AlphaGo Zero

RL for Combinatorial Games

Aayush Borkar

Indian Institute of Technology Bombay

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Table of Contents

Markov Decision Process

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ExpectiMax Tree

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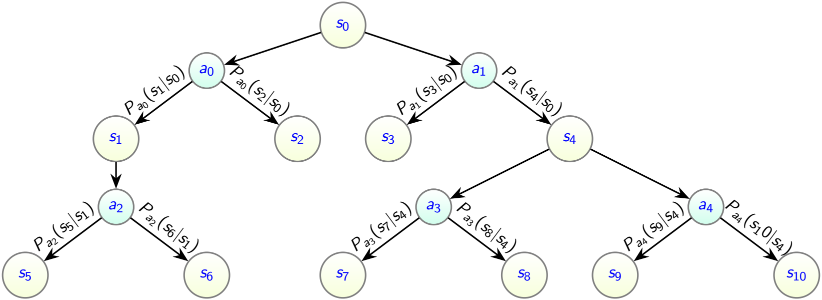

Table of Contents

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- v is a scalar. It is the probability of winning from the current state.

Self-play

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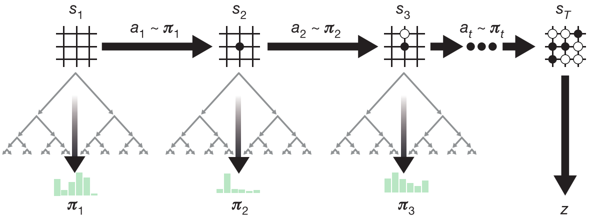

Figure: Self-play in AlphaGo Zero

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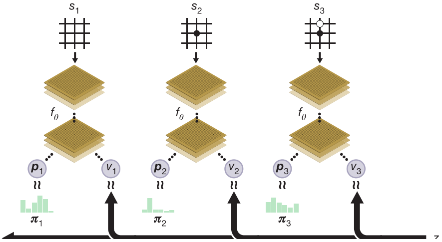
- 

Figure: Neural Network Training in AlphaGo Zero

Table of Contents

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- $P(s,a)$ is the prior probability of selecting action a from state s

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- At each time steps, an action is selected according to the statistics in the search tree, $a_t = \text{argmax}_a \left(Q(s_t, a) + U(s_t, a) \right)$, where $U(s, a)$ comes from the Upper Confidence Bounds algorithm for mutli-armed bandits.

$$U(s, a) = c_{puct} P(s, a) \frac{\sqrt{\sum_b N(s, b)}}{1 + N(s, a)}$$

Higher values of c_{puct} encourage exploration.

Expansion

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- The leaf node is then expanded by adding a new child node for each possible action. Each new edge from L is initialized with

$$N(L,a) = W(L,a) = Q(L,a) = 0, P(L,a) = p_a$$

Backup

The edge statistics are updated in the back-up phase. $N(s,a)$ is incremented by 1, $W(s,a)$ is incremented by v, and $Q(s,a) = \frac{W(s,a)}{N(s,a)}$.

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- τ is a temperature parameter that controls the level of exploration.
- For the first 30 moves of each game, $\tau = 1$.
- For the rest of the game, $\tau \to 0$.

Putting it all together

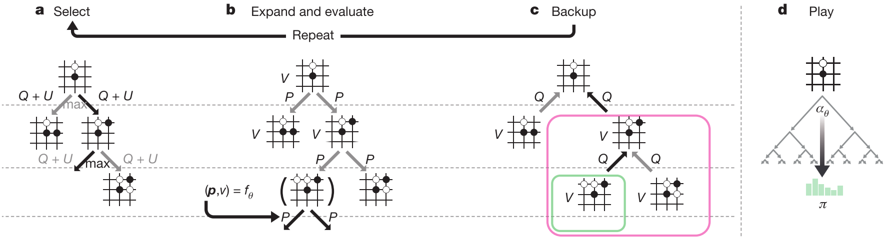

Figure: MCTS in AlphaGo Zero