Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Frequency Response Analysis

1.1 Polar Plot

1.1.1. A position control system is to be designed such that maximum peak overshoot is less than 25 %. Further, appropriate error constant should be 50. For the motor to be used, load and torque speed curve is shown below, where, J_1 = 2 kg-m2 , J_2 = 18 kg-m2 , f_1 = 2 N-m-s/rad, f_2 = 36 N-ms/rad. (Although obvious, consider position as the controlled variable and armature voltage as the manipulated variable.).

(i) Design a lead compensator for the system.

(ii) Design a lag compensator for the system.

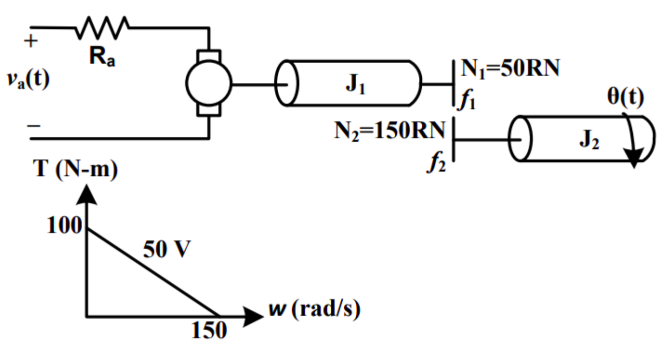

Fig. 1.1.1

Solution: Solving the system shown in 1.1.1,

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From speed-torque curve of DC Motor in figure. Let $\mathbf{T_m}$ be the torque exerted by DC Motor. 1.1.1

$$\mathbf{T_m} = \frac{\mathbf{K_T}}{R_a}V_a - \frac{\mathbf{K_T}.\mathbf{K_v^T}}{R_a}\boldsymbol{\omega_1} \tag{1.1.1}$$

$$\mathbf{T_m} = 2V_a - \frac{2}{3}\boldsymbol{\omega_1} \tag{1.1.2}$$

Variable	Description
$\mathbf{T_m}$	Torque applied by motor on left of J_1
$\mathbf{T_1}$	Torque existing on right of J_1
$\mathbf{T_2}$	Torque existing on left of J_2
$\boldsymbol{\theta_1}$	Position vector of J_1
$\boldsymbol{\theta_2}$	Position vector of J_2
V_a	Armature voltage of DC Motor
$\boldsymbol{\omega_1}$	Angular velocity of J_1
J_1	Moment of Intertia of first load
J_2	Moment of Intertia of second load
f_1	Viscous friction on J_1
f_2	Viscous friction on J_2

TABLE 1.1.1: List of Variables

Change in torque across ends = torque applied on load + viscous friction. On J_1 at one end torque T_m is applied and at the other end T_1 exists.

$$\mathbf{T_m} = \mathbf{T_1} + J_1\ddot{\boldsymbol{\theta}}_1 + f_1\dot{\boldsymbol{\theta}}_1 \tag{1.1.3}$$

Similarly for J_2

$$\mathbf{T_2} = J_2\ddot{\boldsymbol{\theta}}_2 + f_2\dot{\boldsymbol{\theta}}_2 \tag{1.1.4}$$

$$\mathbf{T_2} = \frac{N_2}{N_1}\mathbf{T_1} \text{ (Gear Train Formula)} \tag{1.1.5}$$

$$\boldsymbol{\theta_2} = \frac{N_1}{N_2}\boldsymbol{\theta_1} \text{ (Gear Train Formula)} \tag{1.1.6}$$

Vector/Matrix	Dimension
$\boldsymbol{\theta_1}\&\boldsymbol{\theta_2}$	1X3
$\boldsymbol{\theta}$	2X3
$\mathbf{T_m}$	1X3
$\mathbf{K}$	2X3
$\mathbf{K_v}$	1X3
$\mathbf{K_T}$	1X3

TABLE 1.1.2: Vectors and Matrices

Converting to State Space model

$$\boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta_1} \\ \boldsymbol{\theta_2} \end{pmatrix} \tag{1.1.7}$$

$$3\mathbf{T_m} = \begin{pmatrix} 3J_1 & J_2 \end{pmatrix} \ddot{\boldsymbol{\theta}} + \begin{pmatrix} 3f_1 & f_2 \end{pmatrix} \dot{\boldsymbol{\theta}} \tag{1.1.8}$$

$$\mathbf{T_m} = 2V_a - \begin{pmatrix} \frac{2}{3} & 0 \end{pmatrix} \dot{\boldsymbol{\theta}} \tag{1.1.9}$$

$$\boldsymbol{\theta} = \begin{pmatrix} N_2 & 0 \\ 0 & N_1 \end{pmatrix} K \tag{1.1.10}$$

This is the state space model obtained

$$\ddot{\boldsymbol{\theta}} = \begin{pmatrix} \frac{-13}{6} & 0 \\ 0 & \frac{-5}{3} \end{pmatrix} \dot{\boldsymbol{\theta}} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{6} \end{pmatrix} V_a \tag{1.1.11}$$

$$\dot{\boldsymbol{\theta_2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \dot{\boldsymbol{\theta}} \tag{1.1.12}$$

On solving the above State Space Model

$$V_a(s) = (6s + 10)(s\theta_2(s)) \tag{1.1.13}$$

$$G(s) = K\frac{\theta(s)}{V_a(s)} = \frac{K}{2s(3s+5)} \tag{1.1.14}$$

From Error Constant K = 500

$$G(s) = \frac{\theta(s)}{V_a(s)} = 250\frac{1}{s(3s+5)} \tag{1.1.15}$$

$$\zeta = 0.0695 \tag{1.1.16}$$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 81.6\% \tag{1.1.17}$$

$$\phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4+1}}}\right) \tag{1.1.18}$$

$$\phi_{max} = 39.5^\circ - 7.35^\circ + \text{ correction factor} \tag{1.1.19}$$

$$\phi_{max} = 57^\circ \tag{1.1.20}$$

Specifications	Actual	Expected
OS%	81.6%	25%
ζ	0.0695	0.403
ϕ_m	7.35°	39.5°

TABLE 1.1.3: Table of Specifications

Designing a lead compensator

$$G_c(s) = \frac{1}{a}\frac{s+\frac{1}{T}}{s+\frac{1}{aT}}(a<1) \tag{1.1.21}$$

$$\sin\phi_{max} = \frac{a-1}{a+1} \tag{1.1.22}$$

$$a = 0.1 \tag{1.1.23}$$

$$|G(j\omega_c)| = \frac{1}{\sqrt{a}} = 10dB \tag{1.1.24}$$

$$\omega_c = 5^\circ \text{ (Refer figure 1.1.3)} \tag{1.1.25}$$

$$T = \frac{1}{\omega_c\sqrt{a}} = 0.632 \tag{1.1.26}$$

$$G_c(s) = 10\frac{s+1.6}{s+16} \tag{1.1.27}$$

$$G(s)G_c(s) = 2500\frac{s+1.6}{s(3s+5)(s+16)} \tag{1.1.28}$$

Designing a lag compensator

$$G_c(s) = \frac{1}{b}\frac{s+\frac{1}{T}}{s+\frac{1}{bT}}(b>1) \tag{1.1.29}$$

$$\phi_{max} = 39.5^\circ - 7.35^\circ + correctionfactor \tag{1.1.30}$$

$$\phi_{max} = 45^\circ \tag{1.1.31}$$

ω_c = Frequency at which phase of bode plot of G(s) is -180 +ϕ_{max} i.e. -135 °
$\omega_c = 1.75 rad/sec$ as in Figure 1.1.2
We place the zero at

$$\omega = 0.2\omega_c = 0.35 rad/sec \tag{1.1.32}$$

$$\implies T = 2.85 \tag{1.1.33}$$

codes/ee18btech11001/ee18btech11001_1.py

The magnitude of $G(j\omega)$ at the new gain crossover frequency $\omega_c = 1.75 rad/sec$ is 26 dB as in figure 1.1.3.In order to have ω_c as the new gain crossover frequency, the lag compensator

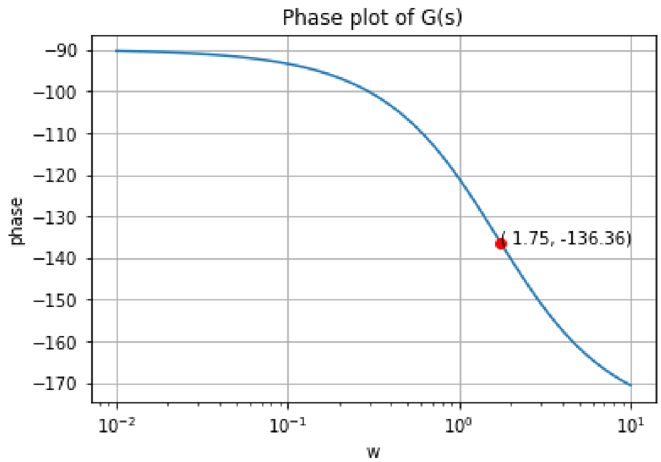

Fig. 1.1.2: Phase plot of G(s)

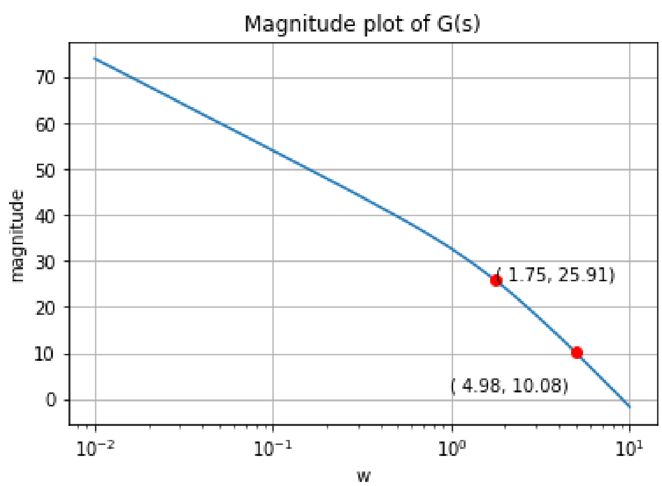

Fig. 1.1.3: Magnitude plot of G(s)

must give an attenuation of -26db at ω_c

$$-20 \log b = -26dB \tag{1.1.34}$$

$$b = 19.95 \approx 20 \tag{1.1.35}$$

$$G_c(s) = 0.05 \frac{s + 0.35}{s + 0.0175} \tag{1.1.36}$$

$$G(s)G_c(s) = 12.5 \frac{s + 0.35}{s(3s + 5)(s + 0.0175)} \tag{1.1.37}$$

Performance Evaluation of compensators
The following code plots the performance curves

codes/ee18btech11001/ee18btech11001_2.py

Figures 1.1.4 and 1.1.5 show the reduced overshoot & settling time for unit step input.

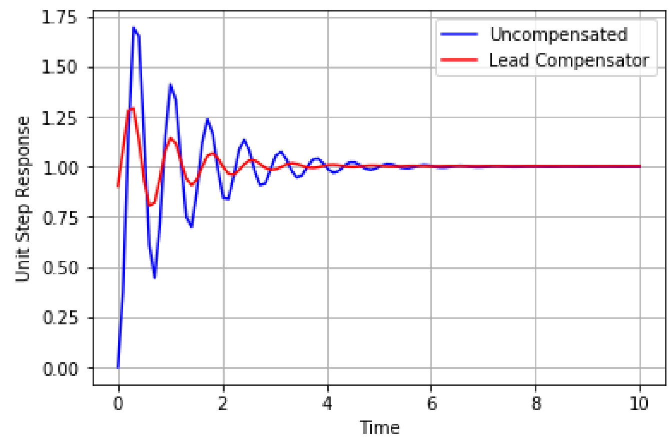

Fig. 1.1.4: Performance of Lead Compensator

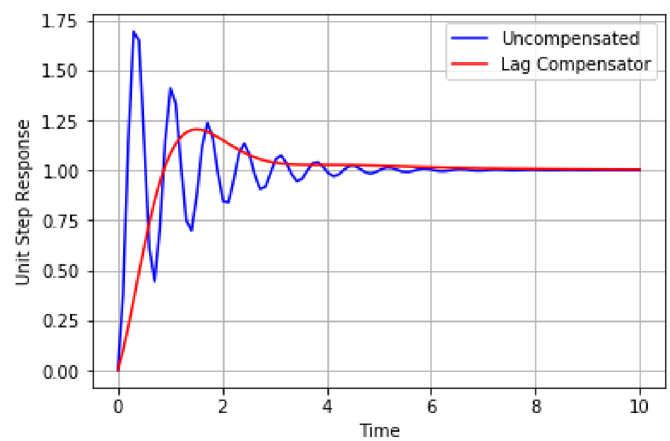

Fig. 1.1.5: Performance of Lag Compensator

Compensator	Actual OS%	Expected OS%
Lead Compensator	26%	25%
Lag Compensator	25%	24%

TABLE 1.1.4: Performance comparison

This verifies that the designed lead and lag compensators work as per specifications.