Assignment 1

AAYUSH GOYAL - EE18BTECH11001

Download all python codes from

https://github.com/aayush2710/EE3025_A1/codes

and latex-tikz codes from

https://github.com/aayush2710/EE3025_A1

1 Problem

(5.3) The system h(n) is said to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

2 Solution

BIBO Stability : For a system to be stable, bounded input should result in bounded output.
If

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty \tag{2.0.1}$$

Then

$$\sum_{n=-\infty}^{\infty} |y(n)| < \infty \tag{2.0.2}$$

Let us assume,

$$\sum_{n=-\infty}^{\infty} |x(n)| < B_x < \infty \tag{2.0.3}$$

$$|y(n)| = \left|\sum_{-\infty}^{\infty} h(k)x(n-k)\right| \tag{2.0.4}$$

$$|y(n)| \leq \sum_{-\infty}^{\infty} |h(k)| \sum_{-\infty}^{\infty} |x(n-k)| \tag{2.0.5}$$

$$|y(n)| \leq B_x \sum_{-\infty}^{\infty} |h(k)| \tag{2.0.6}$$

$$\implies \sum_{n=-\infty}^{\infty} |y(n)| < B_y < \infty \tag{2.0.7}$$

only when h(n) is bounded i.e

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{2.0.8}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| \left|z^{-n}\right|_{|z|=1} < \infty \tag{2.0.9}$$

$$\sum_{n=-\infty}^{\infty} \left|h(n)z^{-n}\right|_{|z|=1} < \infty \tag{2.0.10}$$

$$\sum_{n=-\infty}^{\infty} \left|h(n)z^{-n}\right|_{|z|=1} < \left|\sum_{n=-\infty}^{\infty} h(n)z^{-n}\right|_{|z|=1} \tag{2.0.11}$$

$$\implies |H(z)|_{|z|=1} < \infty \tag{2.0.12}$$

Conclusion : For a stable system, ROC of the system must include unit circle
Given the following difference equation

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2.0.13}$$

Applying Z-Transform

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{2.0.14}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{2.0.15}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{2.0.16}$$

$$\implies Poles = 0, -\frac{1}{2} \tag{2.0.17}$$

ROC of h(n) (Right sided system) lies outside the outermost pole ($\frac{1}{2}$)
H(z) is plotted in z-plane using python via the following script

https://github.com/aayush2710/EE3025_A1/codes/zplot.py

Observe that in figure 0, $|z| = 1$ clearly lies inside ROC.
$\implies$ System is stable.

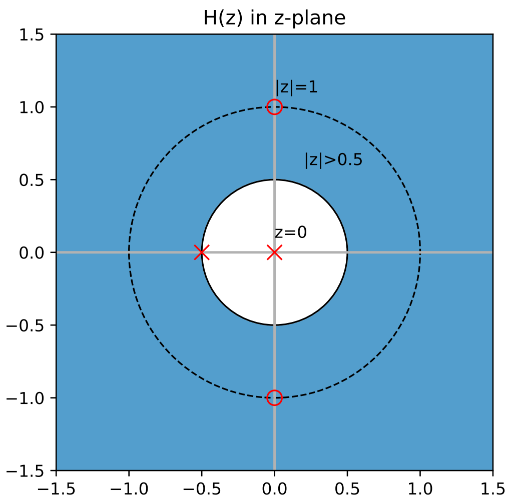

Fig. 0: H(z) in z-plane

3 Verification

Let us assume a bounded input

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{3.0.1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{3.0.2}$$

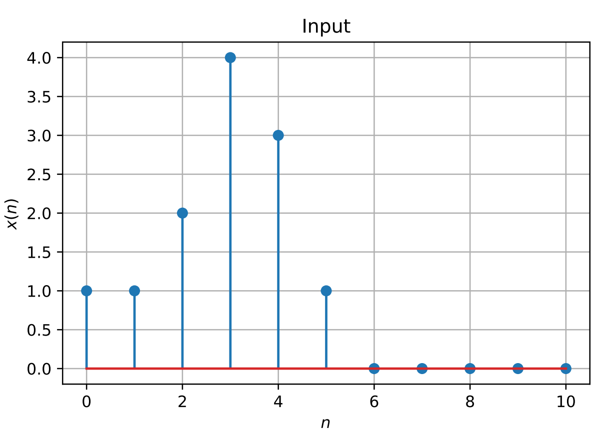

Fig. 0: Input x(n)

Clearly x(n) is absolutely summable
We can calculate y(n) using the difference equation
Use the following code to plot Input, Output and Impulse Response

```
https://github.com/aayush2710/EE3025_A1/codes/
    xyplot.py
```

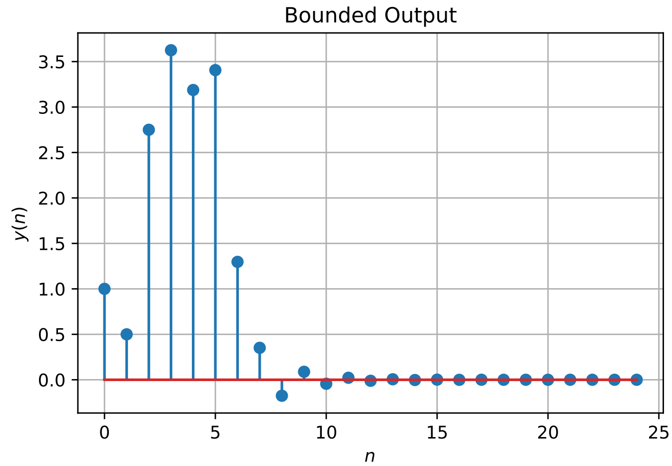

Fig. 0: Output y(n)

Clearly y(n) is absolutely summable as shown in figure 0. Hence Bounded Input results in Bounded Output. This verifies the above solution.

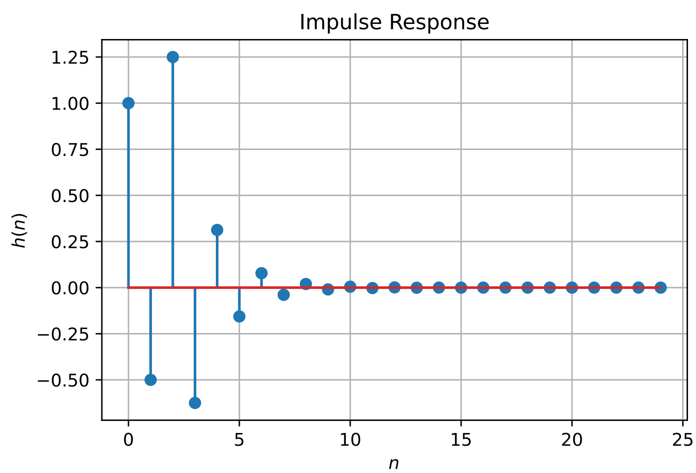

Fig. 0: Impulse Response h(n)

Further Impulse Response $h(n)$ of our system is also bounded as shown in figure 0.