Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Current Amplifier: Example

1.1. An op amp designed to have a low-frequency gain of 10^5 and a high-frequency response dominated by a single pole at 100 rad/s, acquires, through a manufacturing error, a pair of additional poles at 10,000 rad/s.

a) At what frequency does the total phase shift reach 180° ?
b) At this frequency, for what value of β, assumed to be frequency independent, does the loop gain reach a value of unity?
c) What is the corresponding value of closed-loop gain at low frequencies?

Solution:

$$G(s) = \frac{A}{1+\frac{s}{p}} \tag{1.1.1}$$

$$A = \text{Low Frequency Gain} = 10^5 \tag{1.1.2}$$

$$p = 100 \tag{1.1.3}$$

$$G(s) = \frac{10^5}{(1+\frac{s}{100})(1+\frac{s}{10^4})^2} \tag{1.1.4}$$

$$\measuredangle G(j\omega) = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{10^4} \tag{1.1.5}$$

1.2. Calculating the frequency at which the total phase shift reach 180°

At ω_{180}, $\measuredangle G(j\omega_{180}) = -180°$
Also $\omega_{180} >> 100$

$$180° = 90° + 2\tan^{-1}(\frac{\omega_{180}}{10^4}) \tag{1.2.1}$$

$$\tan^{-1}\frac{\omega_{180}}{10^4} = 45° \tag{1.2.2}$$

$$\frac{\omega_{180}}{10^4} = \tan 45° = 1 \tag{1.2.3}$$

$$\omega_{180} = 10^4 rad/s \tag{1.2.4}$$

1.3. Calculating feedback factor β for which loop gain at ω_{180} is unity

$$\text{Loop Gain} = G(s)\beta = 1 \tag{1.3.1}$$

$$\frac{10^5\beta}{\sqrt{1^2+(\frac{\omega_{180}}{10^2})^2}\sqrt{(1+\frac{\omega_{180}}{10^4})^2}} = 1 \tag{1.3.2}$$

$$\beta = 0.002 \tag{1.3.3}$$

1.4. Calculating the closed loop gain at low frequency Let $H(s)$ be the closed loop Transfer Function.

$$H(s) = \frac{G(s)}{1+\beta G(s)} \tag{1.4.1}$$

$$H(s) = \frac{10^5}{1+\beta 10^5 + \frac{s}{100}} \tag{1.4.2}$$

$$|H(s)| = \frac{10^5}{\sqrt{(200)^2+(\frac{s}{100})^2}} \tag{1.4.3}$$

At low frequencies

$$|H(s)| = 500V/V \tag{1.4.4}$$

Parameter	Value
ω_{180}	$10^4 rad/s$
β	0.002
$\lvert H(0)\rvert$	$500V/V$

TABLE 1.4: Obtained Parameters

The following code does all the calculations of

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above equations and returns the parameters

codes/ee18btech11001/code1.py