REVERING MUSINGS ON STRING COMPACTIFICATION (BUT MOSTLY DE SITTER)

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Abstract. These notes are written on the (realistic) string compactifications and the string de Sitter vacua problem. A lot of unanswered questions remain in these regime which are highlighted using a historical canvas and exposition. We discuss also the KKLT proposal and other recent discussions around if there is a de Sitter vacua? In the exposition, we review the Calabi-Yau manifolds, supergravity compactification, flux compactification, moduli stabilization, and all that.

This has been written in the same series where we wrote on the de Sitter quantum gravity and observables, 'Revering Musings on de Sitter and Holography, 2023' [1].

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1. Introduction

Supersymmetry is one of the most controversial topics of online media and we should likely ignore them. SUSY is needed to expand the Standard Model which can possibly solve the issues like hierarchy problems and dark matter. The possible candidates are the Minimal Supersymmetric Standard Model (MSSM) [2] or the non-minimal Supersymmetric Standard Model. However, supersymmetry is not observed in our four dimensional world which means that it is broken at this level. For one doing *realistic*[1] compactification, they must have supersymmetry broken upon compactification. After compactifying, let's say an 11-dimensional supergravity (SUGRA), to

[1] This word has a meaning and understanding of its own in these contexts.

a four-dimensional spacetime, the other extra seven dimensions become 'microscopic' in the sense that it is not observed by us, this is called 'spontaneous compactification' [3]. Additionally, we have metastability conditions in supersymmetry.

The goal of any compactification is to achieve the following.

- Solve the generalized Einstein's equations in string theory
- Contain the standard model $SU(3) \times SU(2) \times U(1)$

Of course, we have obtained tons of theoretical examples of such compactifications found in the literature. Not all of them are realistic or even mimic our real-world scenarios. But some of these are phenomenolgically interesting too, at least historically. We discuss some of them in our paper, too.

The general idea of a compactification is to take our manifold, let us say 10 dimensional $\mathcal{M}_{10}$, which solves the Einstein's equation. We can introduce compactification as

$$\mathcal{M}_{10} = \mathcal{M}_4 \times K \tag{1.1}$$

where $\mathcal{M}_4$ is our effective four-dimensional theory and K is a special manifold called the Calabi-Yau manifold. The reasons why K is a Calabi-Yau manifold are many, but they are of holonomy $SU(n)$ which is important for us. We explain these later in the notes explicitly. In fact, Calabi-Yau manifolds are the favorite mathematics of physicists. They yield very rich ideas of theoretical physics like Mirror symmetry and so on.

Anyway, in doing this compactification, we typically choose a Calabi-Yau manifold K, in this case, say, six-dimensional, with a symmetry group G and $SU(3) \times SU(2) \times U(1) \subset G$ because after compactification $\mathcal{M}_4$ should contain $U(3) \times SU(2) \times U(1)$.

Supergravity solutions emerged as a good way of doing quantum gravity problems. The compactification of 11D supergravity was a classic one as an example. In fact, eleven dimensions are the maximum where one can have supergravity and valid spin 2 gravitons and not more [4].

Superstring compactifications are more general and richer than supergravity compactification. The general hunt (or softly, quest) has been to find why string theory is not observable in our world. The reasons include the breaking of supersymmetry in our world and also that the extra dimensions in string theory are microscopic and as famously written in scientific literature, *hidden*. The five kinds of string theories are just the same theory in different limits of M-theory. These are type I, type IIA, type II_B, $E_8 \times E_8$ and $SO(32)$. The last two are called *heterotic string theory*. From a compactification perspective, the heterotic string theories and type II theories are more promising. $E_8 \times E_8$ case[2] and type IIB has been explored in literature in a more realistic sense than the others, with type IIA getting a little attention relatively. K

[2] In the $E_8 \times E_8$, we embed the SU(3) in the subgroup $SU(3) \times E_6$ and the other E_8 become a hidden sector. More on this has been discussed in Sec.2.3.

having $SU(3)$ holonomy makes it special to be used in string compactifications as well.

Moreover, during compactification, there are scalar fields that are introduced and are massless. This creates a theory with many massless fields that plague our theory. To fix these, we introduce the fluxes in our theory. These fluxes are given by 2-form gauge potentials H and F and the 3-form field strength. The magnetic fluxes, for say type IIB

$$\int_K H_3, \quad \int_K F_3 \tag{1.2}$$

are non-vanishing. Thus a Dirac quantization is established here between electric and magnetic charges. These can be combined into a three-form flux $G_3 = F_3 \tau H_3$ where τ is a complex axiodilaton (explained in Sec. 2.3). Moreover, G_3 is an imaginary self-dual (ISD) which means that $\star G_3 = iG_3$ which are helpful for preserving the supersymmetry and keeping the structure of the Calabi-Yau manifold preserved in these flux compactifications. This is done by introducing a *superpotential* W.

All of these are done to stabilize our moduli fields (massless scalar fields) by which we mean that they gain mass. Now, the superpotential has a contribution from fluxes which is called flux superpotential W_{flux} as well as a non-perturbative superpotential. The reason for introducing the latter is as follows.

There are three types of moduli fields: **complex structure moduli** (that comes from the complex structure on K), **Kähler moduli**, and then the **axiodilaton** τ (which is related to the string coupling). We wish to stabilize these moduli. Introducing a flux superpotential W_{flux}

$$W_{flux} = \int_{K_6} G_3 \wedge \Omega \tag{1.3}$$

where Ω is holomorphic (3,0) from Calabi-Yau manifold K. This flux superpotential stabilized the axiodilaton and complex structure moduli. But at the tree level, Kähler moduli do not get stabilized due to the no-scale property. For this, we introduce the non-perturbative superpotential W_{np} which becomes important to stabilize all the moduli. They are also important from the perspective of Dine-Seiberg problem [5]. We discuss this in detail later.

To say that we have understood string compactification would be an exaggeration, especially in the de Sitter case. Because for the AdS we obtain the vacua. In de Sitter, there is a classical no-go theorem [6] about de Sitter vacua in string theory. This is why quantum corrections and non-perturbative corrections are necessary for the de Sitter solution to exist. But at the classical level, there cannot be a de Sitter solution.

There is an idea that one can take the AdS vacua and then uplift the theory using anti-D3 branes which will create a positive scalar potential. A very famous example of this is the KKLT mechanism [7] which uses non-perturbative corrections to stabilize all the moduli and get a de Sitter vacua solution.

While KKLT is a famous example, it is not a general idea yet. The sociology around de Sitter's solution and if it exists is diverse. With the Swampland Program [8] getting introduced, the phenomenological part of the theory has been given attention using a set of conjectures about quantum gravity with each quantum gravity falling into *landscape* or *swampland*. A theory in the landscape would mean that they satisfy these fundamental conjectures about quantum gravity while with the opposite being true, they fall into swampland. The question becomes, now, if de Sitter solution belongs to swampland or landscape. There are multiple arguments on both sides.

In this paper: We introduce the Kaluza-Klein compactifications in Sec. 2.1 which was historically introduced to unify gravity and electromagnetism. We then discuss the low energy limit of supergravity compactification with $D = 11$. We also explain how a seven-dimensional Calabi-Yau is needed for such compactification. We also explain the need for Rarita-Schwinger fields which replace the Dirac operators in 11D supergravity.

We discuss the string theories and their compactification in Sec. 2.3. In this section, we also discuss a lot about Calabi-Yau manifolds and their special properties. We then discuss the Freund-Rubin compactifications and fluxes. Sec. 3 is about moduli stabilization and flux compactification where we discuss the moduli space. We take the example of type IIB to discuss the flux compactification and moduli stabilization. We discuss the superpotential there within. The section also contains brief discussions of Dine-Seiberg runaway argument and Maldacena-Nuñez no-go theorem. In Sec.3.5, we introduce the anti D3 branes and uplifting to obtain the de Sitter solution. Other helpful notes and reviews include [9–14]

In Sec. 4, we discuss the KKLT solution which is obtained by uplifting the AdS vacua from type IIB using anti D3 branes. We have also included a few computations, all at first order and without correction to provide a typical idea of vacuas, potential and uplifting. Finally, in Sec.5, we discuss the sociological viewpoints on if there exists a de Sitter solution or not.

Moreover, there is an Appendix. A which contains a mathematical discussion about Calabi-Yau manifolds. We discuss some of the motivations regarding the use of these complex manifolds in theoretical physics, especially in mirror symmetry. A future version might contain appended content in this appendix.

2. String compactifications

The starting point for our discussion is to talk about compactifications and supergravity. It is first important to understand the sense in which we want to perform compactifications. Now of course, having $D > 4$ dimensions is not unexpected, as hinted naively from something as simple as the theory of bosonic string theory S_B requiring 26 dimensions + tachyonic problems. In a supersymmetric theory, this becomes resolvable; indeed, adding a fermionic contribution S_{MW} to the bosonic string action $S_B + S_{MW}$ would be a theory with no tachyons taking the GSO projection. This 10-dimensional theory is a good description of a theory with bosons and fermions, but you are still left with the problem of "where do the other 6 dimensions go?"

In the Kaluza-Klein theory, which we will revisit below, one has a $D = 5$ theory for which we want a $D = 4$ compactification. The mathematical argument for this is that the compactification $\mathcal{K}$ is a fibration of the manifold M_D into a compactified manifold M_4, since we want that as the low energy limit:

$$\mathcal{K} : M_D \Rightarrow M_4 \times K^6 \ , \tag{2.1}$$

and we would typically have conditions over the nature of K^6, such as Ricci-flatness or the *moduli* for the compactifications, as we shall see later. The essential idea is that in superstring theory, we want to describe a low energy effective limit on a $D = 4$ manifold $\mathcal{M}_4$, with the additional dimensions compactified onto a suitable $D = 6$ manifold K_6. While we will talk about these compactifications in detail in section 2.2, we will briefly recall how these compactifications work superficially below.

We want a theory of $G_{AB}[\mathbf{g}_D, \mathbf{\Phi}_D]$ with *some*[3] limit compactification to a theory $G_{\mu\nu}[g_4, \Phi_4]$, which is our usual Einsteinian GR. So we end up with something like

$$\mathbf{g}_D = g_4 \times g_{D-4} \ ,$$

and then the question becomes, *how would you know which compactifications yield the right so-and-so limit?* Here, you would start by taking the full Einstein-Hilbert action for the D dimensional theory and split the metric into an ansatz.

2.1. **Kaluza-Klein compactifications.** The first step towards paving the way towards compactifications was from the Kaluza-Klein theory, which unified gravity and electromagnetism in $D = 5$. In order to describe the metric in this geometry, we will split it into the usual 4 dimensional components and an extra component:

$$g_{AB} \sim g_{\mu\nu 5} \ , \tag{2.2}$$

where μ and ν are the usual 4 dimensional components and 5 is the extra dimension. The geometry we are working with is essentially

$$\mathcal{M}_\eta \times S^1 \ ,$$

where S^1 describes the compactification onto a circle of radius r. That is, we have[4] $x^5 \sim x^5 + 2\pi r$.

[3]This will be covered later in section 2.2, but for now we will remain ambiguous as to what it means.

[4]Note here that we take $\mu, \nu = 0, 1, 2, 3$ but x^5 instead of x^4 coordinate component for making things easier.

The general 5D metric ansatz is that you take

$$g_{AB} \sim g_{\mu\nu} + A_\mu + \Phi_{BD} \ , \tag{2.3}$$

where A_μ are the $U(1)$ gauge fields and Φ_{BD} is the Brans-Dicke scalar. In what follows, we will simplify this to be a simple collection of fields ϕ, but the general meaning should be taken to be the above decomposition.

The metric can be written as

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix} \ . \tag{2.4}$$

The usual Einstein-Hilbert action follows with the additional coordinate:

$$S_{EH} = \frac{1}{16\pi G_N} \int \mathcal{R} \sqrt{-\mathbf{g}} \ d^4x dy \ .$$

The field equations for this metric can be solved to give the Einstein equations

$$G_{\mu\nu} = \kappa \phi^2 T_{\mu\nu} \tag{2.5}$$

plus an additional vanishing covariant derivative condition on the field strength tensor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The expanded field equations would then contain the $G_{\mu\nu}$ equations in terms of the modified stress tensor $T_{\mu\nu}$. These can be computed by finding the usual Christoffel symbols in terms of the 5D metric g_{AB}, and expanded into the field equations. Before proceeding though, one remark about the so-called "cylindricity condition".

The cylindricity condition assumes that

$$\frac{\partial g_{AB}}{\partial y} = 0 \ , \tag{2.6}$$

where we are essentially saying that the theory is only dependent on the usual x^μ coordinates. On relaxing this, one would get more complexity in the field equations and would have to compute the Christoffel symbols and the Ricci tensor for the theory in terms of additional fields added into the theory. On top of this, we also assume the vanishing Ricci condition so that $R_{AB} = 0$, due to which the field equations decompose into restricted vacuum solutions.

The field equations then reduce into two sets of equations; the first would be the Einstein tensor in AB components and the second would be computed from the R_{55} components, which become

$$\Box \phi = \frac{\phi^3}{4} F_{\mu\nu} F^{\mu\nu} \ . \tag{2.7}$$

One thing about the conditions we have so far is that the theory has cylindricity. One way to circumnavigate assuming this condition is to take the y terms into compactified geometries that have a very small radius. That is, compactifying into $M_4 \times S^1_r$, we will take the space to have topology $R^4 \times S^1$ with a very small radius. Since our theory is taken to be the usual 4-dimensional gravitational theory with an additional gauge field, we have really two couplings that appear in the action, one for the usual G_N and the other the gauge coupling. In a physical theory, we expect these extra dimensions to live in a compactified space of radius of order of Planck length.

Extending this to higher dimensions (that is, for compactification of more than one extra dimension), we use a similar argument, where we take a 4D manifold $\mathcal{M}_4$ and we compactify the higher dimensions onto some K_{D-4}. For superstring theories, we expect the low energy and weak coupling limits to reduce into the usual 4D Einsteinian gravity plus these extra dimensions compactified onto K_{D-4} with specific constraints imposed on them. In doing these compactifications, one would typically encounter the *moduli* for the compactifications, which are parameters that serve as characterizing the theory. To see an example of this, we will consider a Kaluza-Klein compactification similar to what we saw before, extended to D dimensions, and notice that taking a family of scalar fields Φ_n in compactifications, there are different uniquely specifiable metrics that come along with a family of scalings parametrized by n. That is, the choice of Φ_n gives a unique compactification for the same theory. Consider also that these compactifications require additional data called *fluxes*, which we will discuss next. In this fashion, a compactification is a family of parameters that includes the full theory, the 4D manifold $\mathcal{M}_4$ [5], the fibration manifold K_{D-4}, the moduli and the fluxes.

It is also important to be clear that the Kaluza-Klein procedure that we discussed above is not restricted to $4+1$ dimensions. Indeed, as we shall see below, we will take the full compactification to be like

$$\mathcal{M}_{4+x} \times K_n \ ,$$

so in general the theory does not have the usual $U(1)$ gauge field *only* , but rather a collection of massless fields along with the "external" fields that compose the moduli in the theory.

The overall compactification of the theory depends on the moduli like said before. In this sense, we would take metrics $\mathbf{g}$ with scalings by some λ, which here play the role of the moduli. The collection of all such $\mathbf{g}_\lambda$ forms the *moduli space* $\mathscr{M}$, which is essentially the theory space of all the metrics scaled by these families of parameters. For reasons that we will explain later, we will choose $\mathbf{g}_\lambda$ to be Ricci flat metrics, and the primary interest of compactifications would be with Calabi-Yau threefolds, which we shall discuss in the next subsection. But a general idea is that taking some Ricci flat metric g, scaling by λ will still give you a Ricci flat metric and is, therefore, a moduli parameter. Since these metrics always are specified as a general family of metrics rather than a particular solution, these moduli spaces heavily discretize the resulting compactifications, and the problem of moduli stabilization will be treated with heavy importance. In the 5D KK example taken above, the moduli were simply

[5] This typically also comes with a cosmological constant Λ for AdS, Minkowski or de Sitter spaces, the last of which as we shall see is a rather complicated field.

the collection of ϕ and the corresponding compactifications radius r which is also a parameter that is chosen carefully.

2.2. **Low energy limit of Supergravity compactification.** Let us begin with the classic compactification of supergravity in $11D$. Eleven dimensions is the maximum no of dimensions that one can have in supergravity since dimensions greater than 11 would contain massless particles of spin greater is two which is not possible for a consistent field theory coupled with massless particles [4]. The goal would be to get $SU(3) \times SU(2) \times U(1)$ as the symmetry group in our compacitification to call it *realistic*. And the group $SU(3) \times SU(2) \times U(1)$ contains the symmetry group of Standard model. In fact, the goal behind doing these (supergravity or superstring) compactifications is two-fold:

- These compactifications should solve (generalized) Einstein's equations in string theory.
- These compactifications should be consistent with standard model observations which are based on $SU(3) \times SU(2) \times U(1)$.

Now, we find that the minimum number of dimensions required of a manifold containing the symmetry group $SU(3) \times SU(2) \times U(1)$ is *seven*[6] [15]. In fact, eleven dimensions is a perfect way to do supergravity compactification, since $7+4$ where the latter 4 represent our non-compact space-time dimensions and also 11 is maximum number of dimensions where we have supersymmetry.

So it is almost a perfect example of the eleven-dimensional supergravity through which we can obtain the symmetry $SU(3) \times SU(2) \times U(1)$. Hence, we will use this maximal eleven dimensional supergravity as a case now.

We can argue that the gauge symmetry group $SU(3) \times SU(2) \times U(1)$ is the largest group that can be obtained by the Kaluza-Klein compactification. A very simple example is to consider $CP^2 \times S^2 \times S^1$ whose minimum dimension is seven as mentioned. But there exist *many* manifolds with the same property. To find the number of dimensions, we can calculate $dimG - dimH$ (also G/H) where $dimG$ is the dimensions of $SU(3) \times SU(2) \times U(1)$ and H is the 'maximal' subgroup of G. The maximal subgroup[7] is $SU(3) \times U(1) \times U(1)$ and $dimH = 5$. And dimensions of the symmetry group $SU(3) \times SU(2) \times U(1)$ is 12, so $dimG - dimH = 7$.

An example like $CP^2 \times S^3$ has full symmetry group $SU(3) \times SU(2) \times SU(2)$ which is larger than our symmetry group $SU(3) \times SU(2) \times U(1)$. Another example is $S^5 \times S^2$ which has full symmetry group as $O(6) \times SU(2)$ which is also larger than our gauge group. We can not find any other example, in this scenario of eleven-dimensional

[6] A simple way to see this is that $U(1)$ can at least correspond to a one dimensional circle, $SU(2)$ to a 2-sphere and $SU(3)$ to a CP^2which has four dimensions.

[7] Actually there would be three factors of $U(1)$ which would generate SU(3) and SU(2) and a factor for itself. But we eliminate one factor [15].

supergravity compactification, with a symmetry group larger than the gauge group $SU(3) \times SU(2) \times U(1)$.

Let us say that we are doing a compactification

$$\mathcal{M}_4 \times K \tag{2.8}$$

where K is a seven dimensional manifold with the symmetry group G and $SU(3) \times SU(2) \times U(1) \subset G$. We just saw some examples of K and there could be many more examples for K. These K would be our general type of simply connected manifolds. For example, $CP^2 \times S^2 \times U(1)$ itself, or $(S^5 \times S^3)/U(1)$. Actually, the nature of the two mentioned manifolds are a little bit different mathematically. The paper [15] contains more elaborate examples.

Now we wish to understand the massless leptons and quarks which should arise in the theory of Eq. 2.8. Actually, they are not massless and they acquire a mass only through the Higgs mechanism[8], which will also be a needed feature in our theory. A very starting problem is that the Dirac operators will not yield us massless modes other than four dimensional theory. This is because the Dirac operator $\not{D}$ for extra dimensions contain only massive fermions. Instead we need massless models which corresponds to the eigenvalue of $\not{D}$.

A good substitute for this might be Rarita-Schwinger fields $\psi_{\mu\alpha}$ of spins 3/2 (also called gravitino fields) replacing the Dirac field of spin 1/2 as there is no spin-half field in $11D$ supergravity. Because of the positivity of the Dirac operators, we do not much zero modes, however, Rarita-Schwinger operators do have a wide set of zero modes. Even though Rarita-Schwinger fields are of 3/2 spins, in four dimensional world, they act as 1/2 spins. Precisely, ψ_μ with components $\mu \geq 5$ has spin 1/2 fields. So, the Rarita-Scwhinger equation of higher dimensions will yield us the massless modes of spin 1/2 upon the compactification. The most important thing to remember is that the left-handed and right-handed fermions transform differently in a gauge theory.[9] Moreover, we need to find a Higgs vacuum for which the v.e.v. spontaneously breaks. And other important matters of such a realistic compactification includes a vanishing cosmological constant, CP violation, and so on.

The case of $\mathcal{N} = 1$ is a nicer one than $\mathcal{N} > 1$ because of the obvious phenomenological reasons. If one wishes to a mathematical exploration, then, nonetheless it is a very exciting subject for to look for $\mathcal{N} = 2$ and so on. Especially to understand the algebraic geometric point of view of the compactified manifold K. But, do we want to preserve the supersymmetry in $\mathcal{M}_4$? The answer is yes, in this context, where the fermions modes are absent and supersymmetry can verily map the right bosonic

[8]More appropriately, from the spontaneous breaking of $SU(2) \times U(1)$ to $U(1)$ which is needed for the gauge bosons and fermions to acquire the mass.

[9]This is one of the reasons why $N \geq 2$ is not really helpful for a realistic compactification since they have right-handed and left-handed fermion modes tranform in a same way under the gauge theory. The best possible case is $N = 1$ supersymmetry.

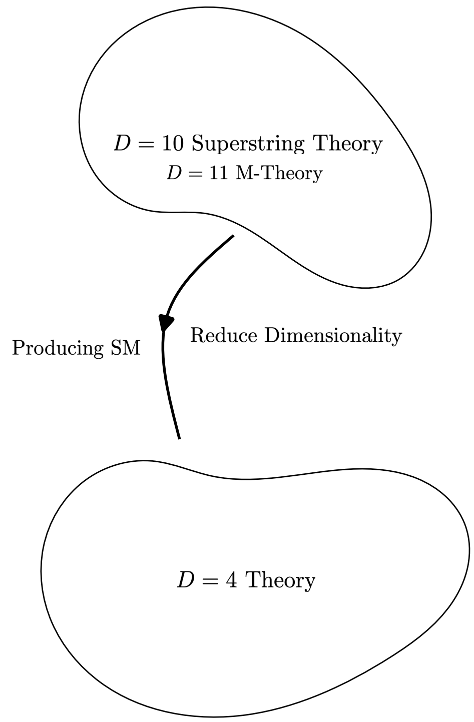

FIGURE 1. The goal of these compactifications is to produce a low-energy effective theory of physics that contains a standard model and solves Einstein's equations.

modes to their corresponding fermions. Before we switch to other types of compactifications, other than supergravity, let us look at a compactification of $D = 11$ and $\mathcal{N} = 1$ supergravity to a $D = 4$ AdS times a S^7 whose isometry group is $SO(8)$. The resultant compactifications has a $N = 8$ supersymmetry in effective theory $D = 4$. S^7 has interesting properties such as 'triality'.[10] An interesting isometrically embedding of S^7 is in $\mathbb{H}P^2$, instead of taking just $\mathbb{R}^8$. Now, when one does a 'squashing' of the S^7, one has non-zero v.e.v for the scalar fields and Higgs mechanism. The group $SO(8)$ breaks further into $SO(5) \times SU(2)$ and the $\mathcal{N} = 8$ reduces to $\mathcal{N} = 1$ supersymmetry. The holonomy is G_2. See [16–18] for relevant discussion on this.

2.3. **Superstring compactification.** So far we have dealt with supergravity and more on the supergravity compactification can be found in [18]. Now we will switch to a discussion of superstring compactification and realize how they are connected to supergravity in $D = 11$ which we have just discussed.

There are five broad categories of superstring theories: Type I, type IIA, type IIB, $E_8 \times E_8$, and $SO(32)$. The last two are called *heterotic string theories*. There is also one more kind of string theory, called M-theory which exists in $D = 11$ and is known

[10]This has its own significance and interesting feature which we do not discuss but see [16].

to be 'dual' to every other string theory [15]. We will first discuss compactification for string theory which is anomaly-free.

The first example we would like to discuss is heterotic string compactifications. Our goal for these compactifications is still the previous ones (see Fig. 1). A very classic example was provided in [19] which was also the first time when requirements of the manifold K were taken in, in the form of Calabi-Yau manifolds. As we noted earlier, cancellation is required for a realistic theory. The theory of $E_8 \times E_8$ and $SO(32)$ does have the anomaly cancellation [20]. We require that the compactification has a realistic mass spectrum, as argued in supergravity case. That also includes a realistic fermion number. Additionally, we will keep the $\mathcal{N} = 1$ supersymmetry unbroken at the compactified scale. The reasons for which will be clear in some time.

Let us again take a theory $\mathcal{M}_{10}$ which has a gauge group $E_8 \times E_8$. We wish to compactify this now as

$$\mathcal{M}_4 \times K \tag{2.9}$$

where K will be our internal manifold. We will reiterate our requirements for K in a moment. The requirement for $\mathcal{M}_4$ is that it is the maximally symmetric solution. At low energy, $E_8 \times E_8$ breaks into a couple of E_8. After choosing a non-trivial connection under which E_8 is not invariant, E_8 is spontaneously broken into E_6 which is more apt. So the standard model will be contained in a single E_6 and the other E_8 will be called a *hidden sector*. As we will argue the K will be a Ricci flat and Kähler manifold. Also, for gauge group E_6, the holonomy of K is $SU(3)$. The reasons for a Kähler structure are provided by the fact that the field strength 3-form obtained from the super Yang-Mills theory H vanishes [19, 21]. Moreover, K will be a compact coset space.

K has holonomy $SU(3)$ which is special. A manifold (complex) with the holonomy $U(3)$ is called Kähler.[11] When in a Kähler manifold, any line element can be written in terms of a Kähler potential. The spin connection of six-dimensional K is $O(6)$ at start which is isomorphic to $SU(4)$ whose two spinors are decomposed in $SU(3)$ as

$$4 \oplus \bar{4} = 3 \oplus \bar{3} \oplus 1 \oplus \bar{1}. \tag{2.10}$$

The spin connection of K is actually $SU(3) \times U(1)$. This means that we must choose a Kähler potential so that the $U(1)$ part of the spin connection is set to zero. In the 1950s, Calabi [22] argued that the spin connects of K will be $SU(3)$ if we choose a Kähler metric carefully, to eliminate the $U(1)$ part. This is done by first defining a field strength $F_{\mu\nu}$ of the $U(1)$ group and then taking a two-dimensional closed surface X in K

$$I_X = \int_S dS^{\mu\nu} F_{\mu\nu} \tag{2.11}$$

[11] We will not be going into much complex geometry details but Kähler manifold is equipped with a Kähler form (which is also mentioned somewhere above) and a hermitian manifold is called Kählerian manifold if the Kähler form is closed ($dH = 0$). The second Betti number of (compact) Kählerian manifold is positive. We will discuss some of these details in appendix A.

and now when $I_X = 0$, the field strength vanishes. $I_X = 0$ is equivalent to saying that there is a vanishing first Chern class[12] for all the two-dimensional X. So, we get the $SU(3)$ holonomy for our Kähler manifold K if the first Chern class vanishes. This is then a classification of compact Kähler manifolds and will be necessary for our purposes. We note that the first Betti number is also zero for K.

Now let us see why $E_8 \times E_8$ is more phenomenologically realistic than the $O(32)$ heterotic gauge group. The holonomy of K is $SU(3)$, as we have noticed many times. We also have a condition that will preserve the supersymmetry, i.e., unbroken supersymmetry. The $SU(3)$ holonomy is then embedded into the $E_8 \times E_8$ and the spin connection to the gauge group. In the embedding of $SU(3)$ in the $O(32)$ theory, the theory does not give any good predictions. The reason is simple: the only subgroup that will commute with the SU(3) in $O(32)$ is $U(1) \times O(26)$ and we have seen the $U(1)$ vanishes for the vanishing of the first-Chern class in K. Then only the representations of $O(26)$ will be considered and they consist of only real vectors. For this reason, we will only consider for a while the $E_8 \times E_8$ case. The embedding of $SU(3)$ is done in the subgroup $SU(3) \times E_6$ of just one copy of E_8. We can ignore the other E_8 and call it a *hidden sector*. Now, this model is more realistic [26] and follows every criteria ($U(1)$ cancellation, unbroken supersymmetry, and embedding of $SU(3)$) mentioned in [19]. We will not get into the phenomenological details of this compactification, lest it is useful as it gives 27 and $\overline{27}$ representation of E_6. The overall takeaway is that the embedding of the spin connection breaks the gauge group of one E_8 to E_6. The other E_8 acts as a hidden sector (which has its own speculations and interpretation that we will not discuss in this paper). This then gives a GUT and $\mathcal{N} = 1$ supersymmetry in $4D$ [19]. In this theory, the GUT scale and Kaluza-Klein scale are the same.

The above presentation only gives up the one generation of leptons and quarks. Moreover, adding more generations becomes a task of defining a more precise Calabi-Yau manifold [27].

We have seen the heterotic string compactification so far. The study of type II compactification requires more tools like branes and moduli stabilization which is our next stop.

2.4. **Freund-Rubin Compactifications.** We will now talk about Freund-Rubin compactifications [28] as an example of the KK compactifications extended to higher dimensions and $D > 5$ compactifications. In order to talk about these, we will first give a very naive overview of *fluxes*. We will give a more detailed discussion in the next section. These help in *moduli stabilizations*, a discussion of which we defer here.

[12] Chern class is a characteristic class. Simply, it is used for classifying the spaces of the unitary group. A first Chern class is here basically a two-form associated with the $U(1)$ group. One can also define the second Chern class (represented with a four-form) and the third Chern class (represented with a six-form), see [23, 24] or [25].

The natural definition of fluxes is to take a compact n-manifold Σ and a field strength F_n and define

$$\Phi = \int_{\Sigma} F_n \ . \tag{2.12}$$

This naturally follows from ordinary electrodynamics where you take compact sections of the usual $D = 4$ spacetime and have a 2-form F_2 that gives the field strength. In the RNS formalism for string theory, we can take the NS-NS sector and the associated Kalb-Ramond field strength B_2. The NS-NS flux here would be

$$F_3 = dB_2 \ . \tag{2.13}$$

We typically have fluxes like

$$F_{n+1} = dC_n \ , \tag{2.14}$$

where C_n is (1) odd-degree for type IIA theories, so C_1, C_3, C_5, etc., and (2) even-degree for type IIB theories, so C_0, C_2, C_4, etc.

In Freund-Rubin compactifications, we take the magnetic fluxes on a $2-$sphere that stabilizes the compactifications, in the following sense. Taking g to be the genus and the corresponding Euler characteristic χ, the case with the $2-$sphere we take is $g = 0$. In the scenario we are dealing with, we take a moduli field R and an associated positive curvature for which the *effective potential* V_{eff} becomes negative. Taking the magnetic fluxes though, one can "stabilize" the negative curvature contribution – yielding the Freund-Rubin vacua. In general, the potential V scales approximately like

$$V \sim \frac{1}{R^4}\left(\frac{2g-2}{R^2} + \frac{N^2}{R^2}\right) \ , \tag{2.15}$$

where the $1/R^2$ factors are due to Weyl rescaling.

The effective potential discussed above describes the full potential contributions at both a classical and quantum level. One reason we worry about these things is to address phenomenological issues with massless scalar fields, which form a part of the compactification data. We will discuss these in greater detail later, especially when we take explicit constructions of the compactification data for more realistic models.

The flux compactification discussed above is a major simplification from a string theoretic perspective. For that matter, the compactification data can be made to be more inclusive of realistic moduli + fluxes + *orientifolds* on manifolds with $g \neq 0$. For a $g = 1$ manifold – $\mathbb{T}^2$ – one can include D-branes and m orientifolds (which contribute to negative T) on $p \in \mathbb{T}^2$, so that the overall contribution scales roughly against $-n/R^4$. Including these $O3$ planes typically is characterized by the inclusion of Calabi-Yau manifolds or "threefolds". The next subject of our discussion would be extensively on moduli stabilization, including fluxes and D-branes in string theory.

Suppose we start with a type IIA theory. The dilaton field ϕ determines the string coupling constant g and the volume R for K_6 are the two main moduli for the theory, which will also include D-branes and O-planes or orientifold-planes. For now, we will not discuss stringy perturbative corrections to g or any instantons. To this theory, one has several fluxes – namely, the full flux terms come from RR fluxes F_n and NS flux H_3. Again, we would just have to take the units of each flux and determine the number of the orientifold planes – how this is done will be explained later. As before, we would take a large number of fluxes similar to the Freund-Rubin model above and one can similarly find the value of N units of the RR fluxes for which the effective potential would yield a vacua.

We will quickly emphasize that this balancing of D3-branes, for instance, with O3-planes is a delicate balance. In $\mathscr{M}$, each point gets "stabilized" by balancing the overall fluxes with the D-branes and O-planes. However, in doing this, we end up encountering the *tadpole* conjecture in the swampland (which we will later discuss in the EFTs discussion) program, where we want the full flux charge q_f to stabilize the moduli but we have to take in the charges from the D-branes and O-planes. This is technically a discussion in F-theory, where one has the *tadpole conjecture*, which states that the $\mathcal{F}$ that stabilize CY 4-folds will actually add to the D3-brane tadpole. This discussion will be less enigmatic later on, but for now, it suffices to understand these as being issues to tackle when we encounter moduli stabilization KKLT de Sitter vacua and more formally, the KKLT de Sitter vacua.

So the full picture is as follows. You have a theory where you have fields (to be acknowledged as *dilatons*), the couplings, the D-branes, and the orientifolds and a necessarily fixed limit on a 4D manifold, for which the external manifold K_6 for instance, is to be found. For this, we have specific conditions like flux compactifications, moduli stabilization, and Ricci flatness criteria. Any such theory reduces to an overall action that is sourced from such fields and field strengths. As an example, for a type IIA theory, you have compactification data that are built of RR and NS-NS sectors, the D-branes and fluxes, orientifolds and the corresponding K_6 to build the overall 10D action. This of course could be written more explicitly by taking each contribution, but for convenience purposes, we will fix the flux terms into $\mathcal{F}$, D-brane contributions into $\mathcal{D}$ and orientifold planes into $\mathcal{O}$. The goal of compactifications is then to fix these to yield physical vacua.

3. Moduli stabilizations and flux compactifications

3.1. **Moduli Spaces.** Now, we will look into the moduli spaces, more specifically to the cases we have been discussion. Then we will need a few extra machineries to describe a more quasi-realistic theory.

A moduli space $\mathscr{M}$ (which is not same as $\mathcal{M}$ which is used to denote the manifold) consists of the scalar fields solutions which are given by the action of the deformation of a metric g of K which preserve the Ricci-flatness. (It can be also called a classifying space of the Ricci-flat metrics.)

We are generally interested in understanding this moduli but since it has a lot of vacuas (possible candidates), it is generically a problem. We will discuss some possible ramifications that could be possible in the next bits of the paper.

Locally, a moduli space $\mathcal{M}$ looks like

$$\mathscr{M} = \mathscr{M}_K \times \mathscr{M}_C \tag{3.1}$$

where $\mathscr{M}_K$ is associated with the Kähler structure deformations of the metric and $\mathscr{M}_C$ is the moduli space associated with the complex structure deformations. The former kind of structure deformations dominates the effective theory of low energy and the gauge couplings at tree level in $\mathcal{N} = 1$ type IIB compactification. While the former, $\mathscr{M}_C$, for the type IIA. Once can also relate the Betti numbers b^2 and b^3 of K with $\mathscr{M}_K$ and $\mathscr{M}_C$, respectively. We have already seen the first Betti number of the manifold K should be zero. Betti number b^2 and b^3 are non-trivial for K (see Appendix. A for some more discussion.)

Let us take an example of the moduli space $\mathscr{M}$. A Calabi-Yau manifold with holonomy $SU(n)$ has complex dimensions n. We have been considering a Kähler manifold with the holonomy $SU(3)$, so the manifolds of interest are the Calabi-Yau three-folds. There are many examples of the Calabi-Yau three-folds that appeared after a good amount of works in algebraic geometry and theoretical physics, for instance, see [29]. We also mentioned some of these constructions, from physics point of view, in section. 2. Counting Calabi-Yau three-folds has been an interesting problem in the theoretical physics and algebraic geometry. The Betti number of the manifold corresponds to the Higgs field, so constructing these examples require a thought about phenomenology. Most examples comes from the complete intersections in higher-dimensional toric varieties [29–31]. For example, the complete intersection of two hypersurfaces of degree 3 in $\mathbf{P}^4$. A very typical example is to consider a quintic hypersurface in $\mathbf{P}^4$. One can take these smooth projective spaces and compute the Hodge numbers, and then Betti numbers. The Betti numbers also correspond to the moduli space, in this context a Calabi-Yau three-fold, of K. The dimension of the moduli space $\mathscr{M}$ of a quintic hypersurface in $\mathbf{P}^4$ is given by first mentioning that there exists 126 coefficients in the quintic polynomials. Now, the automorphism of $\mathbb{P}^4$ is given by $PGL_5(\mathbb{C})$, so these 25 coefficients would be redundant. One then is left with 101 dimensions[13].

The moduli space gives you a good way of understanding EFTs in the sense of the swampland *distance conjecture*, where if take two points $x, y \in \mathscr{M}$ with increasing geodesic distance $s \to \infty$, you get an exponentially massless infinite tower of modes that goes like

$$M \sim m \exp(-\lambda s) \ . \tag{3.2}$$

One reason why this is important is because the de Sitter vacua program, gives you a swampland constraint. The AdS version of this contains a constraint where the scaling in vanishing Λ limit is $m \sim \sqrt{|\Lambda|}$ when taking supersymmetric AdS vacua.

[13]Whenever we write dimensions, we will mean complex dimensions unless otherwise stated.

Similar constraints in de Sitter are of interest. See, for instance, [10] for a good discussion on such aspects of the swampland program.

3.2. **The case of Type IIB.** We briefly talked about the fluxes that appear in closed string theories (Type II and Heterotic string theories) in Sec. 2.4. Now we will be discussing how these fluxes and compactification of the theory gives us more 'nice' solutions which are non-vacuum solutions. The need for this is that vacuum solutions, as we have seen above, are not really apt for defining our world and they are plagued by a moduli space and unbroken supersymmetry. As our world has a broken supersymmetry at low energies.

We will compactify type IIB string theory on an orientifold of a threefold Calabi-Yau K_6. We start by mentioning that there are two fluxes in type IIB, one is sourced by Neveu-Schwarz sector (NS) and another by Dirichlet branes. These are two gauge fields B_2 and C_2 for $p = 1$. These are 2-form gauge potentials and the 3-form field strength can be written as

$$H_3 = dB_2 \tag{3.3}$$

$$F_3 = dC_2 \tag{3.4}$$

The generalized gauge fields sourced by the Dirichlet branes are given by C_{p+1} and field strength F_{p+2} where $p = 1, 3, 5, \cdots$ for Type IIB and $p = 0, 2, 4, \cdots$ for Type IIA. When compactification of the theory on a manifold K, the 3-form field strengths, in this case $p = 1$, are elements of the (co)homology $H^3(K, \mathbb{Z})$. This is also mandated by the Dirac's quantization condition, where the presheaf is $\mathbb{Z}$. This makes the fluxes quantized. We will come to the quantization condition later in the notes. But, to immediately lighten this aspect, note that the 3-forms H_3 and F_3 are generalization of the field strength 2-form F of Maxwell's equations in $D = 4$. If the magnetic charges (monopoles) exist then the magnetic flux

$$\int_{S^2} F \neq 0 \tag{3.5}$$

so similarly, here the flux has to be computed over K of six dimensions. If the magnetic counterparts hold true, then the fluxes

$$\int_K H_3 \text{ and } \int_K F_3 \tag{3.6}$$

are non-vanishing. Then Dirac's quantization puts up a relation between the electric and magnetic charges [32, 33]. For the moment, we will just assume Dirac's quantization in our theory, but we will come to a lengthier discussion of these topics later somewhere.

Anyhow, the quantized fluxes are critical and we will refer to these fluxes as an important ingredient in flux compactification which leads to solving problems like unbroken supersymmetry and massless moduli. Alongside, we have localized sources like D-branes in the theory. Generally, any flux compactification of the above kind has massive moduli. The issue of preserving and breaking of supersymmetry is more crucial and deserves a discussion of its own. But we can assume that these flux compactifications preserve the $\mathcal{N}=0$ or $\mathcal{N}=1$ supersymmetry.

If we may, these vacuas obtained from such compactifications should align with the *landscape* of the quantum gravity. But we can relax that discussion too until we are talking about Swampland program explicitly. Reviews on Swampland program include [34, 35]. However, as is noted in [9], this flux compactification is an approximation in finding the non-vacuum solutions which can give mass to the moduli. No exact solution is known and we generally look for the solutions in the leading order.

From the fact that the field strengths and cycles are coming the cohomology group defined on some manifold K and some presheaf, it is important to note that threading cycles in the compact geometry will depend on the the metric choice on K. When the presheaf is $\mathbb{Z}$, then the data of fluxes will be in a set of integers.

Now, when we have flux charges, and because of Gauss's law, we must also have some negative charges so that

$$Q^{D3}_{loc} + \int H_3 \wedge F_3 = 0 \tag{3.7}$$

where the flux $\int H_3 \wedge F_3$ is positive. We also define another three-form flux

$$G_3 = F_3 - \tau H_3 \tag{3.8}$$

where τ is a complex axiodilaton $\tau = C_0 + ie^{-\phi}$ which combines the RR scalar and the dilation. We can also introduce a five-form flux

$$\tilde{F}_5 = dC_4 + \frac{1}{2}B_2 \wedge F_3 - \frac{1}{2}C_2 \wedge H_3. \tag{3.9}$$

and we impose the self-duality condition on $\tilde{F}_5$ so that $\tilde{F}_5 = \star_{10}\tilde{F}_5$, then the Bianchi identity becomes

$$d\tilde{F}_5 = H_3 \wedge F_3 + \rho^{D3}_{loc} \tag{3.10}$$

where the last term appears for the fact that the source term is charged by D3-branes.

Our first compactification example would be Type IIB compactified on a Calabi-Yau threefold K_6. The reasons for choosing Type IIB over Type IIA include that the latter require more than the fluxes we have introduced like non-geometric fluxes [36] and geometric fluxes. Also Type IIB is more useful when we will discuss KKLT construction. Here we want to keep only $\mathcal{N}=1$ supersymmetry and not more for this flux compactification. So there is a reduction in the supercharges from $\mathcal{N}=2$ to $\mathcal{N}=1$. Here we will have the advantage of O3/O7 orientifolds which we will discuss shortly.

Now, we will define the three-form G_3 in Eq.(3.8) to be imaginary self-dual (ISD) flux which means that the Hodge star $\star$ acts with eigenvalue i on G_3

$$\star G_3 = iG_3. \tag{3.11}$$

The reasons for these ISD fluxes[14] are that they help in preserving the supersymmetry and the structure of the Calabi-Yau manifold. The axiodilaton τ is also important in the definition of G_3. Moreover, ISD flux helps in the moduli stabilization after non-perturbative and quantum corrections by giving a potential contribution called superpotential W. These ISD fluxes give mass to the moduli space of K_6 and the axiodilaton.

3.2.1. *Superpotential Contribution.* Superpotentials help us to break the supersymmetry. In the ISD flux compactification theory, we will get two potential which will determine the scalar potential. One is the Kähler potential and another is the superpotential generated by G_3. The latter superpotential is called the Gukov-Vafa-Witten superpotential

$$W_{flux} = \int_{K_6} G_3 \wedge \Omega \tag{3.12}$$

where[15] G_3 is our three-form, a combination of H_3 and F_3 (and axiodilaton τ) and Ω is a holomorphic $(3,0)$ form on Calabi-Yau threefold K_6. The role of Ω is also important and actually, Ω depends on the moduli of K_6 so it becomes a function of z_i. Varying z_i changes the period of Ω, so $\Omega(z_i)$ has complex structure dependence that will be helpful when being wedged with G_3. The superpotential W_{flux} also depends on the axiodilaton τ cause its appearance in G_3. So when the moduli is fixed through these fluxes, axiodilaton is also affected and it gets stabilized too with moduli. One can write W_{flux} is a symplectic basis as the form

$$W_{flux} = \overline{\Pi} \cdot \Sigma \cdot (\overline{f} - \tau \overline{h}) \tag{3.13}$$

where $\overline{\Pi}$ is the period of $\Omega(z_i)$, $\overline{f}, \overline{h}$ are the integer flux quanta of $F_3, H_3 \in H^3(K_6, \mathbb{Z})$ and Σ is a symplectic matrix, for reference see [9]. Moreover, the superpotential is protected from the non-renormalization theorem and can be purely calculated from topology.

So we can write the Gukov-Vafa-Witten (GVW) superpotential as $W_{flux}(\tau, z_i)$. But this was not the full superpotential because it does not stabilize the Kähler moduli. There is a non-perturbative superpotential as well now which will stabilize it. We will call it W_{np} which we depend on τ, z_i, T_a. Here, T_a is the Kähler moduli where $a = 1, \cdots, h^{(1,1)}$. There will be three sources in this non-perturbative potential W_{np}

$$W_{np} = W_{ED3} + W_{\lambda\lambda} + W_{ED(-1)} \tag{3.14}$$

where the W_{ED3} represents the sum of all the Euclidean D3-branes terms, $W_{\lambda\lambda}$ represents the sum of all gaugino condensate terms and $W_{ED(-1)}$ part is generated by D-instantons. D-instantons' contribution to the non-perturbative superpotential

[14] Similarly, $\star G_3 = -iG_3$, G_3 will be called imaginary self anti-dual (IASD) flux. Here, in the type IIB compactification, the IASD fluxes would not contribute. Then such compactification would be called ISD flux compactification [37].

[15] One can also put up a normalization factor $\sqrt{\frac{2}{\pi}}$ but we will ignore that here.

is seen as a way to control string theory non-perturbatively and there has been a progress in understanding these, see [38, 39] and references there within.

One can in fact write the flux superpotential term W_{flux} as

$$W_{flux} = \text{Polynomials} + \text{Exponential Terms in } \tau \tag{3.15}$$

and these polynomials can be set to zero, in which the case becomes a perturbatively flat potential one. Similarly, the non-perturbative part becomes a sum of all the Pffafian terms. These controls are useful for many things for instance see this vacua construction [9].

So the full superpotential becomes $W = W_{flux} + W_{np}$. We will not expand on this non-perturbative superpotential in this paper and sometimes, one can ignore this non-perturbative superpotential too, since for all the orders of α and g_s corrections the superpotential is just flux superpotential

$$W \approx W_{flux}. \tag{3.16}$$

Anyway, the moduli scalar fields are of three forms in this compactification

$$\mathcal{M} = \mathcal{M}_{K6} + \mathcal{M}_K + \mathcal{M}_\tau \tag{3.17}$$

where $\mathcal{M}_{K6}$ is the moduli spaces on the complex structure on K_6, $\mathcal{M}_K$ means the Kähler moduli spaces, and $\mathcal{M}_\tau$ means the moduli space of axiodilaton. We also saw that these moduli structures have appeared in the flux compactification. The complex structure (z_i) moduli controls the shape of the Calabi-Yau K_6. The Kähler moduli controls the size of the Calabi-Yau K_6 and the axiodilaton moduli which is another type of complex structure controls the string coupling.

One can then split the Kähler potential into three terms as well. At tree level, it given by

$$K_{\text{tree}} = -\ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(-i\int_{X_6} \Omega(z_i) \wedge \overline{\Omega}(\bar{z}_i)\right) - 2\ln\left(\mathcal{V}(T_a, \overline{T}_a)\right). \tag{3.18}$$

But in fact, the beyond tree level, the Kähler potential can be perturbatively understood in the moduli stabilization. Such perturbations are necessary as they contribute to the leading terms in potential V, see [9] for an exposition.

For instance, **the no-scale property** [40] is an important point to mention here. The no-scale property makes the superpotential W not depend on the Kähler moduli at the tree level. What happens is that we can write the scalar potential in a simplified form in SUGRA

$$V = e^K\left(K^{ij} D_i W D_j \bar{W} - 3|W|^2\right) \tag{3.19}$$

and when scalar potential does not depend on the Kähler moduli, then we have an identity called no-scale property

$$K^{ij}\partial_i K \partial_j K = 3 \tag{3.20}$$

and this cancels the $-3|W|^2$ in Eqn. (3.19) and then the reduced scalar potential does not have Kähler moduli terms

$$V_{tree} = e^K(\text{dilaton moduli terms} + \text{complex structure moduli}) \tag{3.21}$$

and because of this, the Kähler moduli remains massless at tree level. The resolution to this, as discussed above, could be to introduce a non-perturbative superpotential W_{np} as was done by KKLT [7]. There are multiple remarks on the no-scale scalar potential (V_{tree}). First, it is positive-definite. The second is that its minimum is zero. Moreover, supersymmetry is broken in the Kähler moduli direction. Third, the Kähler moduli[16] T_a do not get stabilized by this scalar potential. In essence, one must *break* the no-scale property to stabilize all the moduli of the theory including Kähler moduli.

One of the ways to stabilize the Kähler moduli is to introduce the non-perturbative superpotential. These quantum corrections can be found in form of supersymmetric instantons and gaugino condensation contribution. But it is not necessary that every class of Calabi-Yau manifolds will get a non-perturbative superpotential generated. The context of fourfold Calabi-Yau has been given in [39] where progress was made in calculating the non-perturbative superpotential in the M-theory compactification to $\mathcal{N} = 2$ three dimensions on a four-fold K of $SU(4)$ holonomy. This is roughly comparable to $\mathcal{N} = 1$ in four dimensions. The results were also carried to Heterotic string theory and type IIB string theory using F-theory. In this example, the non-perturbative potential could be generated (in cases, *entirely*) from the instantons.

3.3. **Dine-Seiberg Problem.** We will now discuss very quickly the Dine-Seiberg problem of moduli stabilization, which will serve as an intermediate motivation for the problems with constructing dS vacua. A sort of tongue-in-cheek quote from Denef's Les Houches lectures is: *"When corrections can be computed, they are not important, and when they are important, they cannot be computed."* This is in reference to the problem that appears from weakly coupled moduli conditioning, which is naive as follows: for moduli ρ (could be a volume modulus or the inverse string coupling $e^{-\phi}$), we expect that the weakly coupled limit generates a potential like

$$\lim_{\Phi\to\infty} V(\rho) = 0\,. \tag{3.22}$$

However, we only get a runaway or strongly coupled curve for the potential rather than a local minima. To generate these, one requires higher-order corrections. In a sense, this extends into the leading order EFT/swampland discussions where the potential for moduli are a problem. In this fashion, what we expect of dS vacua has to do with the higher order corrections being indicative of strong coupling.

A very important remark we wish to make now is that, as already discussed, Kähler moduli do not get fixed by flux superpotential W_{flux}. That is when the quantum corrections become important and too the Dine-Seiberg problem. Because

[16] Moroever, the Kähler moduli is invariant under pertubative corrections.

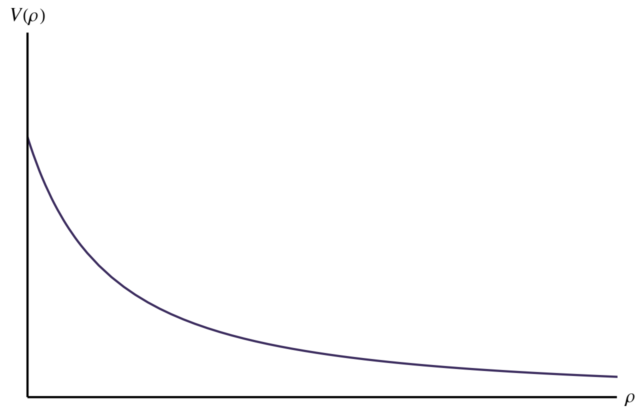

FIGURE 2. The potential $V(\rho)$ decays as ρ increases in the weak coupling limit. If we include the higher order corrections, we get something like Fig. 3

such higher-order corrections take us to the strongly coupled regime and not easy to calculate which has been sloganized in many places including ours.

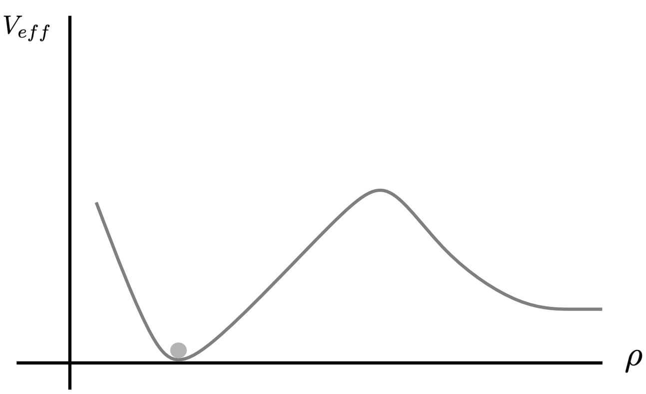

FIGURE 3. According to Dine-Seiberg [5], our string vacuum should be in the strongly coupled regime.

3.4. **Maldacena-Nuñez no-go theorem.** Now we have seen a bit of KKLT construction which promises an uplifted dS vacua from non-perturbative correction in moduli stabilization. But before that, there is a famous result due to Maldacena and Nuñez [6] which goes by the name of 'no-go theorem'. From the Dine-Seiberg, we can believe that our de Sitter vacuum should be in a strong coupling regime. In that way, we would not be running into a *runaway* argument of infinite volume.

Now, for a classical supergravity with a 10D action at two-derivative level (like the ones we have treated in initial parts of this paper where no quantum corrections or non-perturbative corrections are assumed), the compactification does not yield a dS vacuum if the internal manifold is compact, static and without singularities [6]. This no-go theorem is about classical compactification with flux which preserves the supersymmmetry. To circumvent this no-go theorem, one can add quantum corrections or non-perturbative corrections and compactification on an orientifold (localized sources)so that we also do not get a Dine-Seiberg problem. So this is clearly implies that the de Sitter solution is not valid at **classical** level without quantum corrections.

Progress in these no-go theorems have happened over the years and some of these will be relevant in our final discussion in Sec.5. These no-go theorems [41–43] are mostly about heterotic string theories but dualities can extend them to type IIB, M-theory, and such too. A work that we should mention here is about the no-go theorem for the AdS case (Maldacena-Nuñez no-go theorem originally was about Minkowski and de Sitter compactification) when AdS scale is bigger than the Kaluza-Klein scale [44].

A most natural attempt has been done to avoid the classical no-go theorem is to consider a classical compactification on an orientifold and then add quantum corrections or stringy ingredients without which there is no de Sitter vacuum. A successful example of this is the KKLT proposal where a type IIB flux compactification is done and then it considers non-perturbative corrections and then uplifts using anti D3-branes to a dS vacuum. However, such a result does not get entire approval from programs like *Swampland* which we will discuss in the last part of the paper.

Let us make a final comment on the construction of de Sitter vacua by quantum corrections in the SUGRA 10D action. They are not complete and has a few problems as pointed in [45]. For instance, these derivations (and the required corrections) are quite hard to compute. Anyway, flux compactification in type IIB with the engineering that we have described above is more appropriate for moduli stabilization.

3.5. **Uplifting and Moduli Stabilization.** We will now talk about the mixing of D3/$\bar{D}$b-branes, which will pave the way to discussing the KKLT de Sitter vacua construction. Our present state is one where we stabilize moduli for a type IIB compactification onto CY3 and deal with D-branes and O-planes under the constraint of the tadpole cancellation condition. In particular, type IIB compactifications in SUSY AdS will be our focus, which we will uplift into de Sitter (metastable) vacua in later discussion[17]. Schematically, a given superpotential (notationally aligning with most literature) W, this would be independent of the Kahler moduli. As an example, the Gukov-Vafa-Witten superpotential [46] is $V_{GVW} = \int G_3 \wedge \Omega_{3,0}$ is independent of the Kahler moduli and depends solely on the complex structure moduli and dilaton. In the discussion below, a general form for W would be chosen to be

$$W = W_0 + \mathcal{A}e^{i\mathcal{B}\rho} , \tag{3.23}$$

where W_0 is constant and arises from the complex structure moduli and dilaton part of the theory, and $\mathcal{A}$ and $\mathcal{B}$ are constants with ρ the Kahler volume modulus. The

[17] In the next subsection, it will be clear in what sense these conditions allow to circumnavigate the Maldacena-Nunez no-go theorem.

GKP [37] fixing of the Kahler potential into

$$V_K(\rho, \phi) = -3\log\left(\rho + \bar{\rho} - k(\phi)\right), \tag{3.24}$$

for which the *no-scale cancellation*. Simply, the canonical SUGRA potential can be constructed given a Kähler potential like above into

$$V = e^K \left(K^{ij} D_i W D_j \bar{W} - 3|W|^2\right) . \tag{3.25}$$

For the choice of W mentioned above[18], it can be shown that there is a particular way of obtaining a de Sitter vacua albeit metastable. We are clearly skipping a lot of technical details, but we will deconstruct the discussion from AdS vacua into dS vacua gradually. Before doing this, we will also touch on the nature of the Klebanov-Strassler geometry and the uplift scenario when we add in an $\bar{D}3$-brane. It is important to note that we typically expect to find vanishing derivatives of the F-term unless we specifically choose scenarios with directional SUSY-breaking. Once we uplift to de Sitter vacua, this would be a SUSY-breaking compactification with a small positive cosmological constant.

The KS geometry describes, very naively, the geometry of $\mathcal{M}_4 \times \mathcal{C}^6$, where $\mathcal{C}^6$ is a six-dimensional conifold with a tip of finite R. We take it that the complex structure moduli are fixed. The resulting "warp" region that we get here will be used later on when we add in $\overline{D3}$-brane into the theory for tadpole cancellation. The $\overline{D3}$-brane will have a tendency, as we will see below, to move into the throat and the breaking of SUSY thus produced will give us an "uplifted" de Sitter potential.

We first have a few remarks in order. The full potential is composed of the full superpotential and the Kahler potential, where the former receives only non-perturbative corrections, and the latter receives perturbative as well as non-perturbative corrections. This adds to the complexity of stabilizing Kahler potential, and in particular identifying α' corrections to it. However, recently by Moritz et al, it was shown that it is possible to at least find leading order de Sitter vacua, where we restrict solely to the tree-level potential. Further, what we refer to as the "full superpotential" is the GVW superpotential W_{flux} plus the nonperturbative corrections it receives.

A part of the reason that we start with type IIB SUGRA instead of type IIA is also, as mentioned before, that the axiodilaton in type IIB becomes the modulus of elliptic fibration when KK compactifying M-theory into type IIA. From this, the data we would have would be the Calabi-Yau fourfold CY4 with $\mathcal{C}(K^6, \tau)$, and the tadpole cancellation condition would count in D3-branes, $\overline{D3}$-branes and O3-planes. Now, we take the tree-level Kahler potential and find the $\mathcal{N} = 1$ scalar potential V as seen from the canonical SUGRA construction from before. Given this, we expect that there exists a SUSY AdS vacua with some really small negative value $\Lambda = V_0 M_{\rm pl}^2$, where $V_0 < 0$ is the critical point derived from the F-term. The effect of the nature

[18]In the computational example we will take in the KKLT discussion, we will choose to work with a complex scalar T instead of the modulus ρ to a similar functionality.

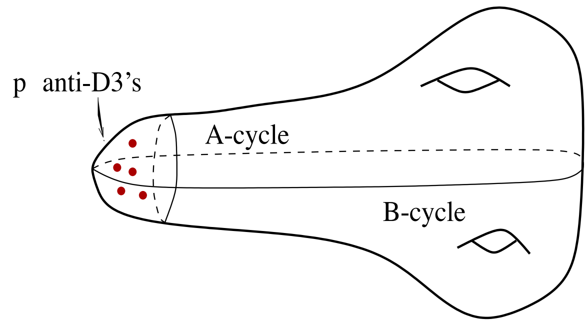

FIGURE 4. An illustration of the conifold geometry (derived from KPV [7]). The M RR-fluxes thread the A-cycle.

of the superpotential is that it receives nonperturbative corrections from D3-branes and D7-brane wrappings via gaugino condensation (which we will not discuss here).

We will now discuss the actual uplifting that we will use in KKLT. The essential thing to remember is that the tadpole cancellation is preserved, so if we add in "too many fluxes" in the sense we will discuss in KKLT, there must be some negative contribution to cancel the extra terms. Flux-wise, we take the following flux units for the conifold: we take, for a Calabi-Yau threefold K, M units of F_3 flux through the A-cycle of K and N units of H_3 flux through the B-cycle of K, and the "counting" to be done here is that for a change between these units of fluxes, a corresponding D3/$\overline{D3}$-brane may be introduced to counter the effect of the change. E.g. as will be appropriate later, if we add too much $\mathcal{F}$, then a corresponding contribution with $n+1$ $\overline{D3}$-branes may be introduced, where n is typically 0. If we take a theory of *only* n $\overline{D3}$ branes at the throat, the geometry of the throat must be determined so that the n $\overline{D3}$-branes settle with least energy. This conifold geometry is typically the Klebanov-Strassler geometry, which looks something like the illustration below.

4. KKLT AND DE SITTER SOLUTIONS

Before we proceed, we have two remarks in order. (1) The KKLT proposal is not the only way to construct a de Sitter string theory vacua, but at least in leading order approximation, it seems to be a viable construction. There are swampland issues with this construction, particularly when taking into account of higher order corrections, but for our purposes we will strictly remain at leading order. (2) In constructing a de Sitter vacua, we expect that the stability of the theories are of some finite order in construction. In this sense, we really seek metastable vacua, and the order of metastability is usually not explicitly computed unless we take precise corrections into account. In any case, doing precise computations is still an open challenge; by using machine learning techniques, one can still find a reasonable sized flux landscape to sample from, but even this is not enough to perfectly determine some key features of the vacua like the metastability order. In order to illustrate the computational complexity of the KKLT construction, consider that at leading order, we have to precisely compute the flux superpotential terms that include Euclidean D3-branes, gaugino condensation terms, the flux terms themselves, then the tree-level Kahler potential, the $\overline{D3}$-brane terms (which at leading order become the KPV potential + subleading terms) and the corrections from WSI.

There are two ingredients in the KKLT construction. The first is the KPV potential, which contributes to the full potential along with the flux superpotential and the nonperturbative corrections to it. For the moment, we will ignore the backreactions to the KS geometry caused by the inclusion of these $\overline{D3}$-branes, but we will consider the corrections to the $\overline{D3}$-potential in later revisions[19]. To begin with, we take the AdS vacua potential that scales by $\sim -\Lambda M_{pl}$ for $|\Lambda| \ll 1$. We then want to uplift this potential into a very small positive value V_0'. Before proceeding, note that the energy-density contribution from the single $\overline{D3}$-brane is

$$V_{\overline{D3}} = \frac{2a_0^4 T_{D3}}{g_s^4} \cdot \frac{1}{(\text{Im } \rho)^3} \ . \tag{4.1}$$

This can be seen as follows: taking the 10d warped geometry with metric

$$ds^2 = e^{2a} g_{\mu\nu} dx^\mu dx^\nu + e^{-2a} g_{\rho\sigma} dy^\rho dy^\sigma \ ,$$

and the warp factor ends up with value $a = a_0$. Going between the string frame and the Einstein frame, we get an overall scaling like

$$M_{\text{pl}}^2 \sim \frac{M_s^8}{g_s^2} \ , \tag{4.2}$$

from which we get an overall dependence of order $\sim 1/g_s^4$. The derivation for the additional factor for the volume modulus is left as an exercise for the reader. (4.1) can be further condensed into the form

$$V_{\overline{D3}} = \frac{\mathcal{D}}{\tau^3} \ , \tag{4.3}$$

where $\tau = (\text{Im } \rho)$ for convenience. In the original KKLT paper, there is an additional prefactor 8 in (4.3) owing for brevity uses, but for our purposes we will drop coefficients pertaining to this factor. The term $\mathcal{D}$ is the key "controller" for finding the vacua landscape, and is the general condensation from whatever the exact potential contribution is. In this condensed form, the full potential is

$$V = V_{\text{flux}} + V_{\overline{D3}} \ . \tag{4.4}$$

The $V_{\overline{D3}}$ is the KPV potential without corrections. Even in a leading order theory, we typically have more data needed, like said before. The computable terms would be:

$$W = W_{\text{GVW}} + W_{\text{GC}} + W_{\text{ED3}} \quad \text{and} \tag{4.5}$$

$$K = K_{\text{tree}} + K_{\text{WSI}} \ . \tag{4.6}$$

[19]KPV corrections appendix to be added in upcoming revisions. [*v1*]

The following "poem" illustrates our position well[20]:

> *First u stabilize the complex structure moduli. Then u stabilize the Kähler moduli. Then u get an AdS (but mebbe no scale separation).*
> *Then u put an anti-D3 [brane] in to uplift to dS. Then u get the money. Then u get the power. Then you get* **[REDACTED]**.

In order to illustrate this scenario, we will use the StringVacua Mathematica package to compute the minima of the F-term. In order to do so, we will fix a simple leading-order example in type IIB supergravity. We will break the setup into two parts, one for the superpotential and one for the Kahler terms.

(1) *Superpotential:* The form our superpotential will take is as usual:

$$W = W_0 + A\exp(-aT) \ , \tag{4.7}$$

where a factors in gaugino condensation and A is a prefactor for the nonperturbative effects. In this example, we will only rely on gaugino condensation contributions and leave out ED3-branes. We take complex scalar

$$T = t + i\tau \ , \tag{4.8}$$

and in this example we will rely on hand-picked values for W_0.

(2) *Kahler potential:* The Kahler potential in this example is

$$K = -3\log\left(T + \overline{T} + \frac{\zeta(\chi)}{\mathcal{U}}\right) \ , \tag{4.9}$$

where $\zeta(\chi)$ is a leading order α'-term that is not fixed and will take insensitive values. It is worth stating that in our example we do not consider explicitly the large-volume scenario with these parameters, which would have to be fixed/bounded.

Then, we have the following parameters:

- **Complex fields:** $\{T\}$
- **Real parts:** $\{t\}$
- **Imaginary parts:** $\{\tau\}$
- **Kahler potential:** Given in (4.9)
- **Superpotential:** Given in (4.7)
- **Parameters:** $\{W_0, A, a, \zeta(\chi)\}$

This is a very simplified example, of course, and we have not added in the KPV potential $V_{\overline{D3}}$ or the α'-corrections. However, since the full scalar potential is (4.4) and the KPV potential is (4.3), we will simply compute the flux terms and add the $V_{\overline{D3}}$ contribution manually. In a realistic calculation, of course, there are parametric bounds on the Klebanov-Strassler throat radius and the corrections beyond leading order that could potentially invalidate the tadpole condition.

[20]We thank Stringking42069 for this work of art.

As a first example, we remove the ζ term from (4.9). We will also fix the W_0 to be equal to -10^{-4}, and $a = 0.1$. We will also further assume that the pfaffian is equal to unity as well, but there are more computations requiring precise evaluation of this. With this, we construct a model from

$$W = -0.0001 + \exp\left(-0.1T\right) \ , \tag{4.10}$$

for which we further evaluate the minima wrt t. Using Stringvacua we get

$$\begin{array}{cc} \text{complex fields} & \{T\} \\ \text{real fields} & \{t\} \\ \text{imaginary fields} & \{\text{tau}\} \\ \text{Kahler potential} & -3\log\left(T^* + T\right) \\ \text{superpotential} & e^{-0.1T} - 0.0001 \\ \text{scalar potential} & \frac{e^{-0.3t}\left(e^{0.1t}\left(0.00166667t^2+0.05t\right)-5.\grave{}*\wedge\text{-}6e^{0.2t}t\cos(0.1\tau)\right)}{t^3} \end{array} \tag{4.11}$$

The minima for this potential would be at $t \to 362$ with minima $-7.20971*10^-27$, and there are no non-zero F-term directions, showing that SUSY is indeed preserved. The potential here is of the form in (4.11), and to this we would add the KPV potential (4.1). For this, we take the value of $\mathcal{D} = 3 \times 10^{-9}$, the same as in the KKLT setup. Then, choosing a suitable parametrization, we take the additional KPV term to go like $\sim \mathcal{D}$, and this would give us a positive value $\sim 2.9 \times 10^{-9}$.

We could instead consider a *racetrack superpotential* rather than the KKLT superpotential of the form (3.23), we can obtain slightly more control over the uplift. In particular, if we consider a twotrack superpotential of the form

$$W = W_0 + A\exp\left(-\frac{2\pi T}{N}\right) + B\exp\left(-\frac{2\pi T}{M}\right) \ , \tag{4.12}$$

where we take in previously hidden gaugino condensation prefactors $a = \frac{2\pi}{N}$ and $b = \frac{2\pi}{M}$, we can obtain a potential for which the control parameters become W_0, A, B, M, N. The prefactors A and B can be taken to be unity for convenience of discussion here. M and N are determined by the gauge group of the gaugino condensation [47]. The minima is then $\sim \{-1.9\hat{}\text{-}15, \{t \to 113.643\}\}$ As usual, we add in an uplift that is controlled by $\mathcal{D}$, for which we take one $\overline{D3}$-brane, and the resulting potential would be uplifted to a small positive value controlled by $\mathcal{D}$. As before, we still typically require the KPV validity constraints with p/M and finite S^2 control. An example of the variance of the potential with and without uplift is shown in figure 5 taking different parameters, where we take constraints on $\mathcal{D}$ and set $p/M \ll 1$. In this case, the uplift "fails" and the minima is still negative:

A few remarks are in order:

(1) We have not constructed a physically realistic uplifting example, in which case we would have to factor in perturbative corrections, worldsheet instanton

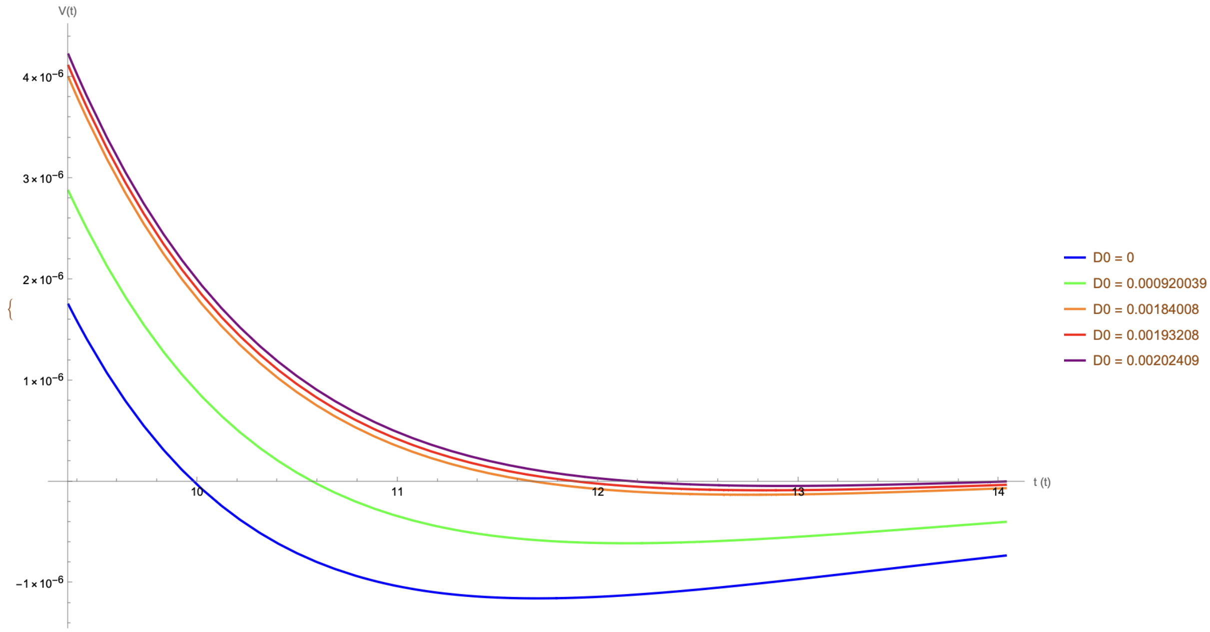

FIGURE 5. Failed KKLT-uplifting in a simplistic scenario.

corrections, backreactions in the throat geometry, and the Kahler potential would have to be evaluated more thoroughly.

(2) We have ignored the explicit form for the nonperturbative superpotential contributions, which would need us to find the ED3- and GC contributions. See for instance the leading order explicit construction of de Sitter vacua by Moritz et al [48].

(3) Our examples have suppressed calculations for the Kahler potential corrections and the KPV corrections/backreactions. For a more realistic computation, we would have to factor in the KPV corrections, which could potentially invalidate the KPV leading potential term [49] and the uplift parameter control, which could lead to runaway decompactifications.

(4) Finally, we also note that the KKLT proposal in itself has physical issues – see Vafa et al [50]. In some sense, it may be more meaningful to work with racetrack superpotentials[21] or the large volume scenario [51].

5. Is there a de Sitter solution?

In this section, we will provide a sociological commentary on the various sides and their arguments on if there exists a de Sitter vacua or not. There have been multiple examples of de Sitter vacua construction, KKLT being a famous one which was covered in Sec. 4, gives an uplift to the AdS vacua. The question if there is a

[21] Which also do not possess a one-shot de Sitter construction due to the bounds of the AdS moduli conjecture.

direct yielding of dS vacua is still open. But as pointed in [45], these all vacuas have problems. For instance, the quantum corrections are impossible to calculate in the strong coupling region or the non-geometric fluxes which have to be introduced as a full string theory must have such backgrounds. However, these non-geometric fluxes are quite hard to handle too. Ref. [45] also mentions about the back reaction of anti D3 branes. Moreover, same paper debated that such failures indicated that not much work had fertilized in string theory on de Sitter and cosmological constant. An important suggestion was made to introduce the dark energy more in string theory. But they (also see [52] for similar thoughts) concluded that the string theory conspires against a de Sitter vacua and there exists **no** solution at all. Is it really true? We would say, we do not know yet. However, the problem of not having a phenomenologically consistent construction is a big one.

Note that we have also ignored some phenomenological aspects of the KKLT constructions, particularly those pertaining to the uplifting in the Klebanov-Strassler setup[22], such as the puffing of $\overline{D3}$-branes into NS5-branes or the curvature backreactions. But even brushing these aside, there are some observations on the EFT side of things.

Moreover, the no-go theorem by Maldacena and Nuñez says that the classical solution of de Sitter does not exist. This troubles the construction as well and leads to believing the de Sitter conjecture in swampland. The swampland criterion [53] states that the potential of the scalar field satisfies a universal bound

$$|\nabla V| \geq \frac{c}{M_{pl}} \cdot V \tag{5.1}$$

where c is an order 1 constant and $V(\phi)$ is the effective potential for a low energy EFT. There exists a refined version of this conjecture which requires that either (**??**) is satisfied or that the minimum of the Hessian eigenvalues are bounded from above with a constant c. This bound was studied cosmologically in [54] and other works include [55–57] and [58]. This conjecture excludes the meta-stable de Sitter vacua. This is called the de Sitter conjecture.

Another related conjecture in swampland is called Trans-Planckian Censorship Conjecture (TCC). The TCC constraints the lifetime of these meta-stable de Sitter vacua. But the best string theory can offer is the meta-stable de Sitter vacua, that too is suggested by [52] that the meta-stable de Sitter belongs to Swampland. So in the 'string theory', a de Sitter does not make sense according to these conjectures and so dS/CFT would not make sense. However, we have seen dS/CFT examples and discussions in different literature, also in non-string theory cases.

Moreover, outside of KKLT and LVS proposal, there are no-go theorems that we did hint in above discussion on Maldacena-Nuñez no-go theorem section which

[22] There also exists a more sophisticated construction involving uplifting using the S-dual setup, but it is beyond the scope of this paper [49].

possibly poses more no-go theorem about de Sitter (even AdS vacua) even in non-classical regime [41, 42, 59].

In conclusion, the swampland program mostly pushes de Sitter vacua, or even meta-stable de Sitter vacua, into swampland. But some examples are also given for the landscape case [60], see also [61].

A few remarks are in order now:
Firstly, the tadpole cancellation is a condition that should be met in any meaningful compactification, so that

$$Q_{flux} + \Delta Q_{O3,\ D3,\ \overline{D3}} = 0\ . \tag{5.2}$$

Bounding to this condition sets constraints on say small g_s, or the distance conjecture implicitly.

Second, it is clear that metastable de Sitter vacua are more likely than stable de Sitter vacua. However, a general problem is that the first step of any KKLT-like uplifting proposal, i.e. to find a SUSY AdS vacua with $|W_0| \ll 1$, is usually not attainable. However there have been papers showing that such values can be found and that the Kahler moduli can be stabilized with a suitable W_{np} as well. In such constructions, not only can you find AdS vacua, but you can uplift it "reasonably" (whatever that means in terms of the throat curvature corrections), and obtain metastable de Sitter vacua. The downside is that the corrections to KPV potential can still invalidate the uplifting, but at leading order or in some limits where the corrections can be suppressed, one indeed does obtain actual dS vacua.

There are many issues in a similar direction, such as destabilized $\overline{D}3$-branes, or invalid throat constraints, or perhaps even that the nature of the conifold geometry could prevent "*small uplifts*", which are those that do not lead to moduli destabilization [62]. A general intrinsic subtlety with the Klebanov-Strassler setup is also that given a finite number of flux vacua and given the tadpole cancellation, there may not be enough "control" over the parameters. In most cases though this is not of concern though since we do not consider "small" compact CY3 geometries.

An observation we would like to quickly go over is that the inclusion of the non-perturbative superpotential term is not necessarily of the KKLT form as said before. In the above setting, we took an N-stack of D7-branes with wraps around a four-cycle, for which the resulting gaugino condensation was a contribution that is roughly $\sim A\exp(-aT)$. Note that we say "roughly" here since we suppress the additional $\mathcal{O}(\exp(-T/N))$ terms for convenience, which don't make the previously seen KKLT form any different though. In a racetrack superpotential theory, we would typically have multiple contributions of this form, which in a two-exponent form would look typically like

$$W = W_0 + \sum \left(A\exp(-aT_i) + B\exp(-bT_i)\right)\ . \tag{5.3}$$

It is possible that these types of superpotentials could give more "control" over the construction, but the exact nature of such KKLT-like constructions is unclear. We say this in double quotes since there haven't been any particular constructions thus far, and this observation is actually in accordance with the AdS moduli conjecture [45].

Something very important to note is that the KKLT solution that we have discussed in this paper is only a particular case of a general class of nonperturbative uplifting constructions that allow de Sitter vacua to be constructed from AdS. So while classically there are no dS or Minkowski string compactifications, we indeed have the liberty to construct them with such corrections. Which *sort* of such constructions would actually give a physically realistic de Sitter vacua though, is an open problem.

There are often discourses about *if there is a de Sitter vacua in string theory* online and offline, some of them become dangerous while some of them are without a satisfactory conclusion. We believe that this problem could be solved if one starts constructing more vacua with better corrections, better control. Perhaps by not going directly into four-dimensional EFT and trying simple examples as suggested by [45].

Finally, in [12], it was shown concretely that there exists a string theoretic de Sitter vacua. However, this paper is too technical to discuss here. So we would like to leave this as an exercise to the reader.

Acknowledgments

Both authors would like to thank multiple discourses on string theory on Twitter. AV would like to thank the multiple hands which go into the harvesting and processing of coffee that we consume everyday, this work was impossible without them. VK would like to thank everyone who works tirelessly to provide the food, drinks, and comforts we often take for granted, especially those whose efforts go severely underappreciated. And Denis Villeneuve for his amazing works of cinema.

Appendix A. Why Calabi-Yau manifolds?

This section will be a mathematically oriented discussion about the Calabi-Yau that we can not afford in a paper on physics.

A Calabi-Yau manifold K is a complex Kähler manifold with a vanishing first Chern class. We saw some of the consequences of this definition and how it is beneficial in supergravity and string compactifications. The history of this particular object is quite interesting from either side of mathematics and physics. In mathematics, it was conjectured by Calabi and proved by Yau [63]. Calabi's conjecture was about that for a compact Kähler manifold K with vanishing first Chern class $c_1(K) = 0$, there exists a unique Ricci flat metric in every Kähler class. Moreover, it made a quite impression in algebraic geometry and differential geometry afterward.

From physics perspective, these manifold solves Einstien's equations with Ricci tensor $R_{\mu\nu} = 0$. They are interesting, or perhaps not really, because they also have a moduli space attached with it since they are complex manifolds and because two complex manifolds can be deformed with each other using **holomorphic** diffeomorphism[23] between them. Upon compactification, these moduli fields appear as massless scalar fields in the theory for which we require a resolution, discussed above in Sec. 3.

A Calabi-Yau manifold is characterized by its the Hodge numbers $h^{p,q}$ which is but the dimension of the complex vector space $H^{p,q}(K)$ on K. And by Hodge's theory, they also determine the Betti numbers. Dolbeaut cohomology and de Rham cohomology are incidentally very much used in discussions about Calabi-Yau manifolds and especially in mirror symmetry. In fact, mirror symmetry, at some abstract level, is a statement about the equivalence of de Rham and Dolbeaut cohomology. Although, here we will introduce a simpler definition[24] of mirror symmetry. Physicists usually see mirror symmetry as an equivalence of theories on Calabi-Yau manifolds and these are related by Hodge numbers of the manifolds. However, mirror symmetry is also equivalent to T-duality which was addressed first by [65]. So a type IIB string theory on a manifold is equivalent to type IIA string theory on the mirror manifold.

It was introduced by Witten [66] that two different sorts of twists done to a quantum field theory are also equivalent to each other by Fourier transform and can be explained by Mirror symmetry too. These twists are A-twist and B-twist. Basically, the A-model on Calabi-Yau manifold Y is equivalent to the B-model on X which is the mirror of Y. However, it would be way out of the scope of these notes to explain them here. A more abstract version of mirror symmetry is provided by Konstevich, called *homological mirror symmetry* [67] which is a conjecture about the equivalence of categories on Calabi-Yau manifold and its mirror.

A Calabi-Yau manifold solves the ten-dimensional supergravity equations in the presence of fluxes, as in type IIB compactification. This particular feature is very favorite to physicists. While in type IIA such a construction should be translated using T-duality [68], however, there has been not much work in this area. Also, see [69] where type IIA flux compactification was studied and moduli stabilized but with O6 planes backreaction. Or perhaps, this paper DeWolfe, Giryavets, Kachru, and Taylor [70] where has been showing that fluxes are sufficient to stabilize all the moduli in type IIA while in type IIB, we also require non-perturbative corrections, at least for a controlled compactification and without a runaway.

[23] In general, we try to vary the complex structure on the manifold, at least for lower dimensional Calabi-Yau manifolds, locally we can say that.

[24] A celebrated discussion of mathematics (mostly algebraic geometry) of Calabi-Yau manifolds and mirror symmetry can be found in [64].

In the above sections, we have seen that in supergravity, Calabi-Yau manifolds were used to preserve the supersymmetry. But the lack of supersymmetry in experiments prompts us to make models where supersymmetry is broken. We also talked about these in the above sections where, say, only half of the supersymmetry is preserved.

Appendix B. Role of Instantons and other Quantum Corrections

We discussed the non-perturbative corrections that are required to stabilize all the moduli fields because the no-scale property makes the flux superpotential not depend on Kähler moduli. Our discussed KKLT mechanism also utilizes the quantum corrections to the superpotential so that Kähler moduli get stabilized. These corrections are better if non-perturbative [45]. So, these non-perturbative corrections (of instantons) are very crucial in KKLT's yielding of de Sitter vacua using uplift.

The non-perturbative correction includes the terms of D-instantons corrections and gaugino condensation.

To be completed in upcoming revisions. [*v1*]

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