# Homework 1

## CptS 317, Spring 2021

## Due Date: February 3, 2021 by 11:59pm Pacific.

## To be submitted on Canvas.

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There are six problems in this homework. Each problem is equally weighted.

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1. Write formal descriptions of the following sets.
   (a) The set containing the numbers 1, 10, and 100
   (b) The set containing all integers that are greater than 10
   (c) The set containing all natural numbers that are less than 10
   (d) The set containing nothing at all
   (e) The set containing the empty string
   (f) The set containing the string abc
2. If $S$ is a set with $s$ elements, how many elements are in the power set of $S$? Explain your answer.
3. Let $S(n) = 1 + 2 + \cdots + n$ be the sum of the first $n$ natural numbers, and let $C(n) = 1^3 + 2^3 + \cdots + n^3$ be the sum of the first $n$ cubes. Prove the following equalities by induction on $n$, to arrive at the curious conclusion that $C(n) = S^2(n)$ (that is $S(n)$, squared) for every $n$.
   (a) $S(n) = \frac{1}{2}n(n+1)$.

(b) $C(n) = \frac{1}{4}(n^4 + 2n^3 + n^2) = \frac{1}{4}n^2(n+1)^2$.

4. The mathematicians Kurt Gödel, Alan Turing, and Alonzo Church were mentioned in the lecture we had on January 22 discussing overview of the theory of computation. Research these three mathematicians (Wikipedia would be a great place to start) and write a brief paragraph describing their lives and their most important contributions to computability theory. Your paragraph should contain a minimum of 50 and a maximum of 100 words.

Problems 5 and 6 require you to provide a Finite Automaton (FA). Each FA should contain information about the following five components (corresponding to the 5-tuple description): i) The set of states $Q$; ii) the alphabet $\Sigma$; iii) the start state; iv) the set of final states $F$; v) the set of transitions $\delta$, which can be either shown in the form of a state diagram (preferred) or a transition table.

5. Give a Finite Automaton for each of the following languages defined over the alphabet $\Sigma = \{0, 1\}$:

   (a) L={ $w$ | $w$ contains the substring 010 }

   (b) L={ $w$ | $w$ ends in 100 }

   (c) L={ $w$ | $w$ has a 1 in its $2^{nd}$ last position, if such a position exists}

6. Give a Finite Automaton for the following language over the alphabet $\Sigma = \{1, 2\}$: L={ $w$ | the sum of the symbols in $w$ is a multiple of 3}

   For example, 2121 is part of the language because the sum of all its symbols equals 6 ($6 \mod 3 = 0$); whereas, 112 is not in the language because it sums up to 4 ($4 \mod 3 = 1$).

   The empty string is **not** accepted by this language.