HW 1

1.

a) $S: \{1, 10, 100\}$ or $\{10^n \mid n = 0, 1, 2\}$

b) $S = \{n \in Z \mid n > 10\}$ where Z is the set of all integers

c) $S = \{n \in N \mid n < 10\}$ or $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

d) $S = \{\phi\}$ or ϕ

e) $S = \{" "\}$ or $\{\varepsilon\}$

f) $S = \{"abc"\}$

2. The number of elements in the power set of $S = 2^S$.

If $S = 0$, there are no elements in S so ϕ is the only subset of S so since $2^0 = 1$, 2^S solves the problem as it gives us the number of elements.

Similarly when $S \geq 1$, we get the number of subsets as the equation 2^S

So in the power set of $S = 2^S$

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3.

a) Given

$$S(n) : 1 + 2 + 3 \ldots + n = \frac{n(n+1)}{2}$$

$$S_n = \frac{1}{2} n(n+1)$$

Basic step:

For $n=1$

$$1 = \frac{1}{2} 1(1+1) = 1$$

S_n is verified to be true for $n=1$

Induction step

let S_k be true

i.e.. $1 + 2 + 3 \ldots + k = \frac{k(k+1)}{2}$

We have to prove that $s(k+1)$ is true

RHS:

$$S_{k+1} = 1 + 2 + 3 \ldots + k + (k+1) = \frac{(k+1)(k+2)}{2}$$

LHS: $1 + 2 + 3 \ldots k + (k+1)$

$$= \frac{k}{2}(k+1) + k+1)$$

$$= (k+1)\left(\frac{k}{2} + 1\right) = \frac{(k+1)(k+2)}{2}$$

$= RHS$

$s(k+1)$ is true.

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By induction S_n is true for all $n \in N$
By principle:

$$1 + 2 + 3 \ldots + n = S(n) = \frac{n(n+1)}{2}$$

1) $C(n) = 1^3 + 2^3 + 3^3 \ldots\ldots n^3$

Prove $= \frac{1}{4} n^2 (n+1)^2$

Basic step: $C(1) = 1^3 = \frac{(1)^2 (1+1)^2}{4}$

$= 1$, which is true

$C(n)$ is true for $n=1$

Induction step:

Assume $C(n)$ is true for $n = k$, where $k \in N$

$$C(k) = 1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$$

If S_k = true, then S_{k+1} should also be

$$S_{k+1} = 1^3 + 2^3 + 3^3 \ldots k^3 + (k+1)^3$$

$$= \frac{(k+1)^2 (k+2)^2}{4}$$

$$C(k+1) = 1^3 + 2^3 \ldots k^3 + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$(k+1)^2 \left\{ \frac{k^2}{4} + k + 1 \right\}$$

$$(k+1)^2 \left(\frac{k^2 + 4k + 4}{4} \right)$$

$$\frac{(k+1)^2 (k+2)^2}{4} = RHS$$

$C(n)$ is true for $n = k+1$

By principle of mathematical induction,

$C(n)$ is true for all $n \in N$

$$C(n) = 1^3 + 2^3 \cdots\cdots + n^3 = \frac{1}{4} n^2 (n+1)^2$$

$$C(n) = \frac{1}{4} n^2 (n+1)^2$$

4. Kurt Gödel was a German who was naturalised in the US. He was a logician, mathematician and philosopher. He contributed the incompleteness theorems and the Gödel numbering and to the continuum hypothesis. Alan Turing was an English mathematician and computer scientist - called the father of A.I. He contributed the Automatic ~~Turing~~ Computing Engine, the Turing machine, and the Turing Test, the origin to Artificial Intelligence. Alonzo Church was an American Mathematecian who contributed the Church-Turing thesis, which is a hypothesis about the nature of computable functions. It was a big leap in computability theory.

5.

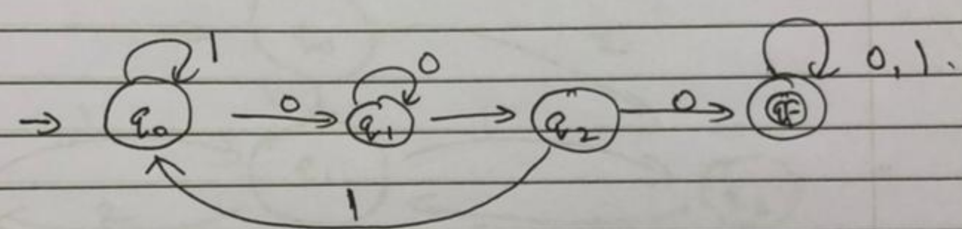
a) $Q = \{q_0, q_1, q_2, q_f\}$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

Final state $= q_f$

δ :



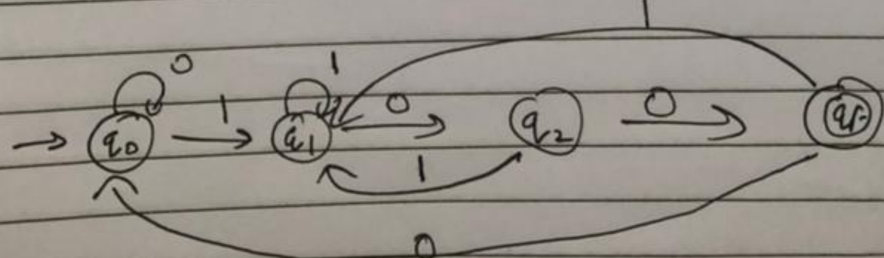
b) $Q = \{q_0, q_1, q_2, q_f\}$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

Final state $= q_f$

δ



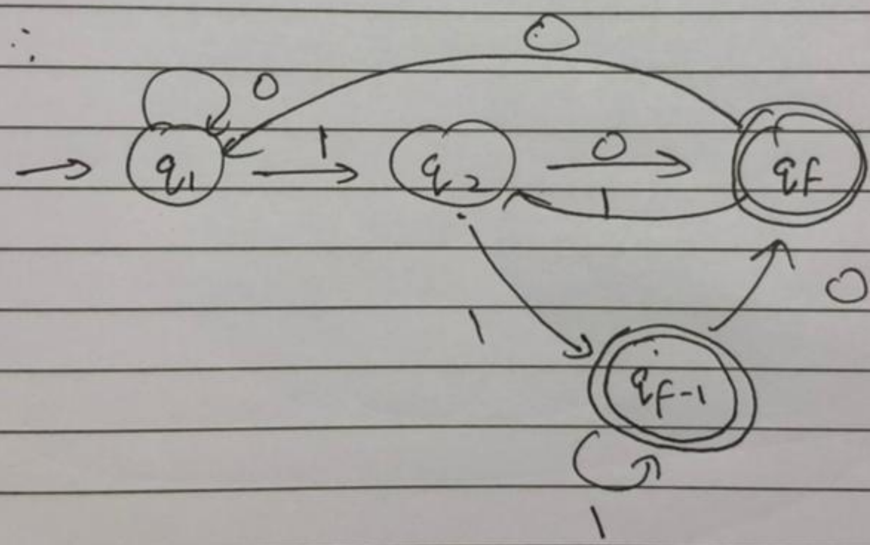
c) $Q = \{q_1, q_2, q_{f-1}, q_f\}$

$\Sigma = \{0, 1\}$

$q_0 = q_1$

Final state $= q_f$

δ :



1. $Q = \{q_0, q_1, q_2, q_f\}$

$\Sigma = \{1, 2\}$

$q_0 = q_0$

$F = q_f$

δ :