Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Frequency Response Analysis

1.1 Polar Plot

1.1.1. A position control system is to be designed such that maximum peak overshoot is less than 25 %. Further, appropriate error constant should be 50. For the motor to be used, load and torque speed curve is shown below, where, J1 = 2 kg-m2 , J2 = 18 kg-m2 , f1 = 2 N-m-s/rad, f2 = 36 N-ms/rad. (Although obvious, consider position as the controlled variable and armature voltage as the manipulated variable.).

(i) Design a lead compensator for the system.
(ii) Design a lag compensator for the system.

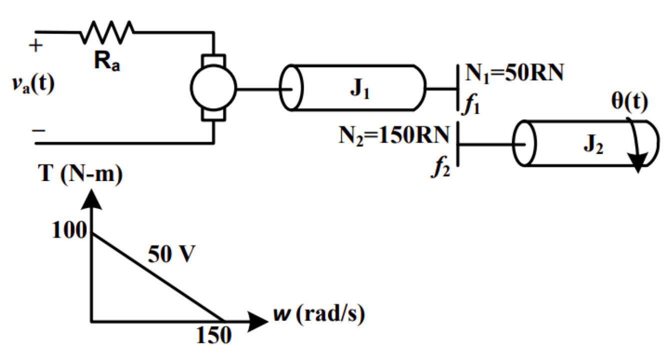

Fig. 1.1.1

Solution: Solving the system shown in 1.1.1,

From load-torque curve of DC Motor

$$T_m = 2V_a - \frac{2}{3}\omega_1 \tag{1.1.1}$$

Let T_1 , ω_1,θ_1 be the Torque, Angular velocity, Angular displacement on J_1

$$T_m = T_1 + J_1\ddot{\theta}_1 + f_1\dot{\theta}_1 \tag{1.1.2}$$

Similarly for J_2

$$T_1 = J_2\ddot{\theta}_2 + f_2\dot{\theta}_2 \tag{1.1.3}$$

$$T_2 = \frac{N_2}{N_1}T_1 \tag{1.1.4}$$

$$\theta_2 = \frac{N_1}{N_2}\theta_1 \tag{1.1.5}$$

On solving the equations (1.1.1) , (1.1.2), (1.1.3), (1.1.4) & (1.1.5)

$$V_a = 6\ddot{\theta}_2 + 10\dot{\theta}_2 \tag{1.1.6}$$

Taking Laplace transform

$$G(s) = K\frac{\theta(s)}{V_a(s)} = \frac{1}{2s(3s+5)} \tag{1.1.7}$$

From Error Constant K = 500

$$G(s) = \frac{\theta(s)}{V_a(s)} = 250\frac{1}{s(3s+5)} \tag{1.1.8}$$

$$\zeta = 0.0695 \tag{1.1.9}$$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 81.6\% \tag{1.1.10}$$

$$PhaseMargin = \phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4+1}}}\right) \tag{1.1.11}$$

$\phi_{max} = 39.5° - 7.35°+$ correction factor
$\phi_{max} = 57°$

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Specifications	Actual	Expected
OS%	81.6%	25%
ζ	0.0695	0.403
ϕ_m	7.35°	39.5°

TABLE 1.1.1: Table of Specifications

Designing a lead compensator Let

$$G_c(s) = \frac{1}{a}\frac{s+\frac{1}{T}}{s+\frac{1}{aT}}(a<1) \quad (1.1.12)$$

$$\sin\phi_{max} = \frac{a-1}{a+1} \quad (1.1.13)$$

$$a = 0.1 \quad (1.1.14)$$

$$|G(j\omega_c)| = \frac{1}{\sqrt{a}} = 10dB \quad (1.1.15)$$

$\omega_c = 5°$ Refer 1.1.3

$$T = \frac{1}{\omega_c\sqrt{a}} \quad (1.1.16)$$

$$T = 0.632 \quad (1.1.17)$$

$$G_c(s) = 10\frac{s+1.6}{s+16} \quad (1.1.18)$$

$$G(s)G_c(s) = 2500\frac{s+1.6}{s(3s+5)(s+16)} \quad (1.1.19)$$

Designing a lag compensator

$$G_c(s) = \frac{1}{b}\frac{s+\frac{1}{T}}{s+\frac{1}{bT}}(b>1) \quad (1.1.20)$$

$$\phi_{max} = 39.5° - 7.35° + correctionfactor \quad (1.1.21)$$

$$\phi_{max} = 45° \quad (1.1.22)$$

ω_c = Frequency at which phase of bode plot of G(s) is -180 +ϕ_{max} i.e. -135 °
$\omega_c = 1.75rad/sec$ as in Figure 1.1.2
We place the zero at

$$\omega = 0.2\omega_c = 0.35rad/sec \quad (1.1.23)$$

$$\frac{1}{T} = 0.35 \quad (1.1.24)$$

$$T = 2.85 \quad (1.1.25)$$

The following code plots the following bode plots

```
codes/ee18btech11001/
    ee18btech11001_1.py
```

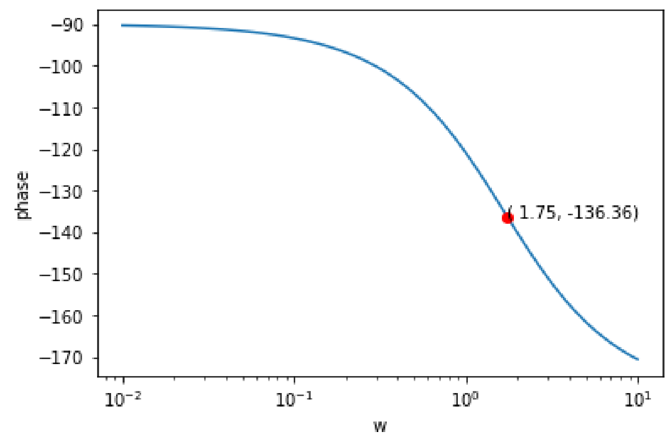

Fig. 1.1.2: Phase plot of G(s)

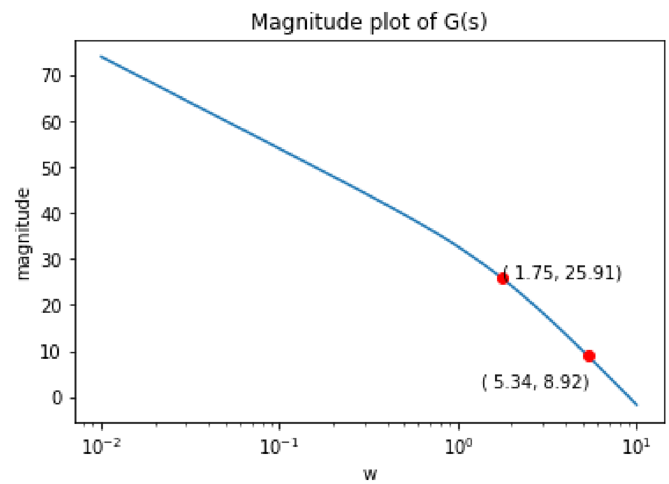

Fig. 1.1.3: Magnitude plot of G(s)

The magnitude of $G(j\omega)$ at the new gain crossover frequency $\omega_c = 1.75rad/sec$ is 26 dB as in figure 1.1.3.In order to have ω_c as the new gain crossover frequency, the lag compensator must give an attenuation of -26db at ω_c

$$-20\log b = -26dB \quad (1.1.26)$$

$$b = 19.95 \approx 20 \quad (1.1.27)$$

$$G_c(s) = 0.05\frac{s+0.35}{s+0.0175} \quad (1.1.28)$$

$$G(s)G_c(s) = 12.5\frac{s+0.35}{s(3s+5)(s+0.0175)} \quad (1.1.29)$$

Performance Evaluation of compensators
The following code plots the performance curves

```
codes/ee18btech11001/
    ee18btech11001_2.py
```

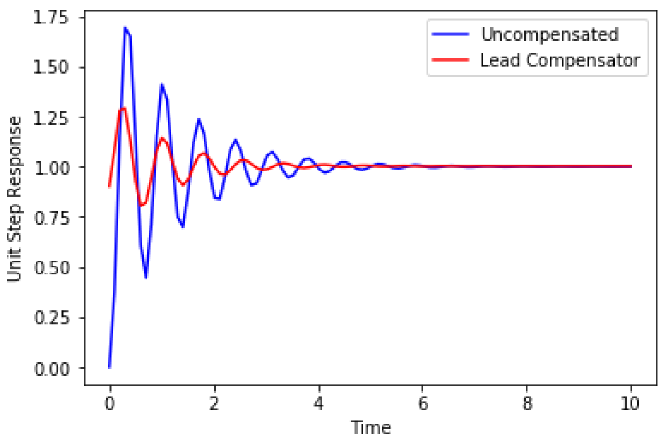

Fig. 1.1.4: Performance of Lead Compensator

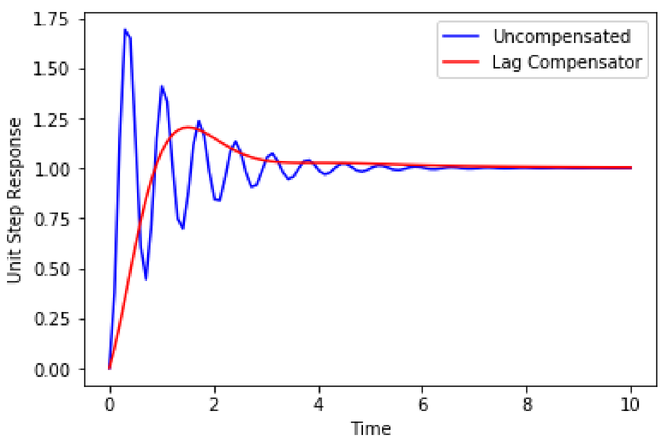

Fig. 1.1.5: Performance of Lag Compensator

Compensator	**Actual OS%**	**Expected OS%**
Lead Compensator	26%	25%
Lag Compensator	25%	24%

TABLE 1.1.2: Performance comparison