

# Control Systems

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### 1 Stability Problem : Example 1

#### 1 STABILITY PROBLEM : EXAMPLE

1.1. An op amp designed to have a low-frequency gain of  $10^5$  and a high-frequency response dominated by a single pole at 100 rad/s, acquires, through a manufacturing error, a pair of additional poles at 10,000 rad/s.

- At what frequency does the total phase shift reach  $180^\circ$  ?
- At this frequency, for what value of  $\beta$ , assumed to be frequency independent, does the loop gain reach a value of unity?
- What is the corresponding value of closed-loop gain at low frequencies?

**Solution:**

$$T(s) = \frac{A}{1 + \frac{s}{p}} \quad (1.1.1)$$

$$G(s) = \frac{A}{(1 + \frac{s}{p})(1 + \frac{s}{p_{error}})} \quad (1.1.2)$$

$$A = \text{Low Frequency Gain} = 10^5 \quad (1.1.3)$$

$$p = 100 \quad (1.1.4)$$

$$p_{error} = 10^4 \quad (1.1.5)$$

$$G(s) = \frac{10^5}{(1 + \frac{s}{100})(1 + \frac{s}{10^4})^2} \quad (1.1.6)$$

$$\angle G(j\omega) = -\tan^{-1} \frac{\omega}{100} - 2 \tan^{-1} \frac{\omega}{10^4} \quad (1.1.7)$$

1.2. Calculating the frequency at which the total phase shift reach  $180^\circ$

At  $\omega_{180}$ ,  $\angle G(j\omega_{180}) = -180^\circ$

Also  $\omega_{180} \gg 100$

$$180^\circ = 90^\circ + 2 \tan^{-1} \left( \frac{\omega_{180}}{10^4} \right) \quad (1.2.1)$$

$$\tan^{-1} \frac{\omega_{180}}{10^4} = 45^\circ \quad (1.2.2)$$

$$\frac{\omega_{180}}{10^4} = \tan 45^\circ = 1 \quad (1.2.3)$$

$$\omega_{180} = 10^4 \text{ rad/s} \quad (1.2.4)$$

1.3. Calculating feedback factor  $\beta$  for which loop gain at  $\omega_{180}$  is unity

$$\text{Loop Gain} = G(s)\beta = 1 \quad (1.3.1)$$

$$\frac{10^5 \beta}{\sqrt{1^2 + \left(\frac{\omega_{180}}{10^2}\right)^2} \sqrt{\left(1 + \frac{\omega_{180}}{10^4}\right)^2}} = 1 \quad (1.3.2)$$

$$\beta = 0.002 \quad (1.3.3)$$

1.4. Calculating the closed loop gain at low frequency Let  $H(s)$  be the closed loop Transfer Function.

$$H(s) = \frac{G(s)}{1 + \beta G(s)} \quad (1.4.1)$$

$$H(s) = \frac{10^5}{1 + \beta 10^5 + \frac{s}{100}} \quad (1.4.2)$$

$$|H(s)| = \frac{10^5}{\sqrt{(200)^2 + \left(\frac{s}{100}\right)^2}} \quad (1.4.3)$$

At low frequencies

$$|H(s)| = 500V/V \quad (1.4.4)$$

Parameter	Value
$\omega_{180}$	$10^4 \text{ rad/s}$
$\beta$	0.002
$ H(0) $	500V/V

TABLE 1.4: Obtained Parameters

The following code performs all the calculations of above equations and returns the pa-

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codes/ee18btech11001/code1.py

### 1.5. Designing the circuit for transfer function $T(s)$

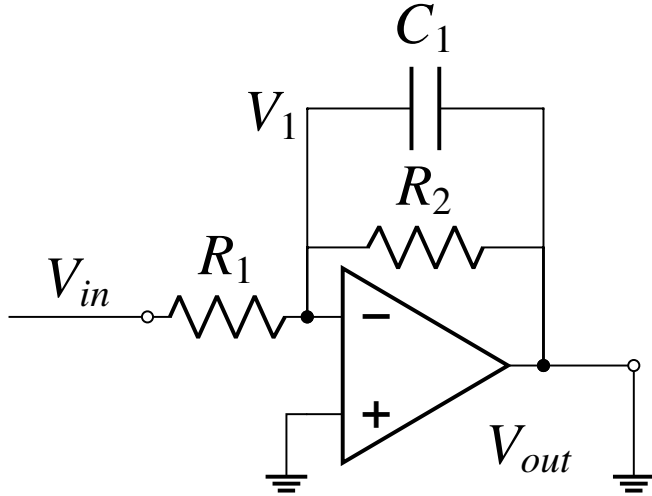


Fig. 1.5: 1

Let us assume Op-Amp to be ideal. So this means  $V_1 = 0$  Applying KCL at node  $V_1$

$$I_{in} = I_{C_1} + I_{R_2} \quad (1.5.1)$$

$$\frac{V_{in}}{R_1} = \frac{V_{out}}{R_2} + C_1 \frac{dV_{out}}{dt} \quad (1.5.2)$$

In Laplace domain

$$\frac{V_{in}(s)}{R_1} = \frac{V_{out}(s)}{R_2} + C_1 s V_{out}(s) \quad (1.5.3)$$

$$\frac{V_{out}}{V_{in}} = \frac{R_2/R_1}{1 + sR_1C_1} \quad (1.5.4)$$

$$T(s) = \frac{10^5}{1 + \frac{s}{100}} \quad (1.5.5)$$

$$\frac{R_2}{R_1} = 10^5 \quad (1.5.6)$$

$$R_1C_1 = \frac{1}{100} \quad (1.5.7)$$

Parameter	Value
$R_1$	$R$
$R_2$	$10^5 R$
$C$	$0.01 R$

TABLE 1.5: Circuit Parameters