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Class 3CO16

ASSIGNMENT → Parameter Estimation

① Mean → θ_1 Variance → θ_2

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

take log

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

for θ_1 diff $\log(L(\theta_1, \theta_2))$ w.r.t θ_1 and set it to zero

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_i) = 0$$

$$\boxed{\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i}$$

M.L.E of θ_1 is simple mean

for θ_2 differentiate w.r.t θ_2 and put it to zero

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

② Binomial Distribution

$b(m, \theta)$

m → no of trials

$\theta = (0,1)$ prob of success

$$L\theta = \prod_{i=1}^{n} f(x_i, n, \theta)$$

PMF

$$f(x, n, \theta) = {}^{n}C_{x}\, \theta^{n} \cdot (1-\theta)^{m-x}$$

$$L(\theta) = \prod_{i=1}^{n} ({}^{n}C_{x_i}) \cdot (\theta^{x_i}) \cdot (1-\theta)^{m-x_i}$$

Take log

$$\log(L(\theta)) = \sum_{i=1}^{n} \log({}^{m}C_{x_i}) + \sum_{i=1}^{n} x_i \log(\theta) + \sum_{i=1}^{n} (m-x_i) \log(1-\theta)$$

$$\frac{d \log(L)}{d\theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{nm} (m-x_i)$$

Multiply both sides by θ and $1-\theta$

$$(1-\theta) \sum_{i=1}^{nm} x_i = \theta \sum_{i=1}^{nm} (m-x_i)$$

$$\theta = \frac{\sum_{i=1}^{m} x_i}{m}$$

MLE of θ for $B(n, \theta)$ is $\bar{X}$ where

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$