TIME SERIES ANALYSIS

Terminology

A time series is a set of observations on the values that a variable takes at different times.

These data are collected at regular intervals, like monthly(eg. CPI), weekly(eg Money supply), quarterly(eg. GDP) or annually(eg. Government Budget).

Some applications where time series is used are statistics, econometrics, mathematical finance, weather forecasting, earthquake prediction and many other application.

Univariate Time Series:

Time series that consists of single observations recorded over regular time intervals.

Eg. Monthly returns data of a single stock.

Cross-Section/Multivariate Data:

These type of data are mainly collected by observing many subjects(such as individuals, firms,countries or regions) at the same point of time or during the same time period.

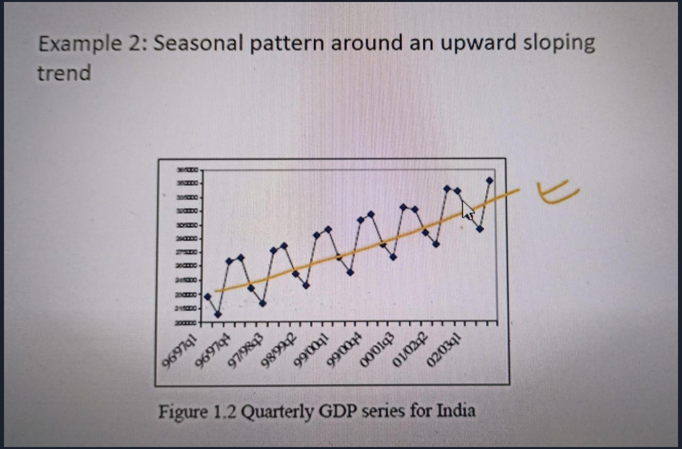
Eg. A analyst wants to know the number of cars a household has bought in the past year. For this he collected sample of 500 families from population & notes the data on how many cars they bought in the past year.

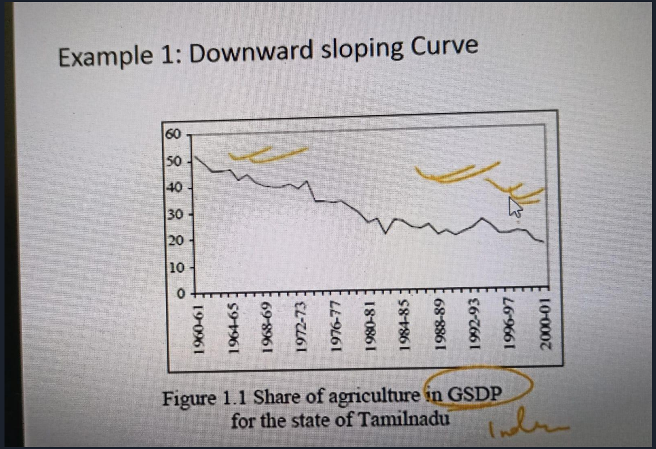
This “cross-sectional” provides a glimpse of population for that duration.

Patterns Emerging In Time Series Data

As time series depends on the frequency of data (hourly, daily, weekly, monthly, quarterly, annually), different patterns emerge in the data set which forms component to be modeled.

The time series may be increasing or decreasing over time with constant slope or there may be patterns around the increasing slope.





Components of Time Series

Patterns in time series is classified into trend,seasonal,cyclical & random components.

Trend: A long term relatively smooth pattern that usually persists for more than one year.

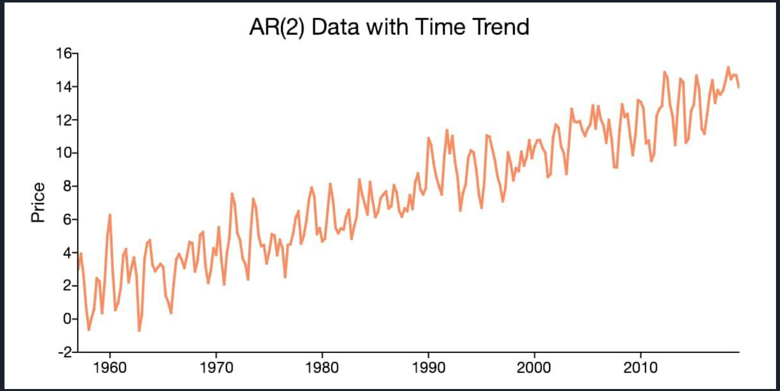
Seasonal: A pattern that appears in regular interval wherein frequency of occurrence is within a year or even shorter. For eg quarterly GDP series for India.

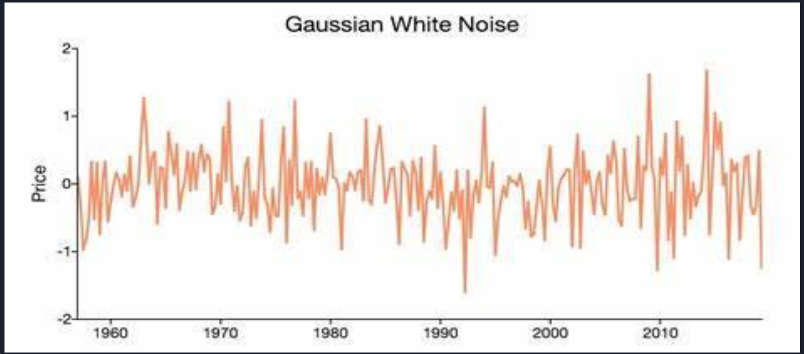
Cyclical: Repeated patterns that appears in a time series but beyond a frequency of 1 year. Cycles are rarely regular and appear in combination with other components.

Eg: Business cycles that records period of economic recession & inflation.

Random: Time series component that is obtained after these three patterns have been extracted out of the series is the random component.

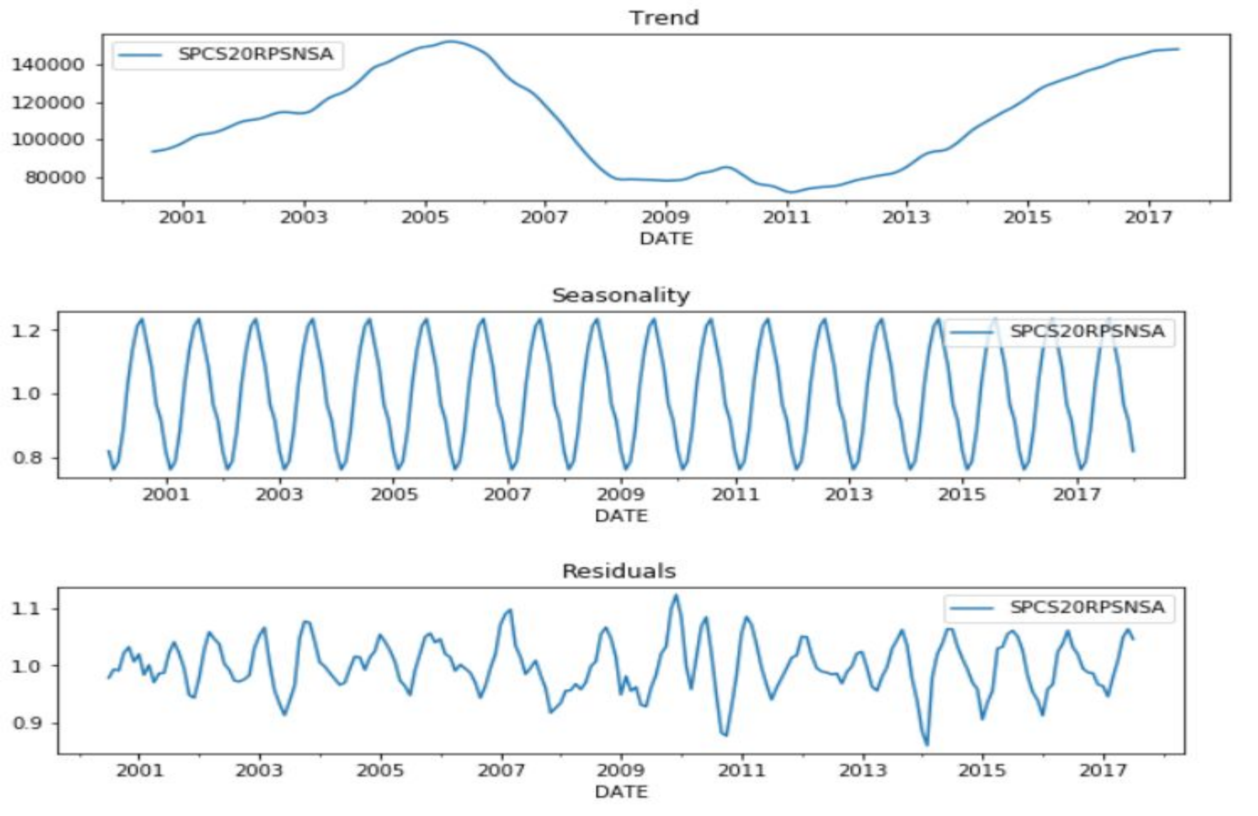
INCREASING TREND





NO TREND

Components of Time Series



Idea behind Univariate time series model

Univariate modelling arises in situation where:-

a) Appropriate economic theory to the relationship between series may not be available & hence one considers only the statistical relationship of given series with its past vale.
b) Sometime when set of explanatory variables may be known it may not be be possible to obtain the entire set of such variables required to estimate a regression model & one would use only single series of dependent variable to forecast future values.

Applications of Univariate Modelling

a) Forecasting inflation rate or unemployment rate.
b) Firms may be interested in demands for their products(soft drinks, 2-wheelers etc) or market share of their product.
c) Used by Housing finance companies to forecast mortgage interest rate and demand for housing loans.
d) Used by Jewel merchants to forecast gold or silver prices.

Different Time Series Process

1. **White Noise**: If a series is purely random in nature it is called white white noise.Let {εt} denote such a series then it has zero mean [E(εt)]=0 has a constant variance [V(εt)]=σ^2] and is uncorrelated [E(εtεs)=0] random variable. (Scatter plot of such series across time indicate no pattern & hence forecasting future values is not possible.)
2. **Autoregressive model:** AR model in which Yt depends only on its own past values Yt-1,Yt-2,Yt-3etc.

   Thus Yt=f(Yt-1,Yt-2,Yt-3,..................εt).

   Common representative of AR model where it depends on p of its past values called AR(p) model represented as

   Yt=β o+ β1Yt-1 +β 2Yt-2+β 3Yt-3+.......................β pYt-p+εt

3.**Moving Average Model (MA Model)**: A MA model is one when Yt, depends only on the random error terms which follows a white noise process.i.e. $Yt=f(\varepsilon t,\varepsilon t-1,\varepsilon t-2,\varepsilon t-3,..............)$

A common representative of MA model where it depends on q of its past values is called MA(q) model & is represented below:

$$Yt=\beta o+\varepsilon t+\phi 1.\varepsilon t-1+\varepsilon\phi 2.t-2+...+\phi q.\varepsilon t-q$$

Error terms εt are assumed to be white noise process with mean 0 & variance σ^2.

4. **Autoregressive Moving Average Model(ARMA model):** The situations where time series may be represented as a mix of both AR & MA models referred as ARMA (p,q).

General form of such time series model, which depends on p of its own past values & q past values of white noise distribution, takes the form:

$$Yt=\beta o+\beta 1.Yt-1+\beta 2.Yt-2+\beta\ 3.Yt-3+.........\beta pYt-p+\varepsilon+\phi 1.\varepsilon t-1+\phi 2.\varepsilon t-1+\phi 3.\ \varepsilon t-3+......+\ \phi q.\varepsilon t-q$$

Stationarity of Time Series

A series is said to be “strictly stationary” if the marginal distribution of Y at time t [p(Yt)] is the same as at any other point in time.

Therefore,

p(Yt)=p(Yt+k) & p(Yt,Yt+k) does not depend on t.

(here, t>=1 & k is any integer)

This implies that the mean, variance and covariance of series Yt are time invariant.

However a series is said to be “weakly stationary” or “covariance stationary” if following conditions are met:

a) $E(Y1)=E(Y2)=E(Y3)=..=E(Yt)=\mu$(a constant).
b) $Var(Y1)=Var(Y2)=Var(Y3)=..=Var(Yt)=Y0$ (a constant).
c) $Cov(Y1,Y1+k)=Cov(Y2,Y2+k)=Cov(Y3,Y3+k)= Yk$, depends only on lag k.

A series which is “non-stationary” can be made stationary after differencing.

A stationary series after being differentiated once is said to be “integrated of order 1 & is denoted by I(1)”.

A stationary series differentiated d times is said to be integrated of order d, denoted by I(d).

A series which is stationary without differencing is denoted/said by I(0).