Control Systems

G V V Sharma*

Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Frequency Response Analysis

1.1 Polar Plot

1.1.1. A position control system is to be designed such that maximum peak overshoot is less than 25 %. Further, appropriate error constant should be 50. For the motor to be used, load and torque speed curve is shown below, where, J_1 = 2 kg-m2 , J_2 = 18 kg-m2 , f_1 = 2 N-m-s/rad, f_2 = 36 N-ms/rad. (Although obvious, consider position as the controlled variable and armature voltage as the manipulated variable.).

(i) Design a lead compensator for the system.

(ii) Design a lag compensator for the system.

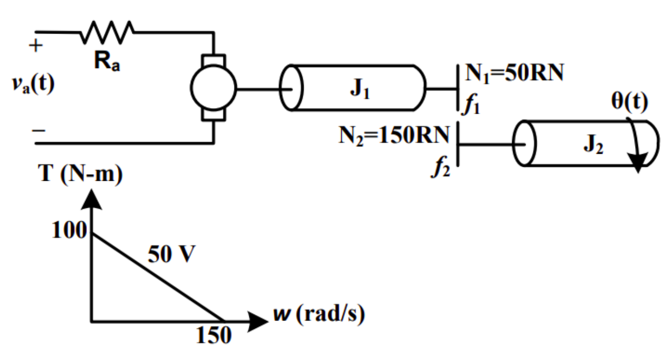

Fig. 1.1.1

Solution: Solving the system shown in 1.1.1, From speed-torque curve of DC Motor in figure. Let $\mathbf{T_m}$ be the torque exerted by DC Motor. 1.1.1

$$\mathbf{T_m} = 2V_a - \frac{2}{3}\boldsymbol{\omega_1} \tag{1.1.1}$$

Let $\mathbf{T_1}$, $\boldsymbol{\omega_1}$,$\boldsymbol{\theta_1}$ be the Torque, Angular velocity, Angular displacement on J_1. Change in torque across ends = torque applied on load + viscous friction. On J_1 at one end torque T_m is applied and at the other end T_1 exists.

$$\mathbf{T_m} = \mathbf{T_1} + J_1\ddot{\boldsymbol{\theta}}_1 + f_1\dot{\boldsymbol{\theta}}_1 \tag{1.1.2}$$

Similarly for J_2

$$\mathbf{T_2} = J_2\ddot{\boldsymbol{\theta}}_2 + f_2\dot{\boldsymbol{\theta}}_2 \tag{1.1.3}$$

$$\mathbf{T_2} = \frac{N_2}{N_1}\mathbf{T_1} \text{ (Gear Train Formula)} \tag{1.1.4}$$

$$\boldsymbol{\theta_2} = \frac{N_1}{N_2}\boldsymbol{\theta_1} \text{ (Gear Train Formula)} \tag{1.1.5}$$

Converting to State Space model

$$\boldsymbol{\theta} = \begin{pmatrix}\boldsymbol{\theta_1}\\ \boldsymbol{\theta_2}\end{pmatrix} \tag{1.1.6}$$

$$3\mathbf{T_m} = \begin{pmatrix}3J_1 & J_2\end{pmatrix}\ddot{\boldsymbol{\theta}} + \begin{pmatrix}3f_1 & f_2\end{pmatrix}\dot{\boldsymbol{\theta}} \tag{1.1.7}$$

$$\mathbf{T_m} = 2V_a - \begin{pmatrix}\frac{2}{3} & 0\end{pmatrix}\dot{\boldsymbol{\theta}} \tag{1.1.8}$$

$$\boldsymbol{\theta} = \begin{pmatrix}N_2 & N_1\end{pmatrix}K \tag{1.1.9}$$

On solving the above State Space Model

$$V_a = 6\ddot{\boldsymbol{\theta}}_2 + 10\dot{\boldsymbol{\theta}}_2 \tag{1.1.10}$$

Taking Laplace transform

$$G(s) = K\frac{\theta(s)}{V_a(s)} = \frac{1}{2s(3s+5)} \tag{1.1.11}$$

From Error Constant K = 500

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

$$G(s) = \frac{\theta(s)}{V_a(s)} = 250\frac{1}{s(3s+5)} \tag{1.1.12}$$

$$\zeta = 0.0695 \tag{1.1.13}$$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 81.6\% \tag{1.1.14}$$

$$\phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4+1}}}\right) \tag{1.1.15}$$

Specifications	Actual	Expected
OS%	81.6%	25%
ζ	0.0695	0.403
ϕ_m	7.35°	39.5°

TABLE 1.1.1: Table of Specifications

$\phi_{max} = 39.5° - 7.35° +$ correction factor
$\phi_{max} = 57°$

Designing a lead compensator

$$G_c(s) = \frac{1}{a}\frac{s+\frac{1}{T}}{s+\frac{1}{aT}}(a<1) \tag{1.1.16}$$

$$\sin\phi_{max} = \frac{a-1}{a+1} \tag{1.1.17}$$

$$a = 0.1 \tag{1.1.18}$$

$$|G(j\omega_c)| = \frac{1}{\sqrt{a}} = 10dB \tag{1.1.19}$$

$$\omega_c = 5° \text{ (Refer figure 1.1.3)} \tag{1.1.20}$$

$$T = \frac{1}{\omega_c\sqrt{a}} = 0.632 \tag{1.1.21}$$

$$G_c(s) = 10\frac{s+1.6}{s+16} \tag{1.1.22}$$

$$G(s)G_c(s) = 2500\frac{s+1.6}{s(3s+5)(s+16)} \tag{1.1.23}$$

Designing a lag compensator

$$G_c(s) = \frac{1}{b}\frac{s+\frac{1}{T}}{s+\frac{1}{bT}}(b>1) \tag{1.1.24}$$

$$\phi_{max} = 39.5° - 7.35° + correctionfactor \tag{1.1.25}$$

$$\phi_{max} = 45° \tag{1.1.26}$$

ω_c = Frequency at which phase of bode plot of G(s) is -180 +ϕ_{max} i.e. -135 °
$\omega_c = 1.75 rad/sec$ as in Figure 1.1.2

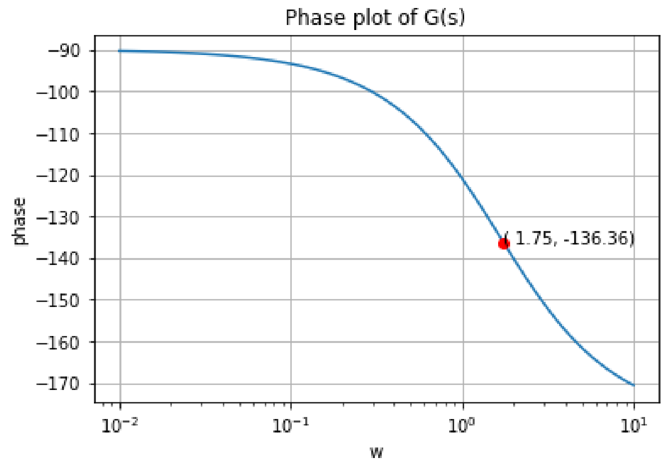

Fig. 1.1.2: Phase plot of G(s)

We place the zero at

$$\omega = 0.2\omega_c = 0.35 rad/sec \tag{1.1.27}$$

$$\implies T = 2.85 \tag{1.1.28}$$

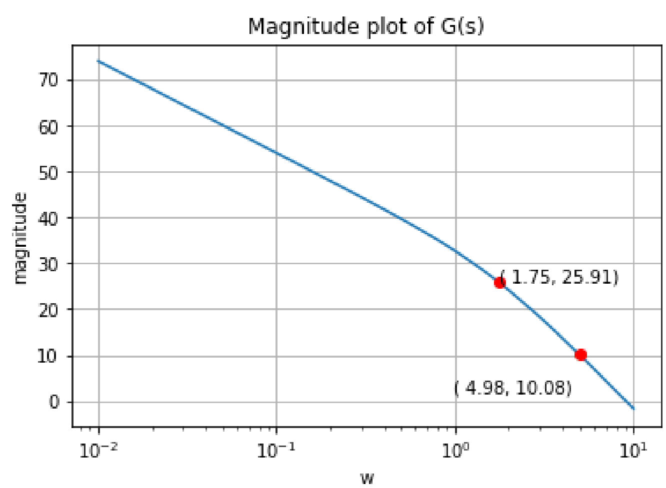

Fig. 1.1.3: Magnitude plot of G(s)

codes/ee18btech11001/ee18btech11001_1.py

The magnitude of $G(j\omega)$ at the new gain crossover frequency $\omega_c = 1.75 rad/sec$ is 26 dB as in figure 1.1.3.In order to have ω_c as the new gain crossover frequency, the lag compensator must give an attenuation of -26db at ω_c

$$-20\log b = -26dB \tag{1.1.29}$$

$$b = 19.95 \approx 20 \tag{1.1.30}$$

$$G_c(s) = 0.05\frac{s+0.35}{s+0.0175} \tag{1.1.31}$$

$$G(s)G_c(s) = 12.5\frac{s+0.35}{s(3s+5)(s+0.0175)} \tag{1.1.32}$$

Performance Evaluation of compensators
The following code plots the performance curves

```
codes/ee18btech11001/ee18btech11001_2.py
```

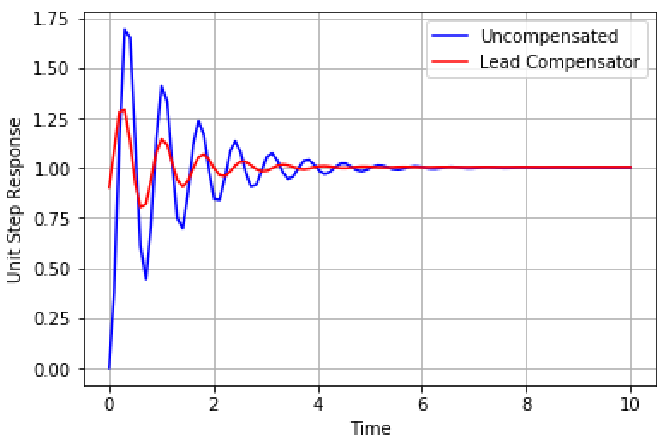

Fig. 1.1.4: Performance of Lead Compensator

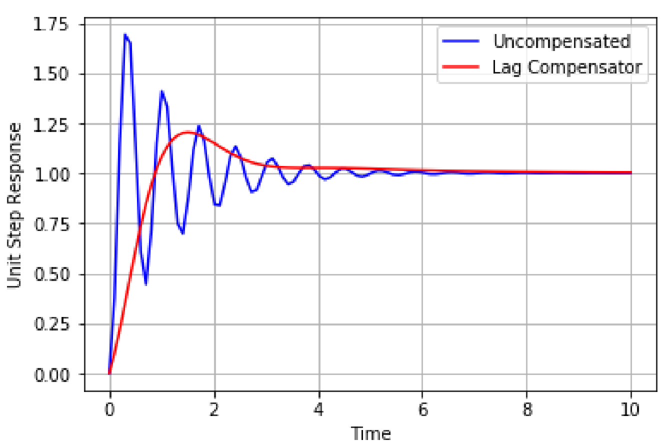

Fig. 1.1.5: Performance of Lag Compensator

The plots in figure 1.1.4 and 1.1.5 show the reduced overshoot and also reduced settling time for a unit step input function.

Compensator	Actual OS%	Expected OS%
Lead Compensator	26%	25%
Lag Compensator	25%	24%

TABLE 1.1.2: Performance comparison

This verifies that the designed lead and lag compensators work as per specifications.