Control Systems

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Contents

1 Stability Problem : Example

1.1. An op amp designed to have a low-frequency gain of 10^5 and a high-frequency response dominated by a single pole at 100 rad/s, acquires, through a manufacturing error, a pair of additional poles at 10,000 rad/s.

a) At what frequency does the total phase shift reach 180° ?
b) At this frequency, for what value of β, assumed to be frequency independent, does the loop gain reach a value of unity?
c) What is the corresponding value of closed-loop gain at low frequencies?

Solution:

$$T(s) = \frac{A}{1+\frac{s}{p}} \tag{1.1.1}$$

$$G(s) = \frac{A}{(1+\frac{s}{p})(1+\frac{s}{p_{error}})} \tag{1.1.2}$$

$$A = \text{Low Frequency Gain} = 10^5 \tag{1.1.3}$$

$$p = 100 \tag{1.1.4}$$

$$p_{error} = 10^4 \tag{1.1.5}$$

$$G(s) = \frac{10^5}{(1+\frac{s}{100})(1+\frac{s}{10^4})^2} \tag{1.1.6}$$

$$\measuredangle G(j\omega) = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{10^4} \tag{1.1.7}$$

1.2. Calculating the frequency at which the total phase shift reach 180°
At ω_{180}, $\measuredangle G(j\omega_{180}) = -180°$
Also $\omega_{180} >> 100$

$$180° = 90° + 2\tan^{-1}(\frac{\omega_{180}}{10^4}) \tag{1.2.1}$$

$$\tan^{-1}\frac{\omega_{180}}{10^4} = 45° \tag{1.2.2}$$

$$\frac{\omega_{180}}{10^4} = \tan 45° = 1 \tag{1.2.3}$$

$$\omega_{180} = 10^4 rad/s \tag{1.2.4}$$

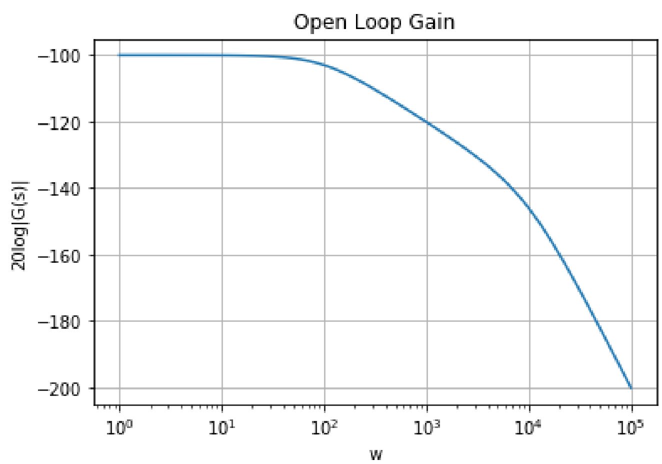

Fig. 1.2: Open Loop Gain

1.3. Calculating feedback factor β for which loop gain at ω_{180} is unity

$$\text{Loop Gain} = G(s)\beta = 1 \tag{1.3.1}$$

$$\frac{10^5\beta}{\sqrt{1^2+(\frac{\omega_{180}}{10^2})^2}\sqrt{(1+\frac{\omega_{180}}{10^4})^2}} = 1 \tag{1.3.2}$$

$$\beta = 0.002 \tag{1.3.3}$$

1.4. Calculating the closed loop gain at low frequency Let $H(s)$ be the closed loop Transfer Function.

$$H(s) = \frac{G(s)}{1+\beta G(s)} \tag{1.4.1}$$

$$H(s) = \frac{10^5}{1+\beta 10^5+\frac{s}{100}} \tag{1.4.2}$$

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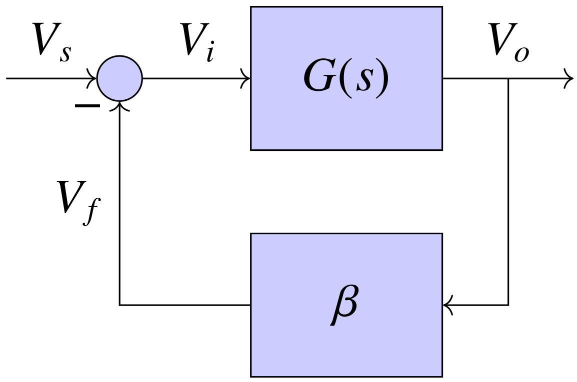

Fig. 1.4: Closed loop circuit

$$|H(s)| = \frac{10^5}{\sqrt{(200)^2 + (\frac{s}{100})^2}} \tag{1.4.3}$$

At low frequencies

$$|H(s)| = 500V/V \tag{1.4.4}$$

Parameter	**Value**
ω_{180}	$10^4 rad/s$
β	0.002
$\lvert H(0)\rvert$	$500V/V$

TABLE 1.4: Obtained Parameters

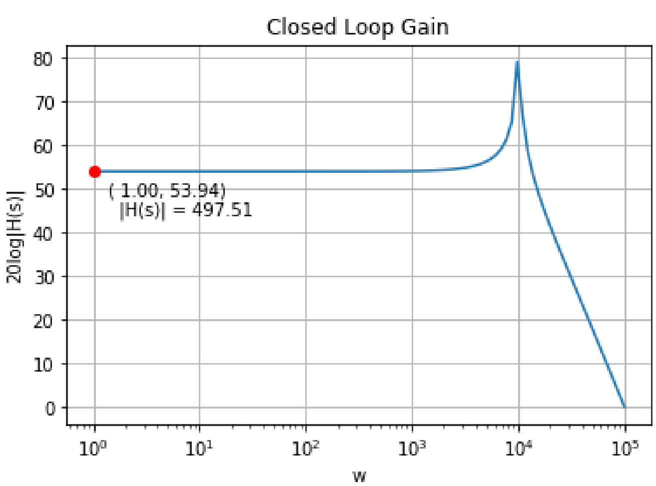

Fig. 1.4: Closed Loop Gain

The following code performs all the calculations of above equations

```
codes/ee18btech11001/code1.py
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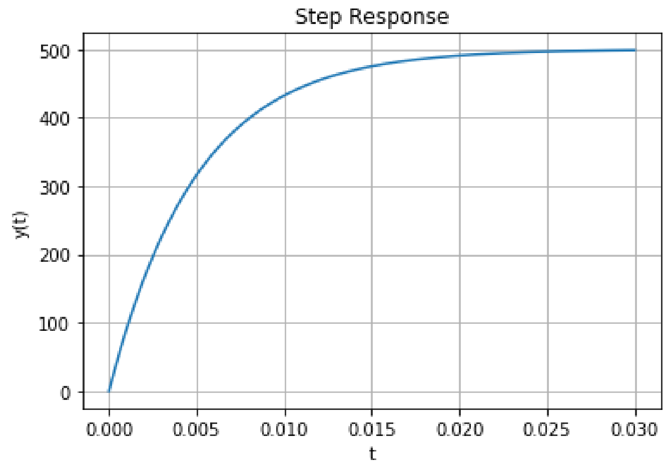

Fig. 1.4: Step Response

The following code plots the open loop gains, closed loop gains and step response to the system

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codes/ee18btech11001/code2.py
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1.5. Designing the circuit for transfer function $T(s)$

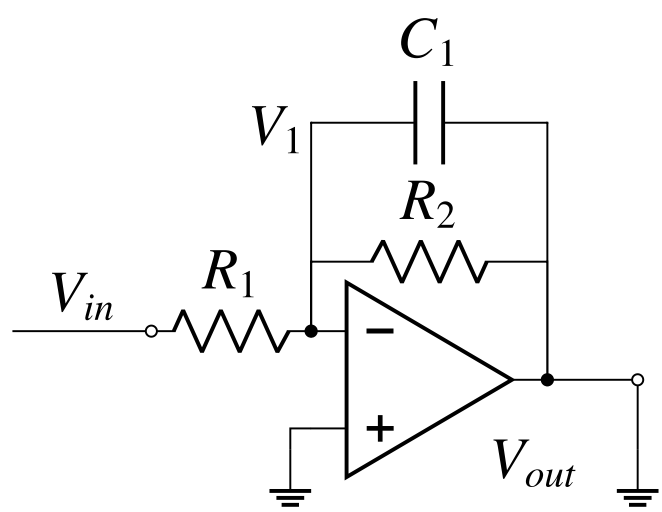

Fig. 1.5: 1

Let us assume Op-Amp to be ideal. So this means $V_1 = 0$ Applying KCL at node V_1

$$I_{in} = I_{C_1} + I_{R_2} \tag{1.5.1}$$

$$\frac{V_{in}}{R_1} = \frac{V_{out}}{R_2} + C_1\frac{dV_{out}}{dt} \tag{1.5.2}$$

In Laplace domain

$$\frac{V_{in}(s)}{R_1} = \frac{V_{out}(s)}{R_2} + C_1 s V_{out}(s) \tag{1.5.3}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_2/R_1}{1 + sR_2C_1} \tag{1.5.4}$$

$$T(s) = \frac{10^5}{1 + \frac{s}{100}} \tag{1.5.5}$$

$$\frac{R_2}{R_1} = 10^5 \tag{1.5.6}$$

$$R_2C_1 = \frac{1}{100} \tag{1.5.7}$$

Parameter	**Value**
R_1	R
R_2	$10^5 R$
C	$10^{-7} R$

TABLE 1.5: Circuit Parameters

1.6. Verification of circuit design through SPICE

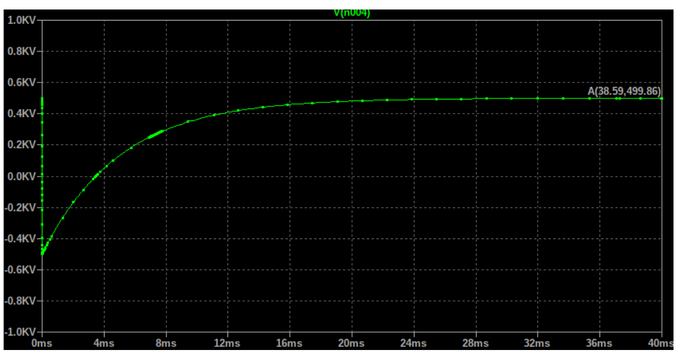

Fig. 1.6: SPICE simulation of circuit 1.5

A SPICE simulation of circuit 1.5 is done by providing a DC input(Unit Step Input). The obtained plot (figure 1.6) is similar to the Step response of the feedback system in figure 1.4. Hence we verify our design is correct.