

AI and ML project

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Problem -

Q) Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, b \cos t)$ and $(1, 0)$, where t is a parameter, is

(A) $(3x-1)^2 + (3y)^2 = a^2 b^2$

(B) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(C) $(3x + 1) + (3y) = a + b$

(D) $(3x + 1)^2 + (3y)^2 = a^2 b^2$

$$\text{Q) } A = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} \quad B = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\text{where } t = \pi \left[0 \quad \frac{2}{100} \quad \frac{4}{100} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 2 \right]$$

Find matrix G that is the centroid?

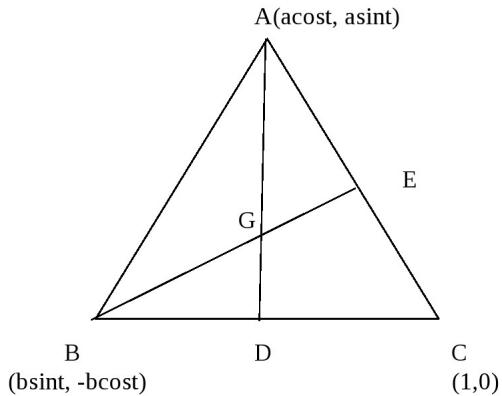


Figure: Figure

Solution through Matrix

$$A = \begin{bmatrix} acost \\ asint \end{bmatrix} \quad B = \begin{bmatrix} asint \\ -acost \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$D = \text{mid point of BC} = \begin{bmatrix} (asint + 1)/2 \\ -acost/2 \end{bmatrix}$$

$$E = \text{mid point of AC} = \begin{bmatrix} (acost + 1)/2 \\ asin/2 \end{bmatrix}$$

$$M_{AD} = (A - D) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad (A \ D) = \begin{bmatrix} (2acost - bsint - 1)/2 \\ (2asint + bcost)/2 \end{bmatrix}$$

$$M_{BE} = (B - E) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad (B \ E) = \begin{bmatrix} (2bsint - acost - 1)/2 \\ (-2bcost - asint)/2 \end{bmatrix}$$

$$N = (n_1 \ n_2) = \begin{bmatrix} -(2asint + bcost)/2 & (2bcost + asint)/2 \\ (2acost - bsint - 1)/2 & (2bsint - acost - 1)/2 \end{bmatrix}$$

$$N^T = \begin{bmatrix} -(2asint + bcost)/2 & (2acost - bsint - 1)/2 \\ (2bcost + asint)/2 & (2bsint - acost - 1)/2 \end{bmatrix}$$

$$P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad P_1 = n_{AD}^T(A) \quad P_2 = n_{BE}^T(B)$$

On calculating P1, P2 and P

$$P_1 = -1/2 (ab + asint) \quad P_2 = 1/2 (ab + bcost)$$

$$P = \begin{bmatrix} -a/2(b + sint) \\ b/2(a + cost) \end{bmatrix}$$

Final Answer

$$\text{Centroid} = G = (N^T)^{-1}(P) = \begin{bmatrix} (acost + bsint + 1)/3 \\ (asint - bcost)/3 \end{bmatrix}$$

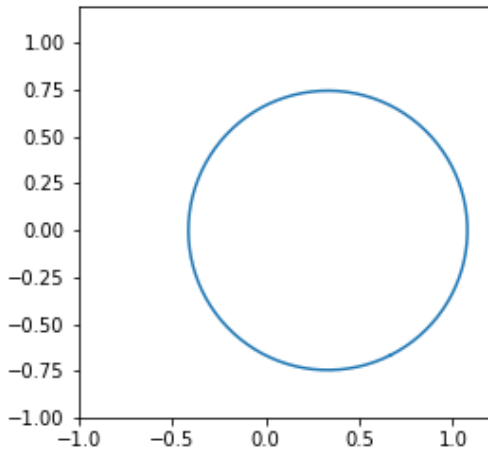


Figure: Locus Plot $(3x - 1)^2 + (3y)^2 = 5$