

Assignment 1

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Download all python codes from

https://github.com/aayush2710/EE3025_A1/codes

and latex-tikz codes from

https://github.com/aayush2710/EE3025_A1

1 PROBLEM

(5.3) The system $h(n)$ is said to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad (1.0.1)$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

2 SOLUTION

BIBO Stability : For a system to be stable, bounded input should result in bounded output.

If

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty \quad (2.0.1)$$

Then

$$\sum_{n=-\infty}^{\infty} |y(n)| < \infty \quad (2.0.2)$$

Let us assume,

$$\sum_{n=-\infty}^{\infty} |x(n)| < B_x < \infty \quad (2.0.3)$$

$$|y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)x(n-k) \right| \quad (2.0.4)$$

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)| \sum_{n=-\infty}^{\infty} |x(n-k)| \quad (2.0.5)$$

$$|y(n)| \leq B_x \sum_{k=-\infty}^{\infty} |h(k)| \quad (2.0.6)$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |y(n)| < B_y < \infty \quad (2.0.7)$$

only when $h(n)$ is bounded i.e

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad (2.0.8)$$

$$\sum_{n=-\infty}^{\infty} |h(n)| |z^{-n}|_{|z|=1} < \infty \quad (2.0.9)$$

$$\sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} < \infty \quad (2.0.10)$$

$$\sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} < \left| \sum_{n=-\infty}^{\infty} h(n)z^{-n} \right|_{|z|=1} \quad (2.0.11)$$

$$\Rightarrow |H(z)|_{|z|=1} < \infty \quad (2.0.12)$$

Conclusion : For a stable system, ROC of system must include unit circle

Given the following difference equation

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \quad (2.0.13)$$

Applying Z-Transform

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \quad (2.0.14)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \quad (2.0.15)$$

$$H(z) = \frac{2(z^2 + 1)}{z(2z + 1)} \quad (2.0.16)$$

$$\Rightarrow \text{Poles} = 0, -\frac{1}{2} \quad (2.0.17)$$

ROC of $h(n)$ (Right sided system) lies outside the outermost pole ($\frac{1}{2}$)

$H(z)$ is plotted in z -plane using python via the following script

https://github.com/aayush2710/EE3025_A1/codes/zplot.py

Observe that in figure 0, $|z| = 1$ clearly lies inside ROC.

\Rightarrow System is stable.

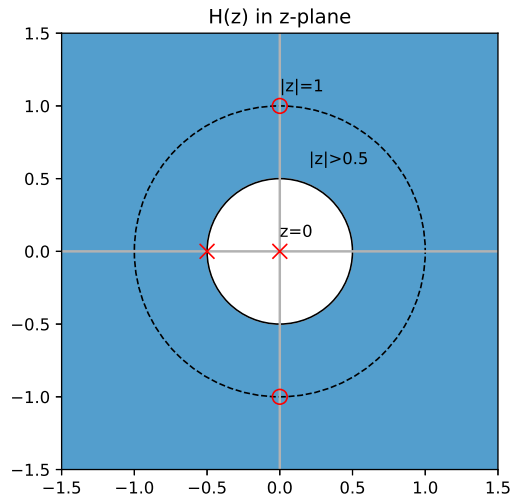


Fig. 0: H(z) in z-plane

3 VERIFICATION

Let us assume a bounded input

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \quad (3.0.1)$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \quad (3.0.2)$$

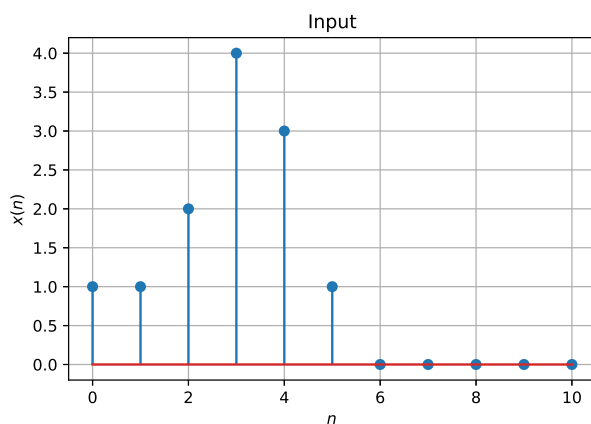
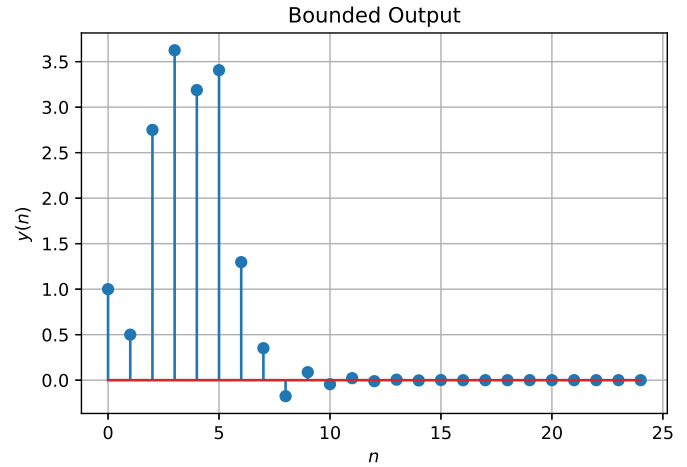


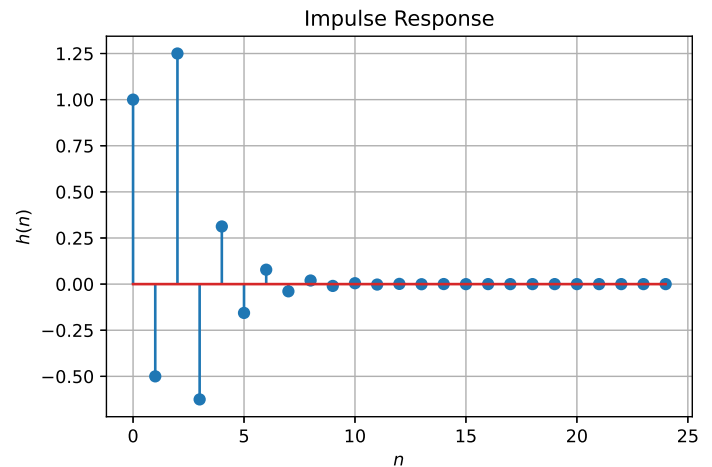
Fig. 0: Input x(n)

Clearly $x(n)$ is absolutely summable
 We can calculate $y(n)$ and plot it using python
 Use the following code to plot Input, Output and Impulse Response

```
https://github.com/aayush2710/EE3025__A1/codes/
xyplot.py
```

Fig. 0: Output $y(n)$

Clearly $y(n)$ is absolutely summable as shown in figure 0. Hence Bounded Input results in Bounded Output. This verifies the above solution.

Fig. 0: Impulse Response $h(n)$

Further Impulse Response $h(n)$ of our system is also bounded as shown in figure 0.