

# Control Systems

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*Abstract*—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

## 1 FREQUENCY RESPONSE ANALYSIS

### 1.1 Polar Plot

1.1.1. A position control system is to be designed such that maximum peak overshoot is less than 25 %. Further, appropriate error constant should be 50. For the motor to be used, load and torque speed curve is shown below, where,  $J_1 = 2 \text{ kg-m}^2$ ,  $J_2 = 18 \text{ kg-m}^2$ ,  $f_1 = 2 \text{ N-m-s/rad}$ ,  $f_2 = 36 \text{ N-ms/rad}$ . (Although obvious, consider position as the controlled variable and armature voltage as the manipulated variable.).

- Design a lead compensator for the system.
- Design a lag compensator for the system.

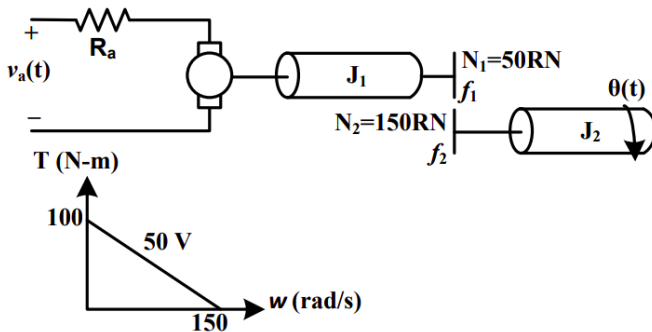


Fig. 1.1.1

**Solution:** Solving the system shown in 1.1.1,

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From speed-torque curve of DC Motor in figure. Let  $T_m$  be the torque exerted by DC Motor.

$$T_m = 2V_a - \frac{2}{3}\omega_1 \quad (1.1.1)$$

Variable	Description
$T_m$	Torque applied by motor on left of $J_1$
$T_1$	Torque existing on right of $J_1$
$T_2$	Torque existing on left of $J_2$
$\theta_1$	Position vector of $J_1$
$\theta_2$	Position vector of $J_2$
$V_a$	Armature voltage of DC Motor
$\omega_1$	Angular velocity of $J_1$
$J_1$	Moment of Intertia of first load
$J_2$	Moment of Intertia of second load
$f_1$	Viscous friction on $J_1$
$f_2$	Viscous friction on $J_2$

TABLE 1.1.1: Performance comparison

Change in torque across ends = torque applied on load + viscous friction. On  $J_1$  at one end torque  $T_m$  is applied and at the other end  $T_1$  exists.

$$T_m = T_1 + J_1\ddot{\theta}_1 + f_1\dot{\theta}_1 \quad (1.1.2)$$

Similarly for  $J_2$

$$T_2 = J_2\ddot{\theta}_2 + f_2\dot{\theta}_2 \quad (1.1.3)$$

$$T_2 = \frac{N_2}{N_1}T_1 \text{ (Gear Train Formula)} \quad (1.1.4)$$

$$\theta_2 = \frac{N_1}{N_2}\theta_1 \text{ (Gear Train Formula)} \quad (1.1.5)$$

Converting to State Space model

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \quad (1.1.6)$$

$$3\mathbf{T}_m = \begin{pmatrix} 3J_1 & J_2 \end{pmatrix} \ddot{\theta} + \begin{pmatrix} 3f_1 & f_2 \end{pmatrix} \dot{\theta} \quad (1.1.7)$$

$$\mathbf{T}_m = 2V_a - \begin{pmatrix} \frac{2}{3} & 0 \end{pmatrix} \dot{\theta} \quad (1.1.8)$$

$$\theta = \begin{pmatrix} N_2 & N_1 \end{pmatrix} K \quad (1.1.9)$$

On solving the above State Space Model

$$V_a = 6\ddot{\theta}_2 + 10\dot{\theta}_2 \quad (1.1.10)$$

Taking Laplace transform

$$G(s) = K \frac{\theta(s)}{V_a(s)} = \frac{1}{2s(3s+5)} \quad (1.1.11)$$

From Error Constant  $K = 500$

$$G(s) = \frac{\theta(s)}{V_a(s)} = 250 \frac{1}{s(3s+5)} \quad (1.1.12)$$

$$\zeta = 0.0695 \quad (1.1.13)$$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 81.6\% \quad (1.1.14)$$

$$\phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}\right) \quad (1.1.15)$$

$$\phi_{max} = 39.5^\circ - 7.35^\circ + \text{correction factor} \quad (1.1.16)$$

$$\phi_{max} = 57^\circ \quad (1.1.17)$$

Specifications	Actual	Expected
OS%	81.6%	25%
$\zeta$	0.0695	0.403
$\phi_m$	7.35°	39.5°

TABLE 1.1.2: Table of Specifications

**Designing a lead compensator**

$$G_c(s) = \frac{1}{a} \frac{s + \frac{1}{T}}{s + \frac{1}{aT}} (a < 1) \quad (1.1.18)$$

$$\sin \phi_{max} = \frac{a-1}{a+1} \quad (1.1.19)$$

$$a = 0.1 \quad (1.1.20)$$

$$|G(j\omega_c)| = \frac{1}{\sqrt{a}} = 10dB \quad (1.1.21)$$

$$\omega_c = 5^\circ \text{ (Refer figure 1.1.3)} \quad (1.1.22)$$

$$T = \frac{1}{\omega_c \sqrt{a}} = 0.632 \quad (1.1.23)$$

$$G_c(s) = 10 \frac{s+1.6}{s+16} \quad (1.1.24)$$

$$G(s)G_c(s) = 2500 \frac{s+1.6}{s(3s+5)(s+16)} \quad (1.1.25)$$

**Designing a lag compensator**

$$G_c(s) = \frac{1}{b} \frac{s + \frac{1}{T}}{s + \frac{1}{bT}} (b > 1) \quad (1.1.26)$$

$$\phi_{max} = 39.5^\circ - 7.35^\circ + \text{correction factor} \quad (1.1.27)$$

$$\phi_{max} = 45^\circ \quad (1.1.28)$$

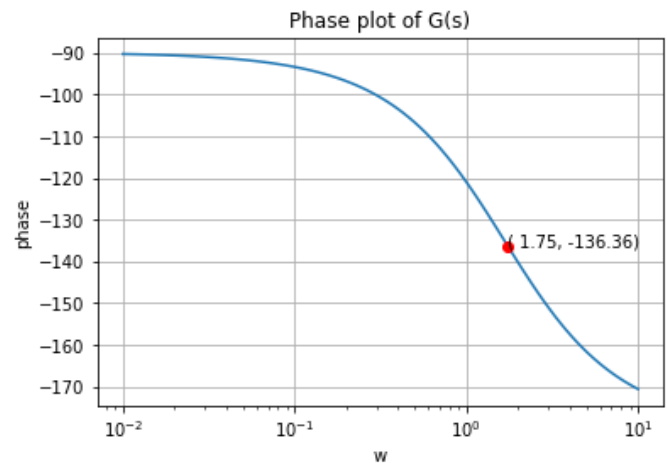


Fig. 1.1.2: Phase plot of G(s)

$\omega_c$  = Frequency at which phase of bode plot of G(s) is  $-180 + \phi_{max}$  i.e.  $-135^\circ$

$\omega_c = 1.75 \text{ rad/sec}$  as in Figure 1.1.2

We place the zero at

$$\omega = 0.2\omega_c = 0.35 \text{ rad/sec} \quad (1.1.29)$$

$$\Rightarrow T = 2.85 \quad (1.1.30)$$

codes/ee18btech11001/ee18btech11001\_1.py

The magnitude of  $G(j\omega)$  at the new gain crossover frequency  $\omega_c = 1.75 \text{ rad/sec}$  is 26 dB as in figure 1.1.3. In order to have  $\omega_c$  as the new

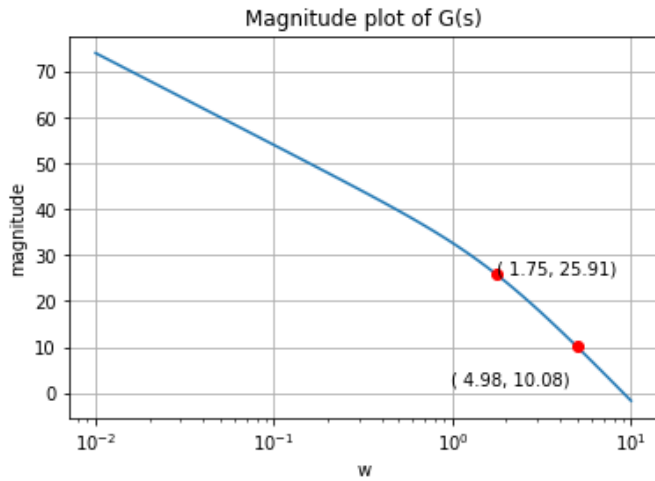


Fig. 1.1.3: Magnitude plot of  $G(s)$

gain crossover frequency, the lag compensator must give an attenuation of  $-26\text{dB}$  at  $\omega_c$

$$-20 \log b = -26\text{dB} \quad (1.1.31)$$

$$b = 19.95 \approx 20 \quad (1.1.32)$$

$$G_c(s) = 0.05 \frac{s + 0.35}{s + 0.0175} \quad (1.1.33)$$

$$G(s)G_c(s) = 12.5 \frac{s + 0.35}{s(3s + 5)(s + 0.0175)} \quad (1.1.34)$$

### Performance Evaluation of compensators

The following code plots the performance curves

```
codes/ee18btech11001/ee18btech11001_2.py
```

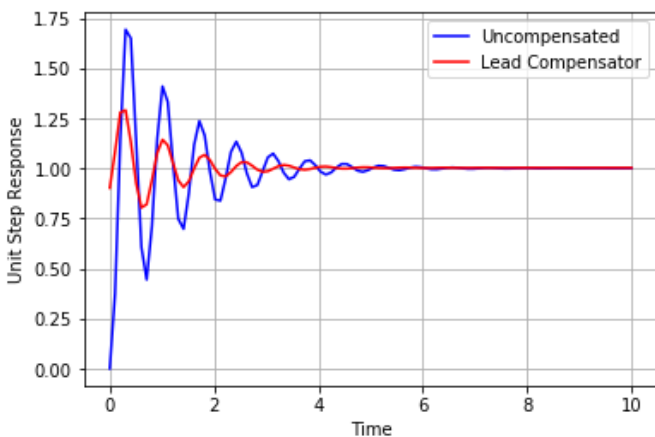


Fig. 1.1.4: Performance of Lead Compensator

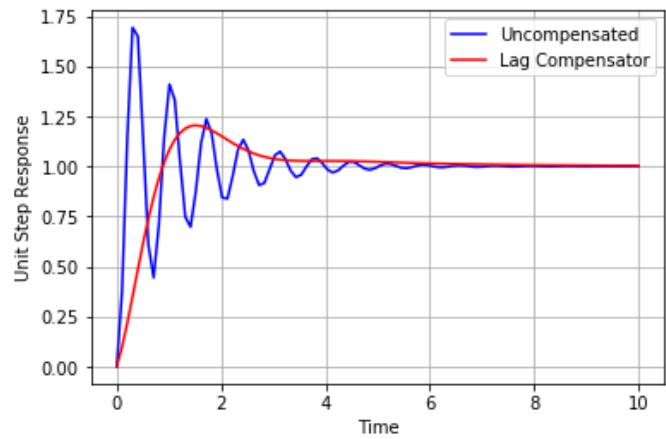


Fig. 1.1.5: Performance of Lag Compensator

Compensator	Actual OS%	Expected OS%
Lead Compensator	26%	25%
Lag Compensator	25%	24%

TABLE 1.1.3: Performance comparison

This verifies that the designed lead and lag compensators work as per specifications.

Figures 1.1.4 and 1.1.5 show the reduced overshoot & settling time for unit step input.