

Assignment 1

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Github repository

<https://github.com/aayush2710/EE4013-Assignment1>

1 PROBLEM

Consider the following Recurrence Relation

$$T(n) = \begin{cases} T(n/2) + T(2n/5) + 7n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases} \quad (1.0.1)$$

Which one of the following options is correct?

- 1) $T(n) = O(n^{\frac{5}{2}})$
- 2) $T(n) = O(n \log n)$
- 3) $T(n) = O(n)$
- 4) $T(n) = O((\log n)^{\frac{5}{2}})$

2 SOLUTION

Let us start by drawing the recurrence tree for the given problem. We can calculate the complexity of each level by summing up the complexity of each node at that level.

For Example : At L1, complexity is

$$7\left(\frac{n}{2}\right) + 7\left(\frac{2n}{5}\right) = 7\left(\frac{9n}{10}\right)$$

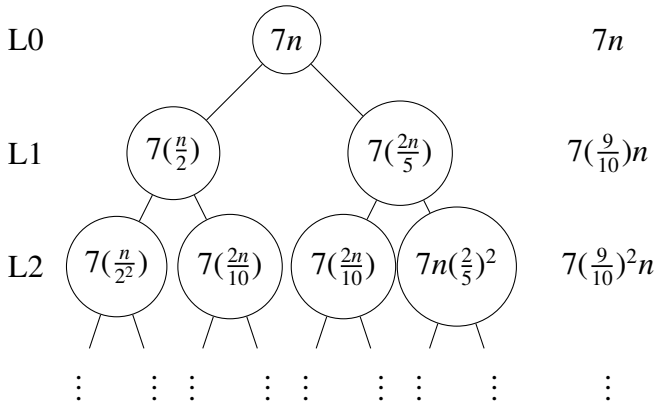


Fig. 1: Recurrence Tree

Length of left most subtree = $\log_2 n$

Length of right most subtree = $\log_{5/2} n$

Rightmost subtree is shortest so it determines the lower bound of time. Whereas the leftmost subtree is the longest, hence it determines the upper bound of time.

Height of the leftmost subtree is : $\log_2 n$ and so

$$T(n) \leq 7n + 7n\left(\frac{9}{10}\right) + 7n\left(\frac{9}{10}\right)^2 + \dots + 7n\left(\frac{9}{10}\right)^{\log_2 n} \quad (2.0.1)$$

$$T(n) \leq 7n \left(1 + \frac{9}{10} + 7\left(\frac{9}{10}\right)^2 + \dots + 7\left(\frac{9}{10}\right)^{\log_2 n} \right) \quad (2.0.2)$$

$$T(n) \leq 7n \frac{1 - \left(\frac{9}{10}\right)^{\log_2 n + 1}}{1 - \frac{9}{10}} \quad (2.0.3)$$

$$T(n) \leq 70n - 70\left(\frac{9}{10}\right) \left(n^{\log_2 \left(\frac{9}{10}\right)}\right) \quad (2.0.4)$$

$$= 70n - 63n^{0.85} \quad (2.0.5)$$

$$\Rightarrow T(n) \leq 70n \quad (2.0.6)$$

$$\Rightarrow T(n) \in O(n) \quad (2.0.7)$$

Height of the rightmost subtree is $\log_{5/2} n$ and so,

$$T(n) \geq 7n + 7n\left(\frac{9}{10}\right) + 7n\left(\frac{9}{10}\right)^2 + \dots + 7n\left(\frac{9}{10}\right)^{\log_{5/2} n} \quad (2.0.8)$$

$$T(n) \geq 7n \left(1 + \frac{9}{10} + 7\left(\frac{9}{10}\right)^2 + \dots + 7\left(\frac{9}{10}\right)^{\log_{5/2} n} \right) \quad (2.0.9)$$

$$T(n) \geq 7n \frac{1 - \left(\frac{9}{10}\right)^{(\log_{5/2} n) + 1}}{1 - \frac{9}{10}} \quad (2.0.10)$$

$$T(n) \geq 70n - 70 \left(\frac{9}{10} \right) \left(n^{\log_{5/2} \left(\frac{9}{10} \right)} \right) \quad (2.0.11)$$

$$= 70n - 63n^{0.89} \quad (2.0.12)$$

$$T(n) \geq n \left(70 - \frac{63}{n^{0.11}} \right) \quad (2.0.13)$$

Since we consider complexity for large n ,
 $\lim_{n \rightarrow \infty} \frac{63}{n^{0.11}} = 0$ So $T(n) \geq 70n$

3 ANSWER

$$\bullet T(n) \in O(n) \quad (3.0.1)$$

$$\bullet T(n) \sim 70n \quad (3.0.2)$$

Hence option 3 is correct

4 VERIFICATION

To verify the theoretical results, I constructed a recursive function with given recurrence relation and measure the execution time for different n .

Here is the plot

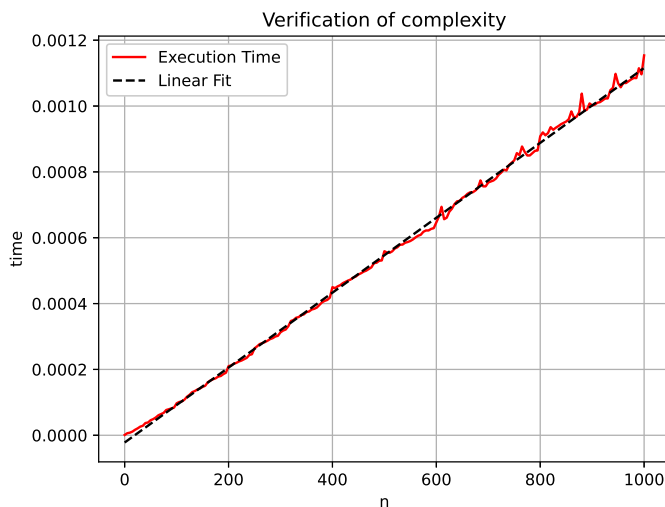


Fig. 2: $T(n)$ vs n

We can clearly observe a linear curve. This verifies that $T(n) \in O(n)$

This plot can be generated through the following python script

<https://github.com/aayush2710/EE4013-Assignment1/code/verification.py>