EE7330: Network Information Theory **2021**

Lecture Notes 7: Fano's Inequality

Instructor: Shashank Vatedka *Scribe: Aayush Goyal*

Disclaimer: *These notes have not been subjected to the usual scrutiny reserved for formal publications. Please email the course instructor in case of any errors.*

7.1 Fano's inequality

If M **&** $\hat{M}$ **are jointly distributed** $P_e = P_r[M \neq \hat{M}]$ then,

- $H(M|\hat{M}) \leqslant H_2(P_e) + P_e log_2|M|$

7.2 Proof

Let $E = \begin{cases} 1 & if \hat{M} \neq M \\ 0 & if \hat{M} M \end{cases}$

$H(E) = H_2(P_e)$

If $M = \hat{M} \implies H(M|\hat{M}) = 0$

If $M, \hat{M}$ are independent $\implies H(M|\hat{M} = H(M)$

$$H(M, E|\hat{M}) = H(M|\hat{M}) + H(E|M, \hat{M}) \tag{7.1}$$

$$H(M, E|\hat{M}) = H(E|\hat{M}) + H(M|E, \hat{M}) \tag{7.2}$$

$$H(M|\hat{M}) = H(E|\hat{M}) + H(M|E, \hat{M}) - H(E|M, \hat{M}) \tag{7.3}$$

Since E is a function of M, $\hat{M}$

$$H(M|\hat{M}) = H(E|\hat{M}) + H(M|E, \hat{M}) - 0 \tag{7.4}$$

Conditionality reduces entropy

$$H(M|\hat{M}) \leqslant H(E) + H(M|E, \hat{M}) \tag{7.5}$$

$$H(M|\hat{M}) \leqslant H_2(P_e) + H(M|E = 0, \hat{M})P_E(0) + H(M|E = 1, \hat{M})P_E(1) \tag{7.6}$$

$$H(M|\hat{M}) \leqslant H_2(P_e) + H(M|\hat{M}, E = 1)P_e \tag{7.7}$$

$$H(M|\hat{M}) \leqslant H_2(P_e) + P_e H(M) \tag{7.8}$$

$$H(M|\hat{M}) \leqslant H_2(P_e) + P_e log_2|M| \tag{7.9}$$