

Control Systems

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Abstract—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

1 FREQUENCY RESPONSE ANALYSIS

1.1 Polar Plot

1.1.1. A position control system is to be designed such that maximum peak overshoot is less than 25 %. Further, appropriate error constant should be 50. For the motor to be used, load and torque speed curve is shown below, where, $J_1 = 2 \text{ kg-m}^2$, $J_2 = 18 \text{ kg-m}^2$, $f_1 = 2 \text{ N-m-s/rad}$, $f_2 = 36 \text{ N-m-s/rad}$. (Although obvious, consider position as the controlled variable and armature voltage as the manipulated variable.).

- Design a lead compensator for the system.
- Design a lag compensator for the system.

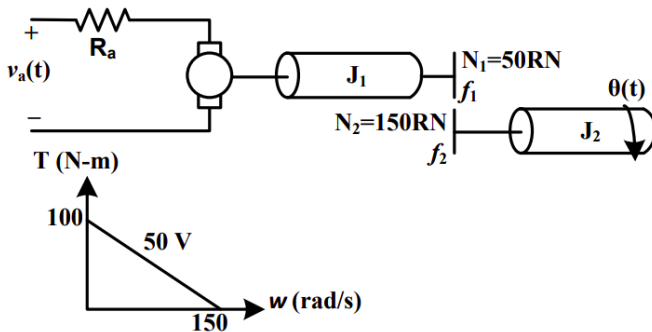


Fig. 1.1.1

Solution: Solving the system shown in 1.1.1,

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From speed-torque curve of DC Motor in figure. Let T_m be the torque exerted by DC Motor.

$$T_m = \frac{K_T}{R_a} V_a - \frac{K_T K_v}{R_a} \omega_1 \quad (1.1.1)$$

$$T_m = 2V_a - \frac{2}{3} \omega_1 \quad (1.1.2)$$

Variable	Description
T_m	Torque applied by motor on left of J_1
T_1	Torque existing on right of J_1
T_2	Torque existing on left of J_2
θ_1	Position vector of J_1
θ_2	Position vector of J_2
V_a	Armature voltage of DC Motor
ω_1	Angular velocity of J_1
J_1	Moment of Intertia of first load
J_2	Moment of Intertia of second load
f_1	Viscous friction on J_1
f_2	Viscous friction on J_2

TABLE 1.1.1: List of Variables

Change in torque across ends = torque applied on load + viscous friction. On J_1 at one end torque T_m is applied and at the other end T_1 exists.

$$T_m = T_1 + J_1 \ddot{\theta}_1 + f_1 \dot{\theta}_1 \quad (1.1.3)$$

Similarly for J_2

$$T_2 = J_2 \ddot{\theta}_2 + f_2 \dot{\theta}_2 \quad (1.1.4)$$

$$T_2 = \frac{N_2}{N_1} T_1 \quad (\text{Gear Train Formula}) \quad (1.1.5)$$

$$\theta_2 = \frac{N_1}{N_2} \theta_1 \quad (\text{Gear Train Formula}) \quad (1.1.6)$$

Vector/Ma- trix	Dimension
$\theta_1 \& \theta_2$	1X3
θ	2X3
T_m	1X3
K	2X3

TABLE 1.1.2: Vectors and Matrices

Converting to State Space model

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \quad (1.1.7)$$

$$3T_m = \begin{pmatrix} 3J_1 & J_2 \end{pmatrix} \ddot{\theta} + \begin{pmatrix} 3f_1 & f_2 \end{pmatrix} \dot{\theta} \quad (1.1.8)$$

$$T_m = 2V_a - \begin{pmatrix} \frac{2}{3} & 0 \end{pmatrix} \dot{\theta} \quad (1.1.9)$$

$$\theta = \begin{pmatrix} N_2 & 0 \\ 0 & N_1 \end{pmatrix} K \quad (1.1.10)$$

This is the state space model obtained

$$\ddot{\theta} = \begin{pmatrix} \frac{-13}{6} & 0 \\ 0 & \frac{-5}{3} \end{pmatrix} \dot{\theta} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{6} \end{pmatrix} V_a \quad (1.1.11)$$

$$\dot{\theta}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \dot{\theta} \quad (1.1.12)$$

On solving the above State Space Model

$$V_a(s) = (6s + 10)(s\theta_2(s)) \quad (1.1.13)$$

$$G(s) = K \frac{\theta(s)}{V_a(s)} = \frac{K}{2s(3s + 5)} \quad (1.1.14)$$

From Error Constant $K = 500$

$$G(s) = \frac{\theta(s)}{V_a(s)} = 250 \frac{1}{s(3s + 5)} \quad (1.1.15)$$

$$\zeta = 0.0695 \quad (1.1.16)$$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 81.6\% \quad (1.1.17)$$

$$\phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}\right) \quad (1.1.18)$$

$$\phi_{max} = 39.5^\circ - 7.35^\circ + \text{correction factor} \quad (1.1.19)$$

$$\phi_{max} = 57^\circ \quad (1.1.20)$$

Specifications	Actual	Expected
OS%	81.6%	25%
ζ	0.0695	0.403
ϕ_m	7.35°	39.5°

TABLE 1.1.3: Table of Specifications

Designing a lead compensator

$$G_c(s) = \frac{1}{a} \frac{s + \frac{1}{T}}{s + \frac{1}{aT}} (a < 1) \quad (1.1.21)$$

$$\sin \phi_{max} = \frac{a - 1}{a + 1} \quad (1.1.22)$$

$$a = 0.1 \quad (1.1.23)$$

$$|G(j\omega_c)| = \frac{1}{\sqrt{a}} = 10dB \quad (1.1.24)$$

$$\omega_c = 5^\circ \text{ (Refer figure 1.1.3)} \quad (1.1.25)$$

$$T = \frac{1}{\omega_c \sqrt{a}} = 0.632 \quad (1.1.26)$$

$$G_c(s) = 10 \frac{s + 1.6}{s + 16} \quad (1.1.27)$$

$$G(s)G_c(s) = 2500 \frac{s + 1.6}{s(3s + 5)(s + 16)} \quad (1.1.28)$$

Designing a lag compensator

$$G_c(s) = \frac{1}{b} \frac{s + \frac{1}{T}}{s + \frac{1}{bT}} (b > 1) \quad (1.1.29)$$

$$\phi_{max} = 39.5^\circ - 7.35^\circ + \text{correction factor} \quad (1.1.30)$$

$$\phi_{max} = 45^\circ \quad (1.1.31)$$

ω_c = Frequency at which phase of bode plot of $G(s)$ is $-180 + \phi_{max}$ i.e. -135°

$\omega_c = 1.75 \text{ rad/sec}$ as in Figure 1.1.2

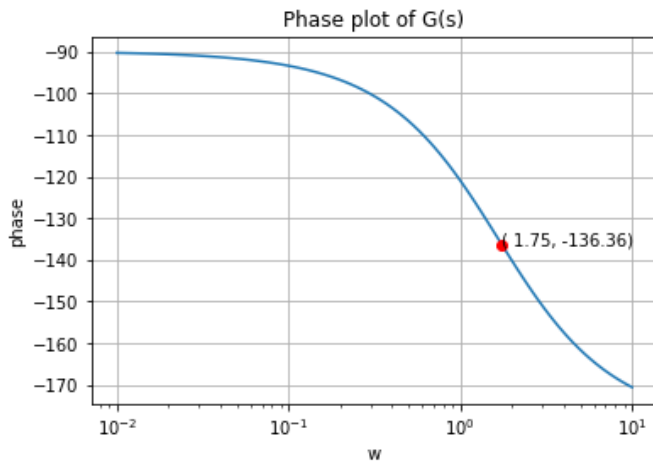
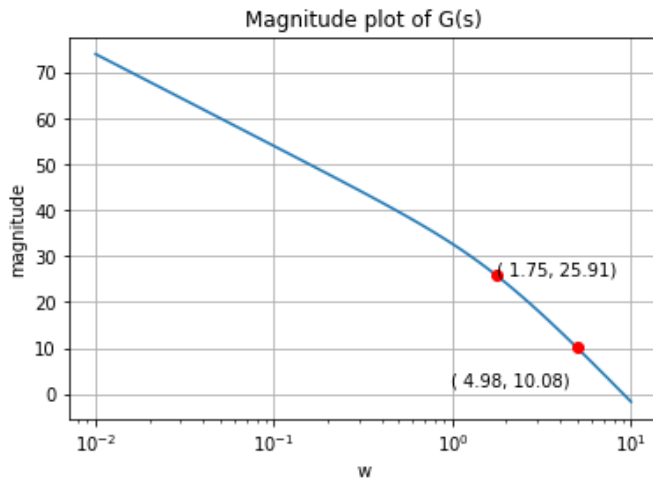
We place the zero at

$$\omega = 0.2\omega_c = 0.35 \text{ rad/sec} \quad (1.1.32)$$

$$\Rightarrow T = 2.85 \quad (1.1.33)$$

codes/ee18btech11001/ee18btech11001_1.py

The magnitude of $G(j\omega)$ at the new gain crossover frequency $\omega_c = 1.75 \text{ rad/sec}$ is 26 dB as in figure 1.1.3. In order to have ω_c as the new gain crossover frequency, the lag compensator

Fig. 1.1.2: Phase plot of $G(s)$ Fig. 1.1.3: Magnitude plot of $G(s)$

must give an attenuation of -26db at ω_c

$$-20 \log b = -26 \text{ dB} \quad (1.1.34)$$

$$b = 19.95 \approx 20 \quad (1.1.35)$$

$$G_c(s) = 0.05 \frac{s + 0.35}{s + 0.0175} \quad (1.1.36)$$

$$G(s)G_c(s) = 12.5 \frac{s + 0.35}{s(3s + 5)(s + 0.0175)} \quad (1.1.37)$$

Performance Evaluation of compensators

The following code plots the performance curves

```
codes/ee18btech11001/ee18btech11001_2.py
```

Figures 1.1.4 and 1.1.5 show the reduced overshoot & settling time for unit step input.

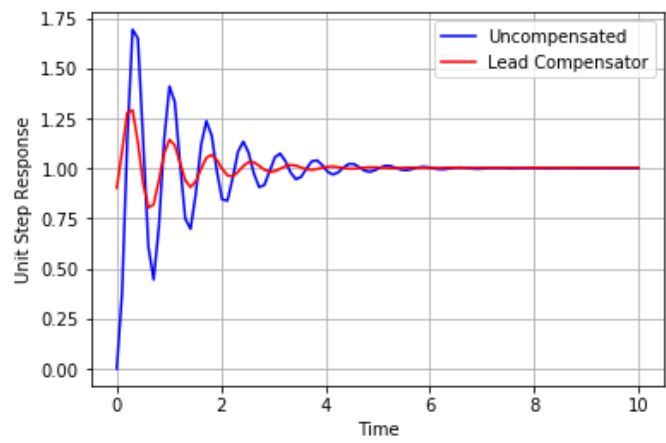


Fig. 1.1.4: Performance of Lead Compensator

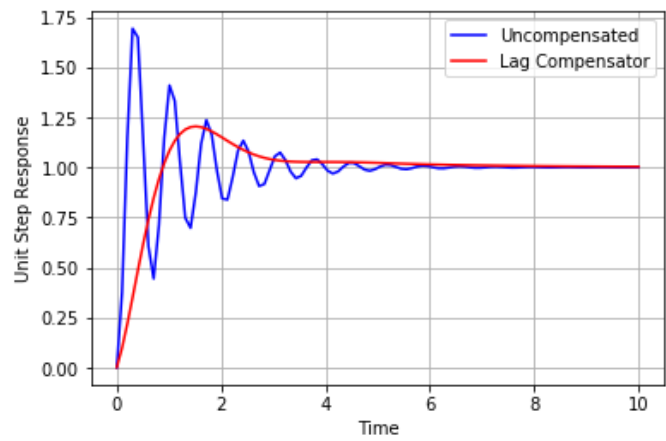


Fig. 1.1.5: Performance of Lag Compensator

Compensator	Actual OS%	Expected OS%
Lead Compensator	26%	25%
Lag Compensator	25%	24%

TABLE 1.1.4: Performance comparison

This verifies that the designed lead and lag compensators work as per specifications.