Stability

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1.1. An op amp designed to have a low-frequency gain of 10^5 and a high-frequency response dominated by a single pole at 100 rad/s, acquires, through a manufacturing error, a pair of additional poles at 10,000 rad/s.

a) At what frequency does the total phase shift reach 180° ?
b) At this frequency, for what value of H, assumed to be frequency independent, does the loop gain reach a value of unity?
c) What is the corresponding value of closed-loop gain at low frequencies?

Solution:

$$G(s) = \frac{G_0}{1+\frac{s}{p}} \tag{1.1.1}$$

Considering manufacturing error

$$G(s) = \frac{G_0}{\left(1+\frac{s}{p}\right)\left(1+\frac{s}{p_{error}}\right)} \tag{1.1.2}$$

$$G_0 = \text{Low Frequency Gain} = 10^5 \tag{1.1.3}$$

$$p = 100 \tag{1.1.4}$$

$$p_{error} = 10^4 \tag{1.1.5}$$

$$G(s) = \frac{10^5}{\left(1+\frac{s}{100}\right)\left(1+\frac{s}{10^4}\right)^2} \tag{1.1.6}$$

$$\measuredangle G(j\omega) = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{10^4} \tag{1.1.7}$$

1.2. Calculating the frequency at which the total phase shift reach 180°
At $\omega_{180}, \measuredangle G(j\omega_{180}) = -180°$
Also $\omega_{180} >> 100$

$$180° = 90° + 2\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) \tag{1.2.1}$$

$$\tan^{-1}\frac{\omega_{180}}{10^4} = 45° \tag{1.2.2}$$

$$\frac{\omega_{180}}{10^4} = \tan 45° = 1 \tag{1.2.3}$$

$$\omega_{180} = 10^4 rad/s \tag{1.2.4}$$

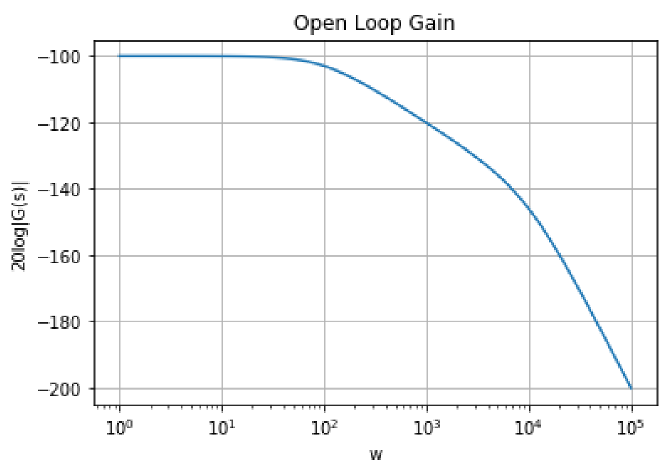

Fig. 1.2: Open Loop Gain

1.3. Calculating feedback factor H for which loop gain at ω_{180} is unity

$$\text{Loop Gain} = G(s)H = 1 \tag{1.3.1}$$

$$\frac{10^5 H}{\sqrt{1^2+\left(\frac{\omega_{180}}{10^2}\right)^2}\sqrt{\left(1+\frac{\omega_{180}}{10^4}\right)^2}} = 1 \tag{1.3.2}$$

$$H = 0.002 \tag{1.3.3}$$

1.4. Calculating the closed loop gain at low frequency Let $T(s)$ be the closed loop Transfer Function.

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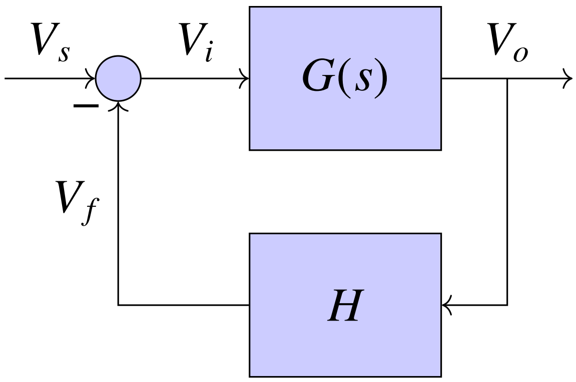

Fig. 1.4: Closed loop circuit

$$T(s) = \frac{G(s)}{1 + HG(s)} \tag{1.4.1}$$

$$T(s) = \frac{10^5}{H10^5 + \left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{1000}\right)^2} \tag{1.4.2}$$

$$T(s) = \frac{10^5}{10^{-10}s^3 + (2x10^{-6})s^2 + 10^{-2}s + 1 + 10^5 H} \tag{1.4.3}$$

At low frequencies

$$T(0) = \frac{10^5}{1 + 10^5 H} \tag{1.4.4}$$

$$|T(0)| = \frac{10^5}{\sqrt{(200)^2}} \tag{1.4.5}$$

$$|T(0)| = 500V/V \tag{1.4.6}$$

Parameter	**Value**
ω_{180}	$10^4 rad/s$
H	0.002
$\lvert T(0)\rvert$	$500V/V$

TABLE 1.4: Obtained Parameters

The following code performs all the calculations of above equations

```
codes/code1.py
```

The following code plots the open loop gains, closed loop gains and step response to the system

```
codes/code2.py
```

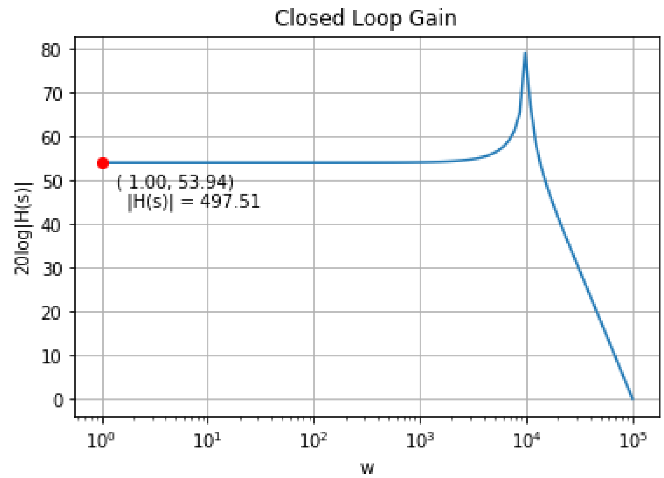

Fig. 1.4: Closed Loop Gain

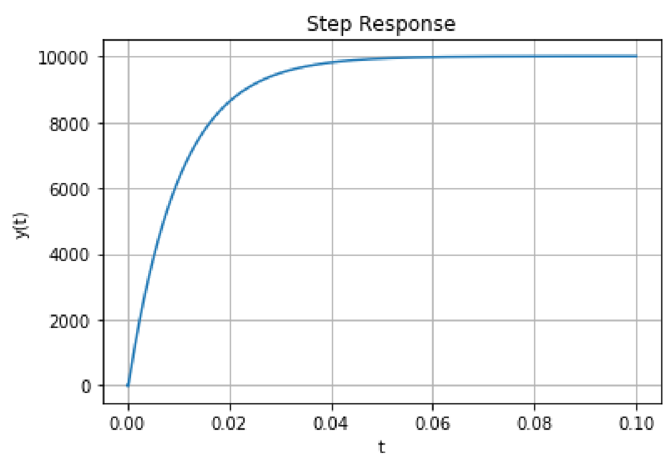

Fig. 1.4: Step Response

1.5. Designing the circuit for transfer function $T(s)$

1) Designing $G(s)$

Let us assume Op-Amp to be ideal. So this means $V_1 = 0$ Applying KCL at node V_1

$$I_{in} = I_{C_1} + I_{R_2} \tag{1.5.1}$$

$$\frac{V_{in}}{R_1} = \frac{V_{o1}}{R_2} + C_1\frac{dV_{out}}{dt} \tag{1.5.2}$$

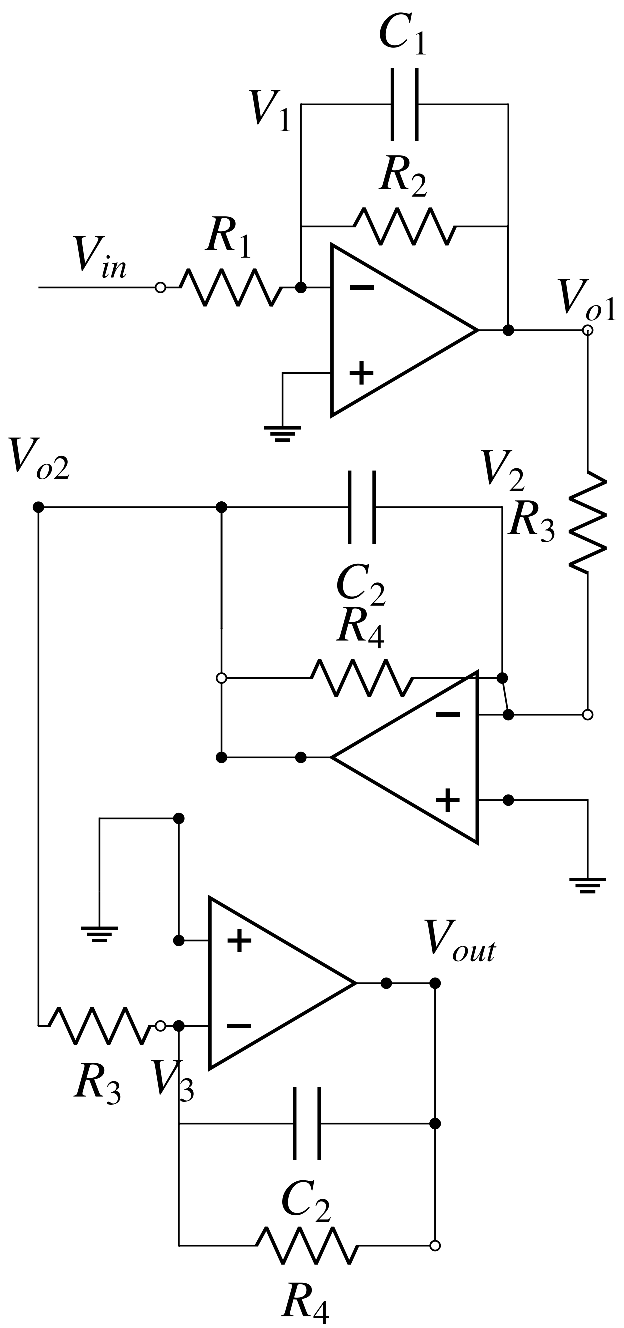

Fig. 1.5: Open Loop design

In Laplace domain

$$\frac{V_{in}(s)}{R_1} = \frac{V_{o1}(s)}{R_2} + C_1 s V_{out}(s) \tag{1.5.3}$$

$$\frac{V_{o1}}{V_{in}} = \frac{R_2/R_1}{1 + sR_2C_1} \tag{1.5.4}$$

$$\frac{V_{o1}}{V_{in}} = \frac{10^5}{1 + \frac{s}{100}} \tag{1.5.5}$$

$$\frac{R_2}{R_1} = 10^5 \tag{1.5.6}$$

$$R_2C_1 = \frac{1}{100} \tag{1.5.7}$$

As shown in 1.5 for the two identical poles at 10000 rad/sec we place similar op amp circuits twice.
Solving the circuit for second pole

$$\frac{V_{o1}(s)}{R_3} = \frac{V_{o2}(s)}{R_4} + C_2 s V_{out}(s) \tag{1.5.8}$$

$$\frac{V_{o2}}{V_{o1}} = \frac{R_4/R_3}{1 + sR_4C_2} \tag{1.5.9}$$

$$\frac{V_{o2}}{V_{o1}} = \frac{1}{1 + \frac{s}{10000}} \tag{1.5.10}$$

$$\frac{R_4}{R_3} = 1 \tag{1.5.11}$$

$$R_4C_2 = \frac{1}{10000} \tag{1.5.12}$$

2) Designing $H(s) = H$

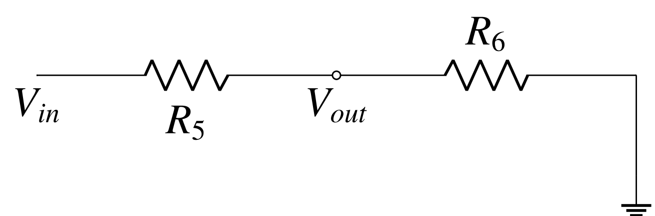

Fig. 1.5: Loop Gain

$$V_{out} = V_{in}\left(\frac{R_6}{R_5 + R_6}\right) \tag{1.5.13}$$

$$\frac{R_6}{R_5 + R_6} = 0.002 \tag{1.5.14}$$

$$R_6 = 0.002R_5 \tag{1.5.15}$$

3) Closed loop design
Figure 1.5 is the final closed loop design for transfer function $T(s)$
The table 1.5 provides the parameters for our circuit design.
The arbitrary parameters can be selected based on practical availability.

1.6. Verification of closed loop circuit design through SPICE
A SPICE simulation of circuit 1.5 is done by providing a DC input(Unit Step Input).
The obtained plot (figure 1.6) is similar to the Step response of the feedback system in figure 1.4. Hence we verify our design is correct.

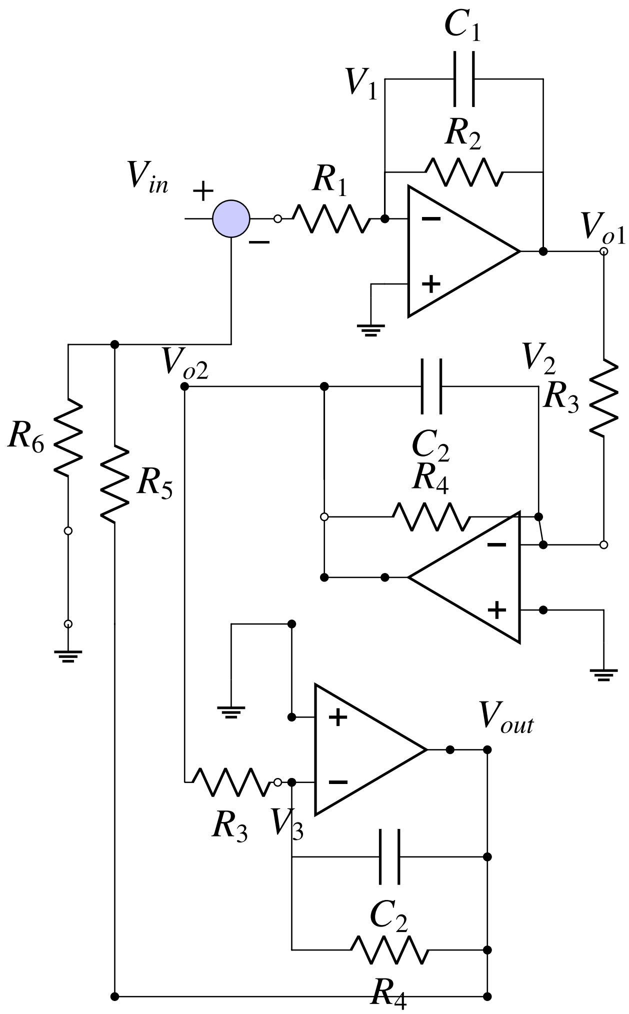

Fig. 1.5: Closed Loop Circuit

Parameter	**Value**
R_1	R
R_2	$10^5 R$
C_1	$10^{-7}/R$
R_3	R'
R_4	R'
C_2	$10^{-4}/R'$
R_5	R''
R_6	$0.002R''$

TABLE 1.5: Circuit Parameters

The following code plots the SPICE simulation results from SPICE plot .dat file and spice .net file

```
codes/spice/plotter.py
```

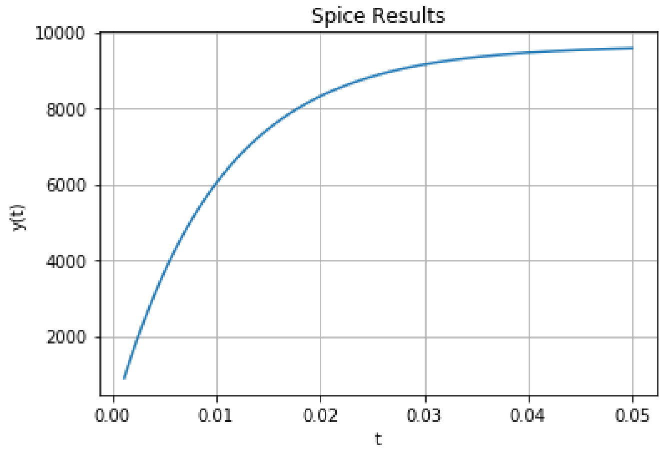

Fig. 1.6: SPICE simulation of circuit 1.5

The following is the spice simulation file

```
codes/spice/spice.net
```

For instructions to run the spice simulation please refer

```
codes/spice/readme.md
```