

# Control Systems

Aayush Goyal\*

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### 1 Stability Problem : Example 1

#### 1 STABILITY PROBLEM : EXAMPLE

1.1. An op amp designed to have a low-frequency gain of  $10^5$  and a high-frequency response dominated by a single pole at 100 rad/s, acquires, through a manufacturing error, a pair of additional poles at 10,000 rad/s.

- At what frequency does the total phase shift reach  $180^\circ$  ?
- At this frequency, for what value of  $\beta$ , assumed to be frequency independent, does the loop gain reach a value of unity?
- What is the corresponding value of closed-loop gain at low frequencies?

**Solution:**

$$G(s) = \frac{A}{1 + \frac{s}{p}} \quad (1.1.1)$$

Considering manufacturing error

$$G(s) = \frac{A}{(1 + \frac{s}{p})(1 + \frac{s}{p_{error}})} \quad (1.1.2)$$

$$A = \text{Low Frequency Gain} = 10^5 \quad (1.1.3)$$

$$p = 100 \quad (1.1.4)$$

$$p_{error} = 10^4 \quad (1.1.5)$$

$$G(s) = \frac{10^5}{(1 + \frac{s}{100})(1 + \frac{s}{10^4})^2} \quad (1.1.6)$$

$$\angle G(j\omega) = -\tan^{-1} \frac{\omega}{100} - 2 \tan^{-1} \frac{\omega}{10^4} \quad (1.1.7)$$

1.2. Calculating the frequency at which the total phase shift reach  $180^\circ$

At  $\omega_{180}$ ,  $\angle G(j\omega_{180}) = -180^\circ$

Also  $\omega_{180} \gg 100$

$$180^\circ = 90^\circ + 2 \tan^{-1} \left( \frac{\omega_{180}}{10^4} \right) \quad (1.2.1)$$

$$\tan^{-1} \frac{\omega_{180}}{10^4} = 45^\circ \quad (1.2.2)$$

$$\frac{\omega_{180}}{10^4} = \tan 45^\circ = 1 \quad (1.2.3)$$

$$\omega_{180} = 10^4 \text{ rad/s} \quad (1.2.4)$$

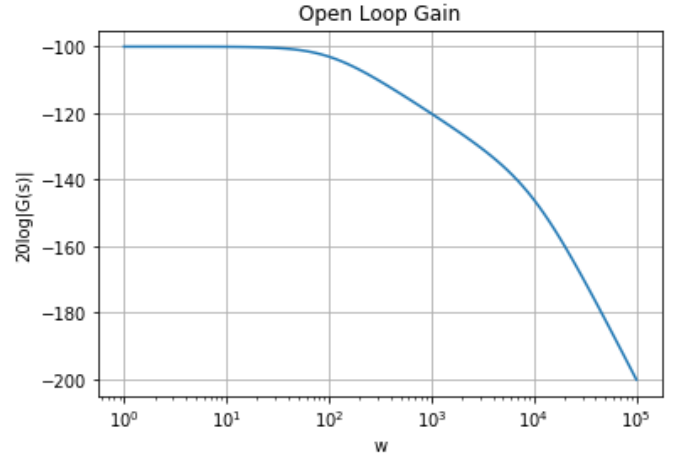


Fig. 1.2: Open Loop Gain

1.3. Calculating feedback factor  $\beta$  for which loop gain at  $\omega_{180}$  is unity

$$H(s) = G(s)\beta = 1 \quad (1.3.1)$$

$$\frac{10^5 \beta}{\sqrt{1^2 + \left(\frac{\omega_{180}}{10^2}\right)^2} \sqrt{\left(1 + \frac{\omega_{180}}{10^4}\right)^2}} = 1 \quad (1.3.2)$$

$$\beta = 0.002 \quad (1.3.3)$$

1.4. Calculating the closed loop gain at low frequency Let  $T(s)$  be the closed loop Transfer Function.

\*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

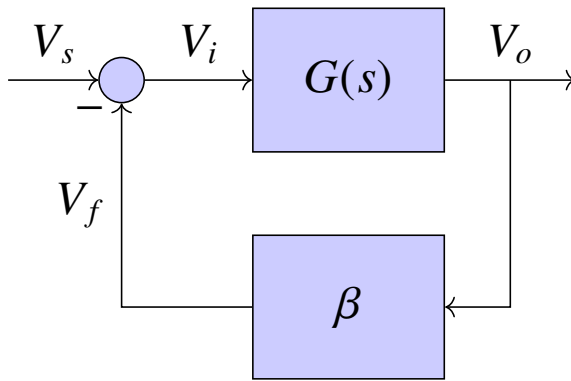


Fig. 1.4: Closed loop circuit

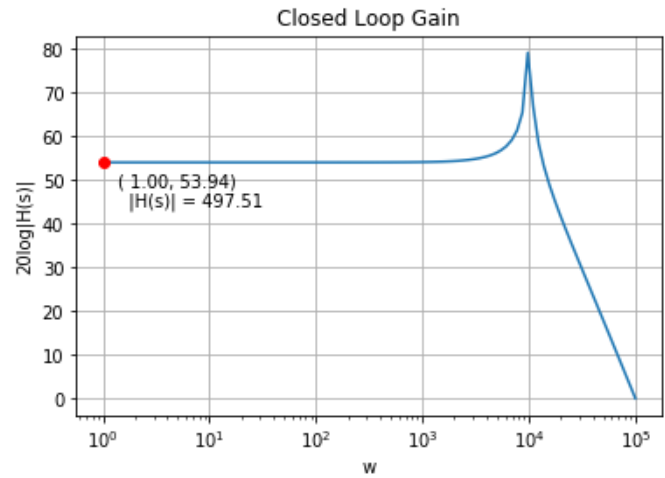


Fig. 1.4: Closed Loop Gain

$$T(s) = \frac{G(s)}{1 + \beta G(s)} \quad (1.4.1)$$

$$T(s) = \frac{10^5}{1 + \beta 10^5 + \frac{s}{100}} \quad (1.4.2)$$

$$|T(s)| = \frac{10^5}{\sqrt{(200)^2 + (\frac{s}{100})^2}} \quad (1.4.3)$$

At low frequencies

$$|T(s)| = 500V/V \quad (1.4.4)$$

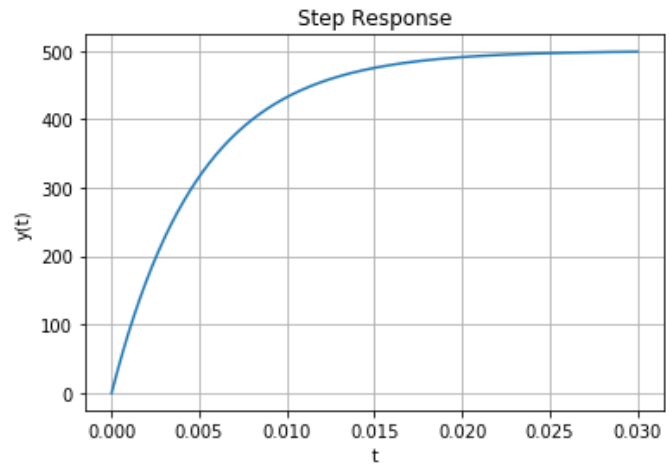


Fig. 1.4: Step Response

Parameter	Value
$\omega_{180}$	$10^4 \text{ rad/s}$
$\beta$	0.002
$ H(0) $	500V/V

TABLE 1.4: Obtained Parameters

The following code performs all the calculations of above equations

```
codes/ee18btech11001/code1.py
```

The following code plots the open loop gains, closed loop gains and step response to the system

```
codes/ee18btech11001/code2.py
```

### 1.5. Designing the circuit for transfer function $G(s)$

Let us assume Op-Amp to be ideal. So this means  $V_1 = 0$  Applying KCL at node  $V_1$

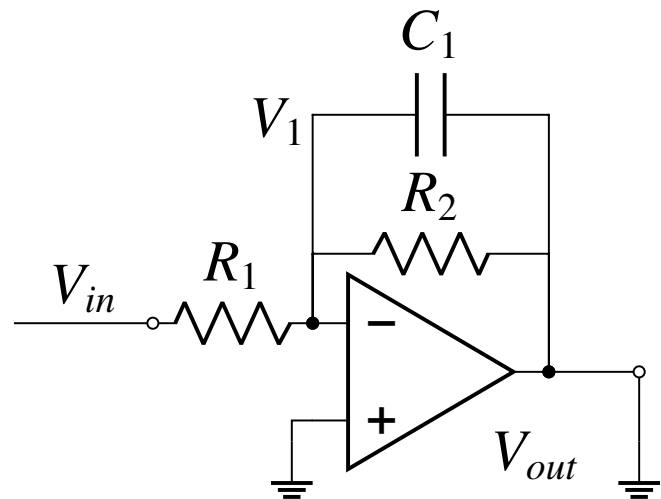


Fig. 1.5: 1

codes/ee18btech11001/spice/plotter.py

$$I_{in} = I_{C_1} + I_{R_2} \quad (1.5.1)$$

$$\frac{V_{in}}{R_1} = \frac{V_{out}}{R_2} + C_1 \frac{dV_{out}}{dt} \quad (1.5.2)$$

In Laplace domain

$$\frac{V_{in}(s)}{R_1} = \frac{V_{out}(s)}{R_2} + C_1 s V_{out}(s) \quad (1.5.3)$$

$$\frac{V_{out}}{V_{in}} = \frac{R_2/R_1}{1 + sR_2C_1} \quad (1.5.4)$$

$$T(s) = \frac{10^5}{1 + \frac{s}{100}} \quad (1.5.5)$$

$$\frac{R_2}{R_1} = 10^5 \quad (1.5.6)$$

$$R_2C_1 = \frac{1}{100} \quad (1.5.7)$$

Parameter	Value
$R_1$	$R$
$R_2$	$10^5 R$
$C$	$10^{-7} R$

TABLE 1.5: Circuit Parameters

## 1.6. Verification of circuit design through SPICE

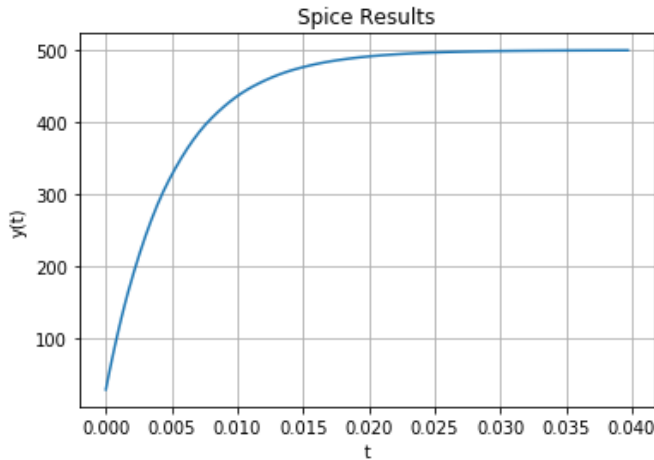


Fig. 1.6: SPICE simulation of circuit 1.5

A SPICE simulation of circuit 1.5 is done by providing a DC input(Unit Step Input). The obtained plot (figure 1.6) is similar to the Step response of the feedback system in figure 1.4. Hence we verify our design is correct.

The following code plots the SPICE simulation results from SPICE plot txt file