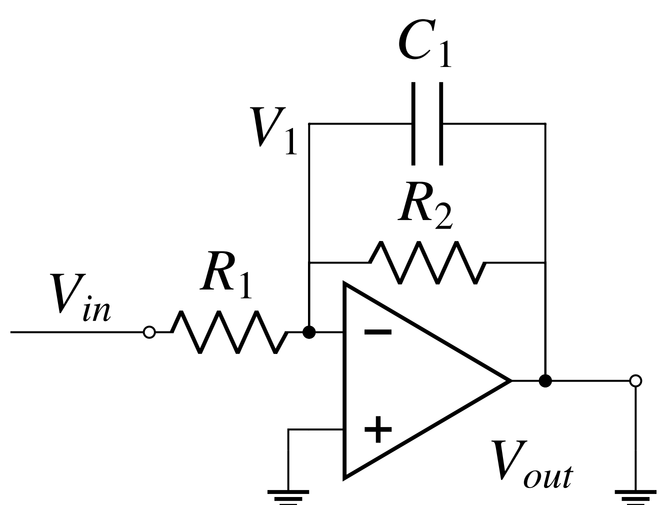

Fig. 1.5: Open Loop design

$$I_{in} = I_{C_1} + I_{R_2} \quad (1.5.1)$$
$$\frac{V_{in}}{R_1} = \frac{V_{out}}{R_2} + C_1\frac{dV_{out}}{dt} \quad (1.5.2)$$

In Laplace domain

$$\frac{V_{in}(s)}{R_1} = \frac{V_{out}(s)}{R_2} + C_1 s V_{out}(s) \quad (1.5.3)$$
$$\frac{V_{out}}{V_{in}} = \frac{R_2/R_1}{1 + sR_2C_1} \quad (1.5.4)$$
$$T(s) = \frac{10^5}{1 + \frac{s}{100}} \quad (1.5.5)$$
$$\frac{R_2}{R_1} = 10^5 \quad (1.5.6)$$
$$R_2C_1 = \frac{1}{100} \quad (1.5.7)$$

2) Designing $H(s) = \beta G(s)$

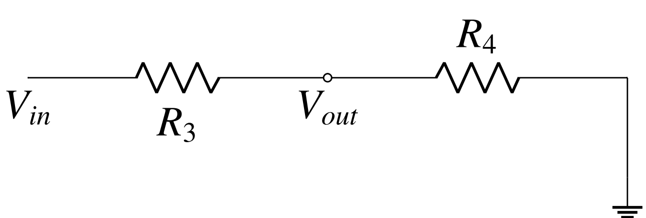

Fig. 1.5: Loop Gain

$$V_{out} = V_{in}\left(\frac{R_4}{R_3 + R_4}\right) \quad (1.5.8)$$
$$\frac{R_4}{R_3 + R_4} = 0.002 \quad (1.5.9)$$
$$R_4 = 0.002R_3 \quad (1.5.10)$$

3) Closed loop design
Figure 1.5 is the final closed loop design for transfer function $T(s)$

Parameter	**Value**
R_1	R
R_2	$10^5 R$
C	$10^{-7} R$
R_3	R'
R_4	$0.002R'$

TABLE 1.5: Circuit Parameters

The table 1.5 provides the parameters for our circuit design

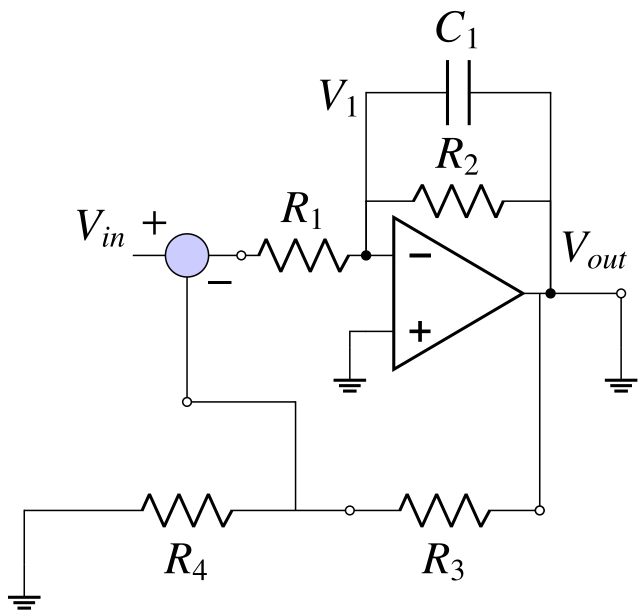

Fig. 1.5: Closed Loop Circuit

1.6. Verification of circuit design through SPICE

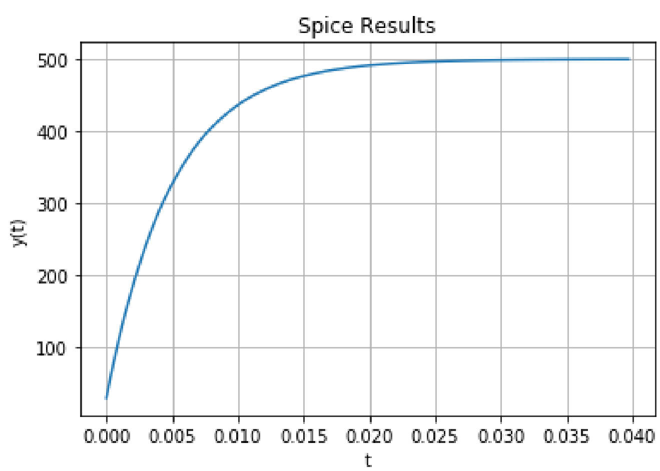

Fig. 1.6: SPICE simulation of circuit 1.5

A SPICE simulation of circuit 1.5 is done by providing a DC input(Unit Step Input). The obtained plot (figure 1.6) is similar to the Step response of the feedback system in figure 1.4. Hence we verify our design is correct.
The following code plots the SPICE simulation results from SPICE plot txt file and spice .net file

```
codes/spice/plotter.py
```

```
codes/spice/spice.net
```

For instructions to run the spice simulation please refer

```
codes/spice/readme.md
```